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Volatility tests and efficient markets

A review essay*

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1. Introduction

About two thirds of *Market Volatility* is a collection of Robert Shiller's papers. The other third consists of excellent nontechnical summaries of the papers and overviews and interpretations of the literature. Most of the book is devoted to volatility tests in the stock (part II, V), bond (part III), and real-estate (part IV) markets. The first and last parts present evidence for alternative popular or psychological models. A valuable appendix gives the basic data series.

The central issue is clearly set out on p. 1 of the introduction: '...what, ultimately, is behind day-to-day movements in prices? Can we trace the source of movements back in a logical manner to fundamental shocks affecting the economy...? Or are price movements due to changes in opinion or psychology, that is, changes in confidence, speculative enthusiasm,...?'

This issue hinges on the interpretation of volatility-test rejections.[1] Do they suggest second-order corrections to a basically correct efficient-markets view, such as better measures of fundamental movements in discount rates or

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[1]Academic debate on this issue also revolves around the other anomalies to simple (frictionless, constant expected return) efficient-market models, including return-forecasting regressions, the Wednesday, January, small-firm and listing effects, the closed-end fund puzzle, the underpricing of initial public offerings, the high volume of trading and its correlation with price volatility, etc. This essay concentrates on volatility tests, because the book concentrates on volatility tests, and because volatility is often seen as the most damning evidence against efficient-market models as a class.

incorporation of frictions like taxes, transactions costs, and market microstructure? Or do they signal a paradigm shift, in which the basic efficient-market structure will be replaced by a model in which fads, fashion, and the psychology of crowds are the driving force behind price changes, constrained only by riskless arbitrage?

Many economists, and most lay readers of academic economics,[2] have misinterpreted volatility tests to provide 'scientific' evidence for the latter view. To them, the tests show that prices are 'too volatile' to be explained by 'efficient markets', i.e., volatility tests reject market efficiency itself (subject at most to technical assumptions that rule out implausible counterexamples), and price volatility is a distinct and more striking phenomenon from anything conventional finance researchers document in their pedestrian examination of expected returns.

This interpretation is wrong. Volatility tests are in fact *only* tests of specific discount-rate models, and they are *equivalent* to conventional return-forecasting (Euler-equation) tests. (Shiller does not disagree: this view is stated clearly in the excellent introduction to part II.) Thus, the bottom line of volatility tests is *not* 'markets are inefficient' since 'prices are too volatile', but simply 'current discount-rate models leave a residual' since '(discounted) returns are forecastable'.

We can still argue over what name to attach to residual discount-rate movement. Is it variation in real investment opportunities not captured by current discount-rate models? Or is it 'fads': waves of irrational optimism and pessimism, 'noise trading', 'feedback trading', or other market inefficiencies? This is an enjoyable argument, which I join: I argue that residual discount-rate variation is small (in a precise sense), and tantalizingly suggestive of economic explanation. I argue that 'fads' are just a catchy name for the residual, and not yet an 'alternative theory' to account for price fluctuations. But these are just arguments: since perfect measurement of investment opportunities or investor sentiment is impossible, neither side will ever have scientific evidence that the other side is wrong.

I also argue that volatility tests have a bleak future, since they are inferior to conventional Euler-equation tests as tests of discount-rate models. This observation, based on the most recent research in the area, is not a criticism of Shiller's work. Volatility tests have a splendid history, even if their future is limited. Volatility tests were one of the first anomalies to simple efficient-market models, some of the first empirical evidence for time-varying expected returns, powerful examples of how important stationarity assumptions are, and the first widely-used rational-expectations tests that took into account unobserved investor information. They provided the context for most of what we have learned about bubbles, sunspots, noise traders, fads, and

[2]See, for example, the *Economist*'s review of *Market Volatility* (June 2 1990, p. 81).

other alternatives to efficient markets. Finally, Shiller's examination of popular and psychological models and his survey evidence are some of the best papers that try to give some content to fads. Therefore, the papers in the book and the excellent overviews and summaries are well worth reading and thinking about, even if in the end the reader is not converted to Shiller's interpretation of the results, and won't 'learn what the October 1987 crash was all about' as the back cover promises.

2. Volatility tests

Volatility tests were first developed[3] for the constant discount-rate present-value model,

$$P_t = \mathrm{E}_t \sum_{j=1}^{\infty} \rho^j D_{t+j}, \tag{1}$$

where P_t is the asset's price, ρ is the discount rate, D_t its dividend or other payoff, and E_t is conditional expectation. We can't test whether a *given* price change is consistent with the present-value model, since we do not observe all the information that agents use to forecast dividends. In particular, it might seem 'implausible' that agents could have enough information that we do not observe to account for large price movements like the October 1987 crash, but this kind of observation is not scientific evidence against the present-value model. However, we *can* check whether price movements are consistent with the present-value model on average, in two ways. The *variance-bounds tests* exploit the fact that $\mathrm{var}(\mathrm{E}(x \mid y)) \leq \mathrm{var}(x)$ for any random variables x and y, so (1) implies

$$\mathrm{var}(P_t) \leq \mathrm{var}\left(\sum_{j=1}^{\infty} \rho^j D_{t+j} \right). \tag{2}$$

Eq. (2) captures the intuition that prices should vary less than the ex-post present values that they should forcast. *Orthogonality tests* follow by multiplying the left-hand side of (1) by any variable observed at time t and taking expectations,

$$\mathrm{E}(Z_t P_t) = \mathrm{E}\left(Z_t \sum_{j=1}^{\infty} \rho^j D_{t+j} \right). \tag{3}$$

[3]To save space, I won't try to properly cite all the papers that have gone into the development and interpretation of volatility tests. Both the book and various reviews [for example, LeRoy (1989)] contain extensive surveys.

Orthogonality tests can also be used to examine volatility. With $Z_t = (P_t - \mathrm{E}(P_t))$, (3) implies

$$\operatorname{var}(P_t) = \operatorname{cov}\left(P_t,\ \sum_{j=1}^{\infty} \rho^j D_{t+j}\right). \tag{4}$$

Intuitively, prices should only vary if they forecast changes in dividends.[4]

The first volatility tests were roughly sample counterparts to eqs. (2)–(4) and resulted in dramatic rejections of the present-value model. The stylized fact behind these rejections is simple, and nicely summarized in a famous plot (pp. 106–107). The discounted sum of dividends is a very smooth series, so its variance or covariance with prices is much smaller than the variance of prices.

However, prices and dividends are not stationary. Therefore, the population moments in eqs. (2)–(4) are not defined, and there is no reason to expect sample moments to satisfy (2)–(4). This point caused a great deal of controversy, and a large literature examined ways of writing analogues to (1)–(4) in terms of stationary variables. Currently, most authors[5] obtain stationary variables by dividing (1) by dividends. Thus, they test a present-value relation between *price/dividend ratios* and *dividend-growth rates*:

$$\frac{P_t}{D_t} = \mathrm{E}_t \sum_{j=1}^{\infty} \rho^j \frac{D_{t+j}}{D_t} = \mathrm{E}_t \sum_{j=1}^{\infty} \left(\prod_{k=1}^{j} \rho \eta_{t+k} \right), \tag{5}$$

where $\eta_t = D_t / D_{t-1}$ represents dividend growth. Variance bounds and orthogonality tests for the price/dividend ratio analogous to eqs. (2)–(4) are easily constructed using (5) in the place of (1).

When expressed in terms of variables that are more plausibly stationary, and in particular in the price/dividend-ratio specification, variance bounds are much less likely to reject.[6] However, orthogonality tests [like (3)–(4)] applied to a price/dividend-ratio model still reject. Essentially, changes in

[4] 'Tests of the present-value relation' or 'tests of iterated Euler equations' are more accurate terms than 'volatility tests', since (3) doesn't have an obvious connection to 'volatility', and 'volatility' is a deliberately vague moment. However, I will defer to the title of the book, and refer to any test of a present-value model as a 'volatility test'.

[5] For example, Campbell and Shiller (ch. 8; 1988), Cochrane (1990), Durlauf and Hall (1989), Mankiw, Romer, and Shapiro (1989).

[6] See LeRoy and Parke (1990) and Cochrane (1990). Shiller criticizes Leroy and Parke's result since they assume a pure random walk for dividends, implying that dividend changes are permanent. My paper allowed arbitrary time-series structure for dividends, so if they were in fact stationary or strongly mean-reverting, the variance bound would be lower. Still, I found that the point estimate of the variance bound was well above the sample variance of the price/dividend ratio.

price/dividend ratios should correspond to changes in dividend-growth forecasts. High price/dividend ratios are associated with higher dividend-growth forecasts in some data sets, but generally not enough to satisfy orthogonality tests.

The volatility tests discussed so far impose a constant discount rate. However, prices will vary, with no change in dividends, if the rate at which dividends are discounted varies over time. In response to this criticism, volatility tests have been generalized to include measures of time-varying discount rates. (Shiller is a leader in this effort. Most other authors are still content to reject the constant discount-rate model.)

A discount rate is a variable γ_t such that the Euler equation

$$1 = \mathrm{E}_t(\gamma_{t+1} R_{t+1}) \tag{6}$$

holds. If $\gamma_t = \rho$, a constant, definition (6) specializes to the constant discount-rate model studied above, and (6) specifies that returns should not be forecastable. However, if γ_t is not constant, (6) specifies that *discounted* returns $(\gamma_{t+1} R_{t+1})$ should not be forecastable. Iterating the definition (6) forward and imposing a transversality condition, we obtain a present-value model that generalizes (1) and (5) to time-varying discount rates,

$$P_t = \mathrm{E}_t \sum_{j=1}^{\infty} \left(\prod_{k=1}^{j} \gamma_{t-k} \right) D_{t+j}, \qquad \frac{P_t}{D_t} = \mathrm{E}_t \sum_{j=1}^{\infty} \left(\sum_{k=1}^{j} \gamma_{t+k} \eta_{t+k} \right). \tag{7}$$

We can construct variance-bound or orthogonality tests with time-varying discount rates by following the construction of (2)–(4) above, using the present-value model (7) in the place of (1).

However, the discount rate γ_t is not directly observable, so one must use some model or proxy for the discount rate to conduct a test. [As we will see below, the present-value model (7) has no content without a discount-rate model.] The first tests assumed that dividends were discounted by an interest rate, perhaps plus a constant risk premium. Campbell and Shiller (ch. 8; 1988) present the state of the art in these tests. Since real interest rates do not vary a great deal, interest-rate-based tests continue to reject the present-value model.

A second kind of model infers discount-rate variation from measures of real investment opportunities (marginal rates of substitution and transformation) in the real economy. For example, the consumption-based asset-pricing model ties discount rates to aggregate nondurable- and services-consumption data: if the representative consumer has utility

$$\mathrm{E}\sum_{t=0}^{\infty}\rho^{t}u(c_{t}),$$

then discount rates are given by

$$\gamma_{t+1}=\rho u'(c_{t+1})/u'(c_{t}).$$

Grossman and Shiller (ch. 21) and Campbell and Shiller (1988) construct volatility tests with the consumption-based model, and their tests continue to reject.

In summary, volatility tests still reject, despite resolution of the statistical controversy surrounding the early tests and despite the introduction of two simple models to capture some of the effects of time-varying discount rates. Therefore, I now turn to what the rejections mean.

3. Why volatility tests looked like tests of efficiency and different from return regressions

At the time of the first volatility tests, the most important 'tests of efficiency' were return-forecasting regressions (less precisely, tests whether stock prices follow a random walk). Their successors are Euler-equation tests [Hansen and Singleton (1982)], which are essentially *discounted* return-forecasting regressions.

For critics of efficient markets, return-forecasting tests suffered two defects. First, it was well understood that they are not pure tests of efficiency, but tests of a joint hypothesis that includes constant discount rates or investment opportunities. Second, they seemed to miss the point. Efficient-markets critics are willing to concede that price changes (returns) are not *predictable*. The issue is, why do prices move so much, given the general absence of news about dividends? Tests of the coefficient in a return-forecasting regression, or tests whether the variation of the *predictable* part of returns is zero, say nothing about the enormous size of the *error term* or the *unpredictable* part of returns. To put the point another way, if there are (unpredictable) 'booms' and 'crashes' that are unrelated to subsequent events, return regressions will never detect them.

But the observation that we cannot account for much price variation is not 'scientific' evidence against efficient markets either. Traders and investors certainly have more information than we do, so we should not expect to account for each and every price change. Given this point, arguments over the size of unexplained price movements are just arguments over how much of an information advantage traders and investors could 'plausibly' have.[7]

Volatility tests seemed to circumvent these problems. Unlike return-forecasting regressions, they bound the allowable magnitude of *unexplained*, *ex-post* volatility, since the left-hand sides of (2) and (4) are unconditional variances. Unlike the observation that prices move unaccountably, they seem immune to unobserved investor information, since the right-hand sides of (2) and (4) are also unconditional moments. For these reasons, it is understandable that readers of early volatility tests interpreted them as tests of efficiency, and volatility as a distinct and more informative phenomenon than return-forecasting regressions. However, as we will see, most authors active in the field no longer hold this view.

4. Bubbles

One of the first alternatives to be considered was bubbles (sometimes called 'rational bubbles'), or violations of the transversality condition. To see how bubbles work, start with the Euler equation with a constant discount rate (the same points hold with time-varying discount rates):

$$1 = \mathrm{E}_t(\rho R_{t+1}) \quad \Rightarrow \quad P_t = \mathrm{E}_t\big(\rho(P_{t+1} + D_{t+1})\big).$$

Iterating this equation forward, we obtain

$$P_t = \mathrm{E}_t \sum_{j=1}^{\infty} \rho^j D_{t+j} + \lim_{T\to\infty} \mathrm{E}_t\big(\rho^T P_{t+T}\big). \tag{8}$$

Thus the present-value relation (1) is derived from the iterated Euler equation *plus* the transversality condition that the second term in (8) is zero. Price paths that satisfy the Euler equation but *not* the transversality condition contain bubbles. An example is $D_t = 0$ for all t, but rather than $P_t = 0$, prices follow $P_{t+1} = P_t \varepsilon_{t+1}/\rho$ with $\mathrm{E}(\varepsilon_{t+1}) = 1$.

Bubbles embody precisely the kind of alternative behavior efficient-markets critics had in mind: prices vary, with no news about dividends, simply because expectations of future prices vary. Returns are unpredictable in a bubble, yet the volatility-test restrictions are violated. For example, the model in the last

[7]The same point is true of the observation that 'nonfundamental' variables such as the volume of insider trading *do* enter return regressions, since the nonfundamental variable may be correlated with the unobserved information. Arguments over unexplained volatility are not limited to finance. For example, cross-sectional wage equations also have low R^2, and 'nonfundamental' variables such as industry dummies and race enter. Labor economists argue whether their residual indicates a fundamental inefficiency such as 'insiders and outsiders' or 'efficiency wages', or whether it is due to 'unobserved heterogeneity', correlated with the nonfundamental variables, in an efficient labor market.

paragraph has arbitrary return variance var(ε_t) and *infinite* price variance, though the present value of dividends is a constant, zero.

However, most authors (including Shiller) do not interpret volatility-test rejections as evidence for bubbles, for two reasons (plus the attractiveness of the alternate view, discussed in the next section, that the Euler equation rather than the transversality condition is violated): First, it turns out that the transversality condition imposes no testable restrictions in a finite sample.[8] A volatility-test rejection shows (at best) that price changes are not justified by subsequent dividends in the sample, so prices must move in response to changing expectations of some event not seen in the sample. In a bubble, that event is the limit of the discounted terminal price. But the bubble alternative cannot be distinguished from changing news about dividends beyond the sample, or extremes of the distribution of dividends that did not occur during the sample (losing wars, etc.). Therefore, we are back to arguing about whether it is 'plausible' that traders and investors have unobserved information about such events.[9,10]

Second, most volatility tests do not actually impose the transversality condition. Since no one has data on the infinite stream of future dividends, it is common to truncate the sum with a terminal price, which can either be the last price in the sample or the k-step-ahead price. In either case, only a finitely iterated Euler equation is tested, for example,

$$P_t = \mathrm{E}_t\left(\sum_{j=1}^{k-1} \rho^j D_{t+j} + \rho^k P_{t+k}\right).$$

Bubbles obviously cannot explain rejection of such volatility tests.

5. Time-varying expected returns: Real or fads?

Shiller and most other efficient-markets critics now emphasize a 'fads' interpretation of volatility-test rejections. In a fad, as in a bubble, the price deviates from the present value of future dividends, due to noise or feedback trading, irrational expectations, or some other inefficiency. However, fad

[8]Flood and Hodrick (1990) survey the bubbles literature, and many of these comments are drawn from their paper.

[9]There is also an equivalent statistical argument that the transversality condition is not testable in a finite sample. In the present-value model (1), prices and dividends are stationary, while in the bubble example the log price has a unit root. This observation generalizes: there is a bubble if *and only if* the log price has a unit root not found in dividends [Hamilton and Whiteman (1985), Diba and Grossman (1988)]. [In (5) and with stationary dividend growth, there is a bubble if and only if the log price/dividend ratio has a unit root.] But a unit root has arbitrarily low power against stationary low-frequency movement in a finite sample [Blough (1991), Cochrane (1991a)], so a bubble test has arbitrarily low power against low-frequency fundamental price movement in a finite sample.

[10]The sampling distribution of volatility-test statistics *is* sensitive to unobserved information, even in large samples. See LeRoy (1989).

price deviations are slowly reversed, where bubble price deviations are expected to last forever. Thus, a fad price rise forecasts a slightly lowering of returns over a long horizon, and implies no violation of the transversality condition.

The finance literature has found confirming evidence that a number of variables, including price/dividend ratios, do in fact forecast small persistent changes in returns [for example, Fama and French (1988)]. But most of the finance literature interprets return forecastability as evidence for slowly changing investment opportunities in the real economy, efficiently reflected in asset markets.[11]

5.1. Volatility tests are equivalent to return regressions

One would suppose that if two literatures come to such different conclusions, there must be something different about the evidence they consider, something along the lines of the view that I outlined in section 3. However, once fads are the alternative, volatility tests are equivalent to long-horizon return-forecasting regressions, so there is in fact no difference in evidence.

At an abstract level, the equivalence of volatility tests and return regressions is easy to see. The ingredients of the present-value model are the Euler equation and the transversality condition. If the alternatives as well as the null impose the transversality condition, then volatility tests only test the Euler equation. In turn, the only testable content of Euler equations is that discounted returns should not be forecastable.

More concretely, consider the following approximate identity:[12]

$$\operatorname{var}\left(\frac{P}{d}\right) \cong \frac{1}{1-\Omega}\sum_{j=1}^{\infty}\Omega^j \operatorname{cov}\left(\frac{P_t}{d_t}, n_{t+j}\right) - \frac{1}{1-\Omega}\sum_{j=1}^{\infty}\Omega^j \operatorname{cov}\left(\frac{P_t}{d_t}, r_{t+j}\right), \tag{9}$$

where $n_t = \ln(D_t/D_{t-1})$, $r_t = \ln(R_t)$, and Ω is a constant slightly less than

[11] This opinion is not unanimous. Poterba and Summers (1988), for example, view time variation in expected returns as evidence for fads. They state that 'it is difficult to think of risk factors that could account for such variation in required returns'. But they explicitly acknowledge that the test itself cannot distinguish whether this fact reflects a failure of the market or of their powers of imagination.

[12] To derive the identity, linearize the identity (10) below by taking a Taylor expansion of the term inside the expectation with respect to n and r, around their unconditional means. This expansion yields

$$\frac{P_t}{d_t} \cong \text{constant} + \frac{1}{1-\Omega}\mathrm{E}_t\left(\sum_{j=1}^{\infty}\Omega^j(n_{t+j} - r_{t+j})\right).$$

[See Cochrane (1990). The linearization is similar to Campbell and Shiller (1988).] Then, multiply both sides by $(P/D - \mathrm{E}(P/D))$ and take unconditional expectations to obtain (9).

one, $\Omega = \exp(\mathrm{E}(n_t) - \mathrm{E}(r_t))$. Eq. (9) is an *identity*, not a model. It is derived by manipulating $1 = R_{t+1}^{-1} R_{t+1}$.

The covariance terms in (9) are the numerators of the slope coefficients β in regressions

$$\sum_{j=1}^{\infty} \Omega^j r_{t+j} = \alpha + \beta \left(\frac{P_t}{D_t} \right) + \varepsilon .$$

Therefore, eq. (9) shows that if price/dividend-ratio forecasts of dividend growth are not sufficient to explain the variance of price/dividend ratios, then price/dividend-ratio forecasts of returns must, *mechanically*, fill the gap.[13] Specifically, the first two terms in (9) are an orthogonality test of the constant discount-rate present-value model. The volatility-test rejection documents that the last term exists, and is therefore *equivalent* to regression evidence that price/dividend ratios forecast the discounted sum of returns.

Long-horizon return-forecasting regressions take the form

$$\sum_{j=1}^{k} r_{t+j} = \alpha + \beta \ln \left(\frac{P_t}{D_t} \right) + \varepsilon_{t+k} .$$

The difference between volatility-test rejections and long-horizon return regressions is that one forms geometrically-weighted sums of returns and the other forms truncated sums of returns. Clearly, both tests provide essentially the same evidence.

These points extend to the volatility tests with measures of time-varying discount rates. The interest-rate-based tests are equivalent to long-horizon excess-return (return minus interest rate) forecasting regressions. The finance literature confirms excess returns are forecastable, but typically interprets that fact to indicate time variation in *risk premia*, induced by time variation in real investment opportunities. The consumption-based tests are equivalent to long-horizon, discounted return-forecasting regressions. The finance literature confirms that returns discounted by the consumption-based model are forecastable, but interprets this fact to indicate that the consumption-based model, using currently available aggregate nondurable and services data and the constant relative risk-aversion utility function, does not fully capture the time variation in investment opportunities available in the real economy. As

[13]This discussion presumes that the transversality condition holds. If it does not, of course, price/dividend ratios need not forecast either dividend growth or returns. Empirically, sample counterparts of the right-hand side of (9) add up to very nearly the sample variance of the left-hand side [Cochrane (1990)]. This confirms the view that return forecastability, not bubbles, accounts for the failure of the constant discount-rate present-value model.

in the constant discount-rate case, there is nothing in the evidence that explains the difference in interpretation.

5.2. *Volatility tests and return regressions are only discount-rate-model tests*

Return-forecasting regressions, and hence volatility tests, are not tests of market efficiency. They are *only* tests of discount-rate models, since one can always construct *some* discount-rate process that rationalizes any return-forecasting regression. A trivial example is $\gamma_t = R_t^{-1}$. Then $1 = \mathrm{E}_t(\gamma_{t+1}R_{t+1})$ by construction.[14]

It is useful to recast this point in present-value terms. We can iterate the identity $1 = R_{t+1}^{-1}R_{t+1}$ forward to obtain the *identity*[15]

$$P_t = \mathrm{E}_t\left(\sum_{j=1}^{\infty}\left(\prod_{k=1}^{j} R_{t+k}^{-1}\right)D_{t+j}\right) \quad \text{or} \quad \frac{P_t}{D_t} = \mathrm{E}_t\left(\sum_{j=1}^{\infty}\left(\prod_{k=1}^{j} R_{t+k}^{-1}\eta_{t+k}\right)\right). \tag{10}$$

Thus the example $\gamma_t = R_t^{-1}$ is also a candidate discount rate that makes the present-value relation (7) hold by construction. Therefore, we cannot ask: 'Is the volatility of prices consistent with the present-value model?' It *always* is, given enough flexibility in the discount rate. The *only* question we can ask is: 'Is the time variation in discount rates implied by the forecastability of returns justified by time variation in real investment opportunities?'

For this reason, the interest-rate-based tests are basically beside the point, as is any test that infers discount-rate variation from asset returns. If the test rejects, critics can point to the failure of the discount-rate model to correctly capture risk premia. As (10) shows, the discount-rate model can always be generalized until the test fails to reject. But if the test fails to reject, this only shows that expected returns on one asset change through time because

[14] Hansen and Richard (1987) and Hansen and Jagannathan (1991) show how to construct a single discount rate that prices a number of assets simultaneously. This is a modern statement of Roll's (1976) observation that the CAPM holds by construction with any mean–variance efficient portfolio as benchmark.

[15] Start with

$$1 = R_{t+1}^{-1}R_{t+1} = R_{t+1}^{-1}\frac{P_{t+1}+D_{t+1}}{P_t} \quad \Rightarrow \quad P_t = R_{t+1}^{-1}(D_{t+1}+P_{t+1}).$$

Iterate forward and impose the transversality condition, to obtain

$$P_t = \sum_{j=1}^{\infty}\left(\prod_{k=1}^{j} R_{t+k}^{-1}\right)D_{t+j} \quad \text{or} \quad \frac{P_t}{D_t} = \sum_{j=1}^{\infty}\left(\prod_{k=1}^{j} R_{t+k}^{-1}\eta_{t+k}\right).$$

This equation, like $1 = R_{t+1}^{-1}R_{t+1}$, holds *ex post* as an identity. Taking conditional expectations yields (10).

expected returns on some other asset or assets change through time. This observation will hardly still efficient-markets critics. In their view, the problem is that expected returns on *all* assets vary too much through time, relative to variation in investment opportunities in the real economy.

Therefore, the interesting question is whether measures of discount-rate variation *inferred from the real economy* can account for volatility-test rejections and return-forecasting regressions. The consumption-based tests start to address this point. However, consumption-based tests will reject (in a large enough sample) if there is *any* residual movement in investment opportunities not captured by the consumption-based model. To obtain evidence against *efficient markets* (not just evidence against the consumption-based discount-rate model), it is not sufficient to 'model', 'proxy', or 'allow for' discount-rate changes, one must suppose that they are *perfectly* measured.

5.3. Summary

In summary, there *is* an unobserved discount-rate process that rationalizes the volatility of prices or, equivalently, the forecastability of returns. Volatility tests and their equivalent return-forecasting regressions document that specific discount-rate models leave a residual. That residual may be due to fads – the discount rates implicit in market prices and returns may not be linked to investment opportunities in the real economy – or it may simply reflect the shortcomings of three simple discount-rate models. Volatility tests and return-forecasting regressions provide no *scientific* evidence that one interpretation of the residual rather than another is correct.

6. The future of volatility tests

There is nothing embarrassing in the fact that volatility tests are only tests of discount-rate models. Most of empirical finance is devoted to testing discount-rate models, so this is a worthy (in some sense its only) pursuit. But if the point is to test discount-rate models, we must ask what advantage volatility tests have over conventional Euler-equation tests. For example, Campbell and Shiller's (1988) volatility test rejects the constant risk premium model

$$E_{t-1}\ln(R_t) = \ln(R_t^f) + \text{constant}.$$

Why not reject this model by running regressions

$$\ln(R_{t+1}) - \ln(R_t^f) = \alpha + \beta Z_t + \varepsilon_{t+1},$$

using the list of variables known to forecast excess returns? Similarly, why use a volatility test to reject the consumption-based model, rather than a conventional Euler-equation test of the moment conditions

$$0 = \mathrm{E}((\gamma_{t+1} R_{t-1} - 1) Z_t), \qquad \gamma_{t+1} = \rho u'(c_{t+1})/u'(c_t),$$

as a string of authors have done following Hansen and Singleton (1982)?

Since the economic content of volatility tests and Euler-equation tests is the same, the argument must be that volatility tests possess some statistical advantage. Perhaps volatility tests have more power against fad-like alternatives (small persistent residual discount-rate changes); perhaps they emphasize lower-frequency aspects of the data that show violations more clearly, or perhaps they are less sensitive to data problems, such as small timing mistakes.

This argument was once plausible, but recent research suggests that single-period Euler equations are still the best way to amass statistical evidence against discount-rate models. The reason is that one can construct Euler-equation tests that are *exactly equivalent* to volatility tests or long-horizon return regressions, and the Euler-equation tests have several important theoretical and econometric advantages.

Hodrick (1990) develops this observation for long-horizon return-forecasting regressions. I start with this case, since the algebra is simpler than for volatility tests. Long-horizon regressions take the form

$$\sum_{j=1}^{k} r_{t+j} = \alpha_k + \beta_k Z_t + \varepsilon_{t+k}, \tag{11}$$

where $r_t =$ log return. Long-horizon regressions can in fact have greater power than single-period regressions,

$$r_{t+1} = \alpha_1 + \beta_1 Z_t + \nu_{t+1}, \tag{12}$$

against a small, slow-moving predictable component in returns. If a rise in Z_t signals a rise in expected returns for many periods, then the forecasted parts add together when one adds the returns up, while the serially uncorrelated forecast error is attenuated.

The regression coefficient $\hat{\beta}_k$ is zero if its numerator is zero,

$$\mathrm{E}\left(\tilde{Z}_t \sum_{j=1}^{k} \tilde{r}_{t+j}\right) = 0, \tag{13}$$

where ~ indicates deviations from means. Z_t and r_t are assumed jointly stationary, so $\mathrm{E}(\tilde{Z}_t \tilde{r}_{t+j}) = \mathrm{E}(\tilde{r}_t \tilde{Z}_{t-j})$. Thus, (13) is equivalent to

$$\mathrm{E}\left(\tilde{r}_t \sum_{j=1}^{k} \tilde{Z}_{t-j}\right) = 0. \tag{14}$$

But (14) implies that $\beta_f = 0$ in the single-period regression

$$r_t = \alpha_f + \beta_f \left(\sum_{j=1}^{k} Z_{t-j}\right) + \delta_t. \tag{15}$$

Thus, we can test *exactly* the same moment as (11) with a *one-period* regression, using a filtered right-hand variable. Intuitively, the filtered right-hand-variable regression (15) has better power than the raw regression (12) because a slow-moving right-hand variable can pick up a slow-moving component on the left-hand side. This increase in power is the same as the increase one obtains by aggregating the left-hand variable in (11).

Furthermore, (15) is far simpler econometrically. Under the null of i.i.d. returns, the error δ_t in (15) is serially uncorrelated, while the error ε_t in (11) inherits the MA(k) structure of the long-horizon returns. To test $\beta_1 = 0$ in (11) one must correct for small-sample bias in the coefficient and for the serial correlation of the error, while neither correction needs to be made in (15).

The same points carry over to volatility tests. Volatility tests are tests of an iterated Euler equation, and we can instead iterate the instruments. As a concrete example, consider the following orthogonality test, formed by multiplying both sides of the present-value model (7) by an instrument Z_t and taking expectations:

$$\mathrm{E}\left(\left(\sum_{j=1}^{\infty}\left(\prod_{k=1}^{j} \gamma_{t+k}\right) D_{t+j} - P_t\right) Z_t\right) = 0. \tag{16}$$

By writing out the sum and shifting the Z's back, one can show that (16) is exactly the same moment as a single-period Euler equation with a combination of past Z's and discount rates as the instrument,

$$\mathrm{E}\left((\gamma_{t+1} R_{t+1} - 1)\left(P_t \sum_{j=0}^{\infty}\left(\prod_{k=0}^{j+1} \gamma_{t-k}\right) Z_{t-j}\right)\right) = 0. \tag{17}$$

Eqs. (16) and (17) are *the same test*.[16] Therefore, the volatility test (16)

[16] The appendix presents the algebra required to derive (17) from (16). This form is the simplest one with which to show the equivalence of volatility and Euler-equation tests, but one can do the same trick with other forms. The appendix gives a few examples that use stationary variables, unlike P_t.

cannot have a power advantage due to more clever exploitation of the null hypothesis, misalignment of data, or emphasis of different frequencies.[17]

This point can also be demonstrated at an abstract level. Single-period moment conditions test *all* of the Euler equation's implications,[18] so each volatility tests *must* be identical to some single-period Euler-equation test, with at most an unusual choice of instruments.

Furthermore, single-period Euler equations have two important advantages over volatility tests. First, the error in the volatility test (16) is serially correlated, requiring bias and standard-error corrections. The Euler-equation error in (17) is serially uncorrelated. Second, volatility tests generally impose statistical or modeling approximations that Euler equations do not. For example, Shiller's papers with John Campbell represent the state of the art, yet they require a log-linear approximation that rules out certain kinds of time-varying risk premia and the effects of third and higher moments, and they only examine one asset at a time. Since Hansen and Singleton (1982), single-period Euler-equation tests have not required any approximations of the model. They can also be used to test the discount-rate model's cross-asset and intertemporal predictions at the same time. (Most discount-rate models fail cross-sectionally at least as badly as they fail intertemporally.)

The example (17) does not imply that one must construct the precise nonlinear combination of instruments that is exactly equivalent to a volatility test in order to obtain power against fad-like discount-rate residuals. All we really know about the alternative is that it includes a small, slow-moving predictable component in returns, so one needs slow-moving instruments. Therefore, instruments that are already slow-moving (dividend/price ratio),[19] or that are linearly filtered to become slow-moving, are likely to do as good a job at documenting the residual as the nonlinearly filtered instrument in (17).

[17] It is not necessarily true that iterated Euler equations and single-period Euler equation with filtered instruments have the same *finite-sample* power against every alternative. Though the moment is the same, the standard errors suggested by each procedure are different. Essentially, the serial correlation corrections one must do with an aggregated left-hand variable [(11) or (16)] under the null can sometimes correct for serial correlation induced by the alternative, that the standard error constructed under the null with an aggregated right-hand variable [(15) or (17)] would miss. This possibility must be balanced against the greater small-sample bias in (11) or (16) and the small-sample unreliability of serial-correlation-corrected standard errors, and one can also constructs standard errors that reflect the serial correlation of the alternatives rather than the null. Hodrick (1990) compares the power of these alternative specifications. The important point is that *either* aggregated left- or right-hand variables can possess better power against fads than nonaggregated variables.

[18] Precisely, the Euler equation $E_t(Z_t(\gamma_{t+1}R_{t+1}-1))=0$ holds *if and only if* the moment conditions $E(Z_t(\gamma_{t+1}R_{t+1}-1))=0$ for every Z_t observed at time t. This is just an equivalent definition of conditional expectation.

[19] The dividend/price ratio is very slow-moving, so once a single lag of the dividend/price ratio is included, further lags do not have much marginal forecast power for returns. Therefore, there is little power advantage in aggregating such a slow-moving right-hand variable. Aggregation does help with less serially correlated right-hand variables, such as returns and interest rates or premia.

For the above reasons, I suspect that single-period Euler equations will dominate volatility tests in future tests of discount-rate models. At best, volatility calculations might be useful *diagnostics*, helpful for clarifying the stylized facts behind Euler-equation rejections and their economic interpretation. But for *testing*, it seems that everything volatility tests can do, Euler equations can do better. (I'm not criticizing Shiller: I include my own best efforts [Cochrane (1990)] in this gloomy assessment.)

7. Interpreting residual discount-rate movement

Volatility tests and their equivalent return forecasting or Euler equations *do* document a small, slow-moving residual in current discount-rate models. While neither test gives 'scientific' evidence whether the residual is due to fads or due to unobserved variation in investment opportunities, it is still important to think about how to interpret the residual. In this section, I give a few reasons why I think the residual indicates imperfections in current discount-rate models rather than evidence for fundamental irrationality by investors. The reasons are 1) the residual seems 'small' in a precise sense, 2) the residual is strongly suggestive of economic explanation, 3) I do not find current independent evidence for investor irrationality convincing, and 4) fad advocates do not seem to take the implications of their views seriously.

7.1. Is the residual 'large' or 'small'?

Volatility tests have been interpreted as rejections of efficiency in part because they are so dramatic: one can excuse a 2% or so mismeasurement of the discount rate as a defect of current models, but a difference between prices and fundamental values (as measured by economists or accountants) of 40% or more seems too large to be rescued by econometric refinements or quibbles about discount-rate models.

But the price and discount-rate errors are exactly equivalent. For example, with constant dividend growth g and discount rate r, the price/dividend ratio is $P/D = 1/(r-g)$. If the price/dividend ratio is 20, $r-g=5\%$. A two-percentage-point discount-rate error to $r-g=3\%$ implies a 66% increase in price. Therefore, the argument can be reversed: dramatic pricing errors can be rewritten as small (if persistent) discount-rate errors to make the same rejection suggest refinement of discount-rate models rather than inefficiency.

In the end, this argument highlights defects of our intuition rather than defects of our models. The *relative* error is roughly the same. If we cannot measure discount rates to better than two percentage points out of 5%, we should *expect* market prices to deviate 40% or more from measures of 'fundamental' value. To get anywhere, we need some objective measure of the size of residuals.

One objective measure is the utility or welfare loss implied by a discount-rate residual, and this measure is 'small'. To make this calculation, suppose that a rejection is in fact due to a fad – suppose that the discount rates implicit in market prices deviate from exactly measured marginal rates of substitution by a few percentage points for long periods, and so prices differ from fundamentals. In the current market structure, this inefficiency does not imply a riskless arbitrage opportunity. All anyone can do about the fad is to change his consumption-growth pattern (savings rate) by a few percentage points, to match variation in the market's discount rate. In Cochrane (1989) I make some explicit calculations of the utility losses from not so adjusting consumption, and find losses on the order of 10¢ per quarter.

Most fad, noise-trader, or feedback-trading models explicitly incorporate small utility costs. Arbitrageurs must not be able to make large short-term profits by trading against the fads, or the fads could not survive.

Small utility losses suggest a basically efficient markets view in two ways. First, one can observe small apparent utility losses if consumers really are optimizing, but in the face of small transactions, information, and other costs that are not included in the discount-rate model. Therefore, small utility losses suggest that the rejection is due to small frictions in a basically efficient market rather than fundamentally irrational investor behavior.

Second, small utility costs suggest that policy might want to treat the market as if it were basically efficient, even if there really are fads. If the utility losses implied by fads are small, the *welfare loss* from fads may also be small: expected utility might not be raised much if *all* consumers readjusted their consumption streams (and producers readjusted their investment decisions) to remove the fads. Ultimately, we want to assess the stock market as an institution. Would replacing it by other institutions raise welfare? Or would the misallocations that will result from closing, taxing, or severely restricting the stock market be worse than the misallocations implied by fads? The same structure that makes fad or noise-trader models immune to arbitrage suggests that the welfare costs of the misallocation of resources they imply may be small as well.

As a second measure of the size of discount-rate residuals, one can calculate moments of unobserved discount-rate processes that explain volatility or return forecastability, and compare those with the moments of 'reasonable' measures of time-varying investment opportunities. For example, I [Cochrane (1990)] constructed the means and standard deviations of discount rates required to explain volatility-test rejections. I found that the standard deviation of the implied discount rates is about five percentage points, when discount rates are measured in annual percent return units. This standard deviation is comparable to that of consumption-growth rates raised to 'reasonable' risk-aversion coefficients, or that of returns on physical investment implied by standard intertemporal production functions

[Cochrane (1991b)].[20] Therefore, the discount rates required to explain volatility-test rejections are not 'too volatile', in the same way that 'too volatile' discount rates are required to explain the unconditional equity premium and term premium [see Hansen and Jagannathan (1991)]. This observation suggests that utility and production functions broadly similar to standard forms *may* be able to explain volatility tests and return regressions.[21]

7.2. *The residual suggests economic explanation*

The time variation in discount rates documented by volatility tests and return regressions occurs over horizons and in response to forecasting variables that are tantalizingly related to business cycles. For example, term premia, default premia, and investment-growth forecast returns, and they forecast GNP growth as well; ex-post stock returns are highly correlated with subsequent investment and GNP growth [Fama (1990), Cochrane (1991b)]. Furthermore, *market-wide* expected returns change over time, not just the expected returns of a few securities.

Of course, fads could *happen* to occur just at the stage of the business cycle at which one would expect changes in investment opportunities. Fads could *happen* to occur for all assets simultaneously, in rough proportion to their betas. But these are obviously not very convincing arguments to the unconverted.

Furthermore, fads seem carefully tailored around the successes of efficient markets, and re-tailored anytime there is a further success. When there *is* fundamental news, the markets react by about the right amount. The puzzle is that prices *also* move then there is no obvious news. Thus, the fad must work *on top of* rational assessment of the news we do see. When discount-rate models are created that explain some price and expected-return variation, the fad must be re-tailored to explain the new, smaller residual. If the fad is due to noise traders, one must assume that their noise trades are highly correlated, *just as if* they had all seen some news that economists missed.

7.3. *Surveys and fad models*

The central problem for fad models is overcoming this charge that they are just a catchy name for a residual. To do so, they require some *independent*

[20] Five percent may seem 'large' to the reader. However, keep in mind that this is the standard deviation of the *ex-post* discount rate γ_t, not the standard deviation of risk-free rates $R^f_t = 1/E_t(\gamma_{t+1})$ or other *ex-ante* discount-rate measures.

[21] However, the discount rates required to explain volatility-test rejections are very highly autocorrelated, unlike consumption growth. I only want to argue that it may be *possible* to write utility or production functions that will explain volatility tests and return regressions, not that it will be *easy*.

measure of investor sentiment, noise trading, etc. 'Unobserved variation in investment opportunities' is also a name for a residual. But a model, such as $\gamma_t = \rho u'(c_{t+1})/u'(c_t)$, has content and can be rejected because it describes an *independent* measure of the discount rate. Skeptics like myself will not be converted from 'we're having trouble modeling discount rates' to 'there are fads' until fad models also generate some rejectable predictions.

Shiller's work on psychological and popular models starts to outline the alternative. Unfortunately, this work has not yet come up with rejectable models, and much of the evidence can be interpreted in a broadly efficient-markets view as well. (However, the work is clearly at an early stage, so these difficulties may be overcome in the future.)

For example, consider 'Investor Behavior in the October 1987 Stock Market Crash: Survey Evidence' (ch. 23). Shiller analyzed questionnaires received from close to 1000 investors immediately following the October 1987 crash. The work is careful, and the results are not surprising. Most investors did not name news of a 'fundamental' event that caused the crash, and they seemed to trade based on past price changes ('feedback trading') rather than on news or even opinions about fundamentals.

This is an interesting observation. But one can read it as a classic example of Hayek's (1945) view of the informational efficiency of decentralized markets. In his view, no one has to know what the 'fundamentals' are, or even how markets work; they only have to know their own tiny piece of information and market prices. And their information consists of items like 'I want to buy a house', 'my firm needs to invest in new machinery', or 'I know an S&L that will buy junk bonds at a high price', items that are unlikely to report as a theory of price movement. Consumers and producers of *all* commodities have very little understanding of price movements. This is no embarrassment, it is how markets are *supposed* to work.

Of course, it is not obvious that Hayek's statements about complete-markets economies hold for dynamic economies with expectations, since rational expectations of future prices must stand in for their unobserved values. This is an open and very active area of theoretical research, but there are some encouraging examples. Grossman (1981) describes an economy in which each agent receives only a tiny bit of 'fundamental' information, but can infer the rest from observing equilibrium prices. Thus the 'market' is efficient, even though no individual has received but a negligible fraction of the information reflected in the market price. And the agents don't have to name the information they receive from prices, or understand any economics. All they need is a decision rule linking observed prices and their own tiny bit of news to the decision to buy and sell, which they can arrive at in a variety of unintelligent ways, and which they can report surrounded by lots of myths. Thus, agents in Grossman's 'efficient' economy might respond to a questionnaire exactly as Shiller's agents did on the aftermath of a big price decline.

7.4. Taking fads seriously

Finally, it is hard to take a fads view to its logical conclusion that stock markets are socially inefficient institutions for allocating capital, one that should be replaced where possible with institutions that provide a more sober and hence more accurate measure of value. For example, the new democracies in Eastern Europe seem to view the institution of a stock market and obtaining a McDonald's franchise as two of the most important requirements for economic growth. Are fads advocates really ready to advise them that stock markets are just driven by irrational waves of optimism and pessimism, so they should forget them and find some other capital market structure?

8. Concluding remarks

Volatility tests once looked like 'scientific' tests of market efficiency, tests of 'volatility', a phenomenon overlooked by conventional finance's focus on expected returns. Now, it is well understood that volatility tests are only tests of discount-rate models, and that their evidence is equivalent to return-forecasting regressions or Euler equations.

Volatility tests and return-forecasting regressions document time variation in discount rates (time variation in expected returns and risk premia) that is not fully explained by current discount-rate models. However, current discount-rate models are quite simple: we have not progressed much past discount rate equals aggregate nondurable consumption growth raised to a power. Therefore, I view the residual as a challenge to the construction of better models of fundamentals: better utility and especially production functions and perhaps better accounting for frictions like taxes and transaction costs. Shiller takes the view that only serious investigation of psychological and popular models featuring fundamental irrationalities will help to explain the discount-rate residual.

Both views face embarrassments. I certainly cannot name the news that caused the October 1987 crash (though I can point out that the dividend/price ratio forecasted a period of low expected returns), or yesterday's change in IBM for that matter. Fads advocates must overcome the charge that they are just naming a residual, a residual with small welfare costs and one that looks tantalizingly suggestive of economic explanation. But these embarrassments do not constitute scientific evidence against the opposing view, though it is a lot of fun to point them out.

I conclude that we can agree to disagree. The evidence is at least as consistent with the view that we only require second-order corrections to efficient-market models as it is with the view that they should be abandoned in favor of fads and fashions. I and others like me whose research is still devoted to extending rational economic models to account for anomalies may, in the end, be wrong, but at least we are not pig-headed in the face of clear contradictory evidence.

These issues are not unique to finance, as our trouble in accounting for price changes is not limited to asset prices. The debate whether individuals can be modeled as rational maximizers, or whether psychological and sociological models are needed, and the debate whether prices reflect marginal products or fashion have been going on a long time. Individuals often do apparently stupid things. Economic theories that ignore this fact are often remarkably successful at explaining market- or aggregate-level phenomena. Ultimately, this is a debate over whether relatively free markets are effective institutions or whether other institutions, typically featuring government control, are more effective. The debate is not likely to end soon.

Appendix: Derivation of (17)

Rewrite the moment condition (16), in terms of

$$P_t^* = \sum_{j=1}^{\infty}\left(\prod_{k=1}^{j}\gamma_{t+k}\right)D_{t+j},$$

as

$$\mathrm{E}(Z_t(P_t^* - P_t)) = 0.$$

We can express $P_t^* - P_t$ as an iteration of terms $(\gamma_t R_t - 1)$ that appear in an Euler equation:

$$\begin{aligned} P_t^* - P_t &= \gamma_{t+1}(D_{t+1} + P_{t+1}^*) - P_t \\ &= \gamma_{t+1}(D_{t+1} + P_{t+1} + P_{t+1}^* - P_{t+1}) - P_t \\ &= P_t(\gamma_{t+1}R_{t+1} - 1) + \gamma_{t+1}(P_{t+1}^* - P_{t+1}). \end{aligned}$$

Recursively substituting,

$$\begin{aligned} P_t^* - P_t = {} & P_t(\gamma_{t+1}R_{t+1} - 1) + \gamma_{t+1}P_{t+1}(\gamma_{t+2}R_{t+2} - 1) \\ & + \gamma_{t+1}\gamma_{t+2}(P_{t+2}^* - P_{t+2}). \end{aligned}$$

Continuing in this way yields

$$P_t^* - P_t = \sum_{j=1}^{\infty}\left(\prod_{k=1}^{j-1}\gamma_{t+k}\right)P_{t+j-1}(\gamma_{t+j}R_{t+j} - 1).$$

Therefore, the moment condition (16) is equivalent to

$$\mathrm{E}\left(Z_t \sum_{j=1}^{\infty}\left(\prod_{k=1}^{j-1}\gamma_{t+k}\right)P_{t+j-1}(\gamma_{t+j}R_{t+j}-1)\right)=0.$$

Writing out the sum and shifting the Z's and P's back, we obtain

$$\begin{aligned}
&\mathrm{E}(Z_tP_t(\gamma_{t+1}R_{t+1}-1)+Z_t\gamma_{t+1}P_{t+1}(\gamma_{t+2}R_{t+2}-1)\\
&\quad +Z_t\gamma_{t+1}\gamma_{t+2}P_{t+2}(\gamma_{t+3}R_{t+3}-1)+\cdots)\\
&=\mathrm{E}(Z_tP_t(\gamma_{t+1}R_{t+1}-1)+Z_{t-1}\gamma_tP_t(\gamma_{t+1}R_{t+1}-1)\\
&\quad +Z_{t-2}\gamma_{t-1}\gamma_tP_t(\gamma_{t+1}R_{t+1}-1)+\cdots)\\
&=\mathrm{E}((\gamma_{t+1}R_{t+1}-1)P_t(Z_t+Z_{t-1}\gamma_t+Z_{t-2}\gamma_{t-1}\gamma_t+\cdots))=0.
\end{aligned}$$

The last equation is (17).

As mentioned in the text, it would be better to run orthogonality tests that do not require the level of prices to be stationary. There are several ways to do this, and to each volatility test one can go through similar algebra to derive the equivalent Euler equation. Here are two examples: First, instead of subtracting P_t from both sides of (7), one can divide by P_t, resulting in

$$\mathrm{E}\left(\left(\frac{\sum_{j=1}^{\infty}\left(\prod_{k=1}^{j}\gamma_{t+k}\right)D_{t+j}}{P_t}-1\right)Z_t\right)=0.$$

Proceeding as above, this orthogonality test is equivalent to the Euler equation

$$\mathrm{E}\left((\gamma_{t+1}R_{t+1}-1)\left(\sum_{j=0}^{\infty}\left(\prod_{k=0}^{j+1}\gamma_{t-k}\right)\frac{P_t}{P_{t-j}}Z_{t-j}\right)\right)=0.$$

Second, start with the price/dividend-ratio–dividend-growth model in (7). Then, corresponding to (16), we have the orthogonality test

$$\mathrm{E}\left(\left(\sum_{j=1}^{\infty}\left(\prod_{k=1}^{j}\gamma_{t+k}\eta_{t+k}\right)-P_t/D_t\right)Z_t\right)=0.$$

Following the same logic, this test is equivalent to the Euler equation

$$\mathrm{E}\left((\gamma_{t+1}R_{t+1}-1)\left(P_t/D_t\sum_{j=0}^{\infty}\left(\prod_{k=0}^{j+1}\gamma_{t-k}\eta_{t-k}\right)Z_{t-j}\right)\right)=0.$$

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