Discount Rates

John H. Cochrane

University of Chicago Booth School of Business

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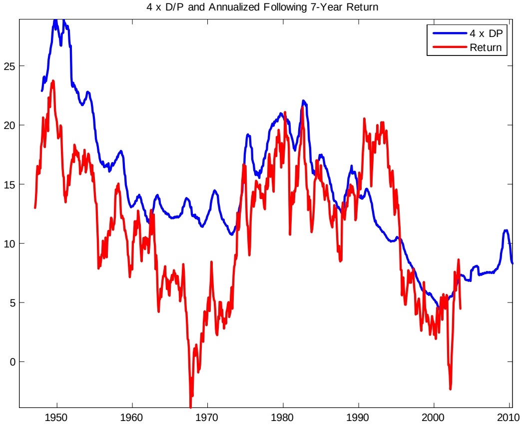
Discount rates

1. Facts: How risk discount rates vary over time and across assets.
2. Theory: Why discount rates vary.
   - "Macro," "Behavioral," "Segmented/institutional," "Liquidity"
3. Applications
   - Portfolio theory, Active/passive management, Accounting, Corporate Finance
4. Apology – see long paper for citation, documentation

Forecasting with DP

Horizon k	b	t(b)	R^2	$\sigma\left[E_t(R^e)\right]$	$\frac{\sigma[E_t(R^e)]}{E(R^e)}$
1 year	3.8	(2.6)	0.09	5.5	0.76
5 years	20.6	(3.4)	0.28	29.3	0.62

$$R^e_{t\to t+k} = a + b\frac{D_t}{P_t} + \varepsilon_{t+k}; \quad \sigma\left[E_t(R^e)\right] \equiv \sigma\left(\hat{b} \times \frac{D_t}{P_t}\right)$$



Long-Horizon Regression Coefficients and Price Volatility

- Identity: $(dp_t \equiv \log(D_t / P_t); \quad \rho = 0.96)$

$$dp_t \approx \sum_{j=1}^{k} \rho^{j-1} r_{t+j} - \sum_{j=1}^{k} \rho^{j-1} \Delta d_{t+j} + \rho^k dp_{t+k}$$

- Long-run regressions, and coefficient identity

$$\sum_{j=1}^{k} \rho^{j-1} r_{t+j} = a + b_r^{(k)} dp_t + \varepsilon_{t+k}^r, \text{ etc.}$$

$$\Rightarrow 1 \approx b_r^{(k)} - b_{\Delta d}^{(k)} + b_{dp}^{(k)}.$$

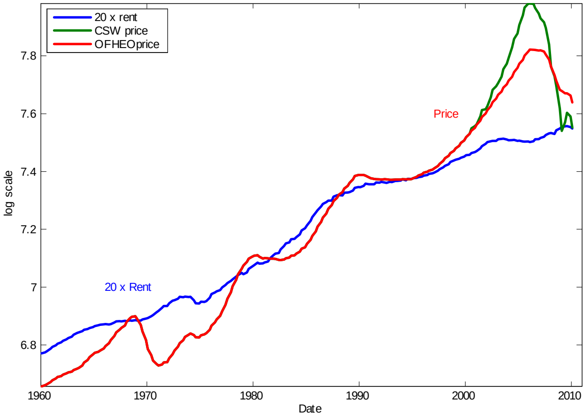
	$b_r^{(k)}$	$b_{\Delta d}^{(k)}$	$b_{dp}^{(k)}$
Direct regression , $k = 15$	1.01	-0.11	-0.11
Implied by VAR, $k = 15$	1.05	0.27	0.22
VAR, $k = \infty$	1.35	0.35	0.00

- Why do prices (p/d) move? 100% (135%!) discount rates, 0% (-35%!) dividend growth

A Pervasive Phenomenon, and cycles

- A pervasive phenomenon:
    1. Stocks. DP $\rightarrow$ Return, not dividend growth
    2. Treasuries. Yield $\rightarrow$ Return, not rising rates
    3. Bonds/CDS. Yield $\rightarrow$ Return, not default
    4. Foreign Exchange. Interest spread $\rightarrow$ Return, not devaluation
    5. Sovereign Debt, Foreign Assets. $\rightarrow$ Return, not repayment, exports
    6. Houses. Price/Rent $\rightarrow$ Return, not rent growth.
- Common element, business cycle association:
  low prices, high returns in recessions. High prices, low returns in booms
- "Bubble?" "Prices too high" $\Longleftrightarrow$ Discount rate "too low"

Houses – Price and Rent



Houses:	b	t	R^2
r_{t+1}	0.12	(2.52)	0.15
Δd_{t+1}	0.03	(2.22)	0.07
dp_{t+1}	0.90	(16.2)	0.90

Stocks:	b	t	R^2
	0.13	(2.61)	0.10
	0.04	(0.92)	0.02
	0.94	(23.8)	0.91

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Multivariate Challenges: More variables

1. Many forecasters. Multiple regression? Common forecasters across assets?

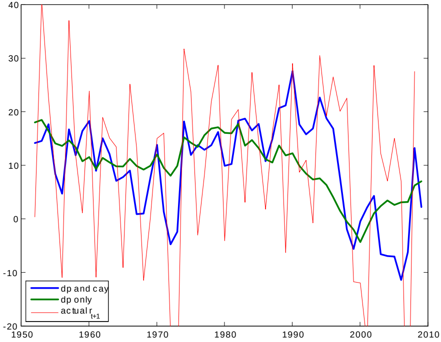
$$\begin{aligned} r_{t+1}^{\text{stock}} &= a_s + b_s \times dp_t + \boxed{c_s \times ys_t} + \boxed{d_s' z_t} + \varepsilon_{t+1}^s? \\ r_{t+1}^{\text{bond}} &= a_b + c_b \times ys_t + \boxed{b_b \times dp_t} + \boxed{d_b' z_t} + \varepsilon_{t+1}^b? \end{aligned}$$

2. Are $E_t(r_{t+1}^i) = b_i \times x_t$ correlated across assets? Factor structure of time-varying expected returns?
3. Relate mean to covariance

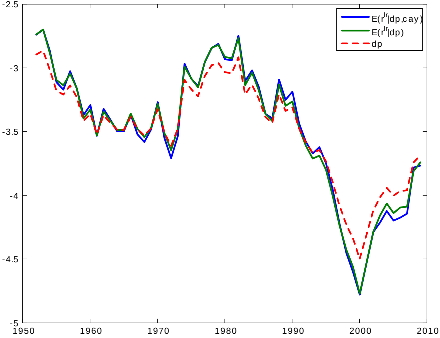
$$E_t\left(r_{t+1}^i\right) = cov_t(r_{t+1}^i \mathbf{f}_{t+1}') \boldsymbol{\lambda}_t$$

4. Can't just run big regressions!
5. Back to prices (price/dividend) – long-run forecasts?

Understanding prices. short and long-run forecasts



$$R_{t+1} = a + b \times dp_t \ \left[+c \times cay_t\right] + \varepsilon_{t+1};$$



$$\sum_{j=1}^{\infty} \rho^{j-1} r_{t+j} = a + b \times dp_t \ \left[+c \times cay_t\right] + \varepsilon$$

The cross section

1. Chaos
2. CAPM $E(R^{ei}) = \beta_i E(R^{em})$
3. Chaos again $E(R^{ei}) = \boxed{\alpha_i} + \beta_i E(R^{em})$ (value)
4. Fama and French

$$E(R^{ei}) = \beta_i E(R^{em}) + \boxed{h_i E(hml) + s_i E(smb)}$$

3. Chaos again

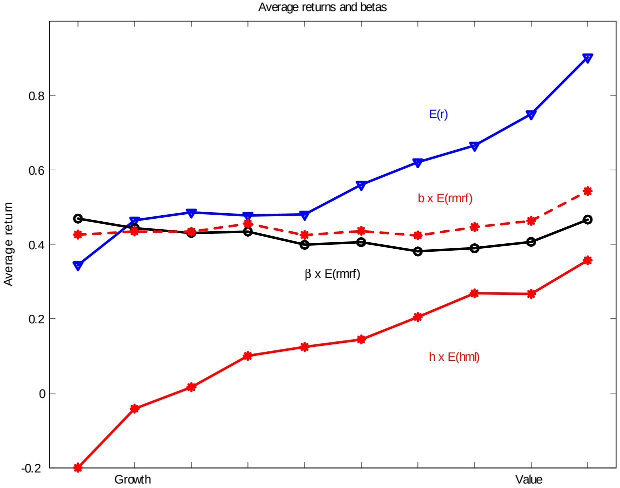
$$E(R^{ei}) = \boxed{\alpha_i} + \beta_i E(R^{em}) + h_i E(hml) + s_i E(smb)$$

(Market, value, size), momentum, accruals, equity issues, beta-arbitrage, credit risk, bond & equity market timing, carry trade, put writing, "liquidity provision,"...

Value effect and factor

4. Fama and French

$$E(R^{ei}) = \beta_i E(R^{em}) + h_i E(hml) + s_i E(smb)$$



Fama - French 10 B/M sorted portfolios. .

Value (size, and bond factors)

4. Fama and French

$$E(R^{ei}) = \beta_i E(R^{em}) + h_i E(hml) + s_i E(smb)$$

   a. Theories (m) only need to explain the factor

$$\begin{aligned} E(R^{ei}) &= ... + h_i E(hml) \text{ (Fama French)} \\ E(hml) &= cov(hml, m) \text{ (Theory)} \end{aligned}$$

   b. Value stocks rise and fall together; mean⇔covariance. (APT). But theories must now explain covariance!
   c. Value betas explain *other* $E(R^e)$ sorts, e.g. sales growth.

5. Chaos again..How to repeat FF?

$$E(R^{ei}) = \boxed{\alpha_i} + \beta_i E(R^{em}) + h_i E(hml) + s_i E(smb)$$

(Market, value, size), momentum, accruals, equity issues, beta-arbitrage, credit risk, bond & equity market timing, carry trade, put writing, "liquidity provision,...

The Multidimensional Challenge

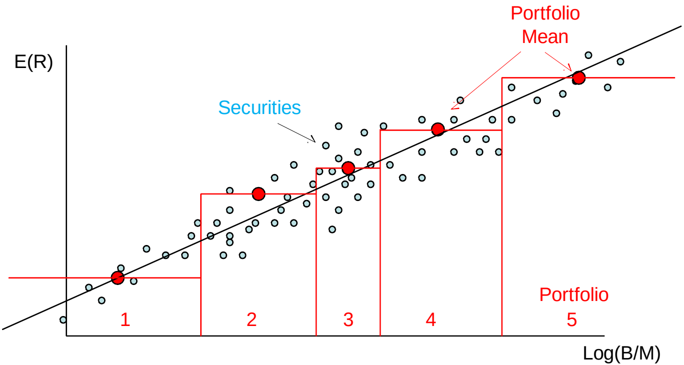
- (Market, value, size), momentum, accruals, equity issues, beta-arbitrage, credit risk, bond & equity market timing, carry trade, put writing, "liquidity provision," ...

1. Which of these are *independently* important for $E(R^e)$? ("multiple regression")
2. Does $E(R^e)$ spread correspond to new factors?
3. Do we need all the new factors? Or again, fewer factors than $E(R^e)$ characteristics?
4. Why do prices move? – Long run.

- How to approach such a highly multidimensional problem?

Asset Pricing on Characteristics/Unification

1. Portfolio sorts are really cross-sectional regressions



$$E(R^{ei}) = a + b\log(b/m_i) + \varepsilon_i;\ \ i = 1, 2, ...N$$

Asset Pricing on Characteristics/Unification

1. Portfolio sorts are really cross-sectional regressions

$$E(R^{ei}) = a + \mathbf{b}'\mathbf{C}_i + \varepsilon_i; \;\; i = 1, 2, ...N$$

2. Time series and cross-section are really the same thing

$$R^{ei}_{t+1} = a + \mathbf{b}'\mathbf{C}_{it} + \varepsilon^i_{t+1}$$

3. Result: Expected return is *a function of characteristics*

$$E(R^{ei}_{t+1}|\mathbf{C}_{it})$$

$$\mathbf{C}_{it} = [\text{size, b/m, momentum, accruals, d/p, credit spread....}]$$

4. Covariance with factors is also *a function of characteristics*

$$\begin{aligned} cov_t(R^{ei}_{t+1}, f_{t+1}) &= g(C_{it}) \\ E(R^e|C) &= g(C) \times \lambda? \end{aligned}$$

Prices?

1. Why ER/β, not p, PV?
2. Long-run / price in the "cross-section"?

$$\sum_{j=1}^{\infty} \rho^{j-1} r^i_{t+j} = a + \mathbf{b}'\mathbf{C}_{it} + \varepsilon^i?$$

3. Prices/long run may simplify.
   3.1 Campbell-Shiller:

$$\sum_{j=1}^{\infty} \rho^{j-1} r_{t+j} = \sum_{j=1}^{\infty} \rho^{j-1} \Delta d_{t+j} - dp_t$$

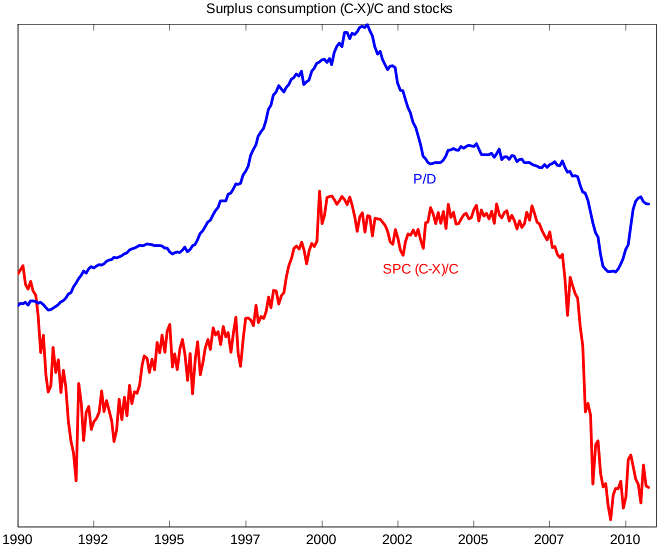
   3.2 One-period:

$$\begin{aligned} R_{t+1} &= \frac{D_{t+1}}{P_t} = \left(\frac{D_{t+1}}{D_t}\right) / \left(\frac{P_t}{D_t}\right) \\ r_{t+1} &= \Delta d_{t+1} - dp_t \end{aligned}$$

Theory classification

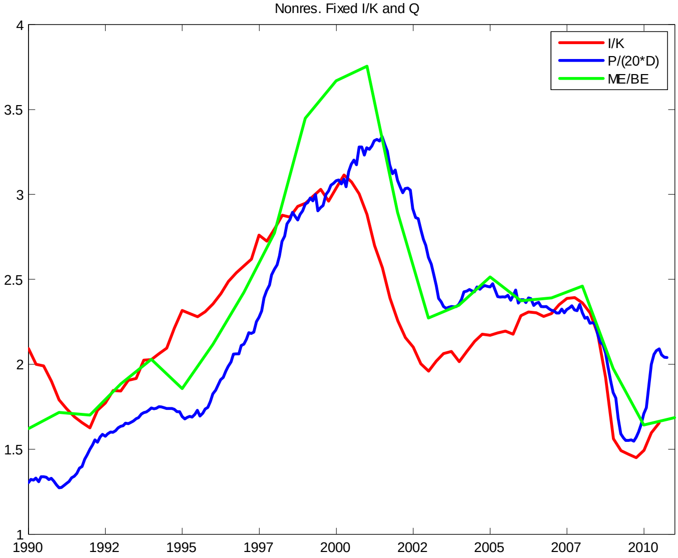
1. Frictionless
   a. Macroeconomics – macro data.
      i. Consumption
      ii. Investment
      iii. Background risks outside income
      iv. General equilibrium.
   b. Behavioral – Irrational expectations.= discount rate.
   c. Finance – $E(R)/\beta$, return-based factors; affine models.
2. Frictions
   a. Liquidity.
      i. Idiosyncratic
      ii. Systemic
      iii. Information trading.
   b. Segmented – Different investors in different markets
   c. Intermediated – Leveraged intermediaries.

Consumption/habits



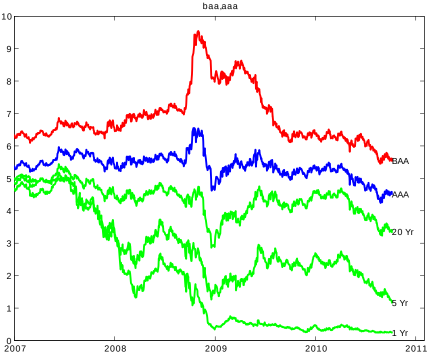
$$X_t \approx k \sum_{j=0}^{\infty} \phi^j C_{t-j} \ ; \ \text{risk aversion}_t = \gamma \frac{C_t}{C_t - X_t}$$

Investment and Q

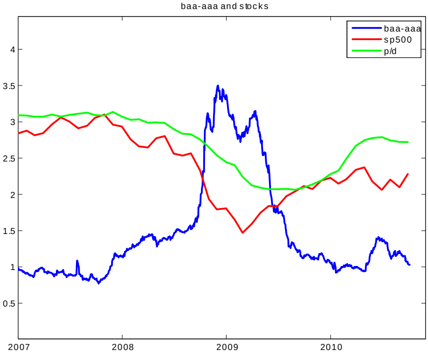


$$1 + \alpha \frac{i_t}{k_t} = \frac{\text{market}_t}{\text{book}_t} = Q_t$$

Challenges for theories



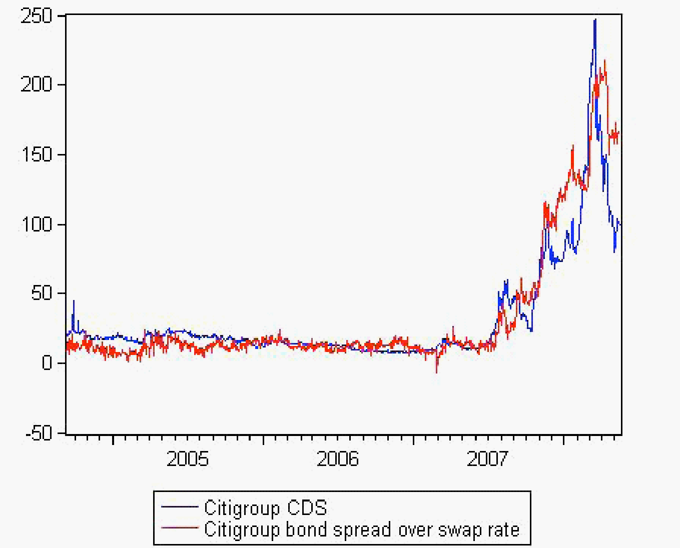
Bond yields



Bonds and stocks

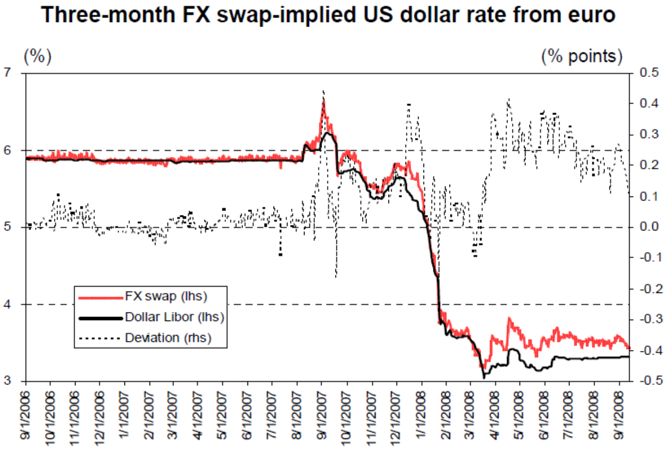
- Pervasive, coordinated risk premium in all markets, especially unintermediated
- Mean returns are associated with comovement.
- Strong correlation with macroeconomics

"Arbitrages"



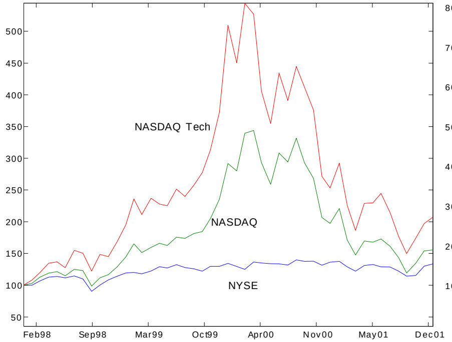
Source: Fontana (2010)

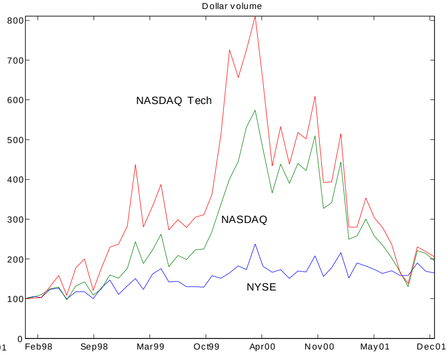
“Arbitrages”



Source: Baba and Parker (2008).

Price and volume in the tech “bubble.”

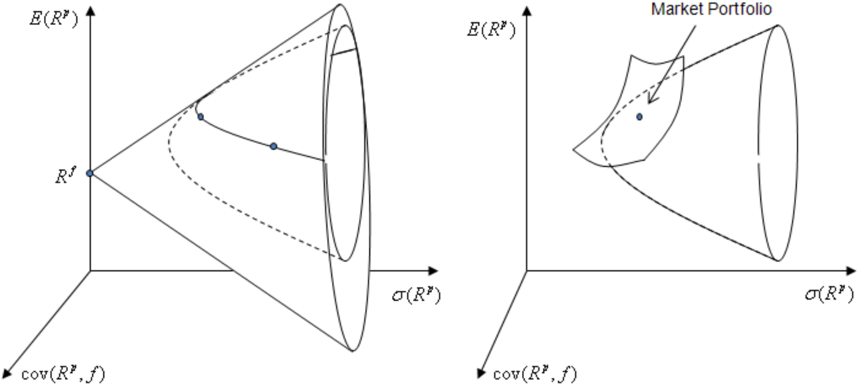




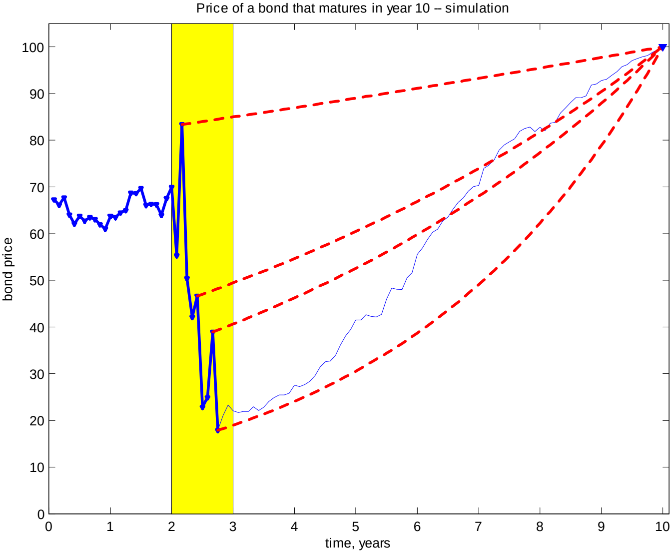
- Price (discount rate) ⇒ Volume? Or some Volume ⇒ Price, like money?
- Why so much information trading?

Portfolio theory with many factors

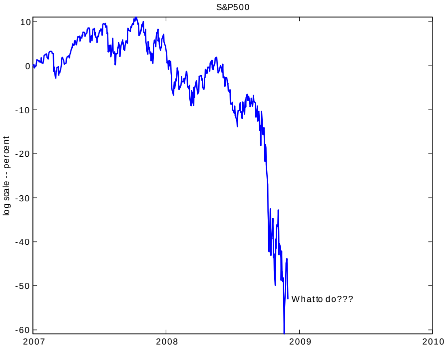
- The average investor must hold the market
- Portfolio theory based on differences

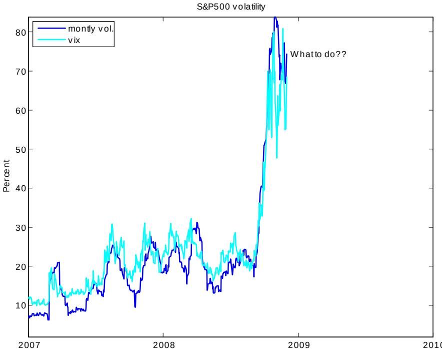


Bonds – a cautionary tale



Stocks (your endowment) in the crisis

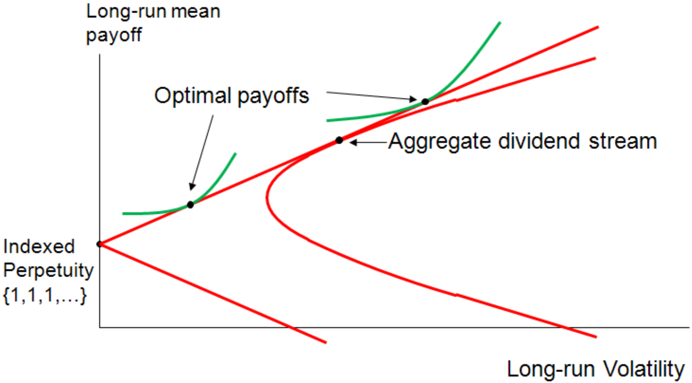




$$\text{share} = \frac{1}{\gamma}\frac{E(R^e)}{\sigma^2(R^e)}. \quad 0.6 = \frac{1}{2}\frac{0.04}{0.18^2} \Longrightarrow \frac{1}{2}\frac{0.04}{\mathbf{0.70}^2} = \mathbf{0.04}???$$

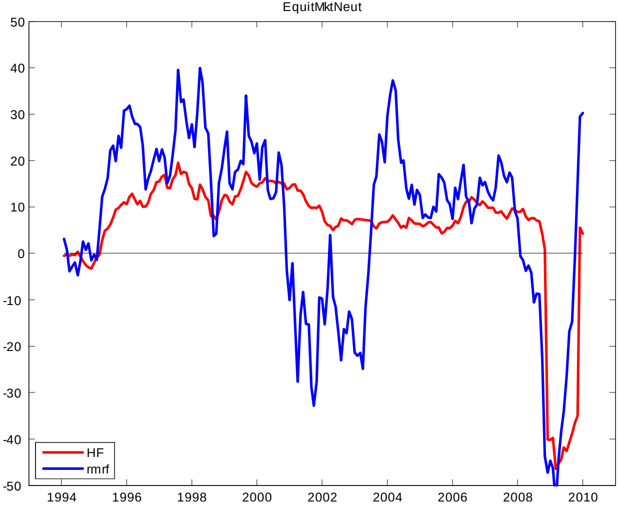
Prices and payoffs: a mean-variance benchmark

If utility is quadratic, $\max_{\{c_t\}} E \sum_{t=0}^{\infty} \delta^t \left(-\frac{1}{2}\right) (c_t - c^*)^2$ and *for any amount of time-varying expected returns,*



"Long run mean" $\tilde{E}(x) = \frac{1}{1-\beta} \sum_{j=0}^{\infty} \beta^j E\left(x_{t+j}\right)$

Alphas, betas, and performance evaluation



$$R_t^{ei} = \alpha_i + \beta_i rmrf_t + h_i hml_t + s_i smb_t + u_i umd_t + \text{vol., carry, beta-arb, issu}$$

Procedures, corporate, accounting, regulation.

- Capital budgeting, valuation

$$\text{value of investment} = \frac{\text{expected payout}}{R^f + \beta \left[E(R^m) - R^f \right]},$$

- Accounting, regulation, capital structure, if prices can change on discount rate news?

Conclusion

- Discount rates vary over time and across assets a lot more than you thought
- Empirical: how. Theoretical: why. Applications: at all.
- We've only started
- How do you ask the right question?

Last word