Michelson-Morley, Fisher, and Occam:
The Radical Implications of Stable Quiet Inflation at the Zero Bound

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Abstract

The long period of quiet inflation at near-zero interest rates, with large quantitative easing, suggests that core monetary doctrines are wrong. It suggests that inflation can be stable and determinate at the zero bound, and by extension under passive policy including a nominal interest rate peg, and that arbitrary amounts of interest-paying reserves are not inflationary. Of the known alternatives, the new-Keynesian model merged with the fiscal theory of the price level is the only simple economic model consistent with this interpretation of the facts.

I explore two implications of this conclusion. First, what happens if central banks raise interest rates? Inflation stability implies that higher nominal interest rates will eventually result in higher inflation. But can higher interest rates temporarily reduce inflation? Yes, but only by a novel mechanism that depends crucially on fiscal policy. Second, what are the implications for monetary policy and the urgency to "normalize?" Inflation stability implies that low-interest rate monetary policy is, perhaps unintentionally, benign, producing a stable Friedman-optimal quantity of money, that a large interest-paying balance sheet can be maintained indefinitely.

The fiscal anchoring required by this interpretation of the data responds to discount rates, however, and may not be as strong as it appears.

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1   Summary and overview

For nearly a decade in US, UK, and Europe, and three decades in Japan, short-term interest rates have been stuck near zero. In the last decade, central banks also embarked on immense open market operations. US quantitative easing (QE) raised bank reserves from $10 billion on the eve of the crisis in Aug 2008 to $2,759 billion in Aug 2014.

The economy's response to this important experiment in monetary policy has been silence. Inflation is stable, and if anything less volatile than before. There is no visually apparent difference in macroeconomic dynamics in the near-zero-rate, large-reserves state than before.

Existing theories of inflation make sharp predictions about this circumstance: Old-Keynesian models, characterized by adaptive expectations, and in use throughout the policy world, predict that inflation is unstable when interest rates do not respond adequately to inflation, ($\phi < 1$ in $i_t = \phi \pi_t$) and so they predict a deflation spiral at the zero bound. It did not happen. Much monetarist thought, $MV = PY$ with V "stable" in the long run, predicts that a massive increase in reserves must lead to galloping inflation. It did not happen.

New-Keynesian models, featuring rational expectations, predict that inflation is stable at the zero bond, and more generally under passive policy. Unless one adds frictions, those models also predict that quantitative easing operations are irrelevant. The observed inflation stability is thus a big feather in the new-Keynesian cap. But standard new-Keynesian models predict that inflation becomes indeterminate when interest rates do not or cannot move in response to inflation. These models have multiple self-confirming equilibria and can jump from one to the other. Therefore, new-Keynesian models predict greater inflation and output volatility at the zero bound. These models also predict a menagerie of policy paradoxes: Productivity improvements are bad, promises further in the future have larger effects today, and reducing price stickiness makes matters worse, without limit.

This is a Michelson-Morley[1] moment for monetary policy. We observe a decisive experiment, in which previously hard-to-distinguish theories clearly predict large outcomes. That experiment yields a null result, which invalidates those theories.

Now, any theory, especially in economics, invites rescue by epicycles. Perhaps inflation really is unstable, but artful quantitative easing just offset the deflation vortex. Or perhaps wages are

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[1] In 1887, Albert A. Michelson and Edward W. Morley set out to measure the speed of Earth through the ether, the substance thought in its day to carry light waves, by measuring the speed of light in various directions. They found nothing: the speed of light is the same in all directions, and the Earth appears to be still. Special relativity follows pretty much from this observation alone.

much "stickier" than we thought, or money is taking a long time to leak from reserves to broader aggregates, so we just need to wait a bit more for unstable inflation to show itself. Perhaps a peg really does lead to indeterminacy and sunspots, but expectations about active ($\phi > 1$) monetary policy in the far future takes the place of current Taylor rule responses to select equilibria. Perhaps we have experienced the proverbial seven years of bad luck, and Japan twenty, so that expectations always featured a quick escape from stuck interest rates. Perhaps the Earth drags the ether along with it.

Occam responds: Perhaps. Or, perhaps one should take seriously the simplest answer: Perhaps inflation can be stable and determinate under passive monetary policy, including an interest rate peg, and with arbitrarily large interest-bearing reserves. Classic contrary doctrines were simply wrong.

We are not left, as Michelson and Morley were, with a puzzle – a set of facts that existing theories cannot account for. Adding the fiscal theory of the price level to to the standard rational-expectations framework, including new-Keynesian price stickiness, we obtain a simple economic model in which inflation can be stable and determinate under passive policy, zero bound, or even a peg, and despite arbitrary quantitative easing. It predicts no spiral, and is consistent with no additional volatility at the zero bound. The model also has a smooth frictionless limit, and resolves new-Keynesian policy paradoxes.

What does this experience, and theoretical interpretation, imply about monetary policy going forward?

First, if inflation is stable at the zero bound, and by extension under an interest rate peg, then it follows that were the central bank to raise interest rates and leave them there, without fiscal shocks, then inflation must eventually *rise*. This reversal of the usual sign of monetary policy has become known as the "neo-Fisherian" hypothesis.

Stability and this form of long-run neutrality are linked, which merits treating them together. If $i_t = r_t + E_t \pi_{t+1}$ is a stable steady state, meaning that once r_t, unaffected by inflation in the long run, settles down, an interest rate peg i will draw inflation π towards it, then raising i and keeping it there must eventually raise π (as long as fiscal policy can and does support the value of government debt, whether "actively" or "passively"). Whether one can accept the second proposition helps to digest the first. Whether the data speak loudly enough on the first illuminates the second.

Stable models have this form of long-run neutrality. Both the standard new-Keynesian model

and its extension with fiscal theory are stable, and in both cases a prolonged interest rate rise raises inflation. The models differ only on determinacy, equilibrium selection, and hence the immediate response to shocks and volatility at the zero bound.

A long-run positive sign is entwined with the standard new-Keynesian equilibrium selection scheme as well. In the standard new-Keynesian model, when the Fed reacts [$\phi > 1$] by raising interest rates more than one for one with inflation, this reaction *raises* subsequent inflation. Inflation then spirals off to infinity, unless the economy jumps to one specific saddle-path equilibrium. This mechanism requires that persistently higher interest rates raise inflation. If persistently higher interest rates eventually lowered inflation, then the path with initially higher inflation would not be ruled out, and the system would remain determinate.

In sum, stability implies long run neutrality, and both flavors of the new-Keynesian model have stability and long run neutrality.

However, higher interest rates might still *temporarily* lower inflation before eventually raising it. I investigate what minimal set of ingredients it takes to produce a negative short-run impact of interest rates on inflation.

This quest has a larger goal. We do not have a simple economic baseline model that produces a negative response of inflation to a rise in interest rates, in our world of interest rate targets and abundant excess reserves. If there is a short-run negative relationship, what is its basic economic nature?

The natural starting place in this quest is the simple frictionless Fisherian model, $i_t = r + E_t \pi_{t+1}$. A rise in interest rates i produces an immediate and permanent rise in expected inflation. In the search for a temporary negative sign I add to this basic frictionless model 1) new-Keynesian pricing frictions 2) backwards-looking Phillips curves 3) monetary frictions. These ingredients robustly fail to produce the short-run negative sign. Even the standard active money ($\phi > 1$) new-Keynesian solutions are Fisherian. You cannot truthfully explain, say, to an undergraduate or policy maker, that higher interest rates produce lower inflation because prices are sticky, or because lower money supply drives up rates and down prices, and our fancy models build on this basic intuition.

One fiscal-theory variation can robustly and simply produce the desired temporary negative sign. If we add long-term debt, then a rise in interest rates can produce a temporary inflation decline. Higher nominal rates lower the nominal present value of long-term debt; absent any change in expected surpluses, the price level must fall to restore the real present value of the

debt. That works, but it is a rather dramatically novel mechanism relative to all standard economic stories and policy discussion. It also remains long-run Fisherian. Protracted interest rate rises eventually raise, not lower, inflation. It cannot give the traditional account of the 1980s.

We are left with a logical conundrum: Either 1) The world really is Fisherian, higher interest rates raise inflation in both short and long run; 2) more complex ingredients, including frictions or irrationalities, are *necessary* as well as sufficient to deliver the negative sign, so this hallowed belief relies on those complex ingredients; 3) the negative sign ultimately relies on the fiscal theory story involving long-term debt – and has nothing to do with any of the mechanisms commonly alluded for it.

The first view is not as crazy as it seems. The VAR evidence for the traditional sign, reviewed below, is weak. Perhaps the persistent "price puzzle" was trying to tell us something for all these decades. Correlations are of little use, as interest rates and inflation move closely together under either theoretical view, at least away from the zero bound. The second view accepts this paper's tentative conclusion – there is no *simple, economic* model behind a negative sign, so that additional complications or frictions are *necessary* to produce it. Either the second or third view rather deeply changes the nature of monetary policy economics.

The second set of policy issues: Is it important for central banks to promptly raise nominal rates and reduce the size of their balance sheets? Should central banks return to rationing non-interest-bearing reserves, and conducting interest rate policy by conventional open market operations? Is it important to swiftly return to active $\phi > 1$ policy rules?

The experience of stable inflation at near-zero interest rates, and this theoretical interpretation, says that we *can* instead live the Friedman optimal quantity of money forever – a large quantity of interest-bearing reserves, low or zero interest rates with corresponding low inflation or even slight deflation.

Whether we *should* do so requires listing (here) and quantifying (eventually) tradeoffs. A steady nominal rate means variations in the real rate will be, but also must be, accompanied by inverse variations in inflation, which in a sticky price context can cause output variation. If the Fed can diagnose real-rate variation, then moving the nominal rate can stabilize inflation. Many arguments for more activist policy remain valid. The argument that without activism inflation will spiral off or become indeterminate is denied, but that is not the only argument for activism. Actual and optimal interest rate policy may not end up looking that different, rising in booms and with inflation, and falling in bad times, as real interest rates plausibly have that pattern.

On the other hand, stability and determinacy open additional possibilities. For example, the Fed could simply target the spread between indexed and non-indexed bonds, and let the level of interest rates vary arbitrarily to market forces. Expected inflation and, with stable fiscal policy actual inflation, will follow.

I address a wide range of common objections. Among others: How can the fiscal theory be consistent with low inflation, given huge debts and ongoing deficits? Fortunately, the fiscal theory does not predict a tight linkage between current debts, deficits and inflation. Discount rates matter as well, and discount rates for government debt are very low. What about other pegs, which did fall apart? Answer: fiscal policy fell apart.

That last observation leads to a final warning. My careful hedging, that an interest rate peg *can* be stable, refers to the necessary fiscal foundations. If fiscal foundations evaporate, that theory warns, and harsh experience reminds us, so can our benign moment of subdued and quiet inflation. Contrariwise, lowering inflation in countries that are experiencing high inflation, along with fiscal and credibility problems, is not a simple matter of lowering interest rates. Without a commitment to the duration of low rates, and without solving the underlying fiscal problems, that strategy will blow up again as it has often in the past.

2   Michelson-Morley

The first part of this paper documents the facts of stable and quiet (opposite of volatile) inflation at the zero bound, the predictions of standard models that inflation should be unstable or volatile at the zero bound, and how the new-Keynesian plus fiscal theory model is consistent with stability and quiet.

2.1   Nothing happened

Figure 1 presents the last 20 years of interest rates, inflation and reserves in the U.S. The federal funds rate follows its familiar cyclical pattern, until it hits essentially zero in 2008 and stays there. In 2008-2009, the severity of the recession and low inflation required sharply negative interest rates, in most observers' eyes and in most specifications of a Taylor rule. The "zero bound" was binding. If you see data only up to the bottom of inflation in late 2010, and if you view inflation as unstable under passive monetary policy, fear of a "deflation spiral" is natural and justified.

But the spiral never happened. Despite interest rates stuck near zero, inflation rebounded
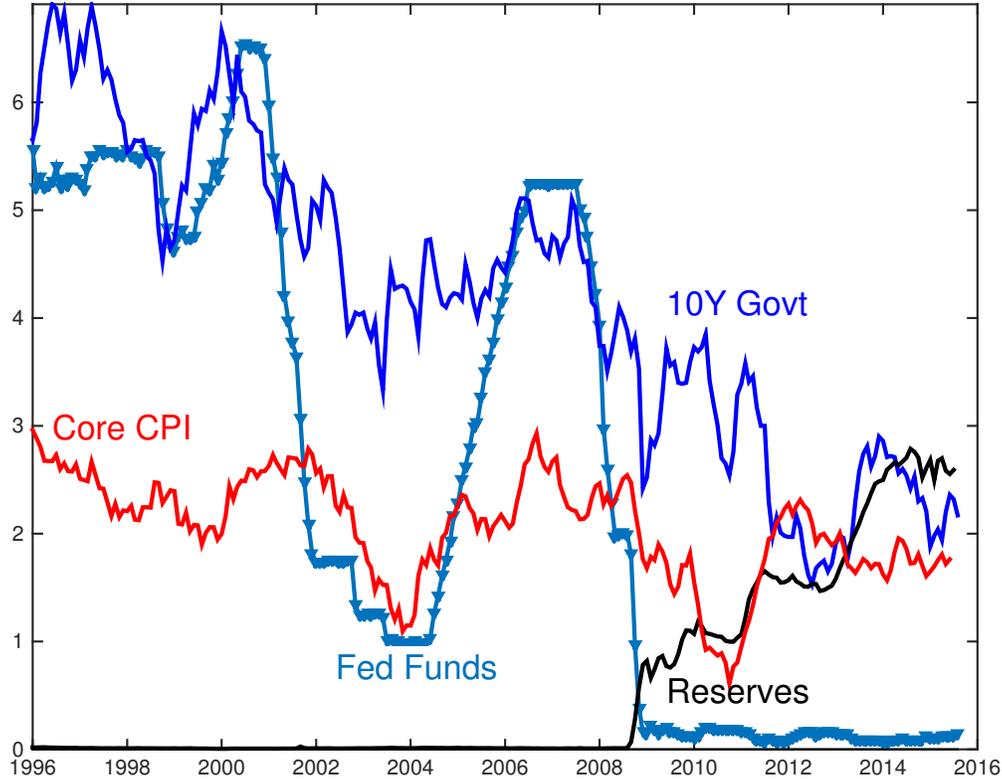
Figure 1: Recent US experience. Core CPI (percent change from a year earlier), federal funds rate (percent), total reserves (trillions) and 10 year Treasury rate (percent).

with about the same pattern as it did following the previous two much milder recessions, then resumed its gradual 20 year downward trend.

After 2012, when the financial crisis and deep recession receded, inflation volatility appears *lower* than it was before 2009, when interest rates could "actively" stabilize inflation, not higher as predicted by new-Keynesian models. More generally, the zero bound does not seem to be an important state variable for stability, determinacy, or any other aspect of inflation dynamics.

The Fed increased bank reserves, from about $50 billion to nearly $3,000 billion, in three quantitative easing (QE) operations, as shown in Figure 1. Once again, nothing visible happened. QE2 is associated with a rise in inflation, but QE1 and QE3 are associated with a decline. And the rise in inflation coincident with QE2 mirrors the QE-free rise coming out of the much milder 2001 recessions.

QE2 and QE3 were supposed to lower long-term interest rates. To the eye, the 20 year downward trend in long term rates is essentially unaffected by QE. Long-term interest rates rose coin-
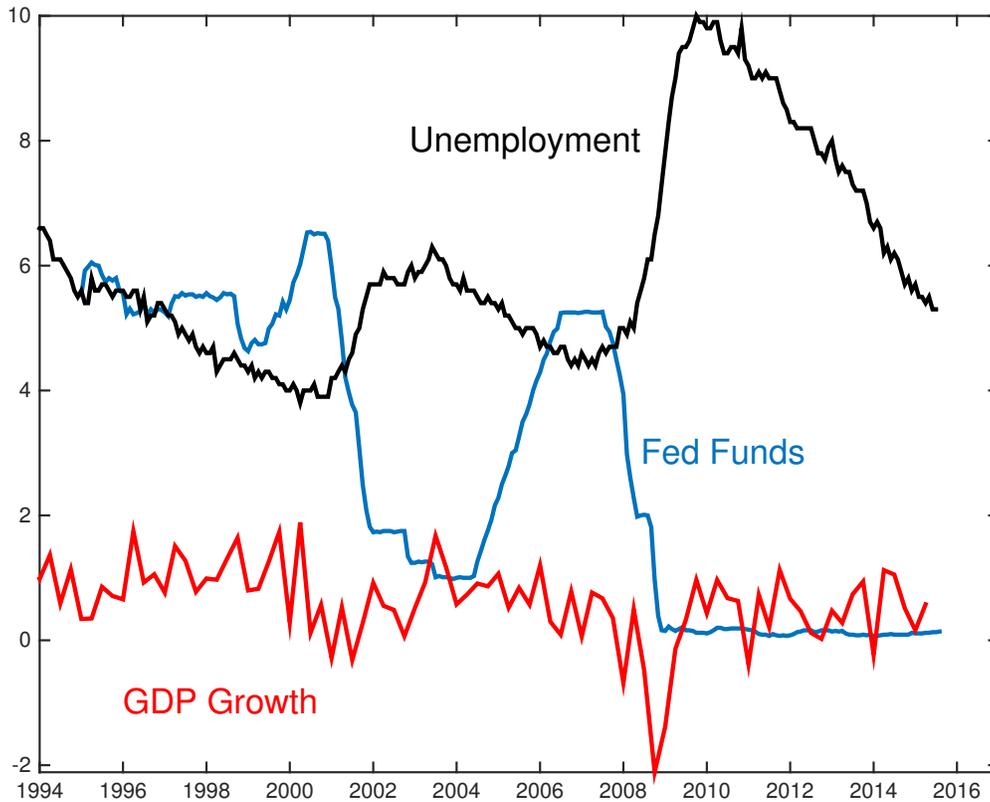
cident with QE2 and QE3 purchases.

Figure 2: US federal funds rate, unemployment rate, and real GDP growth rate.

Figure 2 plots the unemployment rate and GDP growth rate. Together with Figure 1, these figures also show no visible difference in macroeconomic dynamics in and out of the zero rate / QE state, and in particular no increased volatility at the zero bound. Yes, there was a bigger shock in 2008. But the unemployment recovery looks if anything a bit faster than previous recessions. Output growth, though too low in most opinions, is if anything less volatile than before.

Figure 3 tells a similar but longer story for Japan. Japanese interest rates declined swiftly in the early 1990s, and essentially hit zero in 1995. Again, armed with the traditional theory, and seeing data up to the bottom of inflation in 2001, or again in late 2010, predicting a deflation "spiral" is natural. But again, it never happened. Despite large fiscal stimulus and quantitative easing operations, Japanese interest rates stuck at zero with slight deflation for nearly two decades. The 10 year government bond rate never budged from its steady downward trend.

The bottom panel of Figure 3 repeats the story for Europe. Here the spread of low rates and slight deflation is even stronger than in the US.
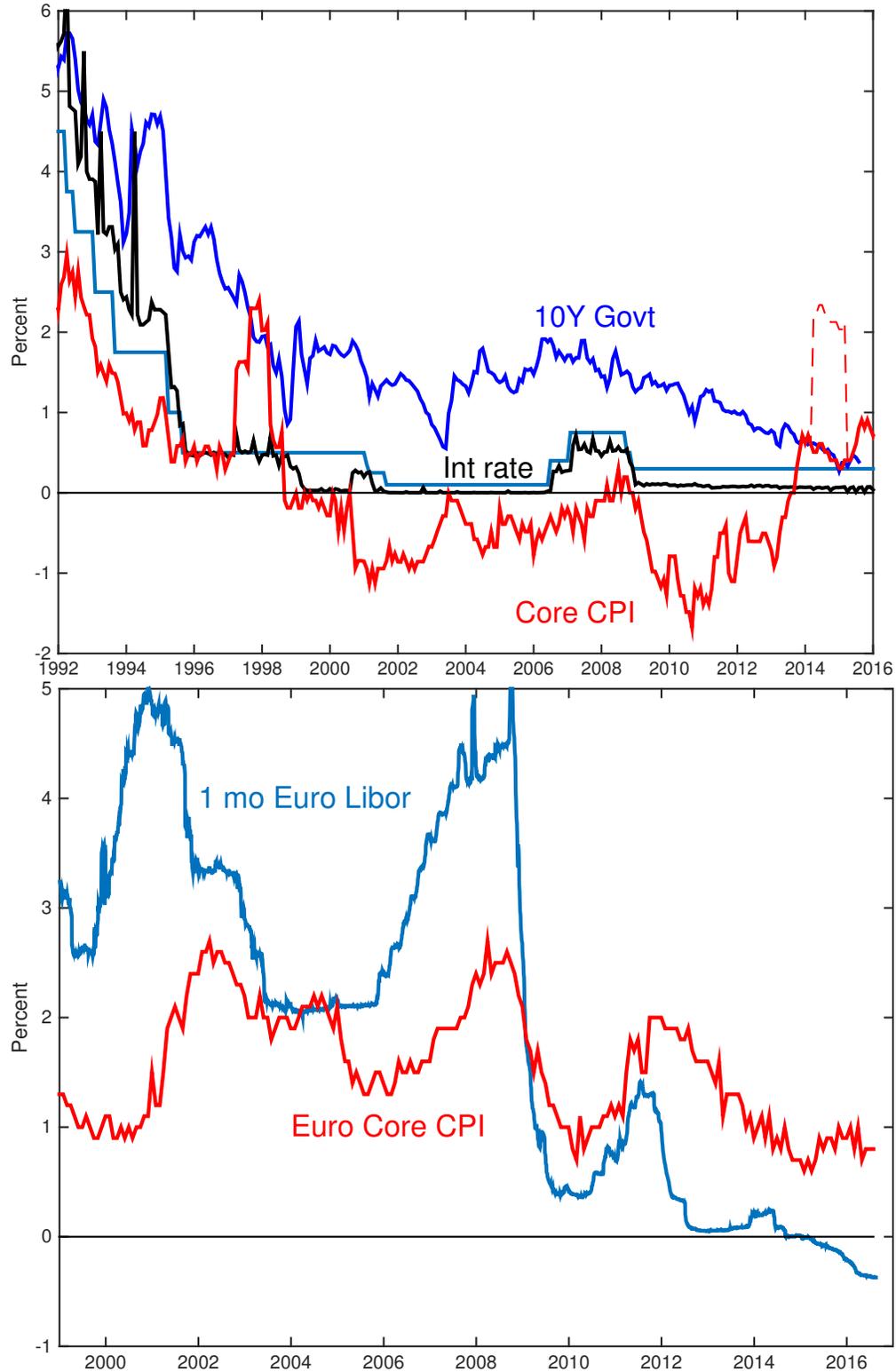
Figure 3: Japan and Europe. Top: Discount rate, Call rate, Core CPI, and 10 year Government Bond Yield in Japan. The thin line presents the raw CPI data. Thick line adjusts the CPI for the consumption tax by forcing the April 2014 CPI rise to equal the rise in March 2014. Bottom: Europe

Both Japan and Europe diverge from the U.S. in the last few years, with less inflation and lower interest rates. But are Japanese and European inflation lower despite their lower or even negative interest rates, or because of them?

2.2   Theories

Old-Keynesian models predict that inflation is unstable at the zero bound, as it is under passive policy or an interest rate peg. The Taylor rule stabilizes an otherwise unstable economy, but when it cannot operate, inflation will spiral out of control. (I use "zero bound" as a short hand. The fact that interest rates were a quarter percent above or below zero is not important to this discussion. The key point is that interest rates no longer move more than one-for-one with inflation in the downward direction.)

New-Keynesian models predict that inflation is stable at the zero bound, as it is under passive policy or an interest rate peg. However, new-Keynesian models are indeterminate at the zero bound. There are many equilibria, and the economy can jump between them following "sunspots" or "self-confirming expectations." Thus, the zero bound, passive interest rate policy, or an interest rate peg lead to extra inflation volatility. In these models, an interest rate rule that responds to off-equilibrium inflation more than one for one destabilizes an otherwise stable economy. The economy is then assumed to jump to the one remaining non-explosive equilibrium. Destabilization removes local indeterminacies. But when this "active" rule cannot operate, sunspot volatility breaks out.

The key distinction between the old and new-Keynesian models is rational vs. adaptive expectations.

The fiscal theory of the price level adds an "active" fiscal policy to the new-Keynesian model. Rather than assume that the Congress will raise or lower taxes or spending to pay off any multiple-equilibrium inflation-induced change in the real value of government debt, we assume that people expect less than perfect adjustment. Now unexpected inflation is determined by the change in present value of primary surpluses. If there is no change in volatility of that present value at and away from the zero bound, then there is no change in the volatility of unexpected inflation. Thus, adding the fiscal theory of the price level to the sticky-price, rational-expectations new-Keynesian framework, we have a theory that is stable at the zero bound, and consistent with the low volatility of inflation.

Since this is review of well-known results, my contribution here is a model that maximizes

simplicity rather than realism or generality.

The three models are hard to tell apart in normal times, when the Taylor rule or active new-Keynesian policy can operate. Interest rates and inflation all move up and down together in all three views. In normal times, the standard active-money new-Keynesian view and the fiscal theory view are observationally equivalent (Cochrane (1998), Cochrane (2011b)). That is why a long zero bound is such an important experiment.

Since these points are known, though perhaps underappreciated due to the technical complexity of realistic models, the pedagogical model here maximizes simplicity and transparency rather than realism or generality.

2.3   A simple model

Consider a Fisher equation, a Phillips curve, a static IS curve, and a Taylor rule for monetary policy:

$$i_t = r_t + \pi_t^e \qquad \text{Fisher} \qquad (1)$$

$$\pi_t = \pi_t^e + \kappa x_t \qquad \text{Phillips} \qquad (2)$$

$$x_t = -\sigma(r_t - r^* - v_t^r) \qquad \text{IS} \qquad (3)$$

$$i_t = \max\left[r^* + \pi^* + \phi\left(\pi_t - \pi^*\right) + v_t^i, 0\right]. \qquad \text{Taylor} \qquad (4)$$

Here i is the nominal interest rate, r is the real interest rate, π is inflation, π^* is the inflation target, r^* is the natural rate, π^e is expected inflation, x is the output gap, v^i is a monetary policy disturbance, and we can call v^r a "natural rate" disturbance. By specifying a static IS equation, without the usual term $E_t x_{t+1}$ on the right hand side, we can solve the model trivially. The same points hold in more general and realistic models.

Eliminating x and r, we reduce the model to the solution of a single equation in π:

$$i_t = \max\left[r^* + \pi^* + \phi\left(\pi_t - \pi^*\right) + v_t^i, 0\right] = -\frac{1}{\sigma\kappa}\pi_t + \left(1 + \frac{1}{\sigma\kappa}\right)\pi_t^e + r^* + v_t^r, \qquad (5)$$

or

$$(1 + \phi\sigma\kappa)\left(\pi_t - \pi^*\right) = (1 + \sigma\kappa)\left(\pi_t^e - \pi^*\right) + \sigma\kappa(v_t^r - v_t^i) \qquad (6)$$

when we ignore the zero bound, or when the interest rate is positive, i.e. when

$$i_t > 0 \leftrightarrow r^* + \pi^* + \phi\left(\pi_t - \pi^*\right) + v_t^i > 0, \tag{7}$$

and

$$\pi_t = (1 + \sigma\kappa)\,\pi_t^e + \sigma\kappa(r^* + v_t^r) \tag{8}$$

when the zero bound binds, and (7) does not hold. Equation (8) is the same solution as obtains from (7) if the Taylor rule picks zero, i.e. with $\phi = 0$, $v_t^i = -(r^* + \pi^*)$.

By substituting $\pi_t^e = \pi_{t-1}$ or $\pi_t^e = E_t\pi_{t+1}$ we recover adaptive vs. rational expectations versions of the model.

2.4 Old-Keynesian

Old-Keynesian models, including much monetarist thought such as Friedman (1968), specify adaptive expectations, $\pi_t^e = \pi_{t-1}$. Substituting that specification in (6) we obtain

$$(\pi_t - \pi^*) = \frac{1 + \sigma\kappa}{1 + \phi\sigma\kappa}(\pi_{t-1} - \pi^*) + \frac{\sigma\kappa}{1 + \phi\sigma\kappa}(v_t^r - v_t^i) \tag{9}$$

For $\phi < 1$, or at a peg $\phi = 0$, the dynamics of this system are *unstable* and *determinate*. The coefficient on lagged inflation is above one. There is only one solution.

In this model the Taylor rule stabilizes an otherwise unstable economy. Raising ϕ to a value greater than one, the coefficient on lagged inflation becomes less than one. But if the Taylor rule cannot or does not act, inflation will spiral away.

If $\phi > 1$ but the economy hits the zero bound, then (8) takes over, in this case

$$\pi_t = (1 + \sigma\kappa)\,\pi_{t-1} + \sigma\kappa(r^* + v_t^r) \tag{10}$$

Now the coefficient on π_{t-1} is greater than one, and a deflation spiral can set in. ((10) is the same as (9) at $\phi = 0$, $v_t^i = -(r^* + \pi^*)$.)

Figure 4 illustrates a typical old-Keynesian spiral prediction. The economy starts at the steady state, $i = i^* = 2\%$, $\pi = \pi^* = 2\%$. The Figure then considers a 3 percentage point decline in the natural rate v^r. I simulate (9) and (10) forward. With the new -3% real rate, the economy needs to find a steady state in which inflation rises 3 percentage points relative to the interest rate. Indeed interest rates fall more than inflation, starting to produce a lower real rate. But soon the interest

rate can fall no more, we switch to (10) dynamics, and then deflation spirals out of control. Once in the trap, deflation keeps spiraling even though the natural rate shock ends.
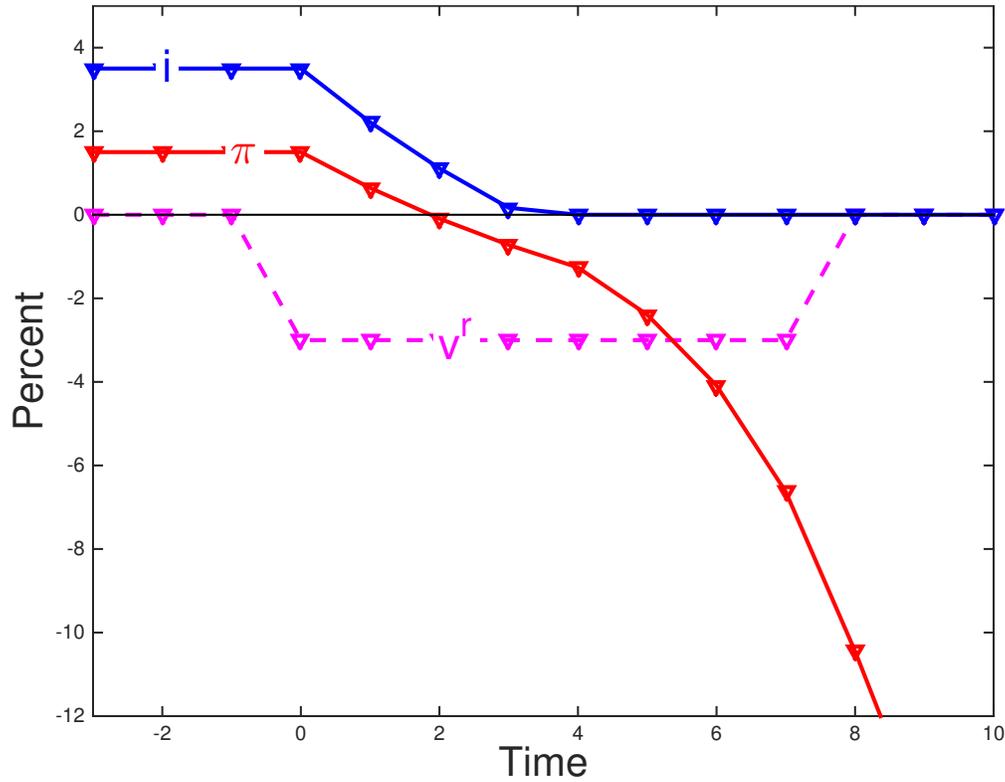

Figure 4: Old-Keynesian spiral at the zero bound.

2.5   New Keynesian

The new Keynesian tradition instead uses rational expectations: $\pi_t^e = E_t \pi_{t+1}$. Substituting this specification into (6), we obtain

$$E_t(\pi_{t+1} - \pi^*) = \frac{1 + \phi\sigma\kappa}{1 + \sigma\kappa}(\pi_t - \pi^*) + \frac{\sigma\kappa}{1 + \sigma\kappa}(v_t^i - v_t^r). \tag{11}$$

For $\phi < 1$, the coefficient on π_t is less than one, so this model is *stable* all on its own, even under an interest rate peg $\phi = 0$ or at the zero bound. Adaptive, backward-looking expectations make price dynamics unstable, like driving a car by looking in the rear-view mirror. Rational, forward-looking expectations make price dynamics stable, as when drivers look forward and veer back on the road without outside help. The new-Keynesian model thus reverses the hallowed doctrine – the first item in the Friedman (1968) list of what monetary policy cannot do –

that interest rate pegs are unstable.

However, the new-Keynesian model with $\phi < 1$ is *indeterminate*. It only ties down expected inflation $E_t\pi_{t+1}$, where the old-Keynesian model ties down actual inflation. To the solutions of this model we can add any expectational error, δ_{t+1}, such that $E_t\delta_{t+1} = 0$, and then write the model's solutions as

$$(\pi_{t+1} - \pi^*) = \frac{1 + \sigma\kappa\phi}{1 + \sigma\kappa}(\pi_t - \pi^*) + \frac{\sigma\kappa}{1 + \sigma\kappa}(v_t^i - v_t^r) + \delta_{t+1}. \tag{12}$$

The δ shocks that index multiple equilibria are "sunspots," or "multiple self-confirming equilibria." In the usual causal interpretation of the equations, small changes in expectations about the future $E_t\pi_{t+j}$ induce jumps between equilibria π_t. Passive policy, a peg or the zero bound induce inflation *volatility*.

In this model, an active policy $\phi > 1$ induces *instability* into an otherwise *stable* model, in order to try to render it locally *determinate* – to select a particular choice of $\{\delta_{t+1}\}$. For $\phi > 1$, expected inflation diverges for all values of inflation π_t other than

$$\pi_t - \pi^* = -\frac{\sigma\kappa}{1 + \sigma\kappa}\sum_{j=0}^{\infty}\left(\frac{1 + \sigma\kappa}{1 + \sigma\kappa\phi}\right)^{j+1}E_t(v_{t+j}^i - v_{t+j}^r). \tag{13}$$

Equivalently, taking $E_t - E_{t-1}$,

$$\delta_t = -\frac{\sigma\kappa}{1 + \sigma\kappa}\sum_{j=0}^{\infty}\left(\frac{1 + \sigma\kappa}{1 + \sigma\kappa\phi}\right)^{j+1}(E_t - E_{t-1})(v_{t+j}^i - v_{t+j}^r). \tag{14}$$

The economy jumps by an expectational error δ_t just enough so that expected inflation does not explode.

This method of inducing determinacy is not entirely uncontroversial (Cochrane (2011b)). That controversy is one motivation to look for another theory. But that controversy is not central here. If have passive policy or a peg $\phi < 1$, we are back to the conclusion of (12): stability – no spirals – but indeterminacy leading to volatility.

New-Keynesian models also predict multiplicity and thus inflation volatility at the zero bound. The dynamics switch to

$$E_t\pi_{t+1} = \frac{1}{1 + \sigma\kappa}\pi_t - \frac{\sigma\kappa}{1 + \sigma\kappa}(r + v_t^r) \tag{15}$$

at the bound. (These $i = 0$ dynamics are also the same as (11) with $\phi = 0$, $v^i = -(\pi^* + r^*)$.)

Figure 5 illustrates the model with a zero bound. The kinked line expresses the equilibrium conditions (11)-(15). As a result of the zero bound, there are two steady states, represented by dots. The arrows represent the dynamics. The right hand steady state is the conventional "active" equilibrium. This is unstable and hence locally determinate. Any other equilibrium spirals away. However, the presence of the zero bound means that interest rates i cannot move downward more than one for one with inflation forever. Therefore, there is a second, stable, "liquidity trap" steady state, with $i = 0$ and slight deflation, at the left.
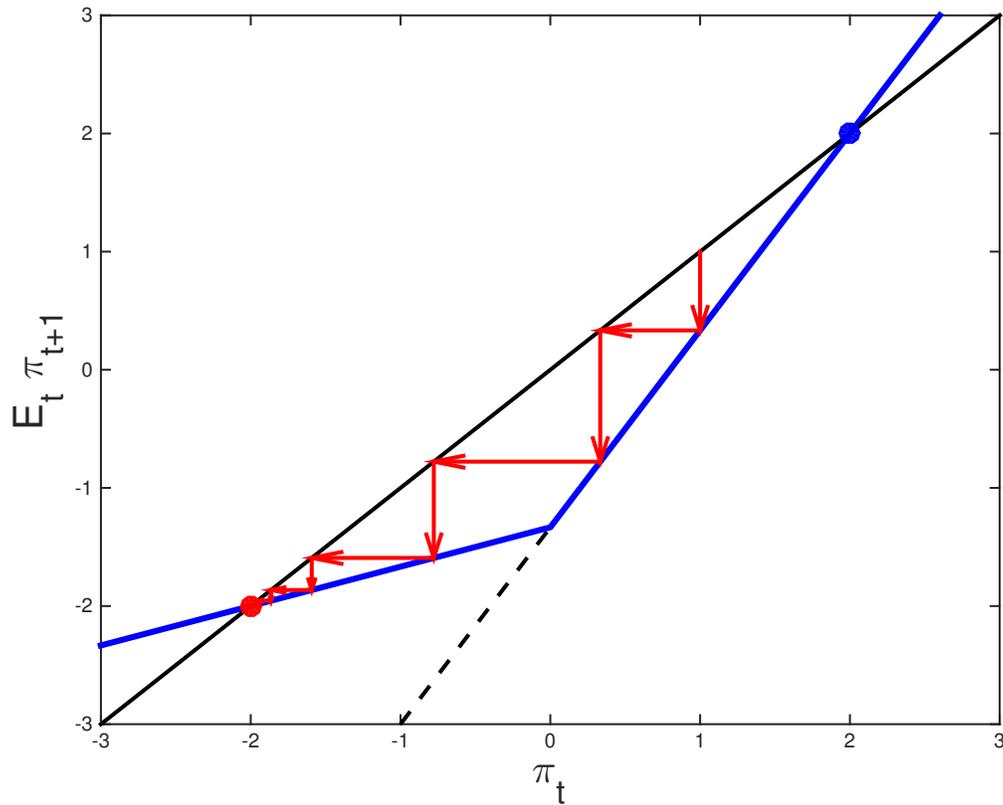

Figure 5: The zero bound in a new-Keynesian model. The kinked line expresses the equilibrium condition (15). The straight line is the 45 degree line. The two dots express the two steady states. The left-hand one is the stable indeterminate "liquidity trap" state. The right-hand steady state is the unstable determinate state. Arrows indicate dynamics. The dashed line shows the policy rule in the absence of the zero bound. $\phi = 2$, $\pi^* = 2\%$, $r = 2\%$, $\sigma\kappa = 2$.

The zero-bound steady state is indeterminate as well as stable. The model does not pin down $\pi_{t+1} - E_t\pi_{t+1}$, so inflation can always jump away from this steady state. Such jumps are expected to revert back to the liquidity trap steady state, so the rule in these models that one throws out explosive equilibria cannot eliminate them.

As a result of the potential extra sunspot volatility of inflation, authors such as Benhabib, Schmitt-Grohé, and Uribe (2002) view the liquidity trap state as a problem, and devote great effort to additional policy prescriptions that governments might adopt to avoid them, despite the fact that absent volatility such steady states would be welfare-improving. Zero interest rates are the Friedman-optimal quantity of money after all, and low inflation reduces the distortions of sticky prices.

The alternative equilibrium shown by arrows in Figure 5 starts away from the zero bound. This equilibrium reminds us that the zero bound is an attractive state for any value of inflation below the active equilibrium on the right. In particular, even sunspot jumps that raise inflation and temporarily relax the zero bound do not allow the economy to escape the trap. And small downward jumps out of the rightmost "active" steady state now converge to the zero bound as well.

In Figure 5, the slide to a liquidity trap can happen all on its own. However, as in the old-Keynesian models, shocks can move us there as well. I discuss this mechanism below.

(The point can be made most simply without pricing frictions, as is the case in Benhabib, Schmitt-Grohé, and Uribe (2002). If we merge the policy rule (4) with the frictionless Fisher equation

$$i_t = r + E_t \pi_{t+1}$$

then inflation dynamics follow

$$E_t \pi_{t+1} = \max \left[\phi \left(\pi_t - \pi^* \right) + \pi^*, -r \right].$$

The frictionless version leads to the same analysis as in Figure 5 but with a horizontal line to the left of the kink. Benhabib, Schmitt-Grohé, and Uribe (2002) also use the fully nonlinear model, but in this range it is visually indistinguishable from the linearization.)

2.6   Fiscal theory of the price level

To show how the fiscal theory of the price level enters this kind of model in the simplest way, I specify one-period or floating-rate debt. Then the government debt valuation equation stating that the real value of nominal debt equals the present value of primary (net of interest) surpluses reads

$$\frac{B_{t-1}}{P_t} = E_t \sum_{j=0}^{\infty} M_{t,t+j} s_{t+j} = E_t \sum_{j=0}^{\infty} \frac{1}{R_{t,t+j}} s_{t+j} = E_t \sum_{j=0}^{\infty} \beta^j s_{t+j}. \tag{16}$$

Here, B_{t-1} is the face value of one-period debt, issued at $t-1$ and coming due at t, P_t is the price level and s_t is the real primary surplus. In the first equality, we discount the future with a general stochastic discount factor M. In the second equality, we discount the future with the ex-post real rate of return on government debt. Either of these statements is valid in general; the latter ex-post as well. The third version specializes to a constant real interest rate with $\beta = 1/(1+r)$.

In general, real interest rate variation affects the present value of surpluses on the right hand side of (16). Models with sticky prices imply variation in real interest rates. I argue below that such real interest rate variation is of first-order importance to understand data, experience, and policy via the fiscal theory. However, the stability and determinacy points are not affected by long-term debt or real rate variation, so I specify constant real rates and short-term debt to make basic points here, then generalize at a cost in algebra below.

Moving the index forward one period, multiplying and dividing (16) by P_t and taking innovations,

$$\frac{B_t}{P_t}(E_{t+1} - E_t)\left(\frac{P_t}{P_{t+1}}\right) = (E_{t+1} - E_t)\sum_{j=0}^{\infty}\beta^j s_{t+1+j}. \tag{17}$$

In this simple setup, *unexpected inflation is determined by innovations to the expected present value of surpluses.*

Indeterminacy is the inability of the standard New Keynesian model to nail down unexpected inflation with passive policy $\phi < 1$, because we can always add any unexpected shock δ to the solution as in (12). Equation (17) shows that the fiscal theory of the price level solves the indeterminacy problem. Unexpected inflation or disinflation revalues outstanding government debt, which requires changes in discounted future surpluses. If people do not expect, say, that an unexpected deflation will be met by more taxes or less spending to finance a windfall to bondholders, then the bondholders try to get rid of overvalued bonds, which raises aggregate demand and hence keeps the price level from falling in the first place.

In sum, the fiscal theory of the price level merged with the new-Keynesian model says that at the zero bound, as under an interest rate peg passive $\phi < 1$ policy, inflation is *stable*, and *determinate*. If there is no change in the volatility of fiscal expectations or their discount rate, there is no change in the volatility of unexpected inflation.

The central bank is far from powerless in the fiscal theory. In this simple frictionless model, the Fed, by setting in interest rate target i_t will set expected inflation $i_t = r + E_t\pi_{t+1}$. Expectations of discounted fiscal policy then select which value of unexpected inflation $\pi_{t+1} - E_t\pi_{t+1}$ occurs. Monetary policy – setting of interest rate targets – remains the central determinant of the path of

expected inflation. Stable fiscal policy expectations just cut own on unexpected inflation volatility. (Cochrane (2014b) explains how the Fed and Treasury can target the nominal interest rate in this model, even with no nominal rigidities, no monetary frictions, no open market operations, by varying the interest the Fed pays on reserves.)

Below, I describe how to integrate fiscal theory with the new-Keynesian model with sticky prices. Again, the fiscal theory ends up just choosing equilibria, and does not alter the equilibrium dynamics. However, time-varying real interest rates now contribute to a time-varying discount rate in the government debt present value relation, so just which equilibrium gets picked is a bit different than in this constant-interest-rate present-value calculation.

Equation (16) *holds* in all models. One cannot "test the fiscal theory" by testing whether this equation holds, and the new-Keynesian model does not operate aloof from fiscal foundations. In the standard new-Keynesian model one assumes that the treasury always adjusts surpluses s_t ex-post to validate (16) for whatever price level emerges by the Fed's equilibrium-selection policies. That assumption means the equation now determines surpluses rather than the price level. But the equation still holds. All unexpected inflations and deflations correspond to changes in expected fiscal policy in both models. If people do not expect the Congress and Treasury to follow "passive" policy, for example raising taxes or cutting spending to validate an disinflation-induced present to bondholders, then the unexpected disinflation won't happen.

2.7   Language

The language used to describe dynamic properties of economic models varies, and I can dispel some remaining confusion by being explicit about my language choices.

I use the words "stable" and "unstable" in their classic engineering sense, to refer to the dynamic properties of the underlying dynamic system. A scalar system $z_{t+1} = Az_t + \varepsilon_{t+1}$ is stable if $|A| < 1$ and unstable if $|A| > 1$. Authors often use "stable" to mean the opposite of "volatile," which I term "quiet," but stability and volatility are distinct concepts. A stable system with large shocks can display lots of volatility.

I use the word "determinate" to mean that an economic model only has one equilibrium. Determinacy is also distinct from volatility and stability, all frequently confused no matter how one names them.

A harder case concerns expectational models with roots greater than one, and a variable that can jump, $E_t(z_{t+1}) = Az_t + v_t$, $|A| > 1$. I continue to use the word "unstable" to describe their

dynamics. However, if one rules out explosions and solves forward, $z_t = E_t \sum_{j=0}^{\infty} A^{-(j+1)} v_{t+j}$, then one could justifiably call the equilibrium path of $\{z_t\}$ "stable" since it always jumps just enough to forestall explosions, and samples show future z expected to revert back after a shock. This behavior is sometimes called "saddle-path stable." I use the term "stationary" to describe this property of *equilibrium* paths, using the word "stable" or "unstable" to describe the properties of the dynamic system, A. The "stability" of a forward-looking system that jumps to offset instability is qualitatively different from that of a system with backward-looking dynamics and no jump variables, in which economic forces slowly push the system back to equilibrium.

The standard three-equation new-Keynesian model with passive $\phi < 1$ monetary policy has dynamics of the form $E_t(z_{t+1}) = Az_t + v_t$ in which one eigenvalue of A is greater than one and unstable, and the other is less than one and stable. One could call such a model "mixed," or develop additional language to describe the relative number of unstable eigenvalues and expectational equations. However, in this case, we uncontroversially solve the unstable eigenvalue forward, using the real transversality condition, and uniquely determining one eigenvector linear combination of variables. Then, the other eigenvector follows a scalar $E_t(z_{t+1}) = Az_t + v_t$ with $A < 1$. I use the language "stable" vs "unstable" and "determinate" vs. "indeterminate" to describe the remaining eigenvector.

I use the word "policy rule" rather than "Taylor rule" to describe $i_t = \phi \pi_t$ with $\phi > 1$ in new-Keynesian models. The latter is really a misnomer. Taylor's rule stabilizes an unstable determinate model, to bring unstable inflation under control; see Taylor (1993), Taylor (1999). In new-Keynesian models, the rule operates to turn a stable model into an unstable one and produce local determinacy, which is an entirely different function.

The "active" and "passive" terminology to describe $\phi > 1$ vs $\phi < 1$ in monetary policy, and surpluses that do not vs. do move exactly enough to validate any inflation that comes along in fiscal policy, comes from Leeper (1991).

I use the word "disturbance" rather than "shock" and roman letters v rather than greek ε as a reminder that "disturbances" can be serially correlated, where "shocks" are unpredictable.

I refer to unstable dynamics such as graphed in Figure 4 as a "spiral," but a downward price level jump followed by recovery, produced by some new-Keynesian zero-bound models such as Werning (2012) or Cochrane (2016b) as a "jump." (See the top panel of Figure 19 for an example.) Some authors use the same word for both. I think this confuses the quite different dynamics of new and old-Keynesian models.

2.8   Monetarism and QE

Monetarist thought took a back seat during the interest-rate targeting period starting in 1982. It largely disappeared from academia, but remains a powerful strain of thought in policy circles and commentary. When Japan hit zero interest rates in the 1990s, monetarist ideas came back quickly in the form of "helicopter money." Ben Bernanke advocated the view most prominently, among other alternative policies (see the review and fascinating discussion in Ball (2016)). Monetarist ideas remain a force behind the analysis of QE (quantitative easing).

Quantitative easing has two parts: The Fed buys bonds or other assets, and issues reserves. Here I consider the question whether larger reserve supply is inflationary. In traditional monetarist thought, whether the increase in the M of $MV = PY$ stems from the Fed buying short-term Treasuries, long-term Treasuries, mortgage-backed securities, or from buying nothing – from helicopter drops, or changes in reserve requirements allowing more inside money – makes no difference.

Much of the current QE discussion takes on a diametrically opposite view: The liabilities (reserves) are irrelevant, but QE "works" by affecting the term and credit spreads in long-term interest rates in segmented bond markets. The asset purchases matter, not the reserve issues. Whether or not QE lowered long term rates by as much as half a percent via this mechanism, for how long, and whether such rate declines in segmented markets had a stimulative effect, is really unrelated to the big issues of inflation stability and determinacy that matter here, so I ignore this question.

Helicopter money combines increased money with a fiscal expansion. From a fiscal theoretic point of view, that such an expansion could cause inflation is not a surprise. In this section, therefore, I only consider increases in reserves accompanied by asset purchases, and hence no direct implied fiscal expansion (ignoring also the Fed's assumption of credit term risk in asset purchases).

In standard new-Keynesian models (before adding extra frictions, see Woodford (2012), QE has no inflationary effect.

In classic monetarist thought, the zero bound is not an important constraint on monetary policy. Yes, the Fed can then no longer control the quantity of money implicitly via an interest rate target. But nothing stops the Fed from buying bonds and issuing reservers at a zero interest rate, and letting MV=PY do its work – as, a monetarist might add, it should have been doing all along anyway.

The behavior of velocity, equivalently of money demand, at zero interest rates is the central issue. Monetarist thought emphasizes the idea that velocity is "stable," at least in the "long run." Even at zero interest rates, or with interest-paying reserves, or in the puzzling situation that reserves pay even more than Treasuries, and even if velocity V decreases somewhat temporarily when M increases, velocity will soon bounce back and persistently more M will lead to more PY.

The contrary view is that at zero interest rates, or when money pays market interest, money and short-term bonds become perfect substitutes. Velocity becomes a correspondence, not a function of interest rates. MV = PY becomes V = PY/M; velocity passively adjusts to whatever split of debt between money and reserves the Fed chooses. The financial system is perfectly happy to hold arbitrary amounts of reserves in place of short-term treasuries. Open market operations have no more effect on spending than open change operations, in which the Fed trades two $10 bills for each $20.

This issue was central to the monetarist vs. Keynesian debates of the 1950s and 1960s. Keynesians thought that at the zero rates of the great depression, money and bonds were perfect substitutes, so monetary policy could do nothing, and advocated fiscal stimulus instead. Monetarists held that additional money, even at zero rates, would be stimulative; and the Fed's failure to provide additional money the great policy error of that decade.

In the postwar era of positive interest rates, with zero interest on reserves, there was really no way to tell these views apart. Now there is, and the experiment is nearly as decisive as the stability of an interest rate peg.

Figure 1 already demonstrated no visible time-series relationship between the massive increase in reserves and inflation. Figure 6 presents a more traditional picture of reserves, scaled by nominal GDP, as a function of their opportunity cost, the difference between the effective Federal Funds rate (the rate at which banks can lend out reserves) and the interest on excess reserves at the Fed. You see a steady decline in reserves from 1980 to 2000, as fewer bank liabilities required reserves and banks became better at avoiding excess reserves. You also see a negatively sloped curve in the periodic recessions. Following tradition, I'll just call this plot a "demand curve" without further ado. I fit the dashed line to the 2000-2007 period, and it gives a conventional semi-elasticity log(reserves/PY) = constant - 0.094 (interest rate). If one extends the line to the vertical axis, it suggests that reserve demand should top out at about 0.12% of GDP, and if reserve velocity is "stable," further increases should lead to increases in nominal GDP.

What does reserve demand do at zero opportunity cost? As Figure 6 shows, we have now run this out-of-sample experiment, on a grand scale – note the numbers on the log scale y axis.
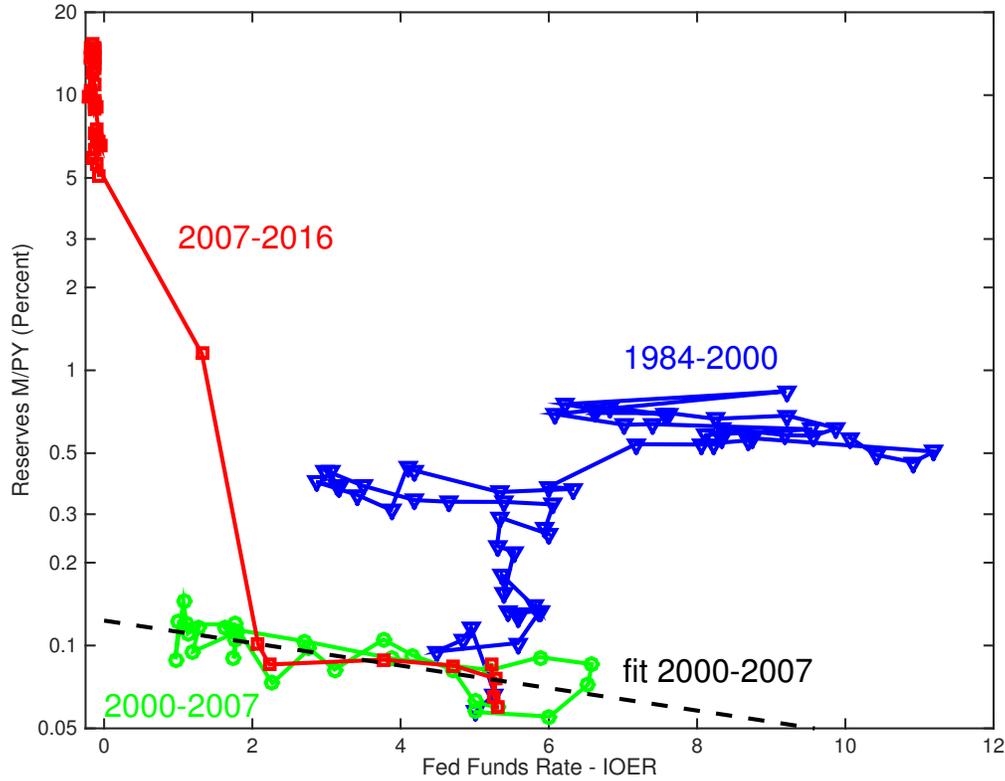
Figure 6: Reserves vs opportunity cost. Y axis: total reserves (Fred series WRESBAL) at end of quarter divided by nominal GDP; log scale. X axis: effective Federal funds rate (Fred FF) less interest on excess reserves (Federal Reserve website policy rates). Sample 1984:1-2016:2. Line fit by OLS 2000:1-2006:4

Reserves increased by two orders of magnitude – from 0.1% of GDP to 15% of GDP – with no visible effect on inflation or nominal GDP.

The answer seems unavoidable: Reserve demand is a correspondence when reserves pay market interest rates; reserves and short-term debt are perfect substitutes; there is no tendency for velocity to revert to some "stable" previous value; arbitrary quantities of zero-cost reserves do not cause inflation. The massive size of the experiment avoids conventional objections – perhaps there was a contemporaneous "velocity shock" such as those alleged to move money demand in the 1980s; perhaps nominal GDP would have fallen had the Fed not increased reserves, and so on.

One may object that reserves are not the relevant M in MV=PY. Figure 7 presents M1, and M2 as percentages of nominal GDP PY, versus the three-month Treasury bill rate. (Since many components of these aggregates pay interest, the three-month Treasury bill rate is not a good
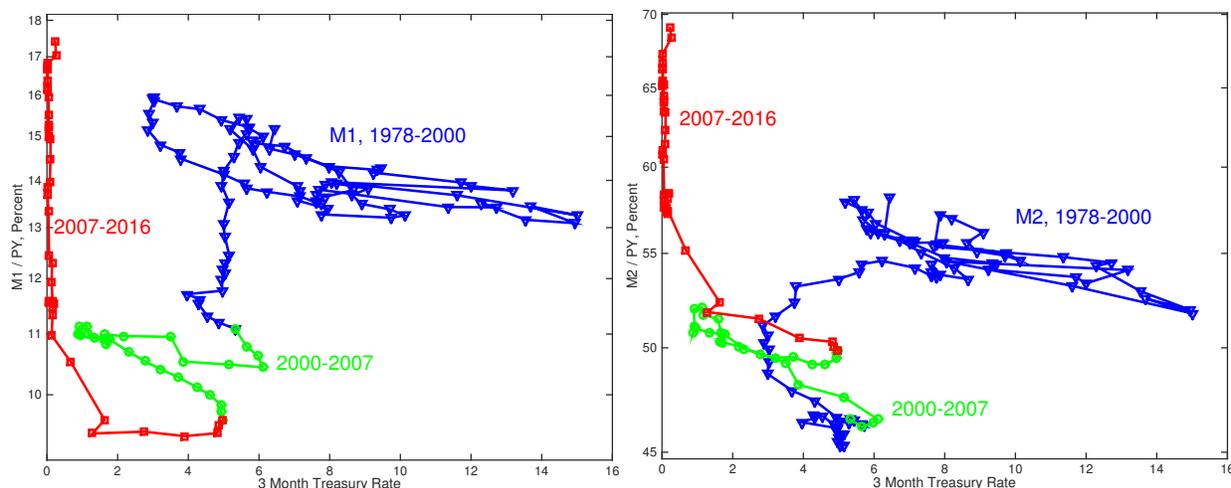
Figure 7: M/PY vs three month treasury bill rate, using M1, and M2.

measure of their opportunity costs, but I'm both following tradition and keeping it simple.) Each aggregate increased since 2007, but less, proportionally, than reserves increased. M1 increased from about 9% of GDP to almost 18%, a bit less than doubling – but not rising by a factor of 100. M2 increased from 50% of GDP to almost 70% of GDP, "only" a 60% rise. Currency (not shown) rose from 5.9% of GDP to 7.5%, a 30% increase.

These are still substantial increases, which if velocity were "stable" should result in equiproportionate rises in nominal income, and eventually the price level.

But even making these plots grants too much. How much inflation a "monetarist" view predicts for the recent period is beside the point. The question for us is whether arbitrary amounts of *reserves*, exchanged for short-term treasuries, cause any inflation; whether open market operations or QE operations affect inflation. Even if one believes that $M2 \times V = PY$ (say), and one claims that a monetarist view does not predict inflation in the current period because reserves did not leak in to M2, that fact only verifies that arbitrary quantities of *reserves* are not inflationary, precisely because they do not leak into M2. That leakage is a central part of the transmission mechanism, and it's not transmitting. When banks are holding trillions of dollars of excess reserves, the money multiplier ceases to operate. So, to argue there is no inflation because M2 did not rise much is precisely to admit that arbitrary quantities of interest-bearing reserves, corresponding to arbitrarily lower quantities of interest-bearing treasuries, are not inflationary.

Figure 6 really only makes a secondary point: What would happen if reserves *were* to leak to larger increases in M1 or M2, or other aggregates? Figure 6 suggests that these aggregates display the same behavior as reserves, only on a smaller (so far) scale – they happily crawl up

the vertical axis. There is nothing in their behavior so far to suggest that this correspondence could not reach the astonishing level that banks' willingness to hold reserves at the expense of treasuries have reached.

Looking back at an 80- year controversy, one may wonder why the "stability" of velocity, even at zero interest cost, and the consequent perfect substitutability of treasuries for reserves, was so controversial. One answer may be that for most of that period, there was no alternative coherent, simple, economic theory of the price level that could hold in that circumstance. In a monetarist world, strike MV=PY and nothing ties down P. So, it is natural to stick with the idea that velocity must be "stable," as otherwise the price level would be indeterminate. To preserve price level control, monetarists, despite otherwise free-market tendencies, were reluctant to endorse financial innovation including lots of inside money, electronic transactions, and interest-bearing money. But now we have an equally simple theory – the fiscal theory – that ties down the price level when money and bonds are perfect substitutes, one that requires no monetary or pricing frictions and thus allows arbitrary financial innovation, and we have a long period of apparent empirical validation that inflation is stable at zero rates despite a massive "monetary" expansion.

My conclusion that abundant interest-bearing reserves do not cause inflation does not address many other objections to the Fed's large balance sheet. One may object to the Fed's assets – its purchases of long-term bonds, of mortgage-backed securities, and of other central bank's purchases of corporate bonds (ECB) and even stocks (BOJ) – both on grounds that independent central banks should not try to directly influence risky asset prices, or on political economy grounds that such policies constitute credit allocation better done (if at all) by politically-accountable Treasuries. This paper is silent on those questions.

3 Fisher

If we grant that inflation is stable at the zero bound, and by implication at an interest rate peg, that observation implies that raising interest rates should sooner or later *raise* inflation, contrary to the usually presumed negative sign. This proposition has become known as the "long-run neo-Fisherian" hypothesis.

This proposition is a form of long-run neutrality under interest rate rules. It basically says that the Fisher equation $i_t = r_t + E_t \pi_{t+1}$ is a stable steady state. "Stable" is a key qualifier. It is a steady state in old-Keynesian models, and interest rates and inflation move together in the

long run. But it is an unstable steady state in those models, so pegging the interest rate leads to spiraling inflation. A stable steady state means that inflation will eventually settle down to a fixed nominal interest rate. (I ignore here a long literature that worries whether long-run real interest rates r are affected by steady state inflation, so the Fisher relationship may not be exactly one for one. All that can be trivially added if needed.)

Both flavors of new-Keynesian model have this implication. The fiscal theory addendum only changes how we think about determinacy and equilibrium selection, and thus the immediate, unexpected-inflation response to a shock. Long-run neutrality is a proposition about long-run expectations in any equilibrium.

Stability and long-run neutrality are news to policy however, which is largely based on old-Keynesian adaptive expectations thinking. If they are true, central banks raising rates will partly cause the inflation they wish to forestall – though in the event will likely congratulate themselves for their prescience. Indeed, that inflation will rise before anticipated interest rate increases (Figure 12 below gives a good example), and the bank will seem to respond to inflation rather than to cause inflation. Stability and long-run neutrality also imply that low inflation at the zero bound is in part due to pedal misapplication – central banks, by keeping interest rates low, partly caused the low inflation that they were trying, wisely or not, to prevent.

But a long-run positive sign leaves open the possibility that higher interest rates *temporarily* lower inflation, and vice versa. The classic belief need not be wrong and may well reflect short-run experience. We can label the contrary proposition that higher interest rates raise inflation even right away as the "short-run" neo-Fisherian hypothesis.

The quest of this section, then, is to find the minimal simple economic model that produces a temporary negative impact of higher interest rates on inflation, validating at least part of the classic belief. (The qualifiers "simple" and "economic" are important.)

The main result is negative. The basic new-Keynesian model produces a uniformly positive sign. One simply cannot say, for example, that sure, the Fisher relation means that raising interest rates raises inflation, but sticky prices overturn that result for a while. They don't. A suite of sensible modifications one might adduce to provide the desired sign do not work. A novel fiscal theory argument with long-term debt produces the desired negative sign, though rather deeply changes one's views of just what monetary policy is and how it works.

Just what the core predictions of new-Keynesian models for the sign of monetary policy turns out to be a delicate question, which is the reason for much of this section and its apparent return

to basics that one would think were well established, but are not. The short preview: the apparent ability of the new-Keynesian model with standard policy rules to deliver a negative response for transitory interest rate shocks is an illusion. In fact the new-Keynesian model has a separate interest rate policy, which governs expected inflation, and an equilibrium selection policy, which can induce an unexpected inflation. By engineering an unexpected disinflation via the latter policy, (and only by this mechanism), the model can deliver a temporary negative sign for any persistence of the interest rate shock, not just transitory shocks. But that's bad news. There is therefore no necessary or logical connection between the Fed's choice to temporarily disinflate and its choice to raise interest rates, which on its own leads uniformly to higher inflation. Such a path remains entirely a Fed choice, not a characterization of how interest rates affect inflation in the economy. Viewed through fiscal theory eyes, which are observationally equivalent, interest rate rises may historically have coincided with fiscal contractions, which generate an unexpected disinflation. But there is no reason that monetary policy alone – interest rate changes that do not have contemporaneous changes in fiscal policy – should therefore lower inflation. In either view, monetary policy – interest rate policy – remains Fisherian.

We are left with three possibilities. 1) The temporary negative sign is not true. The impression that it is true comes from events that feature joint monetary-fiscal contractions, and overly aggressive fishing of VAR specifications. 2) The temporary negative sign is true, but it results from the simple long-term debt fiscal-theory mechanism, and has nothing to do with any of the standard stories. That mechanism also differs sharply in its implications for policy. 3) The temporary negative sign is true, but necessarily relies on novel, complex or non-economic ingredients. Any of these possibilities undermines traditional monetary economics.

3.1   A frictionless benchmark

I start with a simple frictionless model. One may think it obvious that a frictionless model does not deliver the desired negative sign, and one may wish to get on to price stickiness and other variations with more potential to do so. But it turns out those models don't work any better than the frictionless model, and all the conceptual issues can be shown in the very simple frictionless model, with much more transparent algebra.

The model consists of a Fisher equation,

$$i_t = r + E_t \pi_{t+1}, \tag{18}$$

the government debt valuation equation, which implies

$$\frac{B_t}{P_t}(E_{t+1} - E_t)\left(\frac{P_t}{P_{t+1}}\right) = (E_{t+1} - E_t)\sum_{j=0}^{\infty}\beta^j s_{t+1+j}, \tag{19}$$

or, linearizing,

$$(E_{t+1} - E_t)\pi_{t+1} = -(E_{t+1} - E_t)\sum_{j=0}^{\infty}\beta^j s_{t+1+j}/b_t, \tag{20}$$

where b is the real value of debt, and an interest rate policy rule,

$$i_t = (r^* + E_t\pi_{t+1}^*) + \phi(\pi_t - \pi_t^*) \text{ or} \tag{21}$$

$$i_t = r^* + \pi^* + \phi\pi_t + v_t^i. \tag{22}$$

One can derive the first two equations as the two important equilibrium conditions of a complete general equilibrium model with a constant endowment. See Cochrane (2005). The contrast between the two equivalent parameterizations of the interest rate rule, one with a a time-varying inflation target π_t^* and the other with a fixed target π^* and a policy disturbance v_t^i, will turn out to offer important insights. For the purpose of impulse-response functions, the occasionally binding zero bound is not important. This model is "simple," "economic," and consistent with stability at the zero bound.

3.1.1   Response function

Figure 8 presents responses of inflation to an interest rate rise for this model, as given by (18). The model produces a rise in expected inflation starting the period after the rate rise. There is no temporary inflation decline.

This calculation, using (18) alone, does not assume a peg, passive monetary policy, or fiscal theory, nor does it deny an underlying policy rule such as (22). This is a calculation of inflation given a path for the equilibrium interest rate. That interest rate may well be the result of underlying disturbances $\{i_t^*\}, \{\pi_t^*\}$ or $\{v_t^i\}$ with any value of ϕ. Indeed, we can and will use (22) to back out a sequence of disturbances that produce the desired interest rate path.

The response to a given *interest rate* path is a different object than the response to a given path of *policy disturbances* $\{v_t^i\}$. It is more common to plot the latter. But in the end, we are interested in how changes in inflation correspond to changes in *interest rates* which we can directly observe. The path of disturbances $\{v_t^i\}$ can be quite different from the interest rate path,
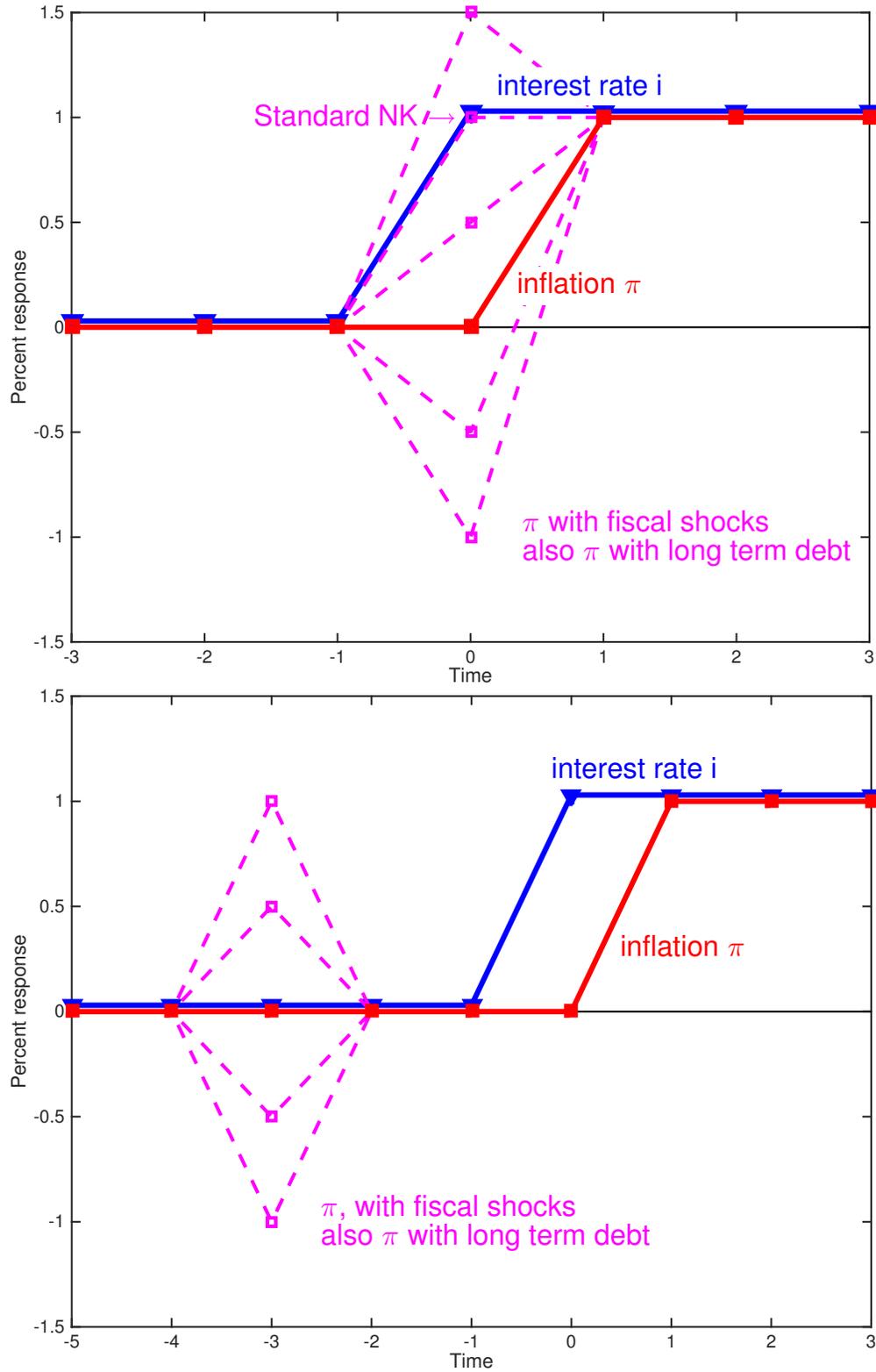
Figure 8: Response of inflation to a permanent interest rate increase. Frictionless model $i_t = r + E_t\pi_{t+1}$. Top: the rise is announced and implemented at time $t = 0$. Bottom: The rise is announced at $t = -3$. The solid inflation line assumes no unexpected inflation on the date of announcement. The dashed lines add unexpected inflation on the announcement date.

as we will see, even having different signs. Solving for the path of inflation given interest rates gives us a more general answer, as the same interest rate path can occur from many different policy disturbance paths, and from both active-fiscal and active-money regimes. It also saves us a reverse-engineering task of finding disturbances that generate the desired path of interest rates. Werning (2012) innovated this clever idea of first finding equilibrium inflation given equilibrium interest rate paths, and then, if needed, constructing the underlying policy rule.

3.1.2   Unexpected disinflation?

By itself, the Fisher equation (18) allows an arbitrary one-period unexpected inflation in the impulse-response function, upwards or downwards, coincident with the announcement of the policy change. The dashed lines in Figure 8 plot some possible response functions that pair an unexpected inflation shock with a rise in interest rates.

Perhaps we can deliver the desired negative sign by pairing a negative unexpected-inflation shock with the interest rate rise, choosing, say, the bottom-most dashed line of Figure 8?

Equation (20) helps us to think about this issue. Each value of unexpected inflation corresponds to a revision in the present value of future surpluses. Disinflations, by unexpectedly raising the value of government debt, must correspond to a higher present value of future surpluses. A 1% disinflation requires that the surplus/debt ratio rises permanently by one percent.

This kind of event may correspond to historical experience. The vast majority of monetary policy changes are reactions to events. It is sensible that fiscal policy reacts to the same events, that the interest rate rises we see historically represent joint monetary-fiscal contractions. One would have to orthogonalize VAR shocks very carefully to measure a monetary policy shock independent of fiscal policy, and no VAR has yet attempted it.

But if that is true, it follows that a pure monetary policy change, consisting of an interest rate rise *without* changes to fiscal policy, would not have a disinflationary effect, and would follow the Fisherian solid line.

This fiscal interpretation and underpinning of a disinflationary surprise is equally valid under "active" as under "passive" monetary policy. Equation (20) holds in classic new-Keynesian analysis under active $\phi > 1$ policy just as it does in the fiscal theory with passive monetary policy. If active $\phi > 1$ monetary policy selects the value of unexpected inflation, and fiscal policy "passively" responds by changing surpluses to pay off corresponding changes in the value of the debt, fiscal policy must still change those surpluses, or the disinflation doesn't happen. Even

"passive" fiscal policy must be voted on by Congress and implemented by the Treasury!

More deeply, the induced fiscal reaction is the *mechanism* by which monetary policy affects aggregate demand and thus inflation, even when fiscal policy is passive. The conventional reading of active monetary - passive fiscal policy regards the passive fiscal assumption as wiping out the fiscal equation (19). Then, monetary policy becomes simply a coordination device between multiple equilibria. But that is not the case. The fiscal equation is still present, and crucial. When monetary policy selects an equilibrium with lower inflation, and fiscal policy "passively" raises surpluses, at the original price level, people find government bonds undervalued. They reduce spending on other things to try to buy government bonds, reducing aggregate demand. The price level declines, eventually raising the value of government bonds to their proper level. In a model with no frictions this is the only way by which a nominal interest rate change affects aggregate demand and hence inflation. If fiscal policy does not cooperate, the aggregate demand does not materialize, and the price level does not change. Monetary policy may be the carrot that leads the fiscal horse that pulls the cart fro place to palce, but the cart does not just jump from place to place on its own.

Thus, one may index multiple equilibria by their fiscal consequences, even if one does not wish to select equilibria by those consequences.

3.1.3 A policy rule with an inflation target

Perhaps motivating the same disinflationary equilibria by active monetary policy rules will make them seem more attractive? Here we compute the responses of inflation and interest rates to a monetary policy *disturbance* ($\{\pi_t^*\}$ or $\{v_t^i\}$) rather than to equilibrium interest rates $\{i_t\}$ directly.

Merging the policy rule (21) with the frictionless Fisher equation (18), treating the government debt valuation equation (20) as passive, determining $\{s_t\}$, and ignoring constants which drop from the impulse response function, we have

$$E_t \left(\pi_{t+1} - \pi_{t+1}^* \right) = \phi \left(\pi_t - \pi_t^* \right). \tag{23}$$

Given the expected sequence of policy disturbances $\{\pi_t^*\}$ then, and $\phi > 1$, any equilibrium $\{\pi_t\}$ will explode going forward other than $\pi_t = \pi_t^*$. Ruling out such forward nominal explosions, we have immediately that monetary policy determines inflation uniquely, on the announcement date as well as dates going forward, with its active policy.

The Fed can achieve any path of inflation it wishes by choice of the process for $\{\pi_t^*\}$. Interest rates will be $i_t = E_t\pi_{t+1}^*$ and unexpected inflation will be $\pi_{t+1}^* - E_t\pi_{t+1}^*$. (King (2000) innovated this insightful parameterization of the model in terms of equilibrium π^* and deviations from equilibrium $\pi - \pi^*$.)

This freedom does little to justify a temporary inflation decline as a characterization of the economy's response to monetary policy, however. An unexpected inflation decline paired with an interest rate increase would be entirely a choice by the Fed. Given this freedom, there is also no evident reason why the Fed should want to make such a pairing, no logical or necessary tie between the unexpected inflation decline and the interest rate rise, which on its own leads to higher later inflation. At best such a pattern would characterize historic Fed preference, but there is no reason the Fed is constrained to follow such a pattern in the future.

Moreover, this freedom opens the door to open-mouth policy. Suppose at date 0 the Fed announces $\pi_0^* = -1$, $\pi_{1,2,3,\ldots}^* = 0$, rather than $\pi_0^* = -1$, $\pi_{1,2,3,\ldots}^* = 1$ (or the equivalent $v_t^i = E_t\pi_{t+1}^* - \pi_t^*$ disturbances). The latter generates a one-period decline followed by the interest rate and inflation rate rise seen in the lowest dashed line of Figure 8. The former generates the one-period decline in inflation by itself, with no change in interest rates at all! All the Fed has to do is to announce the monetary policy disturbance. Then inflation jumps down by just enough that the $\phi\pi_t$ part of the policy rule offsets the disturbance term and interest rates themselves never change.

If the Fed wants disinflation, then, let it just create disinflation by an open-mouth operation, announcing that π_0^* is lower. Why pair an instant disinflation with a subsequent interest rate rise, and higher future inflation? If anything, it would be far more sensible to tie a decline in unexpected inflation to a decline in actual inflation, via a *lower* interest rate.

This inflation-target expression of the policy rule makes clear that the Fed has two independent and separate policy levers in the new-Keynesian active-money world: The Fed has an *interest-rate policy* $i_t^* = E_t\pi_{t+1}^*$, which here sets expected inflation, and the Fed has an *equilibrium-selection policy* $\phi(\pi_{t+1}^* - E_t\pi_{t+1}^*)$, enforced by the threat of inflationary or deflationary explosion (23), which selects which of the multiple equilibria left by interest rate policy the economy will jump to. Since one can choose the expected and unexpected components of a stochastic process separately and freely, interest rate policy is completely independent of equilibrium-selection policy. An open-mouth operation is pure equilibrium-selection policy. And it can be paired with any interest-rate policy. The consequent possibility of open-mouth policy, which recurs in the sticky price models to follow, ought to be disturbing for this whole concept for modeling the

Fed's influence on the economy.

Furthermore, this separation makes it clear that the standard new-Keynesian and fiscal-theory views are observationally equivalent away from the zero bound. In equilibrium, we never observe $\pi_t \neq \pi_t^*$, so we never observe ϕ. Whether the Fed made an open-mouth operation, and fiscal policy followed passively, or whether $\phi = 0$ and fiscal policy led the unexpected inflation cannot be told apart based on interest rate and inflation data. At the zero bound, however, we know that at least the downward ϕ threat cannot be made.

3.1.4 A policy rule with conventional disturbances

Perhaps looking at monetary policy via disturbances v_t^i in (22) rather than π_t^* in (21) will look more reasonable. That parameterization, though algebraically equivalent, is more common in the literature.

The v_t^i parameterization also helps us to track down a puzzle: I showed above that the Fed can engineer a negative disinflationary response π_0 for *any* path of interest rates $i_{0,1,2...}$. Yet the standard wisdom is that new-Keynesian models generate a negative response to transitory monetary policy shocks, and a positive response to persistent monetary policy shocks. As we will see, the standard (false) wisdom comes from specifying a monetary policy disturbance $\{v_t^i\}$ and then limiting the time-series properties of that disturbance.

Repeating the analysis with the v^i paratmerized rule (22), the equilibrium condition is

$$E_t\pi_{t+1} = \phi\pi_t + v_t^i. \tag{24}$$

We solve forward to

$$\pi_t = -\sum_{j=0}^{\infty} \phi^{-j+1} E_t v_{t+j}^i.$$

In the AR(1) case

$$v_t^i = \rho v_{t-1}^i + \varepsilon_t^i$$

we have

$$\pi_t = -\frac{1}{\phi - \rho} v_t^i. \tag{25}$$

We also have either from (18) or (22)

$$i_t = -\frac{\rho}{\phi - \rho} v_t^i. \tag{26}$$

From (26), a positive monetary policy shock v^i sends inflation down. That seems like the result we are looking for. But in (26), that shock also sends interest rates down. Substituting out v^i from (25)-(26), we have

$$\pi_t = \rho i_t. \tag{27}$$

For $\rho > 0$, a higher *interest rate* results entirely in higher inflation. If you have the opposite impression, it comes from confusing the monetary policy disturbance v^i with the interest rate i. They go in opposite directions.

For $\rho = 1$, we recover the permanent interest rate rise of Figure 8. In this case, this standard new-Keynesian solution method produces a perfectly Fisherian response. Not only does $\pi_{1,2,3..} = 1$, as in the solid line, but inflation jumps up immediately in the period of the shock, $\pi_0 = 1$ as well, shown in the second from the top dashed line, marked "standard NK" in Figure 8. We will see the same behavior with price stickiness.

So, if one hoped that simple or reasonable restrictions such as an AR(1) on $\{v_t^i\}$ produce the desired result, in fact they produce the opposite result. And again, even if it worked, it would reflect the Fed's choice of policy disturbance properties, and nothing really about the economy's response to money, interest rates, and so on.

Equation (27) shows a negative effect of interest rates on inflation for $\rho < 0$. Thus we see here a stylized version of the standard false impression that in the new-Keynesian model, persistent interest rates raise inflation while temporary rate rises lower inflation. When we add price stickiness below, the cutoff for a positive vs. a negative effect is between zero and one, allowing a negative effect for reasonable transitory (low but positive ρ) interest rate movements, but leaving a stubbornly positive effect for persistent (high ρ) interest rate movements.

But this all depends on the AR(1) or similar time-series restrictions on the disturbance process v_t^i. For example, suppose we want to engineer the response of Figure 8 with a -1% impact disinflation and a permanent rate rise. We choose $\pi_0^* = -1$, $\pi_{1,2,3,...}^* = 1$. Done. Translated, $v_t^i = E_t \pi_{t+1}^* - \phi \pi_t^*$, i.e. $v_0^i = 1 + \phi$, $v_{1,2,3,...}^i = 1 - \phi$ produces this result. It exists, but it's not an AR(1). Likewise, the model can happily generate the other opposite of conventional wisdom – a positive impact inflation with a transitory interest rate. It can generate an open mouth effect – $v_0^i = 1 + \phi$, $v_{1,2,3...}^i = 0$ produces $\pi_0^* = -1$, $\pi_{1,2,3...}^* = 0$. It's a bit more hidden, but nothing about the conventional parameterization other than the artificial AR(1) restriction ties temporary disinflation to the persistence of subsequent interest rate movements.

3.1.5   Anticipated rate rises

Many interest rate rises are announced or anticipated long in advance. Both new and old-Keynesian models assign strong effects of anticipated interest rate rises. So understanding the effect of anticipated interest rate changes is important for policy and understanding episodes.

VARs study unexpected interest rate movements both by historical habit – VARs were developed under the influence of rational expectations models in which only unanticipated monetary policy had real effects – and by econometric necessity – unexpected movements are more causes of, not responses to, other news about future inflation and output. Announcement shocks not coincident with actual rate movements are also harder to measure.

But for policy and historical analysis, it is important to ask of models what is the effect of an expected policy change.

The bottom panel of Figure 8 shows possible impulse-response functions when the rate rise is announced three periods before it occurs. The response function with no inflation innovation and no fiscal policy shock is the same as before. Anticipated monetary policy, or forward guidance, matter.

Here, however, any disinflation must come only on the date of the shock, the date the policy is announced. Unexpected disinflations must be unexpected; equilibrium selection policy can only select among unexpected movements; fiscal shocks must truly be shocks. This fact raises the bar for interpretation of historical episodes – the model-predicted disinflation will typically start before the actual rate rise. This fact also emphasizes that the disinflation comes from the equilibrium-selection part of policy, not the interest rate part, and not at all from the standard channels in which the interest rate itself affects aggregate demand.

3.2   Long-term debt in the frictionless model

Adding long-term debt produces a stable model in which a rise in interest rates can produce a temporary decline in inflation, with no change in fiscal surpluses. Sims (2011) makes this point in the context of a detailed continuous time model. Cochrane (2016c) shows how to solve Sims' model and boils it down to this central point. Cochrane (2001) analyzes the long-term debt case in detail.

Continue with the frictionless model (18), using a constant real interest rate r and define $\beta \equiv 1/(1 + r)$. In the presence of long-term debt, the government debt valuation equation

becomes

$$\frac{\sum_{j=0}^{\infty} Q_t^{(t+j)} B_{t-1}^{(t+j)}}{P_t} = E_t \sum_{j=0}^{\infty} \beta^j s_{t+j}, \tag{28}$$

where $B_{t-1}^{(t+j)}$ is the amount of zero-coupon debt that matures at time $t + j$, outstanding at the end of time $t - 1$ and thus at the beginning of time t, and $Q_t^{(t+j)} = E_t\left(\beta^j P_t/P_{t+j}\right)$ is the time t nominal price of a j period discount bond.

When the Fed unexpectedly and persistently raises interest rates i_t, it lowers long-term bond prices $Q_t^{(t+j)}$. Debt $B_{t-1}^{(t+j)}$ is predetermined. By assumption, primary surpluses don't change. Hence, *the price level P_t must jump down by the same proportional amount as the decline in the nominal market value of the debt.*

By raising nominal interest rates, the Fed still raises *expected* inflation uniformly, $i_t \approx r_t + E_t\pi_{t+1}$ still applies. Thus, we obtain a downward jump, a one-period disinflation, on the day the higher interest rates are announced and long-term bond prices decline, followed by higher inflation when the higher nominal interest rates occur. The path is exactly the same as the dashed lines Figure 8 showed for contractionary fiscal policy, therefore marked "also with long-term debt." Like a shock to surpluses, the mechanism is straightforward "aggregate demand" or wealth effect of government debt. People try to buy or sell undervalued or overvalued government bonds. They drive down or up the price of everything else until the value of government bonds matches the present value of surpluses.

In a model such as Sims (2011) with costs to swiftly changing prices, the downward jump is replaced by a smeared-out period of disinflation. The jump in these simple models is a guide to the cumulative value of the disinflationary period.

3.2.1   Magnitude?

Just how large a disinflation does this mechanism produce? Is it quantitatively significant, and hence a candidate to understand the apparent patterns in the data, or to guide policy?

Suppose the interest rate $i = i_{t+j}$ is expected to last forever. The bond price is then $Q_t^{(t+j)} = 1/(1 + i)^j$. Consider a geometric maturity structure, $B_{t-1}^{(t+j)} = \theta^j B_{t-1}$, so $\theta = 1$ is a perpetuity and $\theta = 0$ is one-period debt, and a constant surplus $s_t = s$. Now, the government debt valuation equation reads

$$\sum_{j=0}^{\infty} \frac{\theta^j}{(1+i)^j} \frac{B_{t-1}}{P_t} = \frac{1+i}{1+i-\theta} \frac{B_{t-1}}{P_t} = \frac{s}{1-\beta}. \tag{29}$$

The continuous time analogue is prettier. With maturity structure $B_t^{(t+j)} = \vartheta e^{-\vartheta j} B_t$,

$$\vartheta \int_{j=0}^{\infty} e^{-ij} e^{-\vartheta j} dj \, \frac{B_t}{P_t} = \frac{\vartheta}{i+\vartheta} \frac{B_t}{P_t} = \frac{s}{r}. \tag{30}$$

Here $\vartheta = 0$ is the perpetuity and $\vartheta = \infty$ is instantaneous debt. (Algebra in the Appendix.)

Now, suppose interest rates rise permanently and unexpectedly at time t. Denote by i^* the post-shock interest rate, and P_t^* the post-shock price level. Then, dividing (30) for the starred by the nonstarred case,

$$\frac{P_t^*}{P_t} = \frac{i+\vartheta}{i^*+\vartheta}. \tag{31}$$

With this formula, we can get a back of the envelope idea of the size of the disinflation effect and its crucial determinants. The longer the maturity, the stronger the effect. In the most extreme case, pairing this permanent interest rate rise with perpetual debt $\vartheta = 0$ – we have $P_t^*/P_t = i/i^*$. A jump in interest rates from 2% to 3% causes the price level to drop to 2/3 of its previous value, a 33% cumulative disinflation!

However, the US doesn't issue that much long-term debt. Debt out to a 20 year maturity follows a geometric pattern with $\vartheta \approx 0.2$. In this case, a one percentage point interest rate rise implies $P_t^*/P_t = (0.2/0.21) = 0.95$, a 5% drop, or a 5% cumulative disinflation.

Shorter-lived interest rate rises and announcements of future rate rises have less effect still. In the Appendix, I show that an interest rate rise from i to i^* that only lasts M years yields in place of (31),

$$\frac{P_t^*}{P} - 1 \approx \left(1 - e^{-\vartheta M}\right) \left(\frac{i+\vartheta}{i^*+\vartheta} - 1\right). \tag{32}$$

An interest rate rise that lasts 2 years $M = 2$ has only $1 - e^{-\vartheta 2} = 1 - e^{-0.2 \times 2} \approx 1/3$ as large an effect as a permanent increase.

An announcement of a future interest rate rise only affects bonds of maturity longer than the announcement delay. An announcement of an interest rate rise from i to i^* that starts in M years yields in place of (31),

$$\frac{P_t^*}{P_t} - 1 \approx e^{-\vartheta M} \left(\frac{i+\vartheta}{i^*+\vartheta} - 1\right). \tag{33}$$

Thus, a permanent interest rate rise that is announced two years ahead of time has a $e^{-(0.2 \times 2)}$ or about 2/3 as much effect.

Either mechansim gives us about 2-3% cumulative disinflation for a 1% interest rate change, and less if we combine them. This is at least in the right ballpark – not 0.2% and not 20%.

I pursue more careful calibration of this effect to the actual US maturity structure, in the context of a model with sticky prices and long term debt, below.

3.2.2   The answer?

Is this basic mechanism the answer we are looking for to deliver a temporary negative effect of monetary policy on inflation?

In its favor, this basic mechanism unites interest rate policy, forward guidance, and quantitative easing. And all three work in a frictionless model – no monetary frictions, no pricing frictions, and no bond-market segmentation. This is an attractive unification and simplification. One later adds frictions for more realistic dynamics, of course.

Interest rate policy works here only by its effect on long-term bond prices, i.e. by its implied forward guidance. So explicit forward guidance has the same effect. For example, paths such as the temporary disinflations at $t = -3$, shown in dashed lines of the bottom panel Figure 8, are achieved here entirely by "forward guidance," i.e. an announcement at time $t = -3$ that interest rates will rise at time 0.

To see how this mechanism also encompasses quantitative easing (QE), consider a very simple example. Suppose debt $\{B_{-1}^{(j)}\}$ due at $j = 0, 1, 2, ...$ is outstanding at time 0. Suppose further that the government plans neither to sell nor to roll over any more debt. It simply will pay off each coupon $\{B_{-1}^{(j)}\}$ at time j from surpluses at time j.

Now suppose at time 0 the Fed unexpectedly buys back some long term debt. It announces the plan, then buys debt at new bond prices. The price level at each date $j > 0$ is set by the condition that primary surpluses must soak up maturing debt, since by assumption of this simple example no new debt is issued,

$$\frac{B_0^{(j)}}{P_j} = s_j; \; j = 1, 2, 3... \tag{34}$$

Therefore bond prices at time 0 are

$$Q_0^{(j)} = E_0\left[\beta^j \frac{P_0}{P_j}\right] = P_0 E_0\left[\beta^j \frac{s_j}{B_0^{(j)}}\right]. \tag{35}$$

The price level at time 0 is set by the same condition as (34), but on this date bond purchases add some extra cash,

$$B_{-1}^{(0)} - \sum_{j=1}^{\infty}(B_0^{(j)} - B_{-1}^{(j)})Q_0^{(j)} = P_0 s_0. \tag{36}$$

B are the amounts outstanding, so a positive $(B_0^{(j)} - B_{-1}^{(j)})$ corresponds to a debt sale, and a negative value to a purchase.

Substituting the bond price (35) in (36) and rearranging, we have the central result,

$$\frac{B_{-1}^{(0)}}{P_0} = s_0 + \sum_{j=1}^{\infty} \beta^j \frac{B_0^{(j)} - B_{-1}^{(j)}}{B_0^{(j)}} s_j. \tag{37}$$

By unexpectedly buying long-term bonds, the Fed lowers the right hand side of (37), thereby raising P_0, which with price stickiness may stimulate real activity as well. By lowering $B_0^{(j)}$, with P_0 higher, the Fed in (35) raises long-term bond prices and lowers long-term interest rates.

Conversely, by unexpectedly selling long-term bonds, the Fed would engineer a rise in long-term rates, and a decrease in the price level today, exactly the downward price level jump followed by rise in interest rates achieved by the interest rate rise or forward guidance. QE is just the quantity view of the same mechanism.

This model also ties a disinflationary shock to the subsequent interest rate rise, in a way I argued above that standard new-Keynesian depictions of Fed equilibrium selection by interest rate policies do not.

However, there are some important differences between this mechanism and traditional beliefs, and important work on it to be done.

This model is still long-run neutral – persistently higher interest rates eventually raise inflation. This mechanism does not on its own produce the traditional adaptive expectations analysis of the 1980s – doggedly high rates slowly squeeze out inflation. (Figure 10 below illustrates the standard view.) Sims (2011) called this mechanism "stepping on a rake." He views the model as a description of the failed monetary stabilizations of the 1970s, in which interest rate increases produced temporary reductions of inflation that only came back more strongly later.

To produce a successful inflation stabilization, a model of the 1980s, one needs something else. A natural possibility is to view the 1980s as a joint monetary-fiscal stabilization. The interest rate increases of the early 1980s had these temporary effects, but they were paired with fiscal reforms such as the 1982 and 1986 tax act, together with deregulation. Subsequent economic growth was strong, and surpluses surged. The temporary disinflation occasioned by high interest rates turned in to a permanent disinflation with fiscal backing.

This mechanism gives a disinflation when the interest rate rise becomes expected, not when it actually happens. This mechanism operates *only* through expected future interest rates and

by lowering long-term bond prices. The rise in current interest rates is essentially irrelevant, in sharp contrast with standard ISLM or money supply/demand thinking. Operating in a frictionless model, it has nothing to do with Phillips curves, i.e. higher current real rates leading to lower output leading to less pressure on prices and wages. Most deeply, this model does not revive the instability of the old-Keynesian model, which lies behind both traditional activist policy advice and the traditional view of inflation stabilization by doggedly higher interest rates alone. This mechanism offers *nothing* like any story told to undergraduates, FOMC members or the general public about why higher interest rates lower inflation. The fact that it works in a completely frictionless model, though a feature from the view of clarity and simplicity, is a fatal bug for the purpose of describing traditional beliefs.

Nor does this mechanism easily rationalize traditional short-run policy prescriptions. It is not necessarily possible or wise for the Fed to try to control inflation by exploiting this short-run negative sign. Since the negative sign only appears for *unexpected* policy changes, by unexpectedly devaluing the claims of long-term bond holders, systematic policy has limited effect. And getting the timing and dynamics just right are likely to be a challenge. Since the long-run effect is positive and stable, there is a good case here that the Fed should keep interest rates steady based on its long-run inflation goal and real-rate assessment, and not try to micromanage the path of inflation with activist policy exploiting the transitory negative sign.

Finally, this mechanism rests on important and possibly tenuous fiscal foundations. By raising interest rates, the Fed raises future inflation. This is a gift to the Treasury – the Treasury can reduce surpluses and still pay off the promised nominal value of the debt. By fixing surpluses, I assume the Treasury stubbornly refuses the gift. The size and even sign of the effect revolve crucially on how people think the Treasury will react.

In sum, though this model may well be the answer, and may address the data, it is not the answer to every question, and in particular it is not a rationalization of standard beliefs.

To pursue this line, important next steps must follow. First, changes in interest rates with fixed surpluses are a useful textbook, problem-set sort of assumption. But fixed or "exogenous" surpluses are not necessary for the theory, and they are terrible assumptions for policy, econometric, or historical analysis. Just how will the Treasury and Congress respond to inflation? If the Fed, as here, devalues long-term bonds with a promise of future inflation, will the Treasury really take none of that promise, and not lower surpluses even a bit? How do people expect the Treasury and Congress to respond to the same events that occasion the Fed's interest rate rise? These are crucial assumptions to understand how interest rates and QE operations affect inflation.

In particular, we must face the minor embarrassment that this mechanism seems to predict that QE works to produce inflation, whereas I just argued that QE had no visible impact on inflation. The story is flexible enough to account for this QE failure, of course, as the theory describes an unrealistic partial derivative with fixed surpluses and no future rollovers. Here, only QE that corresponds to shortening the Treasury maturity structure counts. MBS purchases have no effect. The Treasury issued debt just as fast as the Fed was buying it, so debt in private hands changed less than we think. If the Fed promises to undo the QE in the future, then the effect vanishes. If the Treasury is expected to roll over the debt when it comes due, then it is at least attenuated. Perhaps this QE was accompanied by changes in fiscal expectations – surely true, but what size and sign? Most deeply, this QE was not in fact accompanied by visible interest rate changes, as Figure 1 makes clear, which are the mark in this model of whether it is effective.

But this smacks of the sort of epicycle argument that I dismissed with Occam's razor. So perhaps these stories of joint fiscal-monetary coordination for excusing the failure of QE also wipe out the disinflationary effects of interest rate rises in other episodes as well, and the negative sign simply isn't true.

Second, of course, one must move beyond the extremely simple model presented here to more detailed models capable of matching dynamics. Sims (2011) is a good example, adding a preference for smooth consumption, a monetary policy rule with output and price reactions and inertia, sticky prices, and a fiscal policy rule that raises surpluses in good times. He produces a hump-shaped inflation response curve in place of my downward jump followed by rise. In a much simpler exercise, I merge this mechanism with a standard simple new-Keynesian model below.

More generally, this mechanism generates important ties between the effects of interest-rate policy, the maturity structure of outstanding debt, and how fiscal policies react to inflation and output and the maturity structure of the debt, that have yet to be faced in a serious quantitative evaluation.

3.3   A simple model of sticky prices

The natural response to the failure of the frictionless model with short-term debt is, well, duh, you need sticky prices to get inflation to go down after an interest rate rise. With sticky prices, a higher nominal rate means a higher real rate, a higher real rate means lower aggregate demand, lower output, and via the Phillips curve lower inflation.

This intuition describes the old-Keynesian adaptive expectations model. But that model is unstable and thus inconsistent with the quiet stable zero bound. We're looking for a model with long-run neutrality on top of a short-run negative sign.

Alas, as we will see here, new-Keynesian models robustly predict higher inflation in response to monetary policy, despite price stickiness.

The points are easiest to see in the very simplified model outlined in section 2.3. The same qualitative results hold in more complex and realistic versions. I verify in particular below that the standard new-Keynesian model with an $E_t x_{t+1}$ term works in the same way as the simple model here.

Omitting constants and the zero bound, which are not relevant here, substituting out the output gap x and real rate r from (1)-(4), we have the equilibrium condition (5)

$$\sigma \kappa \, i_t = -\pi_t + (1 + \sigma \kappa)\pi_t^e + \sigma \kappa \, v_t^r, \tag{38}$$

and the interest rate rule,

$$i_t = \phi \pi_t + v_t^i. \tag{39}$$

3.3.1   Adaptive expectations

In the adaptive-expectations model, $\pi_t^e = \pi_{t-1}$, we solve (38) directly for the response of inflation to the path for interest rates as

$$\pi_t = (1 + \sigma \kappa) \, \pi_{t-1} - \sigma \kappa \, (i_t - v_t^r). \tag{40}$$

If we wish instead to solve for the response of inflation to a monetary policy disturbance, v^i, we substitute the rule (39) for i_t and solve, leading to (9), repeated here:

$$\pi_t = \frac{1 + \sigma \kappa}{1 + \phi \sigma \kappa}\pi_{t-1} - \frac{\sigma \kappa}{1 + \phi \sigma \kappa}(v_t^i - v_t^r). \tag{41}$$

In (40), the coefficient on lagged inflation π_{t-1} is greater than one, and the coefficient on the interest rate i_t is negative. Thus, in response to a sustained rise in interest rates, inflation spirals off negatively. Figure 9 illustrates.

Figure 9 includes the response for less price stickiness, $\kappa = 1$ instead of $\kappa = 1/2$. Sensibly, less sticky prices speed up dynamics. But that just makes the explosion happen faster. The adaptive
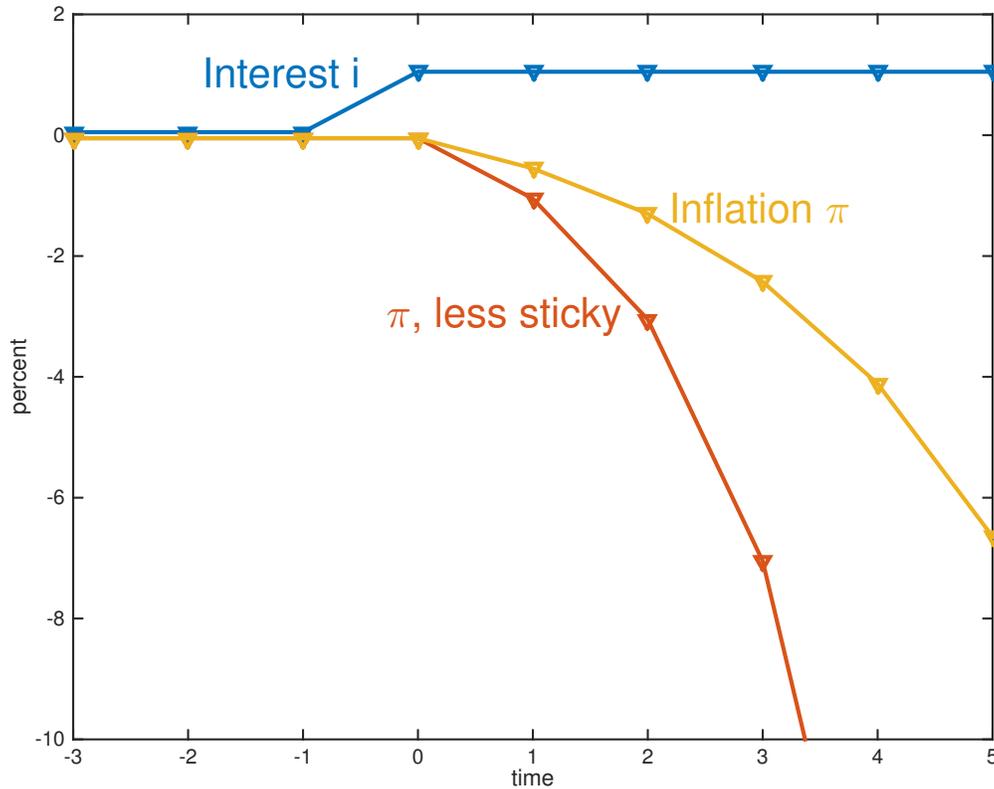
Figure 9: Simulation of a permanent interest rate rise in the simple old-Keynesian model. The baseline uses $\kappa = 1/2, \sigma = 1$. The "less sticky" case uses $\kappa = 1$.

expectations model does not approach the frictionless limit.

This response to *interest rates* is the same for any value of ϕ, and $\phi < 1$ vs. $\phi > 1$ in particular. As in the frictionless model, this response of inflation to interest rates (40) does not assume a time-varying peg $\phi = 0$; it simply assumes that the Fed is following whatever set of shocks is necessary to keep equilibrium interest rates at their assumed value. The rule (41) tells us what path of monetary policy disturbances v_t^i is needed to generate the given path of interest rates. $\phi > 1$ vs. $\phi < 1$ does not determine the stability of this response. $\phi > 1$ just means that the Fed would need an ever-increasing set of shocks v_t^i is necessary to keep interest rates constant.

Why do we not observe this much feared-instability? Because governments and central banks aren't dumb enough to keep interest rates constant forever in the face of inflation. (Though sometimes it takes them a while to catch on.) Likewise, a deflation spiral eventually spurs fiscal stimulus, helicopter drops or other extreme measures.

To illustrate, Figure 10 plots the response of this simple old-Keynesian model to a permanent monetary policy *disturbance* v_t^i, simulating forward equation (41).
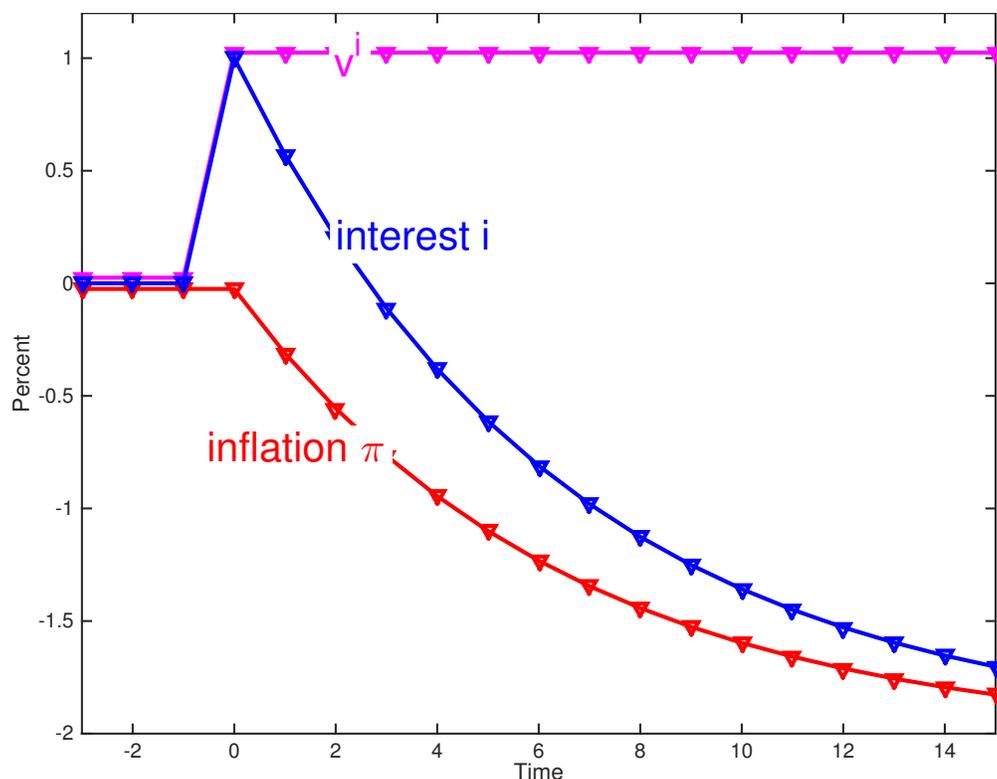
Figure 10: Response to a step-function monetary policy disturbance in the simple old-Keynesian model. The figure plots the response to a permanent 1% increase in v_t^i. $\phi = 1.5$, $\kappa = 0.2$, $\sigma = 1$.

Interest rates rise at first, to get disinflation going, but then quickly follow inflation in order to stop it from going too far. This graph embodies the sequence of events Friedman (1968) (p. 6) described of an interest rate change. It also describes the conventional adaptive-expectations view of the 1980s.

In the long run, interest rates move one for one with inflation. The model is not Fisherian, however, as the Fisher relationship is an unstable steady state, and interest rates must initially rise to get disinflation going. At the zero bound, the Fed is unable to lower interest rates and get the negative of this process going.

In (40) and (41), the natural rate shock v^r enters along with the interest rate and the monetary policy disturbance, respectively. Thus we can read Figure 9 as the response of the economy to a sustained decline in the natural rate when the interest rate does not move, as at the zero bound. The zero bound spiral shown in Figure 4 is the same mechanism – and the absence of such a spiral tells us that an interest rate rise is similarly not likely to have the effect shown in Figure 9.

Likewise, Figure 10 illustrates the reaction of inflation to a permanent decline in the natural

rate with no change in the monetary policy *disturbance*, v^i, i.e. if the Fed allows (and can allow) the interest rate to follow the usual Taylor rule. A decline in the natural rate would set off a protracted decline in inflation. This is the initial path of inflation in Figure 4 before the zero bound binds and the constant interest rate unstable dynamics of Figure 10 take over from the constant disturbance dynamics.

The model (41)-(40) makes no distinction between expected and unexpected disturbances, and there are no forward looking terms. Hence, these responses are the same for anticipated movements as for surprise movements, and anywhere in between, i.e. policies announced at time $t = -3$, for example. The rational-expectations idea that expected and unexpected policy have different effects is not present.

Figure 10 helps to illustrate why it is hard to tell an unstable model, whose central bank is following active policies and not letting instabilities erupt, from a stable model. Equilibrium interest rates and inflation rise and fall together in both cases. Add some noise, and it will be hard to see if the interest rate fall caused the inflation fall or vice versa. That's why the long zero bound is such a telling experiment.

Figure 9 and Figure 10 summarize the classic view of the effects of monetary policy. Alas, the underlying model is inconsistent with the observed stability at the zero bound. That stability means we are looking with only a temporary negative effect of interest rates on inflation.

3.3.2   Rational Expectations

For the new-Keynesian model with rational expectations, $\pi_t^e = E_t \pi_{t+1}$, (38) now implies that the response of inflation to equilibrium interest rates is

$$E_t \pi_{t+1} = \frac{1}{1 + \sigma \kappa} \pi_t + \frac{\sigma \kappa}{1 + \sigma \kappa} \left(i_t - v_t^r \right). \tag{42}$$

Substituting in the policy rule (39), the response to a monetary policy disturbance is (also previously given in (11)),

$$E_t \pi_{t+1} = \frac{1 + \phi \sigma \kappa}{1 + \sigma \kappa} \pi_t + \frac{\sigma \kappa}{1 + \sigma \kappa} (v_t^i - v_t^r). \tag{43}$$

Again, we can calculate the path of inflation corresponding to a given path of equilibrium interest rates in (42), without specifying what path of monetary policy disturbances v_t^i and systematic responses $\phi \pi_t$ produced the interest rate path. Again, that inflation path is independent of the policy rule ϕ, and the set of disturbances v^i that generated the interest rate path, neither

of which is present in (42). Again, (42) does *not* necessarily represent the response to a time-varying peg ($\phi = 0$) – though it can.
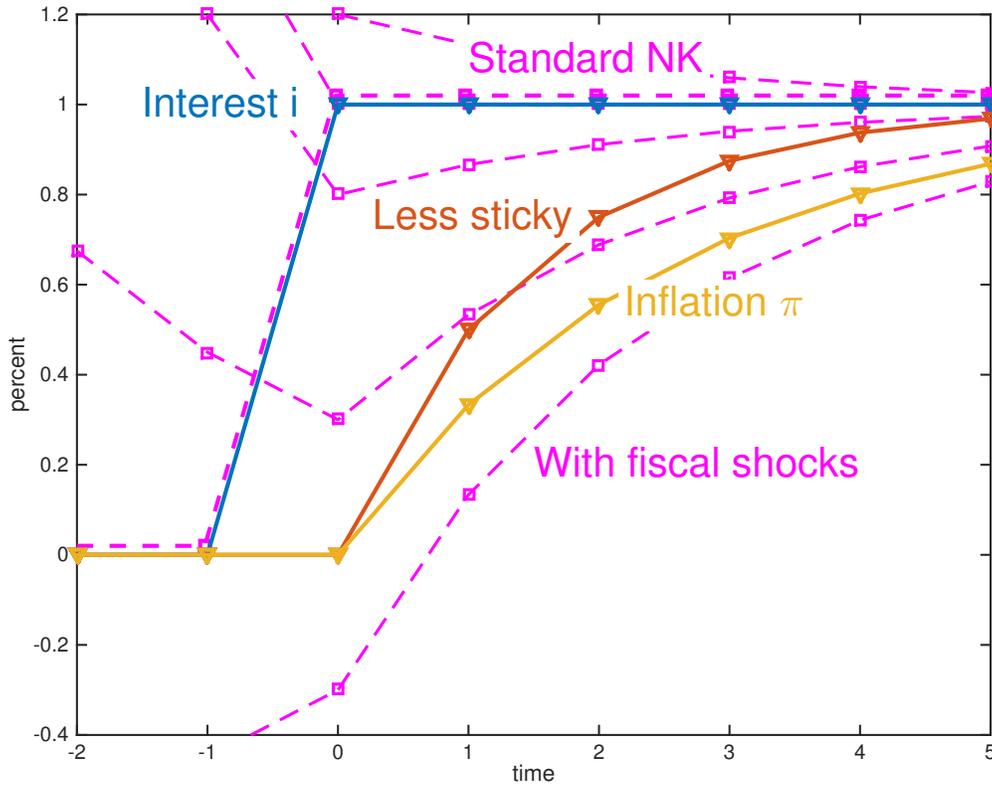

Figure 11: Simulation of an interest rate rise or natural rate fall in the simple new-Keynesian model. The baseline uses $\kappa = 1/2, \sigma = 1$, the "less sticky" case uses $\kappa = 1$. Dashed lines indicate potential multiple equilibria.

In (42) the coefficient in front of i_t is positive, and the coefficient in front of π_t is less than one. The model has a positive, stable inflation response to an increase in interest rates, rather than a negative, unstable response. Figure 11 presents this response. And it is Fisherian, both in the short and long run. The whole reason that we are here – the hope that adding price stickiness to the frictionless model illustrated by Figure 8 would generate a model with a short-run inflation decline – fails, at least so far.

Already, the new-Keynesian model reverses the hallowed doctrine that interest rate pegs are unstable. Now it undoes the widespread presumption that higher interest rates temporarily lower inflation when prices are sticky.

Here too, v_t^r and i_t enter (42) together, so we can read Figure 11 as the response of inflation to a natural rate v^r fall, when interest rates do not or cannot move. The natural-rate fall *raises* inflation. Higher inflation with a fixed nominal rate produces a lower real rate. Thus, infla-

tion accommodates needed changes in the natural real rate, albeit slowly, all by itself, without the need for active Fed action or announcements. Contrariwise, a rise in the natural rate with no change in interest rates leads to a steady decline in inflation, all on its own, to produce the higher real rate with unchanged nominal rates. One might read the history of slowly decreasing inflation during recovery at the zero bound as an instance of this mechanism.

Figure 11 includes the case of less price stickiness, $\kappa = 1$ in place of $\kappa = 1/2$. Again, dynamics happen more quickly. But in this case, unlike the old-Keynesian adaptive expectations model, dynamics smoothly approach the frictionless limit, in which $i_t = r + E_t \pi_{t+1}$ and expected inflation rises immediately to match the rise in nominal interest rate. This is an attractive property.

(Difficulties with the frictionless limit happen in new-Keynesian models when one ties down the equilibria by choices of future inflation and one introduces time - 0 jumps. Then small changes in the future inflation imply large jumps in today's inflation, and those changes get bigger as price stickiness is reduced or the horizon increases. Some of these issues are discussed with Figure 19 below. Cochrane (2016b) discusses the issue at length. *This* calculation smoothly approaches a frictionless limit, in a way the old-Keynesian model above does not. That does not mean that *all* calculations in the new-Keynesian literature smoothly approach frictionless limits. Many do not.)

As in the frictionless model of Figure 8, this rational-expectations sticky-price model only ties down expected inflation, so one can add any unpredictable shock δ_{t+1} to the ex-post versions of (42) and (43), i.e. the version with π_{t+1}, not $E_t \pi_{t+1}$ on the left-hand side. One cannot expect unexpected jumps, so for an impulse-response function, multiple equilibria only introduce the possibility of an unexpected jump on the date people learn the new policy.

Unlike the frictionless case, unexpected jumps have lasting effects. To the solutions of (42) with $\pi_0 = 0$ we can add

$$\pi_t = \left(\frac{1}{1 + \sigma \kappa} \right)^t \pi_0 \tag{44}$$

for any value of π_0.

I indicate such multiple equilibria by dashed lines in Figure 11. On the date of the announcement, inflation could jump to any of the dashed lines, and would be expected to then continue on that line. For example, in the traditional case that the interest-rate rise is a surprise at date 0, inflation at date 0, rather than being 0 (solid line), could jump down to the dashed line at its kink, and then start to rise. In the case of a preannounced rate rise, inflation could jump down to the dashed line at that earlier date. In this way, the graph covers the results of any announcement

date.

As in the frictionless case, we might obtain the transitory negative effect with such a downward jump coincident with the announcement of a rate rise.

Again, each such jump has an associated change in fiscal policy, whether "actively" or "passively" achieved, so I label them "with fiscal shocks." Now, such jumps have protracted effects, and begin to look more like a source of smooth temporary disinflation. But adding fiscal shocks to produce a temporary negative response makes no more sense here than in the frictionless case.

(The solid $\delta = 0$ line corresponds to no change in the present value of surpluses. However, higher nominal rates now imply higher real rates, and therefore a lower present value if surpluses themselves do not change. Thus, the previous definition of "monetary policy" as a change in interest rates with no change in surpluses would include an upward jump in inflation on the date of announcement. As the sign is not going to help us, I leave quantification of this mechanism to the fuller model below.)

Perhaps transitory interest rate changes naturally produce a negative response – the standard wisdom? If the equilibrium interest rate follows a transitory path,

$$i_t = \rho i_{t-1} + \varepsilon_t,$$

the response of inflation to a time-0 interest rate shock is, from (42),

$$\pi_t = \frac{\sigma\kappa}{1+\sigma\kappa} \frac{\rho^t - \left(\frac{1}{1+\sigma\kappa}\right)^t}{\rho - \frac{1}{1+\sigma\kappa}} i_0; \quad t = 1, 2, 3... \tag{45}$$

It's always positive, and typically hump-shaped. The corresponding plot basically just pulls down the right end of Figure 11. Again, this result holds for *any* ϕ, and the response is positive for *any* $\rho > 0$. Transitory interest rate movements are not going to give us a temporary disinflation. We really will have to try to add a downward jump.

3.3.3   Policy disturbances

Perhaps if we specify a policy disturbance sequence rather than an equilibrium interest rate, a downward jump will seem more plausible?

In the new-Keynesian model (43), $\phi > 1$ induces instability. The coefficient on π_t is then

greater than one. We solve (43) forward to

$$\pi_{t+1} = -\frac{\sigma\kappa}{1+\sigma\kappa}\sum_{j=0}^{\infty}\left(\frac{1+\sigma\kappa}{1+\phi\sigma\kappa}\right)^{j+1}E_{t+1}(v_{t+1+j}^i - v_{t+1+j}^r). \tag{46}$$

This solution describes how we pick the multiple equilibrium δ_{t+1}, so it offers hope to pick one of the downward jumps. I dated the equation at $t+1$ to emphasize this point. (We do not similarly solve (41) forward for $\phi < 1$ because there is no jump variable, or undetermined expectation. That response just becomes explosive.)

If the disturbance v_t^i follows an AR(1)

$$v_t^i = \rho v_{t-1}^i + \varepsilon_t,$$

we can solve (46) at time t to give

$$\pi_t = -\frac{\sigma\kappa}{1-\rho+(\phi-\rho)\sigma\kappa}v_t^i. \tag{47}$$

From the policy rule (39), the interest rate follows

$$i_t = \frac{1-\rho(1+\sigma\kappa)}{1-\rho+(\phi-\rho)\sigma\kappa}v_t^i. \tag{48}$$

Both i_t and π_t follow AR(1) responses. Using (48) to substituting out v_t^i in (48), we can express the relation between inflation and interest rates as

$$\pi_t = \frac{\sigma\kappa}{\rho(1+\sigma\kappa)-1}i_t. \tag{49}$$

In the case of a permanent change, $\rho = 1$, these formulas simplify to

$$\pi_{t+1} = -\frac{1}{\phi-1}v^i; \tag{50}$$

$$i_t = -\frac{1}{\phi-1}v^i; \tag{51}$$

$$\pi_t = i_t. \tag{52}$$

The negative sign in (47) and (50) may lead to some optimism: A positive policy disturbance sends inflation down. But it also sends interest rates down, so the relation between interest rates and inflation remains positive. Again, don't confuse the response to a monetary policy shock

with a response to interest rates.

Thus in this standard $\phi > 1$ solution of the new-Keynesian model a permanent ($\rho = 1$) monetary policy shock gives rise to a completely Fisherian response. Inflation rises instantly and follows the interest rate exactly, as shown in the "standard NK" line of Figure 11. It's even more Fisherian than the $\delta = 0$ solutions. Despite price stickiness, this instant response is the same as in the frictionless case of Figure 8.

This solution combines the $\delta = 0$ path illustrated by solid lines in Figure 11 with a unit *positive* δ_0 unexpected inflation shock at time 0. It's not a different solution, it's just one of the multiple equilibria of that original solution. This unexpected inflation shock, which corresponds to a loosening of fiscal policy, drives inflation up immediately and fully. It is the wrong sign to generate our hoped-for disinflation. Standard new-Keynesian models are doubly Fisherian.

Returning to the $\rho < 1$ case, for sufficiently persistent monetary policy, $\rho > 1/(1 + \sigma\kappa)$, the coefficient on the right hand side of (49) remains positive, so higher interest rates correspond uniformly to higher inflation, just as in the permanent case.

For sufficiently transitory monetary policy, however, $\rho < 1/(1 + \sigma\kappa)$, (49) shows that a higher interest rate with an AR(1) decay at rate ρ results in uniformly lower inflation, also following an AR(1) decay, the long-sought traditional sign. (A reader wishing a graph can look ahead at the bottom right panel of Figure 15. Though the calculations in that figure use the full standard model, the results are visually the same as for this simplified model.) Here, also, the monetary policy disturbance v_t^i acts in the same direction as the interest rate.

This result embodies the conventional wisdom that the new-Keynesian model produces a negative response for a transitory policy shock. This sign occurs for $\rho > 0$, unlike the frictionless case in which $\rho < 0$ was necessary for a negative response.

This isn't the answer we're looking for, strictly speaking. It generates a negative response to a temporary interest rate rise, but it does not generate a temporary negative response to a sustained interest rate rise. Still perhaps it is good enough?

At the boundary $\rho = 1/(1 + \sigma\kappa)$, an instructive case reappears – the open-mouth effect. Here, interest rates i_t do not move at all. Inflation simply jumps up or down on the Fed's announcement that a monetary policy shock has occurred. (See the bottom left panel of Figure 15.) With price stickiness, the inflation jump persists with an AR(1) pattern. This case emphasizes how much "monetary policy" in the standard new-Keynesian model is about the Fed's equilibrium-selection powers, not about interest rate movements.

However, the appearance of a link between the persistence and sign of monetary policy is again an artifact of the v_t^i and AR(1) disturbance parameterization. In fact, the Fed can, with a suitable choice of disturbances, select any unexpected inflation consistent with any persistence of equilibrium interest rates.

As in the frictionless case, we can see both facts most easily by parameterizing the policy rule as a time-varying inflation target rather than a v_t^i disturbance,

$$i_t = i_t^* + \phi(\pi_t - \pi_t^*).$$

This parameterization cleanly separates interest-rate policy i_t^* from equilibrium-selection policy $\phi(\pi_t - \pi_t^*)$.

While one can calculate responses to arbitrary disturbances $\{i_t^*\}$, $\{\pi_t^*\}$, the equilibrium interest rate may then not come out to $i_t = i_t^*$ and the equilibrium inflation may not come out to $\pi_t = \pi_t^*$. If we parameterize the disturbances so that they obey the first order conditions of the rest of the model, we have that convenient result. Thus, as the policy rule in the frictionless model satisfied $i_t^* = r + E_t\pi_{t+1}^*$, let the two policy disturbances here satisfy $i_t^* = r_t + E_t\pi_{t+1}^*$. Using (2) and (3) we have

$$r_t = \frac{1}{\sigma\kappa}\left(E_t\pi_{t+1} - \pi_t\right) + v_t^r,$$

so write the policy rule

$$i_t = \left[E_t\pi_{t+1}^* + \frac{1}{\sigma\kappa}\left(E_t\pi_{t+1}^* - \pi_t^*\right) + v_t^r\right] + \phi(\pi_t - \pi_t^*). \tag{53}$$

Substituting the rule (53) in (42) we obtain, rather than (43),

$$E_t\left(\pi_{t+1} - \pi_{t+1}^*\right) = \frac{1 + \sigma\kappa\phi}{1 + \sigma\kappa}(\pi_t - \pi_t^*). \tag{54}$$

Thus, the Fed induces explosive dynamics for any $\pi_t \neq \pi_t^*$, and ruling out such explosions

$$\pi_t = \pi_t^*$$

is the unique equilibrium. Despite sticky prices, the Fed can still achieve any path of inflation it wishes, both expected and unexpected.

Furthermore, the path of unexpected inflation is indepndent from the path of interest rates. There is no necessary tie between the persistence of interest rates and the sign of the inflation

response. Equilibrium interest rates follow

$$i_t = i_t^* = E_t\pi_{t+1}^* + \frac{1}{\sigma\kappa}\left(E_t\pi_{t+1}^* - \pi_t^*\right) + v_t^r. \tag{55}$$

Thus, choosing an interest rate path determines $E_t\pi_{t+1}^*$, but the Fed can independently choose $\pi_{t+1}^* - E_t\pi_{t+1}^*$ the next period.

For example, to produce a response function in which interest rates follow an AR(1),

$$i_t = \rho^t i_0; \quad t = 0, 1, 2, ...$$

The Fed chooses a disturbance $\{\pi_t^*\}$ that results in this AR(1) for $\{i_t^*\} = \rho^t i_0$. From (55),

$$E_0\pi_{t+1}^* = \frac{\sigma\kappa}{1+\sigma\kappa}\rho^t i_0 + \frac{1}{1+\sigma\kappa}E_0\pi_t^*. \tag{56}$$

Iterating forward,

$$E_0\pi_t^* = \left(\frac{1}{1+\sigma\kappa}\right)^t \pi_0^* + \frac{\sigma\kappa}{1+\sigma\kappa}\left[\frac{\left(\frac{1}{1+\sigma\kappa}\right)^{t+1} - \rho^{t+1}}{\frac{1}{1+\sigma\kappa} - \rho}\right] i_0. \tag{57}$$

But the choice π_0^* is unconstrained. The Fed can choose whatever instantaneous response of inflation it wishes, for *any* value of ρ, and still produce the AR(1) interest rate response. Equation (57) is no more or less than the full set of solutions indexed by π_0^*, shown by dashed lines for $\rho = 1$ in Figure 11. We derived the second term directly in (45) as the response to any AR(1) interest rate path with no unexpected inflation. The first term just adds the results of an unexpected inflation, (44). Values of $\pi_0 < 0$ will result in temporary disinflations; values of $\pi_0 > 0$ will speed up the Fisherian response. But there is *no* tie between the persistence of the interest rate response ρ and the sign of the inflation response.

Clearly the i_t^*, π_t^* rule (53) is just a reparameterization of the v_t^i rule (39). For any $\{\pi_t^*\}$ we can construct

$$v_t^i = \frac{1+\sigma\kappa}{\sigma\kappa}E_t\pi_{t+1}^* + \left(\phi - \frac{1}{\sigma\kappa}\right)\pi_t^* + v_t^r$$

and vice versa. So by choosing a suitable $\{v_t^i\}$, the Fed can similarly produce any sign of the inflation response for any persistence or other property of the interest rate response.

Furthermore, as in the frictionless case, it's even easier in the π_t^* parameterization to construct and interpret an open mouth operation. The Fed simply announces that its new inflation

target will be

$$\pi_t^* = \pi_0^* \left(\frac{1}{1 + \sigma\kappa} \right)^t$$

and it happens. From (55), you can verify immediately that the interest rate does not move. (Campbell and Weber (2016) describe similar open-mouth operations.)

Now, perhaps this is our world. Monetary policy at the zero bound has seemed to evolve into central banker statements accompanied by no actual changes in interest rates or asset purchases. Central banks have long moved interest rates in fact by simply announcing a change in rate, with actual open market operations following much later if at all. (For example, see Brash (2002).) These open mouth operations are doubly removed from action, since the central bank can apparently move inflation without even moving interest rates.

Perhaps inflation really has little to do with economics; supply and demand, intertemporal substitution, money, and so forth. Perhaps inflation really is predominantly a multiple-equilibrium question. Perhaps "monetary" policy affects inflation entirely by government officials making statements, with implicit never-observed off-equilibrium threats, that cause jumps from one equilibrium to another, validated by passive fiscal policy. Perhaps changes to interest rates, though economically irrelevant and even counterproductive in the long run, evolved as some sort of communication and signaling equilibrium to indicate a policy shock.

If so, again, sufficient becomes necessary. The quest of this paper – a simple, transparent, baseline economic model of the effect of interest rates on inflation – is over, with a negative result and a disquieting implication for the status of monetary policy in the arsenal of robust and well-understood phenomena.

In sum, the sticky price model works very much like the frictionless model. The intuition that with sticky prices, a higher nominal interest rate produces a higher real interest rate, which depresses aggregate demand, and via the Phillips curve reduces inflation, simply does not describe this model.

3.4   Full new-Keynesian model

The claim that the frictionless and simplified models capture the behavior of real new-Keynesian needs verification. And we need to see how the real model behaves.

I use the standard optimizing sticky-price model,

$$x_t = E_t x_{t+1} - \sigma(i_t - E_t \pi_{t+1}) \tag{58}$$

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t \tag{59}$$

$$i_t = \phi \pi_t + v_t^i \tag{60}$$

$$i_t = i_t^* + \phi(\pi_t - \pi_t^*) \tag{61}$$

where x_t denotes the output gap, i_t is the nominal interest rate, and π_t is inflation. The last two equations give two equivalent parameterizations of an interest rate policy rule.

The solution of this model for a given interest rate path is derived in the Appendix. Inflation and output are two-sided geometrically-weighted distributed lags of the interest rate path,

$$\pi_{t+1} = \frac{\sigma\kappa}{\lambda_1 - \lambda_2} \left[i_t + \sum_{j=1}^{\infty} \lambda_1^{-j} i_{t-j} + \sum_{j=1}^{\infty} \lambda_2^{j} E_{t+1} i_{t+j} \right] + \sum_{j=0}^{\infty} \lambda_1^{-j} \delta_{t+1-j} \tag{62}$$

$$\kappa x_{t+1} = \frac{\sigma\kappa}{\lambda_1 - \lambda_2} \left[\left(1 - \beta\lambda_1^{-1}\right) \sum_{j=0}^{\infty} \lambda_1^{-j} i_{t-j} + \left(1 - \beta\lambda_2^{-1}\right) \sum_{j=1}^{\infty} \lambda_2^{j} E_{t+1} i_{t+j} \right]$$
$$+ \left(1 - \beta\lambda_1^{-1}\right) \sum_{j=0}^{\infty} \lambda_1^{-j} \delta_{t+1-j}, \tag{63}$$

where
$$\lambda_{1,\,2} = \frac{(1 + \beta + \sigma\kappa) \pm \sqrt{(1 + \beta + \sigma\kappa)^2 - 4\beta}}{2}. \tag{64}$$

We have $\lambda_1 > 1$ and $\lambda_2 < 1$. Here, δ_{t+1}, with $E_t \delta_{t+1} = 0$, is an expectational shock indexing multiple equilibria.

Once again, this calculation represents the response of inflation to equilibrium interest rates, i.e. to any disturbances that produce the given response of equilibrium interest rates, with any value of ϕ. I do not assume a time-varying peg $\phi = 0$, nor do I assume active fiscal policy, though the calculation is also valid in those cases. One can, and I will later, substitute $i_t = \phi \pi_t + v_t^i$ to derive the response to policy shocks, or to find the policy shock sequence consistent with a given interest rate path.

3.4.1 Interest rate response

Figure 12 presents the response of inflation and the output gap to a step function rise in the interest rate, using (62)-(63), and choosing the basic solution $\delta_0 = 0$. I use parameters

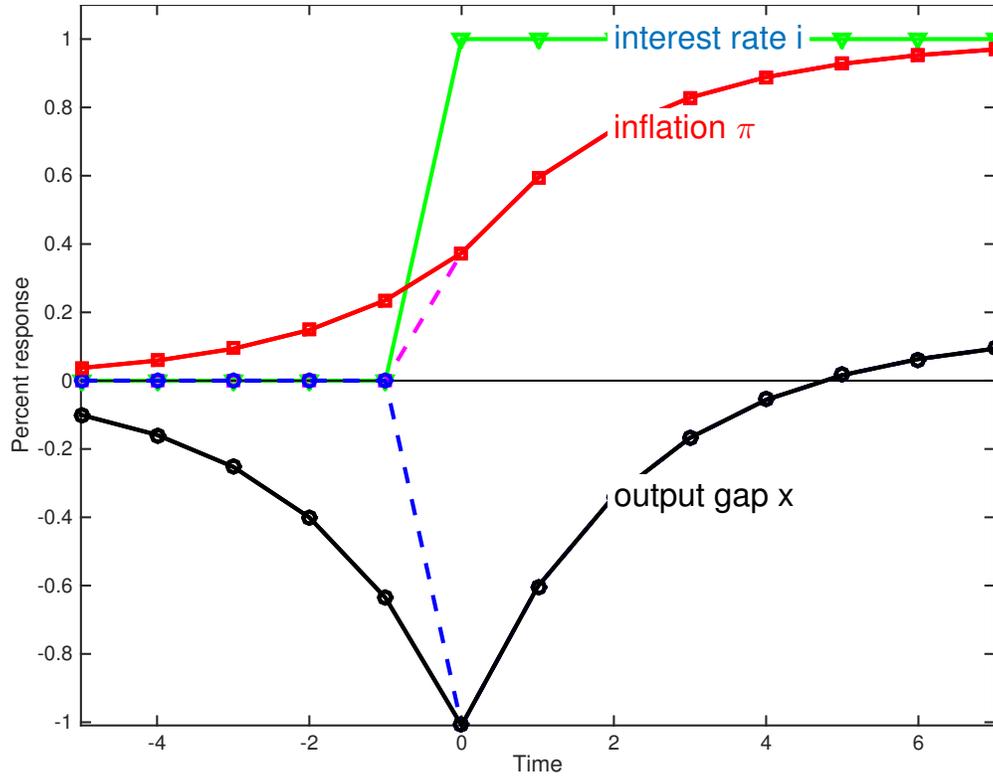$$\beta = 0.97, \kappa = 0.2, \sigma = 1. \tag{65}$$

Figure 12: Response of inflation and output to a step function interest rate change in the standard IS - Phillips curve new-Keynesian model. The solid lines show the response to an expected change. The dashed lines show the response to an unexpected change. Parameters $\beta = 0.97$, $\kappa = 0.2$, $\sigma = 1$.

Inflation rises throughout the episode. Mathematically, that rise is a result of a two-sided moving average with positive weights in (62).

Output declines around the interest rate rise. When the nominal interest rate is higher than the inflation rate, the real rate is high. Output is low when current and future real interest rates are high via intertemporal substitution. Equivalently, the forward-looking Phillips curve (59) says that output is low when inflation is low relative to future inflation, i.e. when inflation is

rising.

Output eventually rises slightly, as the steady state of the Phillips curve (59) with $\beta < 1$ gives a slight increase in the level of output when inflation increases permanently. Using $\beta = 1$, there is no permanent output effect, and all graphs are otherwise visually indistinguishable. The positive inflation effect does not require a permanent output effect.

The solid lines of Figure 12 plot the responses to a pre-announced interest rate rise. The dashed line plots the response to an interest rate rise announced at the same date as the rise, date zero. Announced and surprise interest rate paths are the same after the announcement day. The response to an interest rate change announced at any time before zero jumps up to match the anticipated-policy reaction on the day of announcement. In this way, the solid lines capture the response for any announcement day.

In this class of models, expected monetary policy matters. Expected and unexpected policy have identical effects after the announcement date because the interest rate shock $i_t - E_{t-1}i_t$ does not appear as a separate right hand variable in the model's solutions (62)-(63), as it does in information-based Phillips curves such as Lucas (1972).

Output and inflation move ahead of the expected interest rate rise. "Forward guidance" matters, and outcomes are affected by expectations, even when those expectations are not realized.

In sum, price stickiness smooths the Fisherian response of the frictionless model seen in Figure 8, but does not change its character. One might have hoped that price stickiness would deliver the traditional view of a temporary decline in inflation. It does not.

The model does, however, generate the output decline that conventional intuition and most empirical work associates with monetary policy tightening. Raising interest rates to cool off a booming economy, and lowering interest rates to stimulate a slow economy may still make sense. Doing so just has a different effect on inflation than we might have thought. However, this effect depends on the rather contentious forward-looking Philips curve, which gives lower output when inflation is increasing.

The sign of the response is not affected, and magnitudes not greatly affected, by changes in the parameters. There isn't much you can do to an S shape. The parameters κ and σ enter together in the inflation response. Larger values speed up the dynamics, smoothly approaching the step function of the frictionless model as their product rises. Larger values of β slightly slow down the dynamics. Larger σ on its own gives larger output effects with the same pattern.

3.4.2   Mean-reverting rates

Perhaps transitory interest rate movements produce a negative sign? Figure 13 plots responses
to an AR(1) interest rate shock.
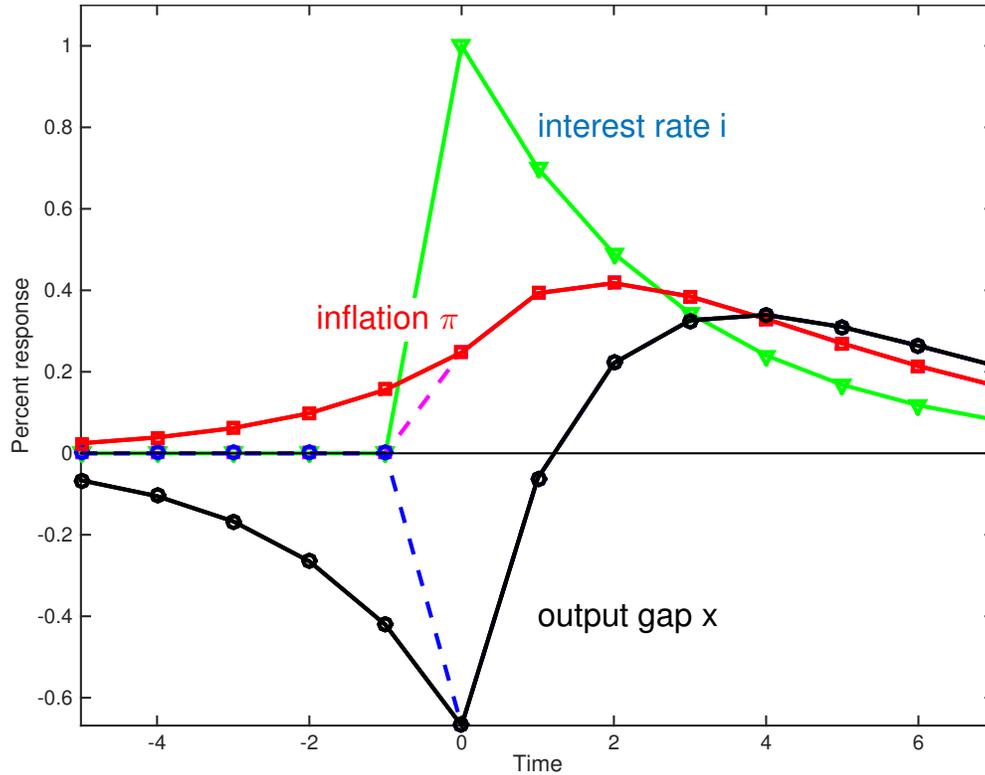
Figure 13: Response of inflation and output to a mean-reverting interest-rate path. Dashed lines
are the response to an unexpected change. Solid lines are the response to an expected change.

The responses in Figure 13 are similar to those of Figure 12 in the short run, with a long-run
return to zero. The weights in the two-sided moving average (62) are positive, and the same
for any interest rate process. They do not give a negative response for any uniformly positive
interest rate path, no matter its time series properties.

Figure 13 serves as an important reminder however: VARs that estimate transitory interest
rate responses do not give us evidence on the long-run Fisher hypothesis. The long zero bound
tells us something that we could not observe in the transitory interest rate changes typical of the
previous era.

3.4.3   Multiple equilibria

There are multiple equilibria, indexed by the expectational shock $\{\delta_t\}$. As first displayed in Figure 8, one might recover a short-run negative inflation response by pairing the announcement of a rate increase with a negative multiple-equilibrium shock δ.

The top panel of Figure 14 plots a range of multiple equilibrium responses to the unanticipated step function in interest rates considered in Figure 12. Each equilibrium is generated by a different choice of the expectational shock δ_0 that coincides with the interest rate shock at date zero. The bottom panel of Figure 14 presents multiple equilibrium responses to an interest rate rise announced at time $t = -3$. These responses are chosen to have the same value of inflation at $t = 0$ as in the top panel. Letters identify equilibrium choices for discussion.

Equilibrium A has a positive additional inflation shock, $\delta_0 = 1\%$. Equilibrium B chooses δ_0 to produce 1% inflation at time 0, $\pi_0 = 1\%$. Equilibrium C chooses δ_0 to have no fiscal consequences, explained below. Between C and D lies the original fundamental equilibrium, with $\delta_0 = 0$, as graphed in Figure 12. Equilibrium D chooses δ_0 to produce no inflation at time 0, $\pi_0 = 0$. Equilibrium E chooses $\delta_0 = -1\%$.

The figure shows graphically that the model may have too many equilibria, but all of them are stable, and all of them are Fisherian in the long run, with inflation converging to the higher nominal interest rate.

In equilibrium B, inflation jumps instantly to the full increase in nominal interest rates, and stays there throughout. Output also jumps immediately to the steady-state value. Thus, despite price stickiness, the model can produce a super-neutral or super-Fisherian response, in which an interest rate rise instantly implies inflation with no output dynamics.

Equilibrium A shows that even more inflation is possible. With a sufficiently large expectational shock, inflation can actually increase by more than the interest rate change, and then settle down, and output can increase as well.

Equilibrium D adds a small negative expectational shock δ_0, so that the initial inflation response is precisely zero. One may be troubled by inflation jumps, since inflation seems to have inertia in the data. It can be inertial in the model as well. (In continuous time, the no-jump equilibrium D is the same as the $\delta = 0$ equilibrium.)

Equilibrium E verifies that the model can produce a temporary decline in inflation coincident with the interest rate rise. Equilibrium E achieves that result by pairing a negative expec-
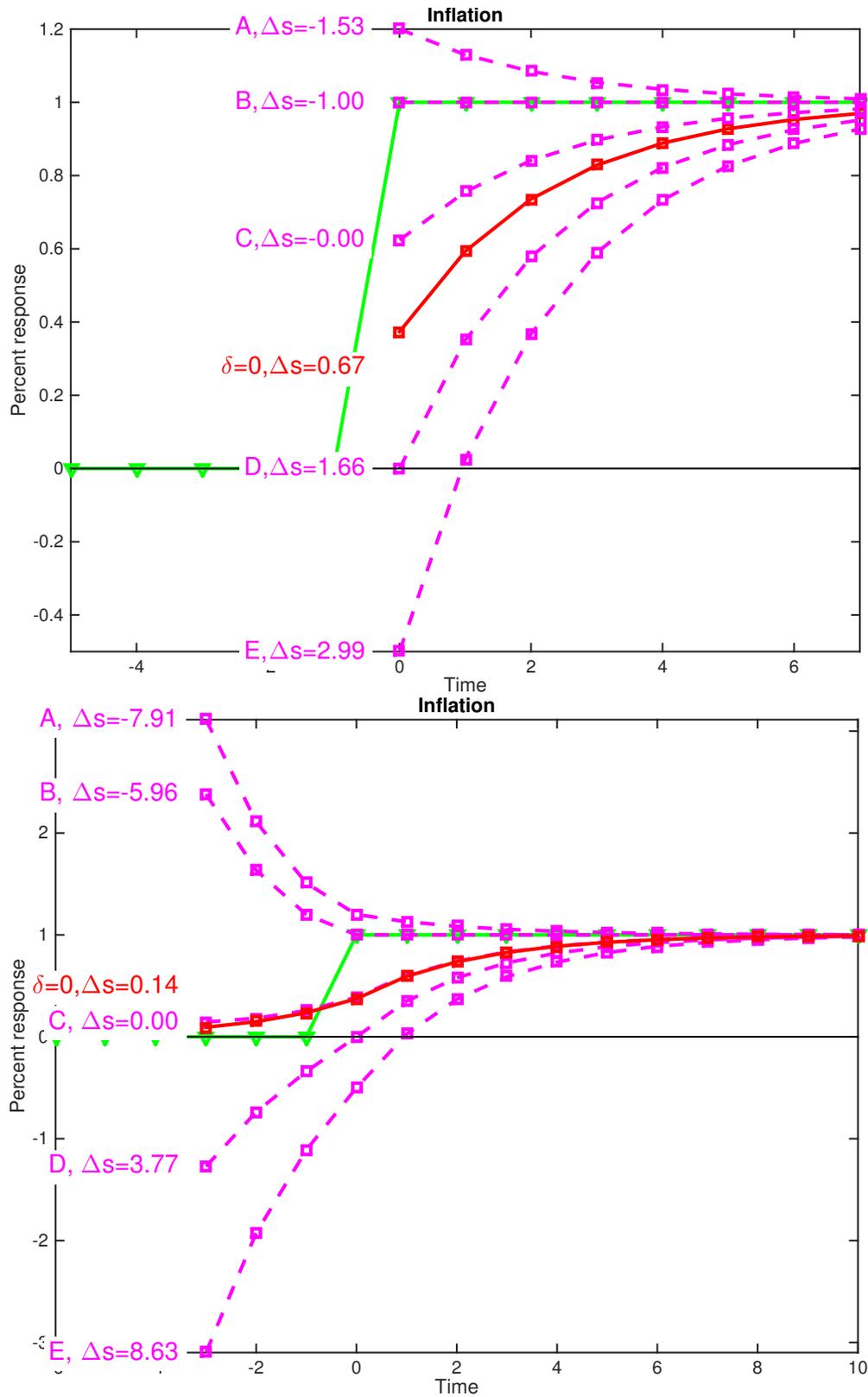
Figure 14: Multiple equilibrium responses to an interest rate rise. Top panel: unexpected rise. Bottom panel: Expected rise. The solid step function gives the interest rate path. Letters identify different equilibria for discussion. The original case is $\delta_0 = 0$.

tational or sunspot shock with the positive interest rate or expected inflation shock. The output responses, not shown line up with the inflation responses, and equilibrium E produces a jump down in output as well, that recovers.

Is there a convincing argument to prefer equilibria such as E, and to view this result as an embodiment of the conventional belief that raising interest rates temporarily lowers inflation?

The issue is not what shock δ_t we will see on a particular date. The question is what shock δ_t we will expect to see *on average*, and *caused by* the Fed's announcement of an interest rate rise.

For that reason, we do not want to fit the correlation of interest rates and unexpected inflation empirically. Our goal is to find economics for an inflation decline, not to fit the most central prediction of monetary economics through a free parameter, the correlation of expected and unexpected inflation shocks. As above, if unexpected disinflation comes from fiscal policy tightening, historically coincident with interest-rate increases, that does not mean that future monetary policy, not coincident with fiscal policy, will have the same effect.

3.4.4   Fiscal index

Each equilibrium choice has a fiscal policy consequence. For each equilibrium choice, then, I calculate the percentage amount by which long-run real primary surpluses must rise or fall for that equilibrium to emerge. Figure 14 presents that number alongside the initial inflation value of each equilibrium.

Again, making this calculation requires no assumption whether fiscal policy is active or passive. We can index equilibria by the "passive" fiscal policy it requires even if we do not select equilibria that way. And, as above, the fiscal expansion or contraction is crucial to producing the aggregate demand that each inflation jump requires.

To make this calculation, I start with the valuation equation for government debt,

$$\frac{B_{t-1}}{P_t} = E_t \left[\sum_{j=0}^{\infty} \beta^j \frac{u'(C_{t+j})}{u'(C_t)} s_{t+j} \right], \tag{66}$$

where B_{t-1} denotes the face value of debt outstanding at the end of period $t-1$ and beginning of period t, P_t is the price level, $u'(C)$ is marginal utility and s_t is the real net primary surplus.

In this case, consumption, equal to output, varies, and real interest rates vary. Higher real interest rates lower the present value of surpluses even when surpluses themselves do not change.

Equivalently, higher real interest rates mean higher debt service costs, which if not met by higher surpluses mean less surplus devoted to repaying debt, and causes inflation. This discount-rate effect is the major change between this model and the frictionless model's analysis of the fiscal consequences of unexpected inflation.

Starting from a steady state with constant surplus s, I calculate the fractional permanent change in surplus Δs, i. e. $s_t = S^{\Delta s}$, that is required of the right hand side of expression (66) for each response function. The calculation is described in the Appendix.

This calculation is simplified in many ways. I specify one-period nominal debt. Here, the objective is to focus on surpluses corresponding to the jumps in the standard model. I study long-term debt in the sticky-price model below. Second, in reality output changes affect primary surpluses, as taxes rise more than spending in booms, and fall more than spending in recessions. We do not need to assume exogenous or fixed surpluses to make these fiscal calculations, or to use the fiscal theory. But some of these effects may represent a change in timing of surpluses – borrowing during recessions that is repaid later during booms – rather than permanent changes that affect the real value of government debt, so adding them in is subtle. Third, inflation also raises revenue due to a poorly indexed tax code. Most of all, perhaps, we could almost as plausibly specify that "monetary policy" changes interest rates without changing the *present value* of the surplus, rather than specify that it does not change surpluses themselves. Realistic monetary - fiscal coordination is not a light topic. A serious calculation of the fiscal impacts of monetary policy requires considerable detail on all these lines. The point here is not quantitative realism, but to capture some of the important effects and to show how one can use fiscal considerations to evaluate different equilibrium possibilities.

The super-neutral equilibrium B in which inflation rises instantly by 1%, also marked "$\Delta s = -1.00$" in Figure 14, corresponds to a 1% decline in long-run surpluses. The 1% jump in inflation devalues outstanding nominal debt by 1%, and since output is constant after the shock there is no real interest rate change. Equilibrium A, with a larger inflation shock, corresponds to a larger than 1% decline in long-run surpluses.

Equilibrium D has no change in inflation at time 0, and so there is no devaluation of outstanding nominal debt. However, the rise in real interest rates means that the government incurs greater financing costs. These costs require a small permanent rise in surpluses.

In between, at equilibrium C, I find the shock δ_0 that requires no change in surpluses at all, so $\Delta s = 0$ by construction. Here, the devaluation effect of an inflation shock just matches the higher financing costs imposed by higher real interest rates. The original equilibrium with no

expectational shock, $\delta_0 = 0$, implies a small but nonzero change in surpluses, to offset the real interest rate effect.

In the frictionless model, with a constant interest rate, all three equilibria are the same and have no inflation shock at time 0. However, at least in this simple calibration, the difference between unexpected inflation C, D, and $\Delta = 0$ is not large. Ignoring real interest rate effects, and discounting surpluses at a constant rate does not make a first-order difference. One does quite well grafting the simple constant-interest-rate FTPL formulas on to the new-Keynesian model.

The difference between equilibria C, $\delta = 0$, and D also punctures one more hope for a negative inflation response. Now, by changing real interest rates, monetary policy has a fiscal effect. Monetary policy changes the present value of surpluses even if it cannot affect surpluses themselves. And this effect is an important part of current (2017) policy discussions. If the Fed were to raise real interest rates 1 %, at 100% debt to GDP ratio, that would raise interest costs and the deficit by 1% of GDP, or nearly $200 billion dollars. This interest-expense channel is a possible fiscal-theoretic channel for the impact of monetary policy, stressed most recently by Sims (2016).

Alas, the sign is wrong for our quest. Raising real interest rates lowers the present value of surpluses, and pushes inflation *up*. C and $\delta = 0$ have positive, not negative, inflation.

Equilibrium E, in which inflation temporarily declines half a percentage point after the interest rate shock, requires an 1.54% rise in permanent fiscal net-of-interest surpluses. Disinflation raises the value of nominal debt, which must be paid. To generate a disinflation coincident with the interest rate rise, we must have a contemporaneous fiscal contraction as well, whether arranged actively or passively. Sticky prices make the needed fiscal contraction larger, not smaller. The fiscal contraction required to produce -1% disinflation is now larger than 1%, because it must overcome the inflationary effect of higher real interest rates.

Though it now produces a prettier, more drawn out response, generating a negative effect of monetary policy by pairing an interest rate rise with a contemporaneous fiscal contraction to produce an unexpected disinflation is no more attractive here than in the frictionless case.

Turning to the anticipated shocks in the bottom panel of Figure 14, we see the effects of multiple equilibria that are stable forward and hence unstable backward. If we want the same inflation variation on date zero, the multiple equilibria have to jump to much larger values on earlier dates. The same sized jumps at time $t = -3$ will imply smaller variation in inflation when interest rates actually rise at $t = 0$.

Larger inflation shocks at time $t = -3$ mean that the fiscal changes required to support most

of the equilibria increase as we move the announcement back in time. For example, the originally super-neutral equilibrium which required a 1% decline in surpluses in Figure 14 now requires a 4.11% surplus decline, because of the larger inflation shock. And equilibrium E, selected to generate a 1% decline in inflation when interest rates rise 1%, now requires a 5.6% permanent rise in fiscal surpluses rather than 1.54%.

The exceptions to this rule are the original equilibrium choice $\delta = 0$, the equilibrium choice C or $\Delta s = 0$ with no fiscal impact, and an equilibrium (not shown) that always chooses no inflation on the announcement date, t-3 in this case. All of these equilibria have *smaller* fiscal impacts as interest rates are announced earlier in time, they all converge to the same point, and they are all stable backward. Cochrane (2016b) argues these feature is useful for equilibrium selection, if one does not want to take a fiscal theory approach. Here, they all lead to Fisherian responses.

Choosing equilibria with no jump in inflation is also an attractive rule. Equilibrium D in Figure 14 has this property, and one can construct an equilibrium with no change in inflation upon announcement for the $t = -3$ shock of the bottom panel of Figure 14. We do not see inflation jumps in the data, and new-Keynesian models are often specified so that inflation must be set one or more periods in advance to reproduce that fact. This choice also is stable, and has limited fiscal impact, as the announcement horizon moves backward. And it leads always to positive subsequent inflation.

In sum, the principles of small fiscal requirements, sensible behavior as announcements come earlier than actual rate changes, or limited jumps in inflation all push one to the view that equilibria near the original $\delta = 0$ equilibrium are sensible, and the others less so.

3.4.5 Policy rules

Perhaps in this full model, spelling out an underlying policy rule can make a disinflationary equilibrium like E more attractive. If monetary policy picks unexpected inflation and fiscal policy is passive, then pairing the announcement of an interest rate rise with a fiscal contraction, as in equilibrium E, might more sense as a description of monetary policy than does viewing the fiscal contraction as a coincidental action by fiscal authorities. More generally, doesn't the standard new-Keynesian model produce a negative sign? That nagging doubt needs to be addressed.

Start with the standard three-equation model, with the standard expression of the policy rule,

(58) (59) and (60), with $\phi > 1$, together with an AR(1) policy disturbance,

$$v_{t+1}^i = \rho v_t^i + \varepsilon_{t+1}^i$$

The government debt valuation equation (66) is still part of the model, but determines surpluses $\{s_t\}$ by passive fiscal policy.

Figure 15 plots the response of inflation and interest rates to an unexpected monetary policy shock v_t^i for this model.
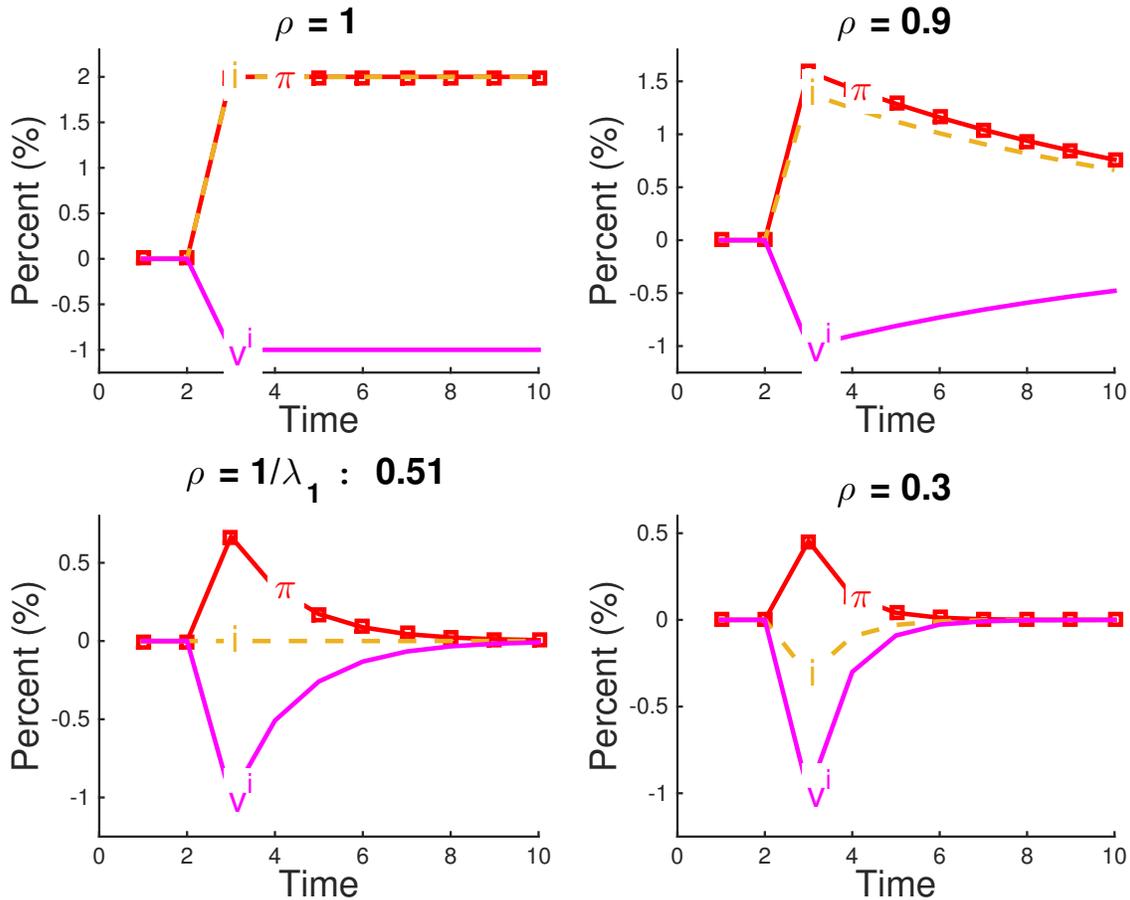
Figure 15: Response of inflation and interest rates to an AR(1) monetary policy shock v^i with persistence ρ in the standard three-equation new-Keynesian model. $\beta = 0.95$, $\kappa = 1/2$, $\sigma = 1$, $\phi = 1.5$.

The top left panel plots the response to a permanent shock, $\rho = 1$. This shock produces an immediate and permanent rise in the equilibrium interest rate. This is the same response as equilibrium choice B of Figure 14. The passive fiscal policy produces the needed 1% fiscal expansion. This standard new-Keynesian exercise produces a super-neutral response, inflation

rising even faster and sooner than the $\delta = 0$ equilibrium, or the equilibrium C with no fiscal response. This is the same response as in the frictionless model and the simple model of Figure 12, which alerts you to the fact that pricing frictions are not central to this model's response.

As in the simpler models, the *disturbance* v_t^i in this parameterization of the policy rule falls though equilibrium interest rates i and inflation π rise. The rule $i_t = \phi\pi_t + v_t^i$ becomes $2 = 1.5(2) - 1$. Inflation has a negative response to the disturbance, though not to actual interest rates. Confusion between the disturbance, which is not measurable in this model, and the path of interest rates may be one reason for a false impression that this standard model delivers a negative sign.

The top right panel of Figure 15 plots the response of inflation to a persistent $\rho = 0.9$ shock. Interest rates rise, inflation rises, and one still sees a Fisherian result. The unexpected inflation shock is still positive, so it is more Fisherian than the response to an AR(1) interest rate rise with no unexpected inflation, graphed in Figure 13. The policy rule is still hurting, not helping, the quest for a negative sign.

The bottom right panel shows that for a sufficiently short-lived shock, $\rho = 0.3 < 1/\lambda_1$, interest rates and inflation finally go in opposite directions. The disturbance v^i exceeds the endogenous response $\phi\pi$, so the negative shock produces negative interest rates. This calculation represents the standard wisdom that a sufficiently temporary shock produces a negative inflation response, and this (at last) is the standard result to reference that the standard model can produce a negative response.

This case combines a swiftly mean-reverting process for the interest rate, as graphed in Figure 13, with a strong contemporaneous fiscal contraction like case E of Figure 14. In (62) if interest rates i mean-revert quickly enough, the central terms will be small. Then, if we add a large enough δ shock at time zero, we produce a negative inflation response.

As in the simple models, however, the appearance of a link between the persistence of interest rates and the sign of the response is an artifact of the parameterization of the policy rule and the AR(1) time-series process for its disturbance. The Fed can produce here too *any* sign together with *any* interest rate persistence by a suitable choice of disturbances. Fundamentally the Fed still has separate and independent interest rate policy and equilibrium-selection policy tools. The negative sign comes entirely from equilibrium-selection policy. Again, this freedom denies our goal – there is no logical link at all between a rise in interest rates and an unexpected disinflation generating a negative sign.

We get a sense of this result already in the bottom left panel of Figure 15, which plots the inflation response in the knife-edge case $\rho = 1/\lambda_1$. In this case, naturally lying between positive and negative interest rate responses, the monetary policy shock is a pure open-mouth operation. The endogenous effect $\phi\pi_t$ just offsets the shock v_t^i so inflation moves with no change at all in interest rates. The Fed just announces the policy shock, inflation moves, and the Fed doesn't actually do anything. This is pure equilibrium-selection.

As in the simple models, these facts are clearer if we parameterize the policy rule by an interest-rate target and an inflation target, (61) rather than a conventional disturbance (60), $i_t = i_t^* + \phi(\pi_t - \pi_t^*)$ rather than $i_t = \phi\pi_t + v_t^i$.

Eliminating x_t from (58)-(59), we have

$$\sigma\kappa i_t = E_t\left[-\pi_t + (1 + \beta + \sigma\kappa)\pi_{t+1} - \beta\pi_{t+2}\right]. \tag{67}$$

It is again convenient to restrict the two disturbances to obey the model first order conditions. Define i_t^* by

$$\sigma\kappa i_t^* = E_t\left[-\pi_t^* + (1 + \beta + \sigma\kappa)\pi_{t+1}^* - \beta\pi_{t+2}^*\right]. \tag{68}$$

Subtract the former from the latter, and use the policy rule $i_t - i_t^* = \phi(\pi_t - \pi_t^*)$,

$$0 = E_t\left[(1 + \sigma\kappa\phi)(\pi_t - \pi_t^*) - (1 + \beta + \sigma\kappa)(\pi_{t+1} - \pi_{t+1}^*) + \beta(\pi_{t+2} - \pi_{t+2}^*)\right]. \tag{69}$$

Factoring the lag polynomial, and we have

$$0 = E_t(1 - \nu_1^{-1}L^{-1})(1 - \nu_2^{-1}L^{-1})(\pi_t - \pi_t^*)$$

with

$$\nu_i^{-1} = \frac{(1 + \beta + \sigma\kappa) \pm \sqrt{(1 + \beta + \sigma\kappa)^2 - 4\beta(1 + \sigma\kappa\phi)}}{2(1 + \sigma\kappa\phi)}$$

For $\phi > 1$, we have $\|\nu_i\| > 1$. The only solution is therefore the forward looking one,

$$\pi_t = \pi_t^*$$

at every date.

The Fed can, by choice of the monetary policy disturbance, obtain any path of inflation it wishes. Again, with the two instruments, expected inflation $E_t\pi_{t+j}^*$ and unexpected inflation $\pi_t^* - E_{t-1}\pi_t^*$, the Fed can independently choose the interest rate path and the unexpected infla-

tion. *There is no link between unexpected inflation and the path of interest rates.*

Examples with unexpected disinflation and persistent rates, or unexpected inflation and transitory rates are as straightforward to calculate here as in the simple model or frictionless model. And again, one can construct $v_t^i = i_t^* - \phi \pi_t^*$ shocks to deliver the same results.

An open mouth policy is just as easy. Suppose the Fed, starting at $i_t^* = 0$, $\pi_t^* = 0$ for $t < 0$, shocks monetary policy for $t \geq 0$ to

$$\pi_t^* = \delta_0 \lambda_1^{-t}. \tag{70}$$

Here, δ_0 is a constant indexing how large the monetary policy shock will be. This is a pure, temporary, change in the Fed's inflation target. Equivalently, suppose the Fed starting at $v_t^i = 0$ for $t < 0$, shocks monetary policy for $t \geq 0$ to

$$v_t^i = -\delta_0 \phi_\pi \lambda_1^{-t}. \tag{71}$$

This is a pure, temporary, monetary policy disturbance. From (62) you can see immediately that this shock produces a jump in inflation, which melts away, and no change in interest rates, as graphed in the lower left hand panel of Figure 15.

A disinflation produced by an unexpected inflation shock, preceding a period of rising interest rates, would be entirely a choice by the Fed, having nothing to do with the economy's response to interest rates.

Again, the heart of the argument is equilibrium selection by making the economy unstable. If $\phi < 1$, so $\lambda_2 < 1$, then there is a family of solutions,

$$E_t(\pi_{t+\tau} - \pi_{t+\tau}^*) = \lambda_2^t(\pi_t - \pi_t^*)$$

and any $\pi_t - E_{t-1}\pi_t$ can occur. But if $\phi > 1$ so $\lambda_2 > 1$, then any deviation of $\pi_t - \pi_t^*$ will explode. Ruling out explosions, it won't happen. Many other equilibrium selection schemes achieve the same purpose, for example see Atkeson, Chari, and Kehoe (2010) and the discussion in the online appendix to Cochrane (2011b).

This construction also verifies that the solution method using equations (62)-(63), solving for inflation given a path of interest rates, does not assume a peg, $\phi = 0$, or fiscal theory. For any ϕ, we can *construct* an active policy rule, a set of i_t^*, π_t^* or a set of v_t^i that generates any of the equilibria displayed in Figure 14.

The fact of adding a policy rule, then, doesn't help us to choose equilibria. It does not link

unexpected inflation to interest rates and expected inflation in a useful way. It does not justify the equilibrium with a disinflationary unexpected inflation married to higher interest rates, our one hope for a negative sign.

3.5   Long term debt and sticky prices

When prices are sticky, nominal interest rate changes imply real interest rate changes, which affect the present value of surpluses. Allowing real interest rate variation and long-term debt, the government debt valuation formula becomes

$$\sum_{j=0}^{\infty} Q_t^{(t+j)} \frac{B_{t-1}^{(t+j)}}{P_t} = E_t \sum_{j=0}^{\infty} \beta^j \frac{u'(C_{t+j})}{u'(C_t)} s_{t+j} = E_t \sum_{j=0}^{\infty} \left(\prod_{k=0}^{j-1} \frac{1}{1+r_{t+k}} \right) s_{t+j}. \tag{72}$$

The first equality is the general formula; the second is an approximation reflecting the linearized nature of the new-Keynesian model we are working with, in which risk premiums do not vary over time.

As before, the only effect of active fiscal policy will be to select an equilibrium, i.e. to determine the value of unexpected inflation. Otherwise, the sticky-price dynamics are unaffected.

To review, in the frictionless model with one-period debt, $Q_t^{(t)} = 1$ is the only bond price on the left side of (72), and real interest rates r on the right are constant. Hence, a change in nominal interest rates with no change in fiscal surpluses leaves (72) unchanged, and there is no jump in the price P_t when the interest rate change is announced.

To a first, expectations-hypothesis, approximation,

$$Q_t^{(t+j)} = E_t \prod_{k=0}^{j-1} \frac{1}{1+i_{t+k}}$$

When we add long-term debt in the frictionless model, a rise in expected future nominal rates i lowers bond prices Q. With nothing else changed in (72), P_t falls.

With one-period debt and sticky prices, now higher nominal interest rates mean higher real interest rates on the right hand side of (72). Higher real rates lower the present value of surpluses, which results in a *positive* shock to the price level P_t, as seen in equilibrium C of Figure 14.

Merging long-term debt and sticky prices adds the last two mechanisms. Higher nominal rates lower bond prices, which results in a lower P_t. But to the extent that higher nominal rates

mean higher real rates, the present value of surpluses on the right hand side of (72) is also lower, which mutes the disinflationary effect. If prices are perfectly sticky, so that real interest rates equal nominal rates $i_{t+k} = r_{t+k}$, then right and left hand sides of (72) move one for one, and there is again no effect.

In sum, we expect that sticky prices will mute the disinflationary effect of an interest rate rise in the presence of long-term bonds. Sticky prices should also provide smoother and more realistic dynamics.

To calculate the response function merging the standard new-Keynesian sticky price model with the fiscal theory and long-term debt, I suppose interest rates start at their 2014 values, and I compute the market value of the debt. I use the 2014 zero-coupon U.S. Treasury debt outstanding provided by Hall and Sargent (2015) for $B_{t-1}^{(j)}$, and the 2014 zero coupon yield curve $\{Y^{(j)}\}$ from Gürkaynak, Sack, and Wright (2007) for bond prices $Q_t^{(j)} = \left(1/Y_t^{(j)}\right)^j$. I calculate the nominal market value of the debt as $\sum_{j=0}^{\infty} Q_t^{(j)} B_{t-1}^{(j)}$.

I then suppose forward rates all rise by the interest rate response function, and I calculate the new nominal market value of the debt. I calculate the present value of an unchanged surplus using the government debt valuation formula (72), and the model implied path of real interest rates. That consideration chooses a single value of unexpected inflation at the time of the shock, equivalently of the multiple equilibria δ_t, on the announcement date. Equations are in the Appendix.

The top left panel of Figure 16 presents the responses of inflation and output to an unexpected and permanent interest rate increase. The devaluation of long-term debt now produces a 1.2% disinflation despite no change in surpluses. Inflation is stable, so eventually rises to meet the long-term interest rate. However, the muliple periods of negative inflation now mean that the *price level* displays a hump-shaped response (not shown). This is the most hopeful graph in this paper for an economically based model that gives the desired response function for monetary policy changes.

In this model, the output gap is related to expected future inflation. After the unexpected downward jump at time 0, the larger expected future inflation produces a sharp -4% output contraction.

Compare to Figure 12, with a +0.4% inflation on the date of inflation rise, and a -1% output gap, or compare to the multiple equilibria in Figure 14. The devaluation effect of long term bonds is exactly equivalent to a fiscal contraction past equilibrium E of figure 14 – again,
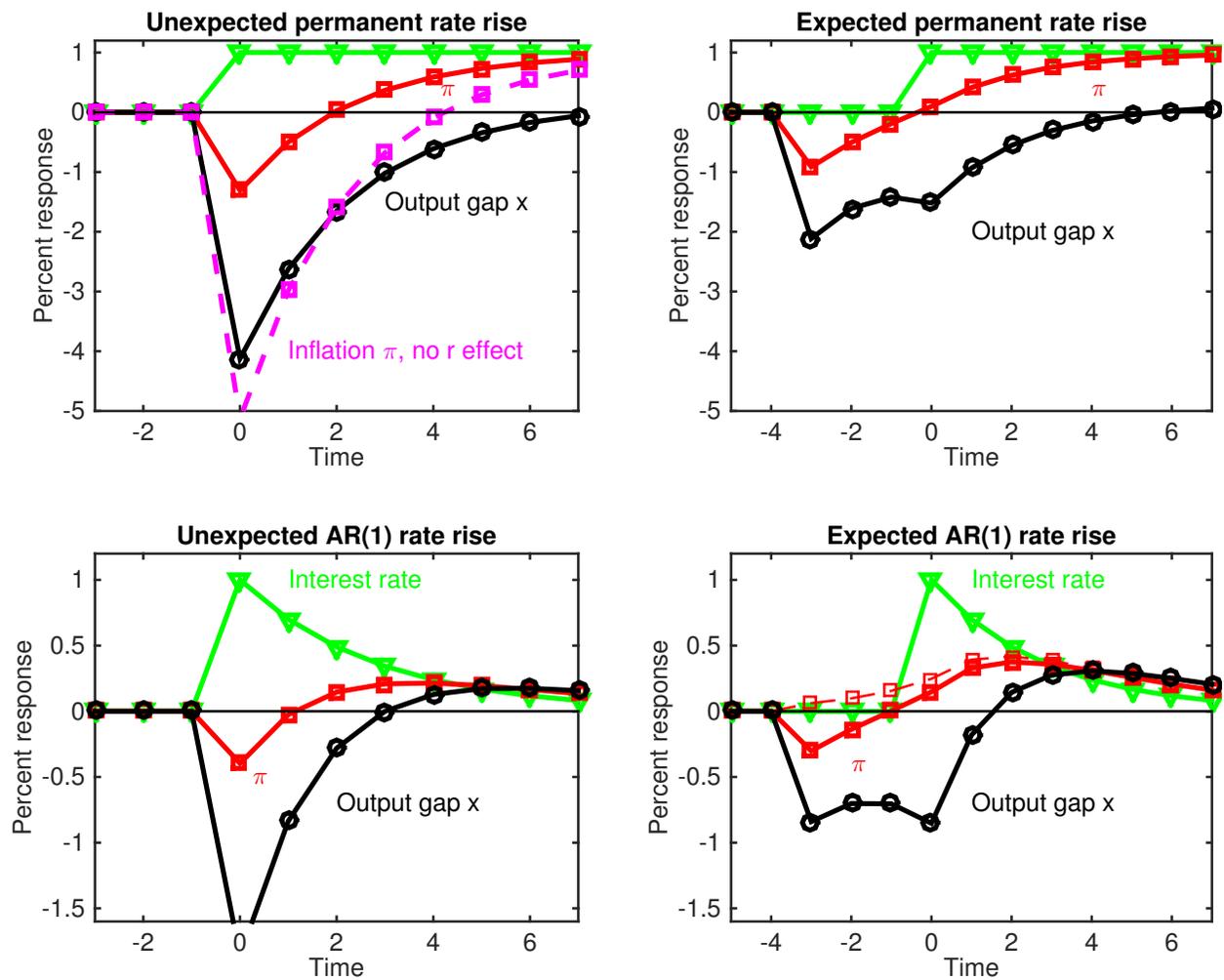
Figure 16: Response to interest rate rises with long-term debt and sticky prices. I use the 2014 maturity structure of the debt to find the jump in price level that implies no change in primary surpluses. The line "inflation, no r effect" in the first panel ignores the effect of rising real rates in devaluing future surpluses. $\rho = 0.7$.

dynamics are the same after the initial inflation shock. The longer period of high real inter-
est rates here drives the output gap down to -4% rather than -1%. (From the IS curve (58),
$x_t = -\sigma E_t \sum_{j=0}^{\infty} (i_{t+j} - \pi_{t+j+1})$.)

The dashed line marked "inflation, no r effect" ignores the change in real interest rates on the
right hand side of (72), to show the effect of sticky prices in moderating the long-term-debt disin-
flationary mechanism. Here, I ignore the effect of rising interest rates in calculating the present
value of the surplus, but otherwise leave model dynamics the same. We see that in the original
calculation, higher real rates substantially lower the present value of surpluses, and make a big
moderating difference to the initial disinflation. Models with this mechanism thus produce *less*
disinflation from nominal interest rate rises when they have *more* sticky prices. However, this
calculation also emphasizes the delicacy of fiscal assumptions. One can read it also as saying
that the fiscal authority is partly passive, agreeing to raise surpluses to pay off higher *real interest
rates* on the debt, though not agreeing to raise surpluses to pay off the real consequences of price
level jumps. Such behavior would induce the larger disinflation.

The top right panel of Figure 16 shows the response when the interest rise is announced three
years in advance. Higher interest rates now only affect bonds with three year or higher maturity.
Thus, the downward inflation rate jump is smaller, only 1%. However, there is still a substantial
period of negative inflation, so the price level response (not shown) continues downward from
period -3 to 0, only then starting to rise. Output suffers a less severe contraction, bottoming out
at 2% not 4%.

As in the frictionless model, fiscal effects happen only on the day of announcement. This is
an important consideration in evaluating this channel. It will not rescue the old-Keynesian view
that the interest rate rise itself sets off the disinflation.

The effects get uniformly smaller as the interest rate rise is expected further and further in
the future. When the interest rate rise is expected after the maturity of the longest bond, the
disinflationary effect vanishes entirely. Thus, this fiscal channel sensibly predicts smaller effects
of expectations further in the future, and does not suffer from the forward-guidance puzzle.

The bottom panels of Figure 16 presents the response to an unexpected (left) and expected
(bottom) AR(1) rate rise, more typical of actual policy movements. The unexpected transitory
rate rise on the left, is the (small) slice of variation that is potentially recovered by VARs. (Though
VARs don't attempt to orthogonalize monetary and fiscal policy shocks, $\Delta s = 0$ as I do here.) The
disinflation effect is now smaller still, less than 0.5% in both cases. The AR(1) interest rate rise
has less effect on longer term bonds than a permanent rate rise.

In sum, long-lived interest rate rises can produce disinflations on the same order of magnitude as the interest rate rises, and thus has the potential to explain the perceived effects of monetary policy.

3.6 Money

Perhaps monetary distortions, in addition to pricing distortions, will give us the traditional result. Perhaps when interest rate increases were accomplished by reducing the supply of non-interest-bearing reserves, that reduction in money and liquidity services produced a temporary decline in inflation. Such a finding would explain traditional beliefs, but it would warn us that raising interest rates by raising the rate paid on abundant excess reserves will not have the same temporary disinflationary effect as history suggests.

I introduce money in the utility function, nonseparable from consumption, so that changes in money, induced by interest rate changes, affect the marginal utility of consumption, and thus the intertemporal-substitution equation.

Woodford (2003) (p. 111) begins an analysis of this specification. But Woodford quickly abandons money to produce a theory that is independent of monetary frictions, and does not work out the effects of monetary policy with money. If theory following that choice now does not produce the desired outcome, perhaps we should revisit the decision to drop money from the analysis.

The detailed presentation is in the Appendix. The bottom line is a generalization of the intertemporal-substitution condition (58), to:

$$x_t = E_t x_{t+1} + (\sigma - \xi)\left(\frac{m}{c}\right) E_t\left[\left(i_{t+1} - i_{t+1}^m\right) - \left(i_t - i_t^m\right)\right] - \sigma\left(i_t - E_t \pi_{t+1}\right). \tag{73}$$

The presence of money in the utility function has no effect on firm pricing decisions and hence on the Phillips curve (59).

Here, $-\xi$ is the interest-elasticity of money demand. A typical number for the semi-elasticity of money demand $d\log(m)/di$, with i in percent, is -0.05 to -0.1 (see Lucas (1988), especially Figure 4, and the last paragraph, and Figure 6 here.) This number suggests values no more than two to four times greater at current interest rates. Since higher elasticity ξ reduces the size of the effects, I use a deliberately low value $\xi = 0.1$. The value m/c is the steady state ratio of real money holdings to consumption. The larger this value, the more important monetary distortions. The

quantity i_t^m is the interest rate paid on money.

Equation (73) differs from its standard counterpart (58) by the middle, change-in-interest rate term. Equation (73) reverts to (58) if utility is separable between money and consumption $(\sigma - \xi) = 0$, if m/c goes to zero, or if money pays the same interest rate as bonds $i = i^m$.

The expression $m/c\,(i_t - i_t^m)$ represents the proportional interest costs of holding money. The middle term following $(\sigma - \xi)$ represents the expected increase or decrease in those costs. An expected increase in interest costs of holding money induces the consumer to shift consumption from the future, when holding the money needed to purchase consumption goods will be relatively expensive, towards the present. It acts just like a lower real interest rate to induce an intertemporal reallocation of consumption.

The presence of expected changes in interest rates brings to the model a mechanism that one can detect in verbal commentary: the sense that *changes* in interest rates affect the economy as well as the level of interest rates.

However, monetary distortions only matter in this model if there is an *expected change in future* interest rate differentials. Expected, change, and future are all crucial modifiers. A higher or lower steady state level of the interest cost of holding money does not raise or depress today's consumption relative to future consumption. An unexpected change in interest costs has no monetary effect at all, since $E_t\,(i_{t+1} - i_t) = 0$ throughout.

The model solution is essentially unchanged. The extra term in the intertemporal substitution equation (73) amounts to a slightly more complex forcing process involving expected changes in interest rates as well as the level of interest rates. One simply replaces i_t in (62)-(63) with z_t defined by

$$z_t \equiv i_t - \left(\frac{\sigma - \xi}{\sigma} \right) \left(\frac{m}{c} \right) E_t \left[\left(i_{t+1} - i_{t+1}^m \right) - \left(i_t - i_t^m \right) \right].$$

The slight subtlety is that this forcing process is the change in *expected* interest differentials. Lag operators must apply to the E_t as well as what's inside. Inflation depends on past expectations of interest rate changes, not to past interest rate changes themselves.

I present results for the traditional specification that the interest on money $i_t^m = 0$, so that increases in the nominal interest rate are synonymous with monetary distortions. This case also generates the largest effects. The top panels of Figure 17 plot the response functions to our expected and unexpected interest rate step with money distortions $m/c = 0,\ 2,\ 4$.
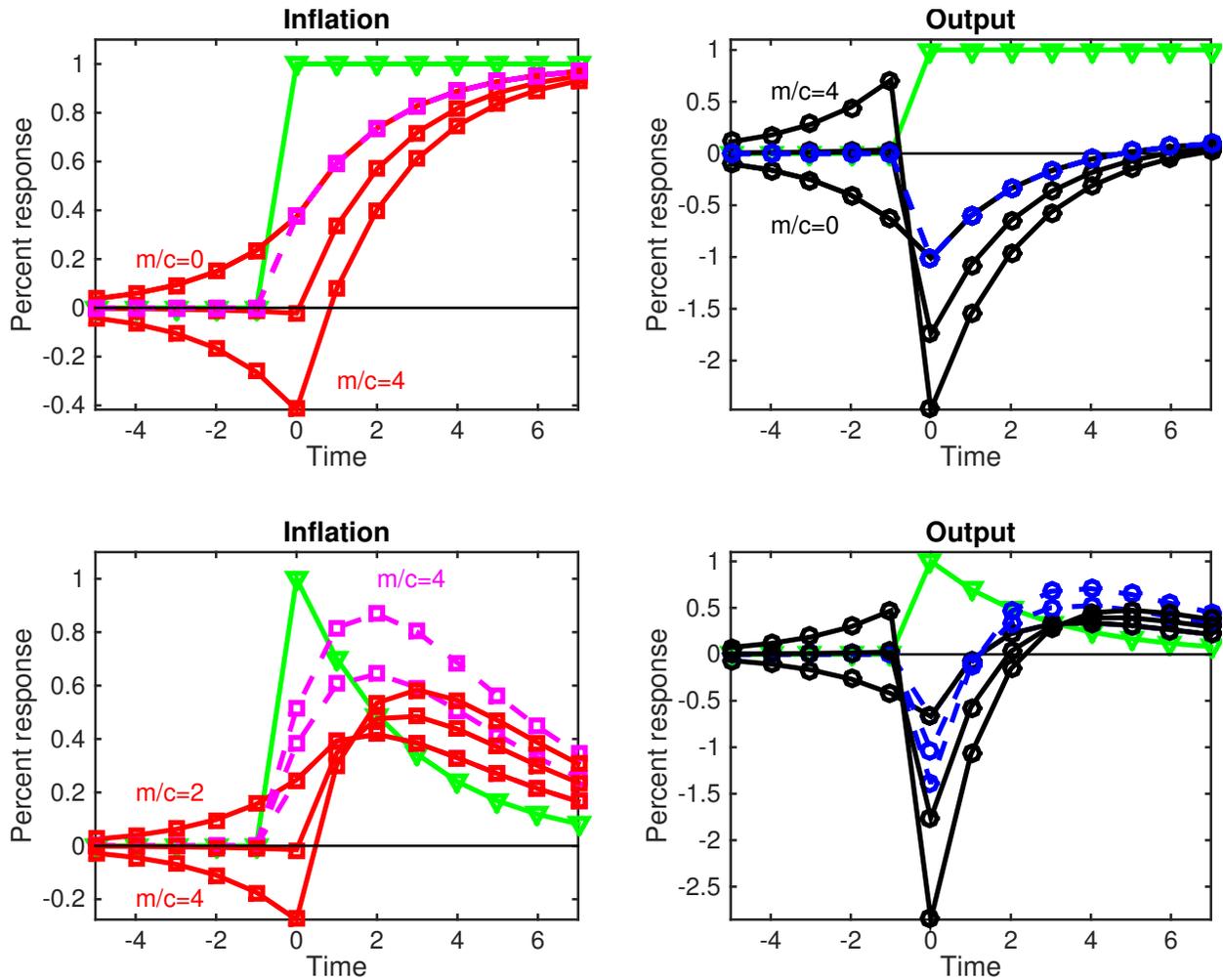
Figure 17: Responses of inflation and output to an interest rate rise; model with money. The three cases are $m/c = 0, \ 2, \ 4$. Solid lines are an expected interest rate rise, dashed lines are an unexpected rise.

For the unexpected interest rate rise, shown in dashed lines, the presence of money makes no difference at all. The dashed lines are the same for all values of m/c, and all the same as previously, and the model remains stubbornly Fisherian. This is an important negative result. In this model based on forward looking behavior and thus intertemporal substitution, money can only affect the response to *expected future* nominal interest rate changes.

The response to an expected interest rate rise, shown in solid lines, is affected by the monetary distortion. As we increase the size of the monetary distortion m/c, inflation is lower in the short run. For $m/c = 4$, we get the desired shape of the impulse response function. The announced interest rate rise produces a temporary decline in inflation, and then eventually the Fisher effect takes over and inflation increases.

Since interest rates are higher after time 0, the consumer has an incentive to shift consumption to times before 0, i.e. to consume when the interest costs of holding the necessary money are lower. Higher output corresponds to decreasing inflation, and vice versa, so this pattern of output corresponds to lower inflation before time 0 and higher inflation afterward.

The $m/c = 4$ curve seems like a success, until one ponders the size of the monetary distortion – non-interest bearing money holdings equal, on average, to four years of output. This model is not carefully calibrated, but $m/c = 4$ is still an order of magnitude or more too large. One may be tempted to look at larger and larger monetary aggregates, but those all pay interest. Interest spreads enter together with m/c in (73), so trading larger m/c for a lower interest spread does not help.

Raising σ, which multiplies m/c in (73), can substitute for a large m/c, though σ also magnifies the last term, which induces Fisherian dynamics. $\sigma = 4$, $m/c = 2$ produces about the same inflation decline as $\sigma = 1$, $m/c = 4$ produced in Figure 17, though it speeds up dynamics as well. Alas, $\sigma = 1$ was already above most estimates and calibrations. A coefficient $\sigma = 4$ implies that a one percentage point increase in the real interest rate induces a four percentage point increase in consumption growth, which is well beyond most estimates. And $m/c = 1$ is already at least twice as big as one can reasonably defend.

Since expected changes in interest rates are the crucial mechanism in this model, perhaps putting in more reasonable interest rate dynamics can revive the desired inflation dynamics? The bottom panels of Figure 17 shows the response function to an AR(1) interest rate path. In response to an unexpected shock, shown in dashed lines, the presence of money uniformly *raises* inflation. The expected decline in interest costs posed by the AR(1) reversion after the shock shifts consumption from the present to the future. Low output corresponds to an expected rise

in inflation. Since the rise in interest rates was unexpected, it has no effect on inflation or output.

The response to an expected interest rate increase now has the same pattern, but less disinflation – the $m/c = 4$ case bottoms out at a bit less than -0.2% in the bottom panel of Figure 17 rather than -0.4% in the top panel.

In sum, these calculations show what it takes to produce the standard view: For an anticipated interest rate rise only, money in the model can induce lower inflation than a model without monetary frictions produces. If we either have very large money holdings subject to the distortion, or a very large intertemporal substitution elasticity, the effect can be large enough to produce a short-run decline in inflation. Adding money to the model in this way has no effect on responses to an unexpected permanent interest rate rise, and thus does nothing to address typical VAR evidence or the widespread view that unexpected interest rate changes have disinflationary effects.

The mechanism is quantitatively small. Relative to the effects of actual changes in real interest rates, the distortions to intertemporal incentives from greater or lesser costs of holding money are second-order.

Also, this mechanism does not give rise to classic intuition. Interest costs of money holdings only affect "demand" if people *expect* higher or lower interest costs in the future than they experience today. The level of interest costs has no effect.

3.7   A backward-looking Phillips curve

Empirically, lags seem important in Phillips curves. The forward-looking Phillips curve (59) specifies that output is higher when inflation is high relative to future inflation, i.e. when inflation is declining. Though all Phillips curves fit the data poorly, especially recently, output is better related to high inflation relative to past inflation, i.e. when inflation is rising (Mankiw and Reis (2002)).

Theoretically, the pure forward-looking Phillips curve is not central. Though it does some violence to the "economic" criterion for the simple baseline theory that we are searching for, we should check if the short or long-run neo-Fisherian conclusions can be escaped by adding past inflation to the Phillips curve. More deeply, one should expect that the sign and stability properties that are central to the argument here are robust to perturbations of the model. And they are.

The simplest approach is to consider a static Phillips curve. This specification is the $\beta \rightarrow$

0 limit of the three equation model (58)-(59). Kocherlakota (2016) provides detailed micro-foundations for a static Phillips curve.

So consider

$$x_t = E_t x_{t+1} - \sigma(i_t - E_t \pi_{t+1}) \tag{74}$$

$$\pi_t = \kappa x_t. \tag{75}$$

The equilibrium is simply

$$E_t \pi_{t+1} = \frac{1}{1 + \sigma\kappa}\pi_t + \frac{\sigma\kappa}{1 + \sigma\kappa}i_t \tag{76}$$

and hence

$$\pi_t = \sigma\kappa \sum_{j=1}^{\infty} \frac{1}{(1 + \sigma\kappa)^j}i_{t-j} + \sum_{j=0}^{\infty} \frac{1}{(1 + \sigma\kappa)^j}\delta_{t-j}. \tag{77}$$

This is *exactly* the same as the dynamics we found for a static IS curve and fully forward-looking Phillips curve, $x_t = -\sigma(i_t - E_t\pi_{t+1})$; $\pi_t = E_t\pi_{t+1} + \kappa x_t$ in equation (42). The dynamics are stable, and inflation responds positively to interest rates throughout. Figure 11 already plots the response function for the static Phillips curve case (74)-(75) - and inflation rises smoothly throughout.

We can even include a backwards-looking accelerationist Phillips curve, which one may feel more realistic, throwing out forward-looking price setting. Consider a Phillips curve based on firm expectations π_t^e,

$$\pi_t = \pi_{t-1}^e + \kappa x_t$$

and were expectations evolve adaptively as

$$\pi_t^e = (1 - \lambda) \sum_{j=0}^{\infty} \lambda^j \pi_{t-j}.$$

Substituting the output gap from the usual intertemporal IS curve (74) ,

$$(\pi_t - \pi_{t-1}^e) = (E_t\pi_{t+1} - \pi_t^e) - \sigma\kappa(i_t - E_t\pi_{t+1})$$

$$(1 + \sigma\kappa)E_t\pi_{t+1} = \pi_t + \pi_t^e - \pi_{t-1}^e + \sigma\kappa i_t$$

$$(1 + \sigma\kappa)E_t\pi_{t+1} = \pi_t + (1 - \lambda)\left[\sum_{j=0}^{\infty} \lambda^j \Delta\pi_{t-j}\right] + \sigma\kappa i_t$$
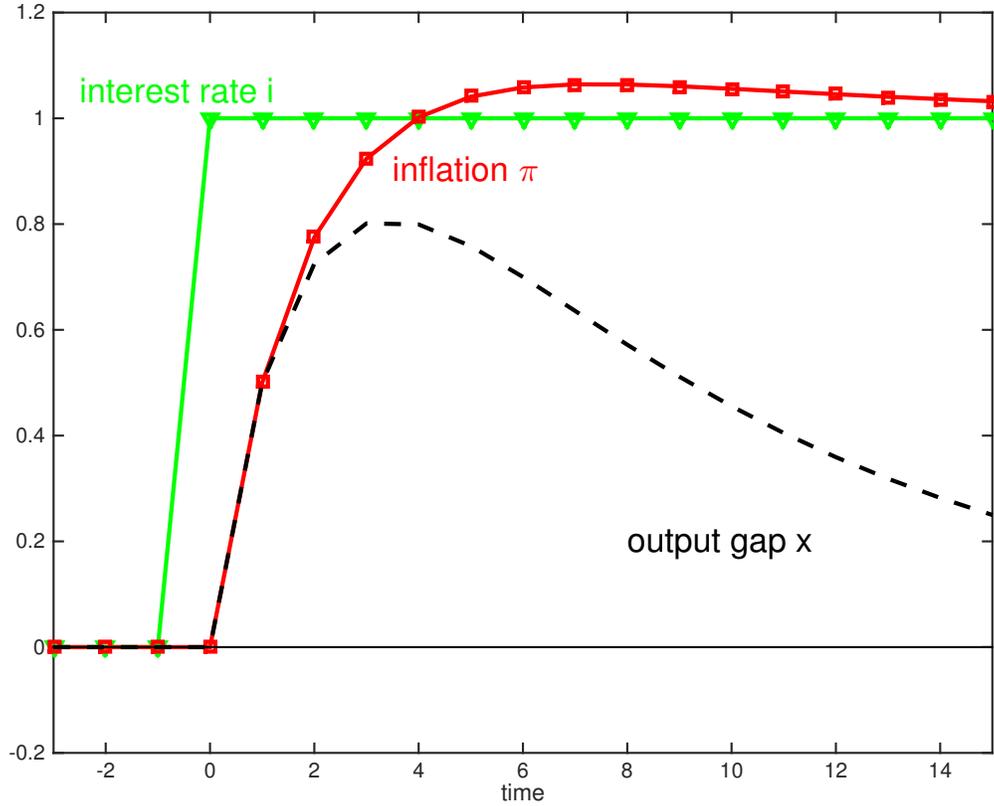
Figure 18: Impluse-response with backward-looking Phillips curve. Model $x_t = E_t x_{t+1} - \sigma(i_t - E_t \pi_{t+1})$, $\pi_t = (1 - \lambda) \sum_{j=0}^{\infty} \lambda^j \pi_{t-j} + \kappa x_t$. $\sigma = \kappa = 1$, $\lambda = 0.9$.

Figure 18 plots the impulse-response function for this model. It is Fisherian throughout. Unlike the standard model with forward-looking Phillips curve in Figure 12, the interest rate rise increases output as well. By giving the "right" positive (old-Keynesian) sign of the relationship between output and inflation, it also gives the "wrong" positive sign of the relationship between interest rates on output, as well as on inflation.

In sum, neither forward-looking IS curve nor forward-looking Phillips curves are essential to producing a short and long run Fisherian response.

4   Policy

To summarize for the purposes of policy implications, the evidence suggests that the zero bound is stable and quiet. There is no deflation spiral and no sunspot volatility. Large interest-paying reserves do not cause inflation. There is a simple economic model, the new-Keynesian model

with fiscal theory, that is consistent with this interpretation. In that theory an interest rate peg or passive $\phi < 1$ policy would also be stable and quiet, as long as fiscal policy retains people's confidence. The evidence suggests that contrary classic doctrines were wrong.

The implication of this fact is that persistently higher interest rates will lead to higher inflation in the long run, a form of long-run neutrality. Is there a negative short run effect? There is as yet no simple economic model for standard beliefs regarding such an effect. There is as yet only the fiscal theory / long-term debt channel, which is far from the views underlying standard policy beliefs. This excursion should at least reduce one's confidence in a simple view that higher interest rates, when they occur, will lower inflation.

If this is right, what are the consequences for policy going forward?

First, we should not unduly fear the zero bound! Much current policy discussion regards the past zero bound as a narrow scrape with the deflation spiral, and argues for a higher inflation target, or dry powder in the arsenal of unconventional monetary policy and large fiscal stimulus to prevent the spiral from breaking out should we return to the zero bound in the next recession or crisis.

Second, we should not unduly fear the large balance sheet, or at least the large interest-paying reserves that a large balance sheet gives rise to. They do not cause inflation. They also have important financial stability benefits. Deposits backed by reserves are less prone to run.

We have discovered that abundant, safe, government-provided, interest-paying electronic money will not cause inflation. The Treasury could equally well provide "reserves" in the form of abundant fixed-value floating-rate highly liquid debt (see Cochrane (2015), Cochrane (2014d)). The Fed does not need to act as the worlds' largest money market fund, transforming longer-term government debt into floating-rate government debt. There is no need to keep Treasury debt artificially illiquid for price-level control. Much current policy discussion by contrast sees large reserves as permanently stimulative, in urgent need of reduction, and many commentators wish for a return to a small amount of non-interest-bearing reserves, in order to ensure control of inflation.

Third, we can live the Friedman (1969) rule and enjoy the Friedman-optimal quantity of moneny. Not only can we have Friedman-optimal interest-bearing reserves, we can have a permanently zero, or very low interest rate, if we wish. Such a rate would not only reduce socially wasteful shoe-leather costs, as Friedman envisaged, it would remove a lot of needless cash management, bill paying and collection hurry, inflation-induced capital income taxation, distortions

due to sticky prices under nonzero inflation, and other distortions.

Policy *can* keep a low rate, insensitive to economic conditions and to inflation. When the real rate rises, inflation will eventually decline to accommodate the real rate, all on its own, and vice versa.

But the policy implications are not so dramatic.

Though the Fed *can* keep a low peg (so long as fiscal policy cooperates), that does not mean that the Fed *should* keep a low peg. In the presence of price stickiness, inflation may take a long time to adjust to the real rate, and output would be affected in the meantime.

The Fed can instead vary the nominal interest rate, raising the nominal rate in good economic times when it thinks the natural rate is higher, and vice versa. Such a policy would result in less inflation variation, and under sticky prices, it would plausibly result in less volatile output as well.

One may distrust the Fed's ability to divine changes in the "natural" rate. Stability opens up an exciting and novel alternative possibility. The Fed could target the spread between indexed and non-indexed debt. It could, for example, decree that one-year Treasury inflation protected securities shall trade at one percentage point lower yield than one-year Treasury bills, and offer to buy and sell at that price differential, ignoring the overall level of interest rates. This strategy would nail down expected inflation but allow the level of the real interest rate to adjust automatically to market forces. It operates much like an expected CPI standard, with CPI futures taking the place of gold to define the value of the currency. If an interest peg is unstable such a standard would not work – the Fisherian steady state would be unstable, and interest and inflation rates would spiral away. But stability implies that expected inflation would have to adjust to the interest-spread target – as always with the fiscal footnote that the Treasury must back the fiscal consequences of large rearrangements of indexed vs. nonindexed debt. $i_t = r_t + E_t\pi_{t+1}$ implies $E_t\pi_{t+1} = i_t - r_t$.

In the other direction, nothing in this analysis denies that the Fed can try to diagnose and offset shocks all over the economy, varying nominal interest rates accordingly, as well as with forward guidance and quantitative easing as analyzed above. It can try to fine-tune the inflation and output path using complex DSGE models, exploiting all the frictions, dynamics, and irrationalities it feels it understands, along with the fiscal/long term debt channel described above. The rules vs. discretion, fine-tune vs. leave the hot/cold water shower handles alone debate can continue undeterred.

Thus, observed policy need not change much. We may continue to see Taylorish responses to output and inflation, plus deviations to respond to other concerns such as exchange rates, financial stability, and so forth.

There are some lessons. Most of all, "active" policy rules are not necessary for inflation stability or determinacy. But the latter were questionable anyway, and unobserved in equilibrium. Writing the rule in the form $i_t = i_t^* + \phi_\pi(\pi_t - \pi_t^*) + \phi_x(x_t - x_t^*)$, where i_t^*, π_t^* and x_t^* represent equilibrium values, the active policy represents only conjectural deviations from equilibrium never seen in equilibrium. *Observed* policy consists of the correlations between i_t^*, π_t^* and x_t^*. Thus, this lesson is important for academic papers and deep foundations, but has little impact on actual policy making and interest rates. Fed officials think in terms of actual interest rates and not active deviations from equilibrium. (see the quote below from Fed Chair Janet Yellen).

More directly relevant to policy, my long and negative search for a simple economic model that delivers one implies that one's faith in the exploitable negative sign of interest rates on inflation should be at least subdued.

We should pay more attention to the fiscal foundations of price stability, but that isn't really the Fed's job. If the time comes for a major disinflation, it will have to come from a joint monetary-fiscal stabilization as before. That could look like the 1980s, inaugurated by high interest rates, or it could look like Sargent's ends of inflations, in which fiscal reform ends inflation and high rates together.

Again affecting economics more than policy, the new-Keynesian plus fiscal theory framework means that monetary economics is now like regular economics. We can start with a simply supply and demand frictionless benchmark, in which the price level is determinate, and there is a role for interest rate policy, forward guidance and quantitative easing. Then we can add frictions to taste, to match dynamics. So far, interest rate based monetary economics could not be built on a frictionless foundation, and even MV=PY relied on a swiftly-vanishing monetary friction.

However, there are some important limits to this analysis and important warnings that must be sounded. It would be simple to interpret these results to say that all a country needs to do to raise inflation is to raise its interest rate, and therefore all a country like Brazil or Turkey needs to do that wishes to lower its inflation rate is to lower its interest rate. That is a dangerous conclusion.

First, such an interest rate move must be persistent and credible. You can't just try the waters. Second, it must wait out a potential move in the other direction, via the long-term debt effect, or

if the many complications discussed below can generate one, via other effects. We have analyzed the simple underlying economic model, but real-world dynamics demand real-world frictions. Third and most importantly the fiscal backing and fiscal coordination must be there especially for disinflation. Lowering nominal rates cannot cure a fundamentally fiscal inflation.

In 2008 the US and Europe did lower interest rates, and lower inflation followed albeit slowly. But the flight to quality of US and European government debt came first. Just why is a topic for later – lower discount rates rather than high expected surpluses are a likely culprit – but it is undeniable that there was a huge shift in demand towards government debt, and interest rates went down on their own. Likewise, many countries have seen monetary stabilization plans of all sorts fall apart when the fiscal cooperation was lacking. The government debt valuation formula is an integral part of the model, and just lowering interest rates will not work with fiscal trouble brewing.

Likewise, it does not follow from the analysis here that the US, Europe and Japan can just peg low interest rates and sleep soundly. The government debt valuation equation holds, in all models. The question is how it holds, and how long it will continue to hold, without requiring a burst of inflation or deflation. A high value of government debt corresponds to high expected surpluses or low discount rates – investors willing to hold government debt despite very poor prospective real returns. At the current moment, low discount rates, revealed in part by low real interest rates – seem like a much more likely source of high values. But low discount rates can evaporate quickly, especially when debt is largely short term and frequently rolled over. A change in discount rate provokes exactly the same sort of unexpected inflation that a change in fiscal surpluses provokes. And like such a change, there is nothing a central bank can do about it.

Some historic interest rate pegs, like exchange rate pegs and the gold standard, lasted a surprisingly long time. Many interest rate pegs fell apart when their fiscal foundations fell apart. With short term debt, that can happen in what feels like "speculative attack" "bubble" or "run" to central bankers. Inflation's resurgence can happen without Phillips curve tightness, and can surprise central bankers of the 2020s just as it did in the 1970s, just as inflation's decline surprised them in the 1980s, and just as its stability surprised them in the 2010s.

5 Occam

There are many ways one could try to save traditional theories in face of the long quiet zero bound. There are many additional ingredients one could add to try to produce a temporary negative inflation response to interest rates, in a model that is consistent with the long quiet zero bound.

In this section, I take up a number of these possibilities. I argue that some popular ones are implausible. But they are mostly logical possibilities, that we cannot disprove with the evidence before us. Most are complex, ex-post patches. Sometimes patches turn out to be correct, as did foreshortening of fast-moving objects, and elliptical planetary orbits. Occam's razor reminds us, however, that more often complex patches fail when there is a clear simple alternative, as did epicycles and ether drift.

5.1 Offsetting instabilities?

Perhaps the economy really is unstable as in old-Keynesian/monetarist models, but fiscal stimulus, inflationary quantitative easing, and deftly-timed forward guidance just offset a deflationary spiral, in Europe and Japan as well as the US. If skillfully walked, the tightrope between abysses looks quiet.

Perhaps. Or perhaps stability and quiet are just what they seem to be.

5.2 Really slow unstable dynamics?

Perhaps the economy really is unstable as in old-Keynesian/monetarist models, but the dynamics are much slower than we previously thought. If so, the deflation spiral is still waiting to break out any day, even in Japan. Likewise, perhaps the "long-run stability" of velocity, even at low interest differentials, is much longer-run than previously thought. If so, velocity will recover and inflation will finally break out.

Perhaps. But perhaps not. First, this speculation is ex-post rationalization. The broad consensus of people using old-Keynesian policy models was and remained throughout that a deflation spiral was a danger. Many monetarists did clearly expect quantitative easing to lead to inflation.

This observation is praise, not criticism. The models clearly made those predictions. People

should commended for offering the advice that their models present. The models were also broadly consistent with prior data. But now the telling experiment has been run, and the models failed. We all understand the dangers of patching a model every time it fails.

Second, patches such as very "sticky" wages are not yet fully worked out, and compared more broadly with macroeconomic or microeconomic data. Job churn is a problem for decade-long stickiness. That is especially so for old-Keynesian models that dominate policy thinking but have vanished from academic journals.

Occam suggests, perhaps not: perhaps the economy is stable at an interest rate peg.

5.3   Sunspot volatility?

Perhaps the new-Keynesian prediction of higher inflation volatility under passive policy can be patched. The next few subsections take up this issue.

Sunspots being ephemeral, "there weren't any sunspot shocks" is an irrefutable ex-post explanation of quiet. And the solution with $\phi < 1$ is not the same as the solution with $\phi > 1$ and added δ_t sunspots, so "sunspots were small" could work as well.

However, this approach involves a bonfire of previous writing. The new-Keynesian literature clearly warns that passive $\phi < 1$ monetary policy causes inflation volatility. That proposition is one of the model's central empirical successes, explaining the greater volatility of the 1970s vs. the 1980s. If we throw out the prediction of higher volatility under passive policy in the 2010s, we are hard pressed not also to throw out that central success.

For example, Clarida, Galí, and Gertler (2000), who found $\phi < 1$ in the 1970s, $\phi > 1$ in the 1980s, attribute the reduction of inflation volatility to that fact, writing (p. 149)

> ...the pre-Volcker [$\phi < 1$] rule leaves open the possibility of bursts of inflation and output that result from self-fulfilling changes in expectations. ... On the other hand, self-fulfilling fluctuations cannot occur under the estimated rule for the Volcker-Greenspan [$\phi > 1$] era since, within this regime, the Federal Reserve adjusts interest rates sufficiently to stabilize any changes in expected inflation.

Again on p. 177, they write

> ... the pre-Volcker rule may have contained the seeds of macroeconomic instability that seemed to characterize the late sixties and seventies. In particular, in the

context of a calibrated sticky price model, the pre-Volcker rule leaves open the possibility of bursts of inflation and output that result from self-fulfilling changes in expectations.

Benhabib, Schmitt-Grohé, and Uribe (2001) likewise write (p. 167)

Perhaps the best-known result in this literature is that if fiscal solvency is preserved under all circumstances,[i.e. passive fiscal policy] ... a passive monetary policy, that is, a policy that underreacts to inflation by raising the nominal interest rate by less than the observed increase in inflation, destabilizes the economy by giving rise to expectations-driven fluctuations.

They write again in Benhabib, Schmitt-Grohé, and Uribe (2002), summarizing a "growing body of theoretical work,"

Taylor rules contribute to aggregate stability because they guarantee the uniqueness of the rational expectations equilibrium, whereas interest rate feedback rules with an inflation coefficient of less than unity, also referred to as passive rules, are destabilizing because they render the equilibrium indeterminate, thus allowing for expectations-driven fluctuations.

(Both sets of authors use "stability" to mean "quiet," i.e. "low volatility," not as I have used the term.)

Benhabib, Schmitt-Grohé, and Uribe (2002) survey many other similar opinions, along with policy prescriptions to avoid the zero inflation state, all motivated by the prediction of extra volatility at that state. Indeed, without extra sunspot volatility, low inflation is welfare-improving in this model. It is both Friedman-optimal, and reduces pricing distortions. The fear of sunspot volatility is the main reason these and other authors have for the effort to find policies that avoid the zero bound and return us to permanently higher inflation.

Is there some feature of policy at the long quiet zero bound that eliminated sunspot shocks, but did not eliminate those shocks in the 1970s? There is much opinion that expectations are "anchored." But anchored by what? And why was that force absent in the 1970s? The 1970s did not lack from promises by Federal Reserve and other government officials. We even had those cute little WIN (whip inflation now) buttons. If anchoring was going to work this time, why did economic researchers not know that fact, and opine not to worry about the zero bound?

5.4   Selection from future actions

In much new-Keynesian zero-bound literature such as Werning (2012) or Eggertsson and Mehrotra (2014), expectations of future active, destabilizing, policy rules take the place of responses to current inflation to select equilibria while interest rates are stuck at zero. In these models, eventually, either deterministically or stochastically, the economy leaves the zero bound. A destabilizing policy rule selects a unique locally-bounded equilibrium in that future state. Modelers then tie equilibria during the zero-rate period to the following equilibria, and thereby eliminate indeterminacies during the zero bound.

One could use this kind of selection scheme to argue that the new-Keynesian model does not, after all, predict sunspot volatility at the zero bound. The point of the literature is different, to match data, or to study policies such as forward guidance and fiscal stimulus at the zero bound, and selection by future active policies just helps authors not to bother with multiple equilibria. But it is a possibility.

Here is a concrete example, using the simple model (1)-(4) with $r^* = 0$. From time $t = 0$ to $t = T$, there is a negative natural rate shock, $v_t^r = -2\%$. At time $t = T$ the natural rate shock passes so $v_t^r = 0, t > T$, provoking a zero-bound exit. The Fed follows a constrained active Taylor rule (4), so interest rates and inflation follow

$$i_t = \max\left[\pi^* + \phi_\pi(\pi_t - \pi^*), 0\right] \tag{78}$$

$$\pi_t = (1 + \sigma\kappa)E_t\pi_{t+1} - \sigma\kappa(i_t - v_t^r). \tag{79}$$

Fiscal policy is passive.

Figure 19 shows possible paths of inflation and interest rate in this model. The thick line in the middle is the selected equilibrium. It fairs in to the inflation target $\pi^* = 2\%$ at $t = T$. The alternative equilibria are de-selected even in the stable region, when $i_t = 0$ for $t < T$, by the fact that they diverge from the inflation target π^* for $t > T$.

This equilibrium-selection scheme has many troubles. As in all active monetary policy rules, "anchoring" of inflation expectations does not occur because the Fed is expected to *stabilize* inflation around the inflation target, but because the Fed is expected to *destabilize* inflation should it diverge from the target. Now this threat is removed from current events to the far future – not "eat your spinach or there won't be dessert," but "eat your spinach or there won't be dessert next year."
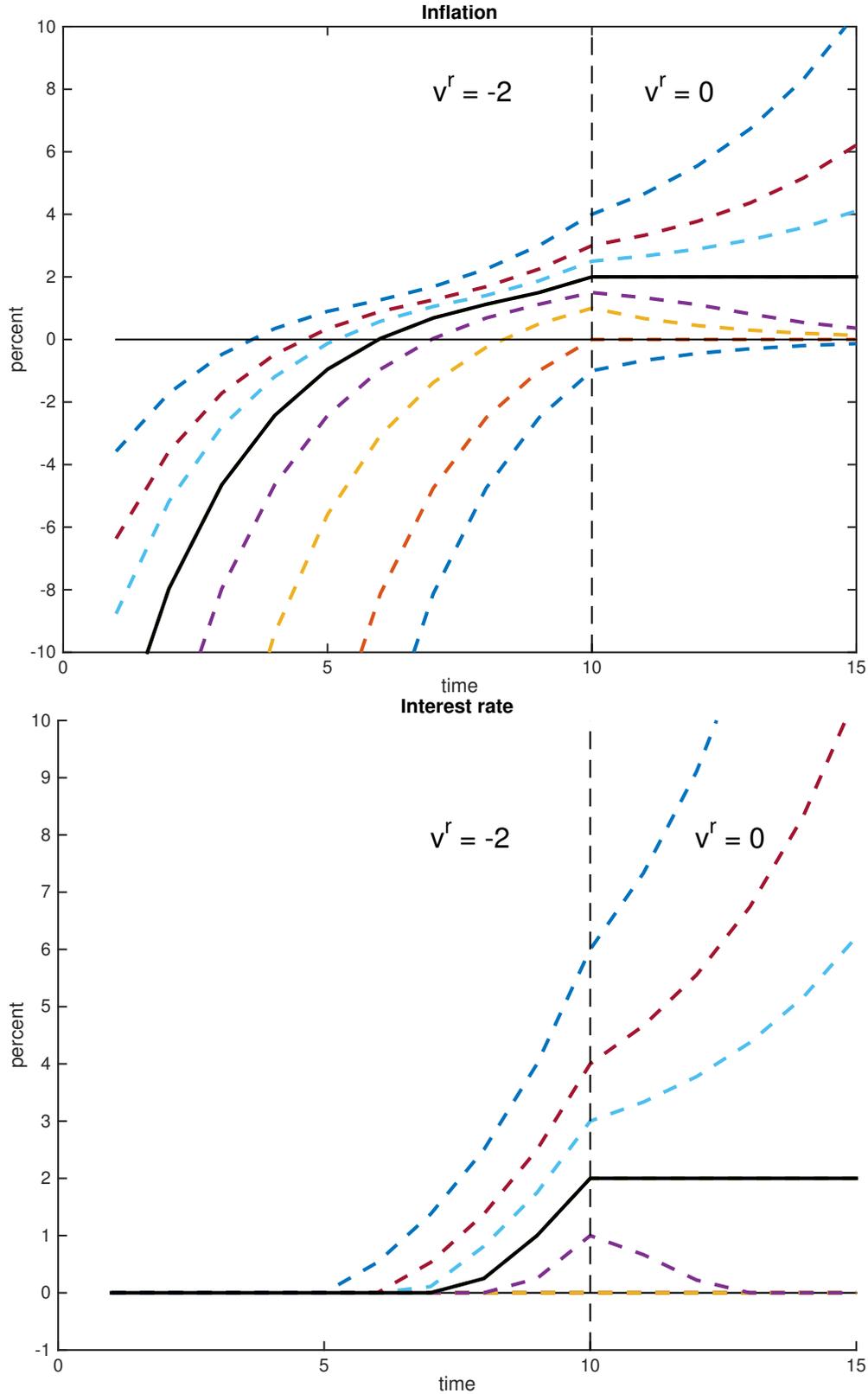
Figure 19: Selection by future and contingent policy rules. Top: Inflation. Bottom: Interest rates. The solid line is the selected equilibrium. The dashed lines are alternative equilibria. There is a natural rate shock $v^r = -2\%$ from time $t = 0$ to $t = T = 10$. The Fed follows a rule $i_t = \max\left[\pi^* + \phi_\pi(\pi_t - \pi^*), 0\right]$. The simple new-Keynesian model reduces to $\pi_t = (1 + \sigma\kappa)E_t\pi_{t+1} - \sigma\kappa(i_t - v_t^r)$. $\sigma = 1$, $\kappa = 1/2$, $\phi = 2$, $\pi^* = 2\%$.

Furthermore, equilibria in which inflation undershoots the time - T target π^* return back to zero inflation and zero interest rates. They are *locally* unstable around the target π^* and thus $\pi_t = \pi^*$ is the only locally bounded equilibrium, but they are not *globally* unstable, so $\pi_t = \pi^*$ is not the only globally bounded equilibrium. The rationale for ruling them out is tenuous.

Figure 19 also illustrates a predictive failure of this model, highlighted by Werning (2012) and Cochrane (2016b). It predicts a jump to deflation at $t = 0$ when the shock hits, which then rapidly improves. This did not happen.

Figure 19 likewise illustrates some of the policy paradoxes of this model, highlighted by Werning (2012), Wieland (2015), and Cochrane (2016b). Small changes in expectations of future inflation at time T on the right move initial time 0 inflation around a lot. The further in the future T is, the larger the effect at time 0. And as price stickiness is reduced, the dynamics happen faster, implying larger deflation and greater effect of such promises.

In a stochastic setting, the sensitivity of inflation at time 0 to small changes in expectations at time T might easily produce greater inflation volatility at the zero bound in this model, relative to normal times that active policy picks equilibria immediately. It's not obvious that, were someone to use this argument to quiet inflation at the zero bound relative to $i_t = \phi \pi_t$ policy, that it would work.

Does all concrete action of monetary policy really vanish, leaving only expectations of far-future off-equilibrium threats behind? Did Japan really avoid deflation in 2001 because people expected some sort of explosive promises around a 2% inflation target to emerge and select equilibria, maybe sometime in 2025 when Japan finally exits zero rates?

Even Fed Chair Janet Yellen Yellen (2016) is unsure that promises of future Taylor rules anchor inflation today:

> ...[H]ow does this anchoring process occur? Does a central bank have to keep actual inflation near the target rate for many years before inflation expectations completely conform? ...Or does ...a change in expectations requir[e] some combination of clear communications about policymakers' inflation goal, concrete policy actions,..., and at least some success in moving actual inflation toward its desired level ...?

Moreover, she clearly states here that anchoring results because a Taylor rule will, in the future, *stabilize* inflation around the target, in the old-Keynesian tradition, not *destabilize* inflation to produce determinacy as shown in Figure 19. If *she* doesn't believe the dynamics of Figure 19, why should people in the economy expect such a thing?

Finally, if now, why not in the 1970s? If inflation is quiet now because people know that after we exit the bound, active policy will return to select equilibria, why did people in the 1970s not know that sooner or later an era of active policy would return, as, the story goes, it did? Working backwards, that expectation should have removed self-confirming fluctuations in the 1970s, and Clarida, Galí, and Gertler (2000) should have found nothing.

This search for equilibrium selection might be more attractive if a simpler solution were not at hand. The fiscal theory picks one equilibrium of the zero-bound new-Keynesian model directly. Expectations are anchored by the present value of future surpluses, not by what people believe the Fed might do to select equilibria after a jump to a higher-interest rate regime. The fiscal theory plus new-Keynesian model does not display policy paradoxes. Each equilibrium corresponds to an innovation in the present value of surpluses, and moving to a larger inflation at time t requires a larger change in the present value of surpluses at time t. The model also has a smooth frictionless limit.

5.5   Seven years of bad luck

Perhaps we weren't really at the zero bound. Perhaps people expected interest rates to recover in the very near future. Perhaps time T in the previous graph was always a year or less, addressing the concerns of the last section. Perhaps we were in a stochastic version of the the right hand, "active," unique locally bounded equilbrium of Figure 5, one that just briefly touched zero. Perhaps the appearance of a zero bound just represents the proverbial seven years of bad luck, or 25 in Japan's case; one bad draw after another.

This view formed the basis of several comments at the conference. As other examples, Swanson and Williams (2014) measure the responsiveness of 1 and 2 year treasury bills to macro news, and conclude that they remained responsive from 2008 to 2010, indicating expectations of active policy and a soon-forthcoming exit, only becoming suggestive of a zero bound in late 2011. That still leaves 2011-2017, however. García-Schmidt and Woodford (2015) write similarly

> ...no central banks have actually experimented with date-based forward guidance that referred to dates more than about two years in the future; and while the period in which the U.S. federal funds rate target has remained at its lower bound has (as of the time of writing) lasted for more than six years, there was little reason for anyone to expect it to remain at this level for so long when the lower bound was reached at the end of 2008.

The first sentence is a bit misleading. The zero bound is usually thought of as a constraint, not a choice. Forward guidance is only a commitment that should the economy improve the Fed will nonetheless keep rates low, but if the economy does not improve one nonetheless expects zero interest rates even without forward guidance.

A similar puzzle occurs in matching a standard new-Keynesian model such as Figure 15, in which only transitory interest rate changes (with an AR(1) response) cause disinflation, to the standard intuition that 1980-1995 represented a disinflation created by a sustained high-interest policy. There, one has to argue that 1980 to 1995 represented 15 years of good luck, continual expectations of a return to high interest rates, continually confounded by events. Bordo, Erceg, Levin, and Michaels (2007), Bordo, Erceg, Levin, and Michaels (2017) document such a view, contrasting the successful high-growth post-civil war deflationary return to a gold standard with the 1980s. They document survey expectations of a return to inflation, and with it higher interest rates, throughout the 1980s. Their main point is to account for the recessions of the early 1980s vs. the post civil war growth, however.

It is possible that long stretches of data do not represent the impulse response function of interest rates to inflation, as expectations were always somewhere else. But each year that passes makes a bad luck story harder to maintain, relative to the simple alternative that maybe we are seeing just what we seem to see, a stable and at least long-run Fisherian response.

5.6   Learning and other selection devices

Perhaps multiple zero-bound new-Keynesian equilibria with passive fiscal policy, as illustrated in the left hand equilibrium of Figure 5, can be ruled out by additional equilibrium-selection rules.

Adding some concept of "learnability" to select equilibria is a popular choice. McCallum (2009a) (also McCallum (2009b)) claims that applying the e-stability concept in Evans and Honkapo-hja (2001) to this situation, the active, right hand equilibrium of Figure 5 is learnable, while the left hand "passive" equilibrium and the multiple equilibria leading to it are not.

Cochrane (2009) disagrees, and argues that learnability leads exactly to the opposite conclu-sion. The parameter ϕ and monetary policy shock are not identified from macroeconomic data in the active equilibrium. The policy rule represents an off-equilibrium threat not measurable from data in an equilibrium. There is no way for a child, observing that eating spinach is always followed by dessert, to learn if not eating spinach would be followed by no dessert. However, in

the stable passive money equilibrium, $\phi < 1$ is measurable. (See also Cochrane (2011a) p. 2-6 for an extended discussion of additional learnability concepts and their ability to prune equilibria.)

That debate concerns whether an individual, waking up in a rational expectations equilibrium, can learn the parameters of that equilibrium, enough to form the proper expectations for his or her own behavior. Christiano, Eichenbaum, and Johannsen (2016) explore a different concept of learning, whether if all people in an economy learn, the resulting equilibrium approaches rational expectations and if so which rational expectations equilibrium. Like McCallum, they conclude that the active, right-hand, equilibrium is learnable, and that the passive zero bound equilibrium, and multiple equilibria leading to it, are not. Likewise, García-Schmidt and Woodford (2015) advocate a concept of convergence to rational expectations that they call "reflective" equilibrium, and claim that active-money equilibria are the limits of such reflective equilibria, and passive-money equilibria are not.

This is not the place for a long analysis, but this view has two problems for the point at hand – whether passive-fiscal new-Keynesian models predicted more volatile inflation during the long period of near-zero rates. First, the argument is too strong. If the left-most, zero-bound, equilibria simply cannot happen, then what do we make of these long stretches at zero interest rates? The only hope, it seems, is to pair this view with the above view that we weren't really at the zero bound, because everyone expected the economy to jump back to a comfortably active region in no more than a year or so, and the appearance of a zero bound is just a sequence of bad shocks. As above, García-Schmidt and Woodford (2015) are explicit on this point. Second, if multiple equilibria under passive policy cannot occur, that conclusion is true of the 1970s as well as the 2010s. We must again throw out Clarida, Galí, and Gertler (2000), Benhabib, Schmitt-Grohé, and Uribe (2002) and related literature.

More generally, Atkeson, Chari, and Kehoe (2010) argue that by using "sophisticated policies," the central bank can prune equilibria at any time, zero bound or no zero bound, and thus eliminate indeterminacies. Similarly, Benhabib, Schmitt-Grohé, and Uribe (2002) and related zero bound literature crafts policies on top of the policy rule that can, they claim, avoid the zero bound and attendant multiple equilibria. But the question is not so much *can* a central bank or government make equilibrium-selection threats – threats understood, believed, and learned by people – but *did* our central banks and governments make such threats, did people know it, and did economists know it, and did not worry about inflation volatility at the zero bound before that volatility failed to materialize.

There are hundreds of other principles one could add to models to select among multiple

rational-expectations equilibria. And they will, in general lead to different results. One might hope is that selection will be robust to which principle one uses, a hope expressed by García-Schmidt and Woodford (2015). Yet here my debate with McCallum is instructive. When researchers with different priors approach this question, they come to diametrically opposed answers. Robustness across papers is subject to selection bias.

At this stage in the debate, one can at least say that the view, don't worry about the zero bound and multiple equilibrium volatility at the zero bound because it can't happen, was not commonplace in the new-Keynesian literature before 2008.

The other way to select equilibria is to remove the assumption of passive fiscal policy, which deletes one simple equation that selects equilibria transparently. It is at least a lot simpler.

5.7   Irrational expectations

Why be so religious about rational expectations? This question reverberated throughout the conference. Bad luck is the same as slow learning, and more generally the analysis in this paper ties stability to rational expectations.

First, it's not so easy. The simple adaptive expectations model gave the traditional sign of the response of inflation to interest rates, but it is unstable at the zero bound. We're looking for a simple model that gives the traditional sign, but is consistent with the quiet lower bound. That's harder to find.

Second, the problems highlighted in this paper stem from the basic sign and stability properties of models. If we had a model with the basic sign and stability properties, then sprinkling in some less than rational expectations to get dynamics right would be more attractive. Putting irrational expectations or other irrationality deeply at the heart of the sign and stability of monetary policy is more worrisome.

Gabaix (2016) is an excellent and concrete example. Gabaix uses a model of rational inattention to argue that people and firms pay less attention to expectations of future income and future prices than they should, modifying the standard three-equation model to

$$x_t = M E_t x_{t+1} - \sigma(i_t - E_t \pi_{t+1}) \tag{80}$$

$$\pi_t = M^f \beta E_t \pi_{t+1} + \kappa x_t \tag{81}$$

$$i_t = \phi \pi_t + v_t^i. \tag{82}$$

where M and M^f are less than one. For sufficiently low M and M^f, Gabaix produces traditional explosive dynamics under a peg, and therefore he produces a negative sign of interest rates on inflation. In this way, Gabaix' model can be seen as a behaviorally micro-founded version of the old - Keynesian model studied above.

Kocherlakota (2017) and McKay, Nakamura, and Steinsson (2016) likewise advocate discounting the future income in the IS curve to (80), though with a less behavioral interpretation. Both authors cite the puzzling power of forward guidance. However, Cochrane (2016b) shows that the forward-guidance puzzle is driven by equilibrium selection rules in new-Keynesian models.

But to get the traditional sign, Gabaix and Kocherlakota must change the stability properties of the model. As one starts to lower M and M^f, nothing happens at all until the eigenvalues cross one. Cochrane (2016a) finds that one needs quite a lot of irrationality, M less than a half, together with substantial price stickiness $\sigma\kappa$ less than about a half, to cross that boundary. Thus, Gabaix's result is bounded away from rationality, and bounded away from the frictionless price limit. A little bit of irrationality or price stickiness will not do.

Gabaix' model also remains unstable, and so does not accommodate the long quiet zero bound without a rather complex patch (section 5.3, and appendix section 9.2). It would be esthetically more pleasing if long-run neutrality were a result of the simple form of a model, and dynamics the result of patches rather than the other way around.

So the model that uses irrational expectations to deliver a temporary negative sign, on top of a frictionless long-run neutral benchmark, has yet to be delivered.

It is certainly not necessary or wise to insist on rational expectations at every data point, and in particular to understand short-run responses to particular historical events far outside the norm of experience – though economic historians remind us that events "far outside" experience, like financial crises, are in fact common.

But we are looking here for the opposite side of that coin – the fundamental, underlying, long-lasting, simple and basic economic nature of monetary policy; the central mechanism on which all of policy analysis depends. Are we really satisfied if that foundation relies crucially on non-economic behavior? Viewed either as introducing irrational expectations or as fundamentally changing our model of intertemporal choice, is it really wise to do such major surgery to economics to accommodate one data point?

The question is not a set of *sufficient* conditions. The question is the minimum *necessary* conditions for the basic sign and stability of monetary policy. If irrationality is *necessary* for

a negative sign, that verifies my tentative conclusion – that there is no such simple *economic* model. And placing irrational expectations so deeply in the foundations of monetary economics, if that is the answer, it paints a revolutionary picture.

Suppose, for example, that a negative effect of interest rates on inflation occurred a few times because people were irrational in their expectations. Lucas' admonition is worth remembering however that if policy makers try to exploit this sort of thing, people sooner or later catch on, and it stops working. A monetary theory whose basic sign and stability depends on irrational behavior is ephemeral.

If it true, then the Federal Reserve should write in its next report to Congress that the sign of the Fed's attempts to control inflation does not rely on simple money supply and money demand rational economics, but instead relies deeply and essentially on a super-rational Federal Reserve offsetting an instability that occurs from people's stupidity, and an interest rate policy that manipulates people's irrationality for their own good. Likewise, if this is the case, economics textbooks need to be rewritten in the same way. An honest advocate for irrational expectations in this paper's context should advocate both.

5.8   More complex models

Since the quiet zero bound point depended on the stability and determinacy properties of models, whether eigenvalues are above or below one and how many expectations are determined, more complex models do not easily change that result.

It is more likely that one can get a temporary negative sign of interest rates on inflation, without adopting the fiscal-theory plus long-term debt mechanism, out of a model consistent with long run stability by adding ingredients to the forward-looking new-Keynesian structure. The general path is fairly clear: a model with two backward-looking roots can produce a response function that is temporarily negative and then positive, without relying on a downward jump.

The models in this paper are also quite simple by the standards of calibrated or estimated new-Keynesian models, such as Smets and Wouters (2003), Christiano, Eichenbaum, and Evans (2005) Woodford (2003) Sections 5.1-5.2, Rotemberg and Woodford (1997), Negro, Giannoni, and Schorfheide (2015), and so forth. (Unfortunately we do not know how these models behave for the experiments of this paper. How do they behave in response to a long-lasting interest rate rise? How sensitive are their predictions to parametric restrictions of the monetary policy shock process?)

So, a natural next step – the sort of thing macroeconomists do all the time when trying to reverse-engineer impulse-response functions – would be to add ingredients such as extensive borrowing or collateral constraints, hand-to-mouth consumers, a lending channel, or other financial frictions, habits, durable goods, housing, multiple goods and other nonseparabilities, novel preferences, labor / leisure choices, production, capital, variable capital utilization, adjustment costs, alternative models of price stickiness, informational frictions, market frictions, payments frictions, more complex monetary frictions, timing lags, individual or firm heterogeneity, and so forth. Going further, perhaps we can add fundamentally different views of expectations and equilibrium, as in Gabaix (2016) and García-Schmidt and Woodford (2015) discussed above, or Angeletos and Lian (2016) (see also the extensive literature review in the latter). I survey a few such models in section 7 below.

But following these paths abandons the qualifier "simple," and with irrational expectations "economic" to our quest for a simple economic model that delivers the basic sign and stability properties of monetary policy. In our quest, this path means that more complex ingredients are *necessary*, not just *sufficient* to deliver the central result. Doing so admits that there is no simple, rational economic model one can put on a blackboard, teach to undergraduates, summarize in a few paragraphs, or refer to in policy discussions to explain at least the signs and rough outlines of the operation of monetary policy–nothing like, say, the stirring and simple description of monetary policy in Friedman (1968).

As with irrationality, if this is the answer, an honest Fed should explain in its next monetary policy report that there is no simple explanation it can give why raising interest rates will combat inflation, that this effect is necessarily a big-black-box outcome, and successful policy is dependent on the Fed's technocratic understanding of the above list. It should explain that without these ingredients, if the economy worked by explainable economic mechanisms, the interest rate lever would move inflation in the opposite direction. And honest textbooks should say the same thing. Necessary vs. sufficient, dynamic wrinkles vs. a basic underlying model, frosting vs. cake, are crucial distinctions.

Such an intellectual outcome would also be unusual in macroeconomics. The standard new Keynesian approach views the complex models or even behavioral modifications as refinements, building on (58)-(59). The refinements help to match the details of model dynamics with those observed in the data, but the simple model is thought to capture the basic message, signs, stability and intuition. Except we just found out it does not. The standard real business cycle approach views complex models as refinements, building on the stochastic growth model, but

that simple model can still capture the basic story. The large multi-equation Keynesian models developed in the 1970s built on simple ISLM models to better match details of the data, but modelers felt that the simple ISLM model captured the basic signs and mechanisms – indeed, many analysts feel the basic ISLM model is better than its explicit computerized elaborations.

And this is healthy. Economic models are quantitative parables, and one rightly distrusts macroeconomic predictions that crucially rely on the specific form of poorly-understood frictions.

So, I stop here, because if we go down this path, we first agree that no simple economic model delivers the desired sign and stability. We agree that the conclusion of this paper is verified.

The world may well have such a negative sign, due to either irrationality or more complex ingredients. Nothing in this analysis denies that possibility, and let me dispel any impression of an unscientific hostility to adding frictions to macro models.

But if complex frictions are necessary for the basic sign and stability, rather than being used to layer real-world dynamics on top of a simple economic model that gets the simple facts right, that circumstance radically changes the nature of monetary policy. And one must admit that the scientific basis on which we analyze policy, and offer advice to public officials and public at large, becomes more tenuous.

5.9   VAR evidence

If theory and experience point to a positive reaction of inflation to interest rates, perhaps we should revisit the empirical evidence behind the standard contrary view. The main formal evidence we have for the effects of monetary policy comes from vector autoregressions (VARs). There are several problems with this evidence.

First, the VAR literature almost always pairs the announcement of a new policy with the change in the policy instrument, i.e. an unexpected shock to interest rates. That habit makes most sense in the context of models following Lucas (1972) in which only unanticipated monetary policy has real effects, and in the context of regressions of output on money, rather than interest rates, in which VARs developed (Sims (1980)).

But in the world, most monetary policy changes are anticipated. VARs may still want to find rare unexpected rate movements, as part of an identification strategy to find changes in policy that are not driven by changes of the Fed's expectations of future output and inflation, but that is a small part of the historical variation. Furthermore, every single interest rate change is de-

scribed by the Federal Reserve as a reaction *to* some other event in the economy. They never say "and we added a quarter percent for the heck of it," or "so that economists could see what would happen."

Moreover, in the models presented here, anticipated monetary policy has strong effects. In particular, the models with money presented here, as in Figure 17, only had a chance of delivering the standard inflation decline if the interest rate rise was anticipated. An empirical technique that isolates unexpected interest rate has great difficulty to find that theoretical prediction.

Second, the analysis of multiple equilibria in Figure 14 found that inflation declines occur when an interest rate rise is paired with a fiscal policy tightening. As discussed there, it is plausible that whatever motivates the Fed to raise or lower interest rates also motivates fiscal authorities to change course. It is plausible therefore that rate shocks in our data set are like equilibrium E of Figure 14. But we want to know what happens if monetary policy moves without coincident fiscal policy changes. VARs have to date made no attempt to orthogonalize monetary policy shocks with respect to fiscal policy, especially expected future fiscal policy which is what matters here.

Third, VARs typically find that the interest rate responses to an interest rate shock are transitory, as are those of Figure 13. As a result, they provide no evidence on the long-run response of inflation to permanent interest rate increases.

Fourth, and most of all, the evidence for a negative sign is not strong, and one can read much of the evidence as supporting a positive sign. From the beginning, VARs produced *increases* in inflation following increases in interest rates, a phenomenon dubbed the "price puzzle" by Eichenbaum (1992). A great deal of effort has been devoted to modifying the specification of VARs so that they can produce the desired result, that a rise in interest rates lowers inflation.

Sims (1992), studying VARs in five countries notes that "the responses of prices to interest rate shocks show some consistency - they are all initially positive." He also speculates that the central banks may have information about future inflation, so the response represents in fact reverse causality. Christiano, Eichenbaum, and Evans (1999) take that suggestion. They put shocks in the order output (Y), GDP deflator (P), commodity prices (PCOM), federal funds rate (FF), total reserves (TR), nonborrowed reserves (NBR), and M1 (M) (p. 83). With this specification (their Figure 2, top left), positive interest rate shocks reduce output. But even with the carefully chosen ordering, interest rate changes have no effect on inflation for a year and a half. The price level then gently declines, but remains within the confidence interval of zero throughout. Their Figure 5, p.100, shows nicely how sensitive even this much evidence is to the shock identification

assumptions. If the monetary policy shock is ordered first, prices go up uniformly. The inflation response in Christiano, Eichenbaum, and Evans (2005) also displays a short run price puzzle, and is never more than two standard errors from zero.

Even this much success remains controversial. Hanson (2004) points out that commodity prices which solve the price puzzle don't forecast inflation and vice versa. He also finds that the ability of commodity prices to solve the price puzzle does not work after 1979. Sims (1992) was already troubled that commodities are usually globally traded, so while interest rate increases seem to lower commodity prices, it's hard to see how that could be the effect of domestic monetary policy.

Ramey (2015) surveys and reproduces much of the exhaustive modern literature. She finds that "The pesky price puzzle keeps popping up." Of nine different identification methods, only two present a statistically significant decline in inflation, and those only after four or more years of no effect have passed. Four methods have essentially no effect on inflation at all, and two show strong, statistically significant positive effects, which start without delay. Strong or reliable empirical evidence for a short-term (within 4 years) negative inflation effect is absent in her survey.

The Christiano, Eichenbaum, and Evans (1999) procedure may seem fishy already, in that so much of the identification choice was clearly made in order to produce the desired answer, that higher interest rates lead to lower inflation. Nobody spent the same effort seeing if the output decline similarly represented reverse causality, Fed reaction to news of future output, because the output decline fits priors so well. Uhlig (2006) defends this imposition of priors on identification. *If* one has strong theoretical priors that positive interest rate shocks cause inflation to decline, then it makes sense to impose that view as part of shock identification, in order to better *measure* that and other responses. (Uhlig's eloquent introduction is worth reading and contains an extensive literature review.)

But imposing the sign only makes sense when one has that strong theoretical prior; when, as when these papers were written, existing theory uniformly specifies a negative inflation response and nobody is considering the opposite. In the context of this paper, when theory specifies a positive response, when only novel and as yet unwritten theories produce the negative sign, and we are looking for empirical evidence on the sign, following identification procedures that implicitly or explicitly throw out positive signs does not make sense. And even imposing the sign prior, Uhlig like many others finds that "The GDP price deflator falls only slowly following a contractionary monetary policy shock."

With less strong priors, positive signs are starting to show up. Belaygorod and Dueker (2009) and Castelnuovo and Surico (2010) find that VAR estimates produce positive inflation responses in the periods of estimated indeterminacy. Belaygorod and Dueker (2009) connect estimates to the robust facts one can see in simple plots: Through the 1970s and early 1980s, federal funds rates clearly lead inflation movements. Dueker (2006) summarizes.

And even all of this evidence comes from the period in which the Fed kept a small amount of non-interest-bearing reserves, the money multiplier plausibly bound, and the Fed implemented interest-rate changes by changing the quantity of reserves. We want to know how inflation will behave in the current regime, in which the Fed will change interest rates by changing the interest it pays on super-abundant reserves, with no open market operations at all.

6 Fiscal theory objections

I collect here quick answers to some of the most common theoretical and empirical objections to fiscal theory arguments. Many of the these objections came up at the conference. Though each theoretical objection is addressed elsewhere (see in particular Cochrane (2005), Cochrane (1998)) since they reappear, it is worth uniting the relevant responses briefly in one place.

The simplest summary: The government debt valuation equation underlying the fiscal theory is the same as the present value relation underpinning the theory of finance. Any theoretical or empirical objection one has to the fiscal theory is exactly the same objection to price equals present value of dividends. If one is a "budget constraint," so is the other. If one is a knife-edge case that "collapses under the tiniest reasonable perturbation," so is the other. If one is only sustained by strange off-equilibrium promises, so is the other. If one requires "exogenous" surpluses, the other requires "exogenous" dividends. (Neither is the case.) Yes, we do not have an easy independent measure of expected surpluses and discount rates, so we do not have an easy fiscal-theory "test" by calculating an independent prediction for the price level. Yes, we do not have an easy independent measure of expected dividends and discount rates, so we do not have an easy present-value model "test" by calculating a prediction for stock prices. Each equation *holds*, almost trivially – all they need is the absence of arbitrage. One can always discount by expected returns. Thus, neither is testable by itself, and neither allows an easy rejection by looking at time series. No, the fiscal theory cannot be easily invalidated by noting that times of high deficits and debt are not always associated with inflation. No, the present value model cannot be easily invalidated by noting that times of high or low dividends and book values do

not always correspond to high or low prices. Yes, stock market values are often puzzling given available dividend forecasts, and discount rates are a bit nebulous. Yes, the currently high value of government debt is puzzling given available surplus forecasts, and discount rates are a bit nebulous.

These facts have not kept the present value model from being useful, and at the cornerstone of finance – and its controversies – for decades. Expect of the fiscal theory no less, but also no more. The fiscal theory is, like the present value model, not easily dismissed nonsense. But the fiscal theory, like the present value model, will likely pose challenges to understand the data, and warring and hard-to-test stories. It will not be easy either way.

6.1   Exogenous surpluses?

The fiscal theory of the price level does not require that surpluses are "exogenous," just as price = present value of dividends does not require that dividends are "exogenous."

Though it is often useful to think through models and events by thinking of the price level, stock price, or consumption, as determined "given" surpluses, dividends, income, and discount rates, the latter quantities are all economically endogenous and co-determined. Equilibrium conditions do not have a causal structure, as useful as it is to think that way at times.

In particular, the fiscal theory still determines the price level if one models surpluses resulting from proportional taxes. If $P_t s_t = \tau P_t y_t$, then the right hand side of the government debt valuation equation becomes τy_{t+j} in place of s_{t+j}. The only P is still on the left hand side and thus still determined. Including taxes and spending that respond to output and inflation all can leave the basic theory intact. These modifications may well change the quantitative predictions a lot, of course, but those are for another day. Only in the knife-edge case that surplus responses to the price level P_t on the right just cancel those on the left, if the supply and demand curves overlap, then the theory fails.

Like the present value equation, the government debt valuation equation is not a "budget constraint." The government is not assumed to be a large actor that can "threaten to violate its budget constraint at off equilibrium prices." Nobody can violate budget constraints. The government debt valuation equation is an equilibrium condition, deriving ultimately from consumer's first order and transversality conditions. If consumers valued government debt directly, perhaps wishing to wallpaper living rooms with it, then the government would not have to repay debt, and the valuation equation would not hold. It is not a "budget constraint" of the government

any more than the present value equation is a "budget constraint" on businesses, that forces them to raise dividends in response to prices. For this reason I call the central equation the "government debt valuation equation" rather than the common but misleading term "government intertemporal budget constraint."

6.2 Tests?

The government debt valuation equation holds in equilibrium in all regimes – "active" or "passive." Therefore active and passive regimes are observationally equivalent. There is no test for Granger causality, no regression of surpluses on debt or interest rates on inflation, no relationship among time series drawn from the equilibrium of an economy, that can distinguish fiscally active and passive, or monetary active and passive regimes. That fact drives the style of analysis in this paper – only by looking across regimes, or from experiments such as the zero bound episode, can we tell theories apart.

Like price = present value of dividends, the government debt valuation holds under very weak conditions – absence of arbitrage, law of one price. A test necessarily involves auxiliary hypotheses, about discount rates, cash flows, information, etc.

Therefore, one does not productively address data or distinguish models by "testing" the government debt valuation equation, any more than one can "test" price = present value of dividends, or "test" whether MV=PY describes how money causes inflation or how nominal GDP drives money demand. There is a reason Friedman and Schwartz wrote a whole book, not one definitive Granger-causality test. That's not bad news, it just places fiscal theory exactly in the frustrating but productive realm of the rest of macroeconomics and finance.

6.3 Responses to debt, on and off equilibria

In particular, the fiscal theory does not require that fiscal surpluses are set "independently" of outstanding debt, an assumption cited in Christiano' critique at the conference. No. Fiscal theoretic governments borrow money, pay it off, and pay back more when debts are larger, just as one would expect.

In a passive-fiscal regime, the government raises surpluses to pay back debts arising from *any* cause. If a bubble or sunspot causes the price level to decline 50%, the government doubles taxes to repay that windfall to bondholders. An active-fiscal government ignores this kind of

debt revaluation. But it does respond to other increases in debt, in particular raising surpluses to pay off debts accumulated from fighting wars and recessions, or rising real interest rates.

This distinction between on-equilibrium and off-equilibrium responses underlies the new-Keynesian active-money view, so anyone happy with new-Keynesian equilibrium selection should be happy with the fiscal theory as well. Following King (2000), write the policy rule in a new-Keynesian model as

$$i_t = i_t^* + \phi_\pi(\pi_t - \pi_t^*) \tag{83}$$

where i_t^*, π_t^* are equilibrium values the Fed desires to select. As before, this is just a convenient rewriting of the usual rule $i_t = \phi\pi_t + v_t^i$. As we have seen, a threat $\phi > 1$ selects $\pi_t = \pi_t^*$, $i_t = i_t^*$ as the unique locally bounded equilibrium. How a central bank's observed interest rate i_t^* responds to observed equilibrium inflation π_t^* is a separate issue from how the central bank responds by off-equilibrium $(i_t - i_t^*)$ to off-equilibrium inflation $(\pi_t - \pi_t^*)$. In general – without further restrictions on the policy rule – correlations among equilibrium values are separate from how the central bank responds to a sunspot inflation.

The same argument applies to the fiscal theory. The government can commit to repaying any "on-equilibrium" debt, but to defaulting or inflating away "off-equilibrium" debt. Write

$$\frac{B_{t-1}}{P_t} = E_t \sum_{j=0}^{\infty} \beta^j s_{t+j}. \tag{84}$$

If surpluses respond to *any* change in debt,

$$s_t = \hat{s}_t + \alpha(B_{t-1}/P_t) \tag{85}$$

then, yes, fiscal policy is "passive," surpluses adjust on the right hand side of (84) for any price level, and the government debt valuation equation drops out of equilibrium determination. But if we write the fiscal rule in form analogous to (83)

$$s_t = \hat{s}_t + \alpha(B_{t-1}/P_t^*) \tag{86}$$

where $*$ denotes the government's desired equilibrium, then we see exactly the same response in equilibrium – the government pays off its debts – but no response to off-equilibrium price level bubbles or sunspots. As Taylor-rule off-equilibrium interest rate rises select a unique equilibrium, so fiscal-rule off-equilibrium inaction selects a unique equilibrium. And there is no way to tell apart (85) from (86) using data from a given equilibrium, since $P_t = P_t^*$ in equilibrium.

Specification (86) is reasonable. A government following (86) pays off debts it incurs, for example, to finance wars or fight recessions. If the government wants to borrow real resources and keep a stable price level P^*, then such commitment is vital. But a government could and arguably should refuse to accommodate bubbles and sunspots in the price level.

This is also a reasonable model of the gold standard. By following a gold standard, the government announces that it will raise surpluses to pay back debts at the gold value of the currency, no more and no less. And we have seen cases in which governments do not always rally surpluses to unexpected inflations and deflations. For example, the US in the 1930s went off the gold standard and defaulted on the gold clause in its bonds after a sharp deflation.

6.4 Understanding data

The second and more interesting category of objections (since it has not been extensively answered in a long literature) is, how can the fiscal theory match data and historical episodes? In this analysis, the obvious unanswered question is, *why* is the present value of surpluses so high, and why is it so quiet (not volatile)? I have been careful to write that the zero bound or an interest rate peg *can* be stable and quiet, not that either *is* stabile and quiet. Quiet depends on quiet expectations of fiscal policy.

The short answer is, I don't know. I don't attempt a separate measurement of expected surpluses, and of their discount rate, to produce an independent measure of what the price level or volatility of inflation should be by the fiscal theory.

We know this will not be a trivial exercise. Despite the half century in which the same present value model has dominated finance, attempts to independently measure expected dividends and discount rates, to compute the present value model's prediction for stock prices, and to compare those actual prices have not been productive. And yet the present value model remains the indispensable organizing principle for understanding asset values.

The analysis in this paper stops with the claim that the fiscal theory model *can* account for the quiet zero bound, in a way that the other models cannot. I do not claim it is proved, passes additional tests, such as an independent measurement of surpluses and discount rates would provide, or addresses every other event in history. Those would be nice, but not necessary for the claim.

The fiscal theory is at least consistent with the fact that the zero bound does not appear to be a state variable for inflation dynamics. The low volatility of revisions to the present value of

surpluses, and low volatility of inflation before and after the zero bound are the same. So there is no particular fiscal theory puzzle about the zero bound. We only need to account for the fact that the volatility of surplus and discount rate expectations did not change much. In fact, there is a glimmer that inflation volatility seems lower at the zero bound than before, so perhaps quiet nominal rates are leading to quieter real rates and therefore quieter present values of debt and inflation.

6.5   What about Japan? And Europe? And the US?

Since the valuation equation holds, our question is how. Does it require such laughable forecasts and discount rates that it is a useless abstraction? The answer is no.

Japan's debt approaches 200% of GDP, and 20 years of deficit spending have not produced desired inflation. Doesn't the fiscal theory predict hyperinflation for Japan? Europe is north of 100% debt to GDP ratio, and the US fast approaching. Present value of surpluses? What surpluses? The CBO's deficit forecasts have even primary deficits exploding.

The fiscal theory does not predict that high debts or high deficits must correlate with high inflation. The discounted present value of surpluses relative to debt is a far different object than todays raw debt or deficit.

Despite awful projections, it is not unreasonable for bond markets to believe – for now – that the western world's debt problems will be solved successfully. The CBO's deficit forecasts are "if something doesn't change" forecasts, and include straightforward policies that can turn surpluses around – mild pro-growth economic policies, mild entitlement reforms. The same angst over debt-fueled inflation emerged in the late 1970s and early 1980s – Sargent and Wallace (1981) "unpleasant monetarist arithmetic" being only the most famous example. The US largely repaid greater debts after WWII, and the UK in the 1800s. A debt crisis leading to inflation will be a self-inflicted wound, not an economic necessity.

More importantly, in my view, the fiscal theory needs to digest the main lesson of asset pricing – discount rates vary a lot, and are vitally important to understand valuations.

Real interest rates are zero or negative throughout the western world, even at very long horizons. From this perspective, the fiscal theory puzzle is not the lack of *inflation*. The fiscal theory puzzle is the lack of *deflation*. Consider a constant discount rate r and an economy growing at

rate g, and hence primary surpluses $s_t = \tau y_t$ growing at g. Then,

$$\frac{B_t}{P_t} = E_t \int_{j=0}^{\infty} e^{-rj} s_{t+j} dj = E_t \int_{j=0}^{\infty} e^{(g-r)j} dj s_t = \frac{s_t}{r-g} \tag{87}$$

or, the ratio of surplus to real value of the debt is $r - g$. The discount rate r matters as much as g. If r declines, the value of the debt rises, so P declines.

So, to understand low current inflation, the salient fact is the extraordinarily low expected real return on government debt. Debt is valuable despite poor fiscal prospects, not because of great ones. This little calculation also highlights the warning above – $1/(r-g)$ is very sensitive to r and g. A little higher r not accompanied by g could change our quiet inflation quickly.

6.6   What about 2008? And cyclical inflation?

Discount rate variation is likely also to be crucial to understanding episodes such as 2008, and the cyclical correlation of inflation.

Fall 2008. Output falls sharply. Deficits expand into the trillions. The growth slowdown and continuing entitlement problems make future deficits seem even worse. And inflation... falls. How is that consistent with the government debt valuation equation? In every recession, low inflation correlates with large deficits, and vice versa. Isn't the sign wrong?

Again, it is a mistake to confuse current with expected future deficits. A government fighting a war borrows today and simultaneously commits to higher future taxes to pay off the debt – precisely because it does not want to create inflation, and it does want to raise revenue from its bond sales. Our government's fiscal stimulus programs always include at least lip service to future deficit reduction. The surplus should *not* be modeled as an AR(1), but as a series in which a dip today portends a rise in the future. (Cochrane (2001) pursues this point in detail.) Contrariwise, "helicopter drop" plans to deliberately create inflation come with commitments not to repay the debt.

But future surpluses also didn't get a lot *better* in 2008. The answer to a sharp disinflation must then be the discount rate. Both nominal and real interest rates dropped sharply in 2008. A "flight to quality" further lowered the expected return of government bonds relative to corporate bonds. People were trying to hold more government bonds – and to hold less private assets and to spend less to get government bonds.

This mechanism helps as well to understand the general cyclical correlation of inflation. In

any recession, output falls, deficits rise, and yet inflation falls. Why? In part, people understand that current deficits correspond to larger future surpluses. But the most important part of the effect is likely that people are willing to hold claims to the same surplus at lower rates of return in a recession than they are in a boom.

6.7   What about 1951?

What about all the failed interest rate pegs? The plausible answer, and warning: Fiscal policy.

For example, Woodford (2001) analyzes the US peg of the 1940s and early 1950s. He credits fiscal-theoretic mechanisms for the surprising stability of the interest rate peg – it did last a decade, and more if you count zero rates in the Great Depression. In his view, it fell apart when the Korean war undermined fiscal policy. Other countries whose pegs fell apart after WWII motivating Friedman (1968) were facing difficult fiscal problems. Most historic pegs were enacted along with price controls, exchange controls, and monetary controls as devices to reduce interest payments on the debt (that was explicit in the US case) and to help otherwise difficult fiscal policy. In most historic pegs, central banks were trying to hold down rates that otherwise wanted to rise, by lending out money to banks at low rates, and with financial repression to force people to hold government debt they did not want to hold. Our central banks are taking *in* money from banks who can't find better opportunities elsewhere, thus apparently holding interest rates above what they otherwise want to be, in the face of overwhelming demand for government debt. Countries whose pegs fell apart had problems financing current deficits. We have doomsaying forecasts of deficits decades from now. The lessons of historical pegs under vastly different fiscal circumstances do not necessarily apply to all pegs or zero bound episodes.

Interest rate pegs, like exchange rate pegs, live on top of solvent fiscal policy or not at all. For this reason too I have been careful to state the conclusion as "an interest rate peg *can* be stable."

None of this discussion is proof, nor is it intended as such. The point is that *there exists* a vaguely plausible reconciliation of the data with the government debt valuation equation; that merely citing the episodes does not *disprove* the fiscal theory.

7   Literature

The doctrines of inflation instability or indeterminacy are longstanding. Milton Friedman (1968) gives the classic statement that an interest rate peg is unstable, and that higher interest rates lead

to temporarily lower inflation. Friedman writes (p.5) that Monetary policy "cannot peg interest rates for more than very limited periods..." Friedman's prediction also comes clearly from adaptive expectations: (p. 5-6):

> "Let the higher rate of monetary growth produce rising prices, and let the public come to expect that prices will continue to rise. Borrowers will then be willing to pay and lenders will then demand higher interest rates-as Irving Fisher pointed out decades ago. This price expectation effect is slow to develop and also slow to disappear."

Friedman stressed the effect of money growth on output rather than interest rates, via ISLM's IS or modern intertemporal substitution. But the bottom line dynamics from interest rate to inflation does not depend on this view of the mechanism. The very Keynesian model of Figure 9 captures completely Friedman's description of inflation instability and interest rate policy under adaptive expectations.

Friedman was heavily influenced by recent history of his time:

> These views [ineffectiveness of monetary policies] produced a widespread adoption of cheap money policies after the war. And they received a rude shock when these policies failed in country after country, when central bank after central bank was forced to give up the pretense that it could indefinitely keep "the" rate of interest at a low level. In this country, the public denouement came with the Federal Reserve-Treasury Accord in 1951, although the policy of pegging government bond prices was not formally abandoned until 1953. Inflation, stimulated by cheap money policies, not the widely heralded postwar depression, turned out to be the order of the day.

We do not know how Friedman might have adapted his views with our recent 8 years of experience, or Japan's 20 in mind, rather than the inflations of the immediate WWII aftermath.

Taylor (1999) provides a clear statement that old-Keynesian (backward-looking, adaptive expectations) models are unstable, and that Taylor rules induce stability.

The doctrine that inflation is *indeterminate* under an interest rate peg or passive policy, under rational expectations, started with Sargent and Wallace (1975). The basic point: fixing i_t with $i_t = r_t + E_t \pi_{t+1}$ leaves $\pi_{t+1} - E_t \pi_{t+1}$ indeterminate. Their point is quite different from Friedman's instability. Indeterminacy, instability, and volatility are distinct concepts, frequently confused.

The fiscal theory of the price level goes back to Adam Smith:

> "A prince who should enact that a certain proportion of his taxes should be paid in a paper money of a certain kind might thereby give a certain value to this paper money" – Wealth of Nations, Book 2, Ch. II.

Monetary economists have long recognized the importance of monetary-fiscal interactions. The modern resurgence and deep elaboration owes much to Sargent and Wallace (1981) and Sargent (2013). Leeper (1991) shows how the fiscal theory can uniquely determine the price level under passive monetary policy. Sims (1994) clearly states that the fiscal theory and rational expectations overturn Friedman's doctrine of instability, as well as Sargent and Wallace's indeterminacy:

> "A monetary policy that fixes the nominal interest rate, even if it holds the interest rate constant regardless of the observed rate of inflation or money growth rate, may deliver a uniquely determined price level."

The observation that interest rate pegs are stable in forward-looking new Keynesian models also goes back a long way. Woodford (1995) discusses the issue. Woodford (2001) is a clear statement, analyzing interest rate pegs such as the WWII US price support regime, showing they are stable so long as fiscal policy cooperates.

Benhabib, Schmitt-Grohé, and Uribe (2002) is a classic treatment of the zero-rate liquidity trap. They note that the zero bound means there must be an equilibrium with a locally passive $\phi_\pi < 1$ Taylor rule, with multiple stable equilibria. However, they view this state as a pathology, not a realization of the optimal quantity of money and optimal (low) level of markups, and devote the main point of their paper to escaping the "trap" via fiscal policy.

Schmitt-Grohé and Uribe (2014) realize that inflation stability means the Fed could raise the peg and therefore raise inflation. To them this is another possibility for escaping a liquidity trap. They write

> "The paper... shows that raising the nominal interest rate to its intended target for an extended period of time, rather than exacerbating the recession as conventional wisdom would have it, can boost inflationary expectations and thereby foster employment."

The standard model here with a forward-looking Phillips curve disagrees that raising inflation raises employment, but that is not a robust feature.

Belaygorod and Dueker (2009) and Castelnuovo and Surico (2010) estimate new-Keynesian DSGE models allowing for switches between determinacy and indeterminacy. They find that the

model displays the price puzzle – interest rate shocks lead to rising inflation, starting immediately – in the indeterminacy region $\phi_\pi < 1$, as I do.

The possibility long periods of low rates cause deflation, so raising interest rates will raise inflation, has had a larger recent airing in speeches and the blogosphere. (See Williamson (2013), Cochrane (2013, 2014c), Smith (2014).) Minneapolis Federal Reserve Chairman Narayana Kocherlakota (2010) suggested it in a famous speech:

> "Long-run monetary neutrality is an uncontroversial, simple, but nonetheless profound proposition. In particular, it implies that if the FOMC maintains the fed funds rate at its current level of 0-25 basis points for too long, both anticipated and actual inflation have to become negative. Why? It's simple arithmetic. Let's say that the real rate of return on safe investments is 1 percent and we need to add an amount of anticipated inflation that will result in a fed funds rate of 0.25 percent. The only way to get that is to add a negative number – in this case, 0.75 percent.
>
> To sum up, over the long run, a low fed funds rate must lead to consistent–but low–levels of deflation."

To be clear, Friedman (1968) disagrees. Friedman views the Fisher equation as an unstable steady state. Kocherlakota, seeing recent experience, views it as a stable one. Friedman's "long-run neutrality" is that a permanent rise in M gives rise to a permanent rise in P, but a permanent rise in i will lead to explosive P and π. A permanent rise in $\dot{M}$ gives rise to a permanent rise in $\dot{P}$ and therefore of both i and π, but not the other way around. Kocherlakota's "long-run neutrality" is that a permanent rise in i will lead to a permanent rise in π. The difference between a stable and an unstable steady state is key.

Cochrane (2014b) works out a model with fiscal price determination, an interest rate target, and simple k-period price stickiness. Higher interest rates raise inflation in the short and long run, just as in this paper, but the k-period stickiness leads to unrealistic dynamics.

Following Woodford (2003), many authors also tried putting money back into sticky-price models. Benchimol and Fourçans (2012) and Benchimol and Fourçans (2015) study a CES money in the utility function specification as here, in a detailed model applied to the Eurozone. They find that adding money makes small but important differences to the estimated model dynamics. Ireland (2004) also adds money in the utility function. In his model, money also spills over into the Phillips curve. (See p. 974.) However, he finds that maximum likelihood estimates lead to very small influences of money, a very small if not zero cross partial derivative u_{cm}.

Ireland's Phillips curve comes from quadratic adjustment costs. Andrés, López-Salido, and Vallés (2006) find a similar result from a Calvo-style pricing model. Their estimate also finds no effects of money on model dynamics.

Many authors have noted that the stability, absence of deflation, and subsequent quiet (lack of volatility) of inflation is a puzzle, along with other puzzles in accounting for the great recession. Homburg (2015) points out Japan's "benign liquidity trap" and the puzzle it poses for standard models. He advocates a repair with pervasive credit constraints.

Hall (2011) analyzes many features leading to the large fall in output. As for inflation, he argues for the "near-exogeneity" of the rate of inflation.

Quite a few recent contributions fall in the category of section 5.8 that adds a long list of ingredients and frictions to match recent experience and address this and related theoretical puzzles.

Eggertsson and Mehrotra (2014) is a good example of the "secular stagnation" view. They select equilibria by expectations of active policy once the zero bound passes, as analyzed in section 5.4 here. In their model, the stagnation would be cured if the Fed would only commit to a pure peg – expectations of return to active policy are the central problem. (See Cochrane (2014a).)

Christiano, Eichenbaum, and Trabandt (2014) use a detailed nonlinear new-Keynesian model, designed to match quantity variables in the slow recovery as well as inflation. They have four big shocks – a "financial wedge," a "consumption wedge," a TFP shock, and changes in government consumption. Their monetary policy includes an active rule with zero bound constraint, forward guidance about keeping interest rates low after the recovery, and, in the taxonomy of this paper, a return to active policy which selects equilibria.

Farhi and Werning (2017) add agent heterogeneity, incomplete markets, uninsurable idiosyncratic risk, occasionally-binding borrowing constraints, and bounded rationality in the form of level-k thinking. They are however not aimed at explaining the quiet zero bound or reviving the negative sign but to "a powerful mitigation of the effects of monetary policy" and the forward guidance puzzle.

Negro, Giannoni, and Schorfheide (2015) is an ambitious attempt to account for quiet inflation during the great recession, combining more frictions and ingredients than I can compactly summarize here. For their central purpose, understanding why inflation did not fall more, the key assumption is the form of Phillips curve and measurement of marginal costs. For the issues

here, they rely on expectations of future active policy to select equilibria at the zero bound, and many years of bad luck.

8   Concluding comments

The observation that inflation has been stable or gently declining and quiet at the zero bound is important evidence against the classical view that an inflation is unstable at the zero bound, and by implication at an interest rate peg, and the new-Keynesian view that these lead to sunspot volatility. If an interest rate peg is stable, then raising the interest rate should raise inflation, at least eventually.

Conventional new-Keynesian models predict that inflation is stable. Adding the fiscal theory of the price level, or related rules for selecting nearby equilibria, removes indeterminacies and sunspots and leads to a very simple monetary model consistent with our recent experience.

Those models also predict that raising interest rates will raise inflation, both in the long and short run. My attempts to escape this prediction by adding money, backward looking Phillips curves, multiple equilibria all fail.

The new-Keynesian model plus fiscal theory and long-term debt does produce a temporary negative inflation response to unexpected interest rate increases. It is "simple," and "economic," but quite novel relative to standard monetary models. It does not produce the standard, adaptive-expectations view of a permanent disinflation such as the 1980s, nor does it justify policy exploitation of the negative sign, especially in systematic, Taylor-principle form.

It is likely that the negative sign can be found by adding more complex ingredients, including abandoning rational expectations. But that path makes those assumptions necessary, at the foundation of monetary economics, rather than sufficient, small perturbations that get dynamics right.

This paper was also a search for a simple model that captures the effects of monetary policy, but overcomes the critiques of active and instability-inducing Taylor rules in forward-looking models. The new-Keynesian plus fiscal theory model in this paper satisfies that criterion.

A review of the empirical evidence finds weak support for the standard theoretical view that raising interest rates lowers inflation, and much of that evidence is colored by the imposition of strong priors of that sign.

I conclude that a positive reaction of inflation to interest rate changes is a possibility we, and

central bankers, ought to begin to take seriously. At least our faith in a stable exploitable negative relationship coincident with the actual raising of rates ought to be weakened.

The fact of quiet inflation and the theory here rehabilitate interest rate pegs as a possible policy. We can live the Friedman rule of low interest, low inflation, and enormous reserves. Real policies may choose a time-varying peg if central banks think they can offset shocks (v_t^i here may react to v_t^r), and may desire headroom of higher inflation to do that. But there is no need to fear catastrophe of inflation from the former policy configuration.

However, that quiet depends on fiscal foundations. The large demand for government debt which provides the fiscal foundations of this quiet (under either "active" or "passive" views), driven by low discount rates not strong fiscal policies, could evaporate as unexpectedly as it arrived.

References

Andrés, Javier, J. David López-Salido, and Javier Vallés. 2006. "Money in an Estimated Business Cycle Model of the Euro Area." *The Economic Journal* 116 (511):457–477. URL http://dx.doi.org/10.1111/j.1468-0297.2006.01088.x.

Angeletos, George-Marios and Chen Lian. 2016. "Forward Guidance without Common Knowledge." *Manuscript* URL http://economics.mit.edu/files/12277.

Atkeson, Andrew, Vardarajan V. Chari, and Patrick J. Kehoe. 2010. "Sophisticated Monetary Policies." *Quarterly Journal of Economics* 125 (1):47–89.

Ball, Laurence. 2016. "Ben Bernanke And The Zero Bound." *Contemporary Economic Policy, Western Economic Association International* 34:7–20.

Belaygorod, Anatoliy and Michael Dueker. 2009. "Indeterminacy, Change Points and the Price Puzzle in an Estimated DSGE Model." *Journal of Economic Dynamics and Control* 33 (3):624–648.

Benchimol, Jonathan and André Fourçans. 2012. "Money and risk in a DSGE framework: A Bayesian application to the Eurozone." *Journal of Macroeconomics* 34:95–111.

———. 2015. "Money and monetary policy in the Eurozone: an empirical analysis during crises." *Macroeconomic Dynamics* Forthcoming.

Benhabib, Jess, Stephanie Schmitt-Grohé, and Martín Uribe. 2001. "Monetary policy and multiple equilibria." *American Economic Review* :167–186.

———. 2002. "Avoiding Liquidity Traps." *Journal of Political Economy* 110 (3):535–563.

Bordo, Michael D., Christopher J. Erceg, Andrew T. Levin, and Ryan Michaels. 2007. "Three Great American Disinflations." Working Paper 12982, National Bureau of Economic Research. URL https://www.federalreserve.gov/pubs/ifdp/2007/898/ifdp898.pdf. Also International Finance Discussion Papers 898. Board of Governors of the Federal Reserve System (U.S.).

———. 2017. "Policy Credibility and Alternative Approaches to Disinflation." *Research in Economics* Forthcoming.

Brash, Donald T. 2002. "Inflation Targeting 14 Years On." URL http://www.bis.org/review/r020121b.pdf. Speech given at 2002 American Economics Association Conference.

Campbell, Jeffrey R. and Jacob P. Weber. 2016. "Open Mouth Operations." *Manuscript, Federal Reserve Bank of Chicago* .

Castelnuovo, Efrem and Paolo Surico. 2010. "Monetary Policy, Inflation Expectations and The Price Puzzle." *The Economic Journal* 120:1262–1283. URL doi:10.1111/j.1468-0297.2010.02368.x.

Christiano, Lawrence, Martin Eichenbaum, and Charles Evans. 1999. "Monetary policy shocks: what have we learned and to what end?" In *Handbook of Macroeconomics*, edited by Michael Woodford and John B. Taylor. Amsterdam: North-Holland, 65–148.

———. 2005. "Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy." *Journal of Political Economy* 113, (1–45).

Christiano, Lawrence J., Martin Eichenbaum, and Benjamin K. Johannsen. 2016. "Does the New Keynesian Model Have a Uniqueness Problem?" *Manuscript* .

Christiano, Lawrence J., Martin S. Eichenbaum, and Mathias Trabandt. 2014. "Understanding the great recession." Working paper, National Bureau of Economic Research. URL http://www.nber.org/papers/w20040.

Clarida, Richard, Jordi Galí, and Mark Gertler. 2000. "Monetary Policy Rules and Macroeconomic Stability: Evidence and Some Theory,." *Quarterly Journal of Economics* 115:147–180.

Cochrane, John H. 1998. "A Frictionless View of U.S. Inflation." *NBER Macroeconomics Annual* 13:323–384. URL http://www.jstor.org/stable/4623752.

———. 2001. "Long term debt and optimal policy in the fiscal theory of the price level." *Econometrica* 69:69–116.

———. 2005. "Money as Stock." *Journal of Monetary Economics* 52:501–528.

———. 2009. "Can learnability save new-Keynesian models?" *Journal of Monetary Economics* 56 (8):1109 – 1113. URL http://www.sciencedirect.com/science/article/pii/S0304393209001470.

———. 2011a. "Appendix B from John H. Cochrane, "Determinacy and Identification with Taylor Rules"." *Journal of Political Economy* 119:565–615. URL http://www.journals.uchicago.edu/doi/suppl/10.1086/660817.

———. 2011b. "Determinacy and Identification with Taylor Rules." *Journal of Political Economy* 119:565–615.

———. 2013. "What if we got the sign wrong on monetary policy?" URL http://johnhcochrane.blogspot.com/2013/12/what-if-we-got-sign-wrong-on-monetary.html. "Grumpy Economist" Blog Post.

———. 2014a. "Comments on Eggertsson and Mehrotra, "A Model of Secular Stagnation"." *manuscript* URL http://faculty.chicagobooth.edu/john.cochrane/research/papers/NBER_EFG_Cochrane.pdf.

———. 2014b. "Monetary Policy with Interest on Reserves." *Journal of Economic Dynamics and Control* 49:74–108.

———. 2014c. "The neo-Fisherian Question." URL http://johnhcochrane.blogspot.com/2014/11/the-neo-fisherian-question.html. "Grumpy Economist" Blog Post.

———. 2014d. "Toward a run-free financial system." In *Across the Great Divide: New Perspectives on the Financial Crisis*, edited by Martin Neil Baily and John B. Taylor. Stanford, CA: Hoover Press, 197–249. URL http://faculty.chicagobooth.edu/john.cochrane/research/papers/across-the-great-divide-ch10.pdf.

———. 2015. "A New Structure For U. S. Federal Debt." In *The $13 Trillion Question: Managing the U.S. Government?s Debt*, edited by David Wessel. Brookings Institution Press, 91–146.

———. 2016a. "Comments on "A Behavioral New-Keynesian Model" by Xavier Gabaix." *Manuscript* URL http://faculty.chicagobooth.edu/john.cochrane/research/papers/gabaix_comments.pdf.

———. 2016b. "The New-Keynesian Liquidity Trap." *Manuscript* URL http://faculty.chicagobooth.edu/john.cochrane/research/papers/zero_bound_elsarticle-harv.pdf.

———. 2016c. "Stepping on a Rake: Replication and Diagnosis." *manuscript* URL http://faculty.chicagobooth.edu/john.cochrane/research/papers/sims_replication.pdf.

Dueker, Michael J. 2006. "The Price Puzzle: An Update and a Lesson." *Economic Synopses* 4:1. URL https://research.stlouisfed.org/publications/es/06/ES0624.pdf.

Eggertsson, Gauti and Neil Mehrotra. 2014. "A Model of Secular Stagnation." *Manuscript* .

Eichenbaum, Martin. 1992. "Comment on Interpreting the macroeconomic time series facts: the effects of monetary policy." *European Economic Review,* 36 (5):1001–1011.

Evans, George W. and Seppo Honkapohja. 2001. *Learning and Expectations in Macroeconomics.* Princeton, N. J.: Princeton University Press.

Farhi, Emmanuel and Iván Werning. 2017. "Monetary Policy, Bounded Rationality, and Incomplete Markets." Working Paper 23281, National Bureau of Economic Research. URL http://www.nber.org/papers/w23281.

Friedman, Milton. 1968. "The Role of Monetary Policy." *The American Economic Review* 58:1–17.

———. 1969. "The Optimum Quantity of Money." In *The Optimum Quantity of Money and Other Essays.* Chicago, IL: Aldine, 1–50.

Gabaix, Xavier. 2016. "A Behavioral New Keynesian Model." *Manuscript* .

García-Schmidt, Mariana and Michael Woodford. 2015. "Are Low Interest Rates Deflationary? A Paradox of Perfect-Foresight Analysis." Working Paper 21614, National Bureau of Economic Research. URL http://www.nber.org/papers/w21614.

Gürkaynak, Refet S., Brian Sack, and Jonathan H. Wright. 2007. "The U.S. Treasury yield curve: 1961 to the present." *Journal of Monetary Economics* 54 (8):2291 – 2304. Data at https://www.federalreserve.gov/econresdata/researchdata/feds200628.xls.

Hall, George J. and Thomas J. Sargent. 2015. "A History of U.S. Debt Limits." *Manuscript* URL http://people.brandeis.edu/~ghall/papers/debt_limit_12_08_2015.pdf.

Hall, Robert E. 2011. "The Long Slump." *American Economic Review* 101:431–469.

Hanson, Michael. 2004. "The 'Price Puzzle' Reconsidered." *Journal of Monetary Economics* 51. URL http://www.sciencedirect.com/science/article/pii/S0304393204000959.

Homburg, Stefan. 2015. "Understanding Benign Liquidity Traps: The Case of Japan." *German Economic Review* URL http://diskussionspapiere.wiwi.uni-hannover.de/pdf_bib/dp-557.pdf.

Ireland, Peter. 2004. "Money's Role in the Monetary Business Cycle." *Journal of Money, Credit, and Banking* 36 (6):969–983. URL http://irelandp.com/pubs/role.pdf.

King, Robert G. 2000. "The New IS-LM Model: Language, Logic, and Limits." *Federal Reserve Bank of Richmond Economic Quarterly* 86 (3):45–104.

Kocherlakota, Narayana. 2010. "Inside the FOMC." URL https://www.minneapolisfed.org/news-and-events/presidents-speeches/inside-the-fomc. Speech.

———. 2017. "Comment on Cochrane, NBER Macro Annual 2017." *Manuscript* URL https://docs.google.com/viewer?a=v&pid=sites&srcid=ZGVmYXVsdGRvbWFpbnxrb2NoZXJsYWtvdGEwMDl8Z3g6N2VmZjM3NjQ2MWEwMjgxNw.

Kocherlakota, Narayana R. 2016. "Fragility of Purely Real Macroeconomic Models." *Manuscript* .

Leeper, Eric M. 1991. "Equilibria Under 'Active' and 'Passive' Monetary and Fiscal Policies." *Journal of Monetary Economics* 27:129–147.

Lucas, Robert E. 1972. "Expectations and the Neutrality of Money." *Journal of Economic Theory* 4:103–124.

Lucas, Robert E. 1988. "Money Demand in the United States: A Quantitative Review." *Carnegie-Rochester Conference Series on Public Policy* 29:137–167.

Mankiw, N. Gregory and Ricardo Reis. 2002. "Sticky Information versus Sticky Prices: A Proposal to Replace the New Keynesian Phillips Curve." *Quarterly Journal of Economics* 117:1295–1328.

McCallum, Bennett T. 2009a. "Inflation determination with Taylor rules: Is new-Keynesian analysis critically flawed?" *Journal of Monetary Economics* 56 (8):1101 – 1108. URL http://www.sciencedirect.com/science/article/pii/S0304393209001305.

———. 2009b. "Rejoinder to Cochrane." *Journal of Monetary Economics* 56 (8):1114 – 1115. URL http://www.sciencedirect.com/science/article/pii/S0304393209001408.

McKay, Alisdair, Emi Nakamura, and Jón Steinsson. 2016. "The Power of Forward Guidance Revisited." *American Economic Review* 106:3133–3158.

Negro, Marco Del, Marc P. Giannoni, and Frank Schorfheide. 2015. "Inflation in the Great Recession and New Keynesian Models." *American Economic Journal: Macroeconomics* 7:168?196. URL http://dx.doi.org/10.1257/mac.20140097.

Ramey, Valerie. 2015. "Macroeconomic Shocks and Their Propagation." Manuscript.

Rotemberg, Julio J and Michael Woodford. 1997. "An optimization-based econometric framework for the evaluation of monetary policy." *NBER macroeconomics annual* 12:297–346.

Sargent, Thomas J. 2013. *Rational Expectations and Inflation (Third Edition).* Princeton University Press. URL http://www.jstor.org/stable/j.ctt2jc97n.

Sargent, Thomas J. and Neil Wallace. 1975. "Rational Expectations, the Optimal Monetary Instrument, and the Optimal Money Supply Rule." *Journal of Political Economy* 83:241–254.

———. 1981. "Some Unpleasant Monetarist Arithmetic." *Federal Reserve Bank of Minneapolis Quarterly Review* 5:1–17.

Schmitt-Grohé, Stephanie and Martín Uribe. 2014. "Liquidity Traps: An Interest-Rate-Based Exit Strategy." *The Manchester School* 81 (S1):1–14. URL doi:10.1111/manc.12065.

Sims, Christopher A. 1980. "Macroeconomics and reality." *Econometrica* :1–48.

Sims, Christopher A. 1992. "Interpreting the macroeconomic time series facts: The effects of monetary policy." *European Economic Review* 36 (5):975–1000.

———. 1994. "A simple model for study of the determination of the price level and the interaction of monetary and fiscal policy." *Economic Theory* 4 (3):381–399. URL http://dx.doi.org/10.1007/BF01215378.

———. 2011. "Stepping on a Rake: The role of fiscal policy in the inflation of the 1970s." *European Economic Review* 55:48–56. URL doi:10.1016/j.euroecorev.2010.11.010.

———. 2016. "Fiscal Policy, Monetary Policy and Central Bank Independence." *Jackson Hole* URL https://www.kansascityfed.org/~/media/files/publicat/sympos/2016/econsymposium-sims-paper.pdf.

Smets, Frank and Raf Wouters. 2003. "An Estimated Dynamic Stochastic General Equilibrium Model of the Euro Area." *Journal of the European Economic Association* 1 (5):1123–1175.

Smith, Noah. 2014. "The Neo-Fisherite Rebellion." URL http://noahpinionblog.blogspot.com/2014/04/the-neo-fisherite-rebellion.html. Noahpinion Blog Post.

Swanson, Eric T. and John C. Williams. 2014. "Measuring the Effect of the Zero Lower Bound on Medium- and Longer-Term Interest Rates." *American Economic Review* 104:3154–85.

Taylor, John B. 1993. "Discretion versus policy rules in practice." *Carnegie-Rochester Conference Series on Public Policy* 39:195 – 214. URL http://www.sciencedirect.com/science/article/pii/016722319390009L.

———. 1999. "The Robustness and Efficiency of Monetary Policy Rules as Guidelines for Interest Rate Setting by the European Central Bank." *Journal of Monetary Economics* 43:655–679.

Uhlig, Harald. 2006. "What are the effects of monetary policy? Results from an agnostic identification procedure." *Journal of Monetary Economics* 52:381–419.

Werning, Iván. 2012. "Managing a Liquidity Trap: Monetary and Fiscal Policy." *Manuscript* URL https://dl.dropboxusercontent.com/u/125966/zero_bound_2011.pdf.

Wieland, Johannes. 2015. "Are Negative Supply Shocks Expansionary at the Zero Lower Bound?" *Manuscript* .

Williamson, Steven. 2013. "Phillips Curves and Fisher Relations." URL http://newmonetarism.blogspot.com/2013/12/phillips-curves-and-fisher-relations.html. Blog Post.

Woodford, Michael. 1995. "Price-Level Determinacy Without Control of a Monetary Aggregate." *Carnegie-Rochester Conference Series on Public Policy* 43:1–46.

———. 2001. "Fiscal Requirements for Price Stability." *Journal of Money Credit and Banking* 33 (669-728).

———. 2003. *Interest and prices*. Princeton N. J.: Princeton University Press.

———. 2012. "Methods of Policy Accommodation at the Interest-Rate Lower Bound." *Manuscript* URL http://www.columbia.edu/~mw2230/JHole2012final.pdf.

Yellen, Janet. 2016. "Macroeconomic Research After the Crisis." Tech. rep., Federal Reserve Board of Governors. URL http://www.federalreserve.gov/newsevents/speech/yellen20161014a.htm. Speech given at "The Elusive 'Great' Recovery: Causes and Implications for Future Business Cycle Dynamics" 60th annual economic conference sponsored by the Federal Reserve Bank of Boston, Boston, Massachusetts.

9 Algebra Appendix to "Michelson-Morley, Fisher, and Occam: The Radical Implications of Stable Quiet Inflation at the Zero Bound."

9.1 Formulas for delayed and temporary rate rises

Here I work out the algebra for impulse response functions of the fiscal theory with long term debt model, with an announcement M years ahead of the interest rate rise, and an interest rate rise that only lasts M years, in both continuous and discrete time, Equations (32)-(33).

An interest rate rise from i to i^* that only lasts M years, continuous time:

$$\left[\vartheta \int_0^M e^{-i^*j}e^{-\vartheta j}dj + \vartheta \int_M^\infty e^{-i^*M-i(j-M)}e^{-\vartheta j}dj\right]\frac{B_t}{P_t^*} = \frac{s}{r}$$

$$\vartheta \left[\frac{e^{-(i^*+\vartheta)M}}{i+\vartheta} + \frac{1-e^{-(i^*+\vartheta)M}}{i^*+\vartheta}\right]\frac{B_t}{P_t^*} = \frac{s}{r}$$

$$\frac{P_t^*}{P_t} = \left(\frac{e^{-(i^*+\vartheta)M}}{i+\vartheta} + \frac{1-e^{-(i^*+\vartheta)M}}{i^*+\vartheta}\right) / \left(\frac{1}{i+\vartheta}\right)$$

$$\frac{P_t^*}{P_t} - 1 = \left(1-e^{-(i^*+\vartheta)M}\right)\left(\frac{i+\vartheta}{i^*+\vartheta}-1\right) \approx \left(1-e^{-\vartheta M}\right)\left(\frac{i+\vartheta}{i^*+\vartheta}-1\right)$$

An announcement of an interest rate rise from i to i^* that starts in M years, continuous time:

$$\left[\vartheta \int_0^M e^{-ij}e^{-\vartheta j}dj + \vartheta \int_M^\infty e^{-iM-i^*(j-M)}e^{-\vartheta j}dj\right]\frac{B_t}{P_t^*} = \frac{s}{r}$$

$$\vartheta \left[\frac{e^{-(i+\vartheta)M}}{i^*+\vartheta} + \frac{1-e^{-(i+\vartheta)M}}{i+\vartheta}\right]\frac{B_t}{P_t^*} = \frac{s}{r}$$

$$\frac{P_t^*}{P_t} = \left(\frac{e^{-(i+\vartheta)M}}{i^*+\vartheta} + \frac{1-e^{-(i+\vartheta)M}}{i+\vartheta}\right) / \left(\frac{1}{i+\vartheta}\right)$$

$$\frac{P_t^*}{P} - 1 = e^{-(i+\vartheta)M}\left(\frac{i+\vartheta}{i^*+\vartheta}-1\right) \approx e^{-\vartheta M}\left(\frac{i+\vartheta}{i^*+\vartheta}-1\right)$$

An interest rate rise from i to i^* that only lasts M years, discrete time:

$$\left[\sum_{j=0}^{M-1}\frac{\theta^j}{(1+i^*)^j} + \sum_{j=M}^\infty \frac{\theta^M}{(1+i^*)^M}\frac{\theta^{(j-M)}}{(1+i)^{(j-M)}}\right]\frac{B_{t-1}}{P_t^*} = \frac{s}{1-\beta}$$

$$\left[\frac{1 - \left(\frac{\theta}{1+i^*}\right)^M}{1 - \frac{\theta}{1+i^*}} + \frac{\left(\frac{\theta}{1+i^*}\right)^M}{1 - \frac{\theta}{1+i}} \right] \frac{B_{t-1}}{P_t^*} = \frac{s}{1 - \beta}$$

Thus,

$$\frac{P_t^*}{P_t} - 1 = \left(\frac{1 - \left(\frac{\theta}{1+i^*}\right)^M}{1 - \frac{\theta}{1+i^*}} + \frac{\left(\frac{\theta}{1+i^*}\right)^M}{1 - \frac{\theta}{1+i}} \right) \Big/ \left(\frac{1}{1 - \frac{\theta}{1+i}} \right) - 1$$

$$\frac{P_t^*}{P_t} - 1 = \left(1 - \left(\frac{\theta}{1+i^*} \right)^M \right) \left(\frac{\frac{1}{1 - \frac{\theta}{1+i^*}}}{\frac{1}{1 - \frac{\theta}{1+i}}} - 1 \right)$$

$$\frac{P_t^*}{P_t} - 1 = \left(1 - \left(\frac{\theta}{1+i^*} \right)^M \right) \left(\frac{1+i^*}{1+i_t} \frac{1+i-\theta}{1+i^*-\theta} - 1 \right)$$

$$\frac{P_t^*}{P_t} - 1 \approx \left(1 - \theta^M \right) \left(\frac{1+i-\theta}{1+i^*-\theta} - 1 \right)$$

An interest rate rise from i to i^* that starts in M years, discrete time:

$$\left[\sum_{j=0}^{M-1} \frac{\theta^j}{(1+i)^j} + \sum_{j=M}^{\infty} \frac{\theta^M}{(1+i)^M} \frac{\theta^{(j-M)}}{(1+i^*)^{(j-M)}} \right] \frac{B_{t-1}}{P_t^*} = \frac{s}{1 - \beta}$$

$$\left[\frac{1 - \left(\frac{\theta}{1+i}\right)^M}{1 - \frac{\theta}{1+i}} + \frac{\left(\frac{\theta}{1+i}\right)^M}{1 - \frac{\theta}{1+i^*}} \right] \frac{B_{t-1}}{P_t^*} = \frac{s}{1 - \beta}$$

Thus,

$$\frac{P_t^*}{P_t} - 1 = \left(\frac{1 - \left(\frac{\theta}{1+i}\right)^M}{1 - \frac{\theta}{1+i}} + \frac{\left(\frac{\theta}{1+i}\right)^M}{1 - \frac{\theta}{1+i^*}} \right) \Big/ \left(\frac{1}{1 - \frac{\theta}{1+i}} \right) - 1$$

$$\frac{P_t^*}{P_t} - 1 = \left(\frac{\theta}{1+i} \right)^M \left(\frac{1}{1 - \frac{\theta}{1+i^*}} - \frac{1}{1 - \frac{\theta}{1+i}} \right) \left(\frac{1}{1 - \frac{\theta}{1+i}} \right)$$

$$\frac{P_t^*}{P_t} - 1 = \left(\frac{\theta}{1+i} \right)^M \left(\frac{1+i^*}{1+i} \frac{1+i-\theta}{1+i^*-\theta} - 1 \right)$$

$$\frac{P_t^*}{P_t} - 1 \approx \theta^M \left(\frac{1+i-\theta}{1+i^*-\theta} - 1 \right)$$

9.2   Sticky-price model solution

Here I derive the explicit solutions (62)-(63), for inflation and output given the equilibrium path of interest rates. The simple model (58)-(59) is

$$x_t = E_t x_{t+1} - \sigma(i_t - E_t \pi_{t+1})$$

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t.$$

The model with money generalizes the IS equation only, to (73)

$$x_t = E_t x_{t+1} + (\sigma - \xi)\left(\frac{m}{c}\right) E_t\left[(i_{t+1} - i_{t+1}^m) - (i_t - i_t^m)\right] - \sigma\left(i_t - E_t \pi_{t+1}\right).$$

We can treat the two cases simultaneously by defining

$$z_t \equiv i_t - \left(\frac{\sigma - \xi}{\sigma}\right)\left(\frac{m}{c}\right) E_t\left[(i_{t+1} - i_{t+1}^m) - (i_t - i_t^m)\right]$$

and writing the IS equation as

$$x_t = E_t x_{t+1} - \sigma\left(z_t - E_t \pi_{t+1}\right).$$

One must be careful that lags of z_t are lags of expected interest rate changes, not lags of actual interest rate changes.

Expressing the model in lag operator notation,

$$E_t(1 - L^{-1})x_t = \sigma E_t L^{-1}\pi_t - \sigma z_t$$

$$E_t(1 - \beta L^{-1})\pi_t = \kappa x_t$$

Forward-differencing the second equation,

$$E_t(1 - L^{-1})(1 - \beta L^{-1})\pi_t = E_t(1 - L^{-1})\kappa x_t$$

Then substituting,

$$E_t(1 - L^{-1})\left(1 - \beta L^{-1}\right)\pi_t = \sigma\kappa E_t L^{-1}\pi_t - \sigma\kappa z_t$$

$$E_t\left[(1 - L^{-1})\left(1 - \beta L^{-1}\right) - \sigma\kappa L^{-1}\right]\pi_t = -\sigma\kappa z_t$$

$$E_t\left[1 - (1 + \beta + \sigma\kappa)L^{-1} + \beta L^{-2}\right]\pi_t = -\sigma\kappa z_t.$$

Factor the lag polynomial

$$E_t(1 - \lambda_1 L^{-1})(1 - \lambda_2 L^{-1})\pi_t = -\sigma\kappa z_t$$

where

$$\lambda_i = \frac{(1 + \beta + \sigma\kappa) \pm \sqrt{(1 + \beta + \sigma\kappa)^2 - 4\beta}}{2}.$$

Since $\lambda_1 > 1$ and $\lambda_2 < 1$, reexpress the result as

$$E_t\left[(1 - \lambda_1^{-1}L)(1 - \lambda_2 L^{-1})\lambda_1 L^{-1}\pi_t\right] = \sigma\kappa z_t$$

$$E_t\left[(1 - \lambda_1^{-1}L)(1 - \lambda_2 L^{-1})\pi_{t+1}\right] = \sigma\kappa\lambda_1^{-1} z_t$$

The bounded solutions are

$$\pi_{t+1} = E_{t+1}\frac{\lambda_1^{-1}}{(1 - \lambda_1^{-1}L)(1 - \lambda_2 L^{-1})}\sigma\kappa z_t + \frac{1}{(1 - \lambda_1^{-1}L)}\delta_{t+1}$$

where δ_{t+1} is a sequence of unpredictable random variables, $E_t\delta_{t+1} = 0$. I follow the usual practice and I rule out solutions that explode in the forward direction.

Using a partial fractions decomposition to break up the right hand side,

$$\frac{\lambda_1^{-1}}{\left(1 - \lambda_1^{-1}L\right)\left(1 - \lambda_2 L^{-1}\right)} = \frac{1}{\lambda_1 - \lambda_2}\left(1 + \frac{\lambda_1^{-1}L}{1 - \lambda_1^{-1}L} + \frac{\lambda_2 L^{-1}}{1 - \lambda_2 L^{-1}}\right).$$

So,

$$\pi_{t+1} = \frac{1}{\lambda_1 - \lambda_2}E_{t+1}\left(1 + \frac{\lambda_1^{-1}L}{1 - \lambda_1^{-1}L} + \frac{\lambda_2 L^{-1}}{1 - \lambda_2 L^{-1}}\right)\sigma\kappa z_t + \frac{1}{(1 - \lambda_1^{-1}L)}\delta_{t+1}$$

or in sum notation,

$$\pi_{t+1} = \sigma\kappa\frac{1}{\lambda_1 - \lambda_2}\left(z_t + \sum_{j=1}^{\infty}\lambda_1^{-j}z_{t-j} + \sum_{j=1}^{\infty}\lambda_2^j E_{t+1}z_{t+j}\right) + \sum_{j=0}^{\infty}\lambda_1^{-j}\delta_{t+1-j}. \tag{88}$$

We can show directly that the long-run impulse-response function is 1:

$$\frac{1}{(1 - \lambda_1^{-1})(1 - \lambda_2)}\frac{\sigma\kappa}{\lambda_1} = -\frac{\sigma\kappa}{(1 - \lambda_1)(1 - \lambda_2)}$$

$$= -\frac{\sigma\kappa}{(1 - (\lambda_1 + \lambda_2) + \lambda_1\lambda_2)} = -\frac{\sigma\kappa}{(1 - (1 + \beta + \sigma\kappa) + \beta)} = 1.$$

Having found the path of π_t, we can find output by

$$\kappa x_t = \pi_t - \beta E_t \pi_{t+1}.$$

In lag operator notation, and shifting forward one period,

$$\kappa x_{t+1} = E_{t+1} \left[(1 - \beta L^{-1}) \pi_{t+1} \right]$$

$$\kappa x_{t+1} = \frac{\sigma \kappa}{\lambda_1 - \lambda_2} E_{t+1} \left[(1 - \beta L^{-1}) \left(1 + \frac{\lambda_1^{-1} L}{1 - \lambda_1^{-1} L} + \frac{\lambda_2 L^{-1}}{1 - \lambda_2 L^{-1}} \right) z_t \right] + E_{t+1} \frac{(1 - \beta L^{-1})}{(1 - \lambda_1^{-1} L)} \delta_{t+1}.$$

We can rewrite the polynomials to give

$$\kappa x_{t+1} = \frac{\sigma \kappa}{\lambda_1 - \lambda_2} E_{t+1} \left[\frac{1 - \beta \lambda_1^{-1}}{1 - \lambda_1^{-1} L} + \frac{(1 - \beta \lambda_2^{-1}) (\lambda_2 L^{-1})}{1 - \lambda_2 L^{-1}} \right] z_t + E_{t+1} \left[\frac{1 - \beta \lambda_1^{-1}}{1 - \lambda_1^{-1} L} \right] \delta_{t+1}.$$

(In the second term, I use $E_t \left[\beta L^{-1} \delta_{t+1} \right] = 0$) or, in sum notation,

$$\kappa x_{t+1} = \frac{\sigma \kappa}{\lambda_1 - \lambda_2} \left[\left(1 - \beta \lambda_1^{-1} \right) \sum_{j=0}^{\infty} \lambda_1^{-j} z_{t-j} + \left(1 - \beta \lambda_2^{-1} \right) \sum_{j=1}^{\infty} \lambda_2^j E_{t+1} z_{t+j} \right] +$$

$$+ \left(1 - \beta \lambda_1^{-1} \right) \sum_{j=0}^{\infty} \lambda_1^{-j} \delta_{t+1-j}.$$

9.3 Impulse response function – explicit solution

The solution () is

$$\pi_{t+1} = \frac{\sigma \kappa}{\lambda_1 - \lambda_2} \left(i_t + \sum_{j=1}^{\infty} \lambda_1^{-j} i_{t-j} + E_{t+1} \sum_{j=1}^{\infty} \lambda_2^j i_{t+j} \right) + \sum_{j=0}^{\infty} \lambda_1^{-j} \delta_{t+1-j}$$

$$\lambda_1 = \frac{(1 + \beta + \sigma \kappa) + \sqrt{(1 + \beta + \sigma \kappa)^2 - 4\beta}}{2}$$

$$\lambda_1 = \frac{(1 + \beta + \sigma \kappa) - \sqrt{(1 + \beta + \sigma \kappa)^2 - 4\beta}}{2}$$

While it is straightforward to calculate and simulate the solution for a given path of interest rates, it is useful also to have a formula for the response to a step function. We want to find the impulse-response function to $i_t = 0$, $t < 0$, and $i_t = i$, $t = 0, 1, 2, \dots$ The interest rate rise is

announced at time $-M$, so only $\delta_{-M} \neq 0$. That response is,

$$t < -(M+1): \ \pi_{t+1} = 0$$

$$-(M+1) \leq t \leq 0: \pi_{t+1} = \frac{\sigma \kappa}{\lambda_1 - \lambda_2} \left(\frac{\lambda_2^{-t}}{1 - \lambda_2} \right) + \lambda_1^{-(t+1+M)} \delta_{-M}$$

$$0 < t: \pi_{t+1} = \frac{\sigma \kappa}{\lambda_1 - \lambda_2} \left(\frac{1}{1 - \lambda_2} + \frac{\lambda_1^{-1}(1 - \lambda_1^{-t})}{1 - \lambda_1^{-1}} \right) + \lambda_1^{-(t+1+M)} \delta_{-M}$$

Proceeding in the same way, the solution for x is

$$\kappa x_{t+1} = \frac{\sigma \kappa}{\lambda_1 - \lambda_2} \left((1 - \beta \lambda_1^{-1}) \sum_{j=0}^{\infty} \lambda_1^{-j} i_{t-j} + (1 - \beta \lambda_2^{-1}) E_{t+1} \sum_{j=1}^{\infty} \lambda_2^{j} i_{t+j} \right) + (1 - \beta \lambda_1^{-1}) \sum_{j=0}^{\infty} \lambda_1^{-j} \delta_{t+1-j}$$

so the impulse-response function to $i_t = 0, t < 0$, and $i_t = i, t = 0, 1, 2, ..$announced at time $-M$, is,

$$t < -(M+1): \ x_{t+1} = 0$$

$$-(M+1) \leq t \leq -1: \kappa x_{t+1} = \frac{\sigma \kappa}{\lambda_1 - \lambda_2}(1 - \beta \lambda_2^{-1}) \frac{\lambda_2^{-t+1}}{1 - \lambda_2} + (1 - \beta \lambda_1^{-1})\lambda_1^{-(t+1+M)}\delta_{-M}$$

$$0 \leq t: \kappa x_{t+1} = \frac{\sigma \kappa}{\lambda_1 - \lambda_2} \left((1 - \beta \lambda_1^{-1}) \frac{1 - \lambda_1^{-(t+1)}}{1 - \lambda_1^{-1}} + (1 - \beta \lambda_2^{-1}) \frac{\lambda_2}{1 - \lambda_2} \right) + \lambda_1^{-(t+1+M)}\delta_{-M}$$

The interest rate is then

$$r_t = i_t - E_t \pi_{t+1}.$$

For the impulse-response function, the expected and actual values are the same, except at $-M$, where though $\pi_{-M} \neq 0$, $E_{-M-1}\pi_{-M} = 0$. Hence,

$$t \leq -(M+1): \ r_t = 0 \tag{89}$$

$$-M \leq t < 0: r_t = -\frac{\sigma \kappa}{\lambda_1 - \lambda_2} \left(\frac{\lambda_2^{-t}}{1 - \lambda_2} \right) - \lambda_1^{-(t+1+M)}\delta_{-M} \tag{90}$$

$$t = 0: r_t = i - \frac{\sigma \kappa}{\lambda_1 - \lambda_2} \left(\frac{\lambda_2^{-t}}{1 - \lambda_2} \right) - \lambda_1^{-(t+1+M)}\delta_{-M} \tag{91}$$

$$0 < t: r_t = i - \frac{\sigma \kappa}{\lambda_1 - \lambda_2} \left(\frac{1}{1 - \lambda_2} + \frac{\lambda_1^{-1}(1 - \lambda_1^{-t})}{1 - \lambda_1^{-1}} \right) - \lambda_1^{-(t+1+M)}\delta_{-M} \tag{92}$$

9.4   Three-equation model solution

I solve the three-equation model of Figure 15 by standard methods, incorporating the Taylor rule in to monetary policy rather than conditioning on the equilibrium interest rate and then constructing the underlying Taylor rule. Both methods give the same answer, but a conventional calculation is more transparent in this case, and it verifies that both approaches give the same answer.

While one can solve the model quickly via matrix techniques, here I use lag operator techniques to write the solution for inflation analytically.

The model is

$$x_t = E_t x_{t+1} - \sigma(i_t - E_t \pi_{t+1})$$
$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t$$
$$i_t = \phi \pi_t + \hat{v}_t^i$$
$$v_t^i = \rho v_{t-1}^i + \varepsilon_t^i$$

Substituting the Taylor rule,

$$x_t = E_t x_{t+1} - \sigma(\phi \pi_t + v_t^i - E_t \pi_{t+1})$$
$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t$$

Expressing the model in lag operator notation,

$$E_t(1 - L^{-1})x_t = \sigma E_t \left(L^{-1} - \phi\right) \pi_t - \sigma v_t^i$$

$$E_t(1 - \beta L^{-1})\pi_t = \kappa x_t$$

Forward-differencing the second equation,

$$E_t(1 - L^{-1})(1 - \beta L^{-1})\pi_t = E_t(1 - L^{-1})\kappa x_t$$

Then substituting into the first equation,

$$E_t(1 - L^{-1})\left(1 - \beta L^{-1}\right) \pi_t = \sigma \kappa E_t \left(L^{-1} - \phi\right) \pi_t - \sigma \kappa v_t^i$$

$$E_t \left[1 - \frac{1 + \beta + \sigma\kappa}{1 + \sigma\kappa\phi} L^{-1} + \frac{\beta}{1 + \sigma\kappa\phi} L^{-2} \right] \pi_t = -\frac{\sigma\kappa}{1 + \sigma\kappa\phi} v_t^i.$$

Factor the lag polynomial

$$E_t (1 - \lambda_1 L^{-1})(1 - \lambda_2 L^{-1}) \pi_t = -\frac{\sigma\kappa}{1 + \sigma\kappa\phi} v_t^i$$

where

$$\lambda = \frac{1 + \beta + \sigma\kappa \pm \sqrt{(1 + \beta + \kappa\sigma)^2 - 4\beta(1 + \phi\sigma\kappa)}}{2(1 + \sigma\kappa\phi)}$$

These lag operator roots are the inverse of the eigenvalues of the usual transition matrix. The system is stable and solved backward for $\lambda > 1$; it is unstable and solved forward for $\lambda < 1$.

The standard three-equation model uses $\phi > 1$ so both roots are unstable, $\lambda_1 < 1$ and $\lambda_2 < 1$. Then, we can write

$$E_t (1 - \lambda_1 L^{-1})(1 - \lambda_2 L^{-1}) \pi_t = -\frac{\sigma\kappa}{1 + \sigma\kappa\phi} v_t^i$$

$$\pi_t = -E_t \frac{1}{(1 - \lambda_1 L^{-1})(1 - \lambda_2 L^{-1})} \frac{\sigma\kappa}{1 + \sigma\kappa\phi} v_t^i$$

$$\pi_t = E_t \frac{1}{\lambda_1 - \lambda_2} \left(\frac{-\lambda_1}{1 - \lambda_1 L^{-1}} + \frac{\lambda_2}{1 - \lambda_2 L^{-1}} \right) \frac{\sigma\kappa}{1 + \sigma\kappa\phi} v_t^i$$

$$\pi_t = \frac{\sigma\kappa}{1 + \sigma\kappa\phi} \frac{1}{\lambda_1 - \lambda_2} E_t \left(-\lambda_1 \sum_{j=0}^{\infty} \lambda_1^j v_{t+j}^i + \lambda_2 \sum_{j=0}^{\infty} \lambda_2^j v_{t+j}^i \right)$$

Using the AR(1) form of the disturbance v^i,

$$\pi_t = \frac{\sigma\kappa}{1 + \sigma\kappa\phi} \frac{1}{\lambda_1 - \lambda_2} \left(-\lambda_1 \sum_{j=0}^{\infty} \lambda_1^j \rho^j + \lambda_2 \sum_{j=0}^{\infty} \lambda_2^j \rho^j \right) \hat{\imath}_t$$

$$\pi_t = \frac{\sigma\kappa}{1 + \sigma\kappa\phi} \frac{1}{\lambda_1 - \lambda_2} \left(-\frac{\lambda_1}{1 - \lambda_1 \rho} + \frac{\lambda_2}{1 - \lambda_2 \rho} \right) v_t^i$$

$$\pi_t = \frac{\sigma\kappa}{1 + \sigma\kappa\phi} \frac{1}{\lambda_1 - \lambda_2} \left(\frac{\lambda_2 (1 - \lambda_1 \rho) - \lambda_1 (1 - \lambda_2 \rho)}{(1 - \lambda_1 \rho)(1 - \lambda_2 \rho)} \right) v_t^i$$

$$\pi_t = -\frac{\sigma\kappa}{1 + \sigma\kappa\phi} \left(\frac{1}{(1 - \lambda_1 \rho)(1 - \lambda_2 \rho)} \right) v_t^i$$

Thus, to produce Figure 15, I simply simulate the AR(1) impulse-response, for $\{v_t^i\}$, calculate π_t by the last equation, and calculate $i_t = \phi\pi_t + v_t^i$.

9.5   Impulse-response with long-term debt and price stickiness

I develop the exact nonlinear formulas for the value of surpluses and a linear approximation. The linear approximation turns out to be quite accurate in this application.

An interest rate rise from time $t = 0$ onwards is announced at time $t = -M$. I calculate for each value of the inflation shock δ_{-M} the percent change in a constant surplus corresponding to that shock. Writing the surplus as $se^{\Delta s}$, the value of nominal debt before the shock satisfies

$$\sum_{j=0}^{\infty} \left(\prod_{k=0}^{j-1} \frac{1}{1+f^{(k)}} \right) \frac{B^{(j)}}{P_{-M}} = \sum_{t=-M}^{\infty} \beta^{t-M} s. = \frac{1}{1-\beta} s.$$

where $\{B^{(j)}\}$ is the observed maturity structure of the debt, and the observed forward rates are $f^{(j)}$, ($f_t^{(j)}$ is the forward rate at time t for loans from $t+j$ to $t+j+1$; $f_t^{(0)} = i_t$ is the one-period interest rate). After the shock, nominal interest rates increase by i, the price level jumps from P_{-M} to P_{-M}^*, with

$$e^{\pi_{-M}^*} = e^{\pi_{-M}+\delta_{-M}} = \frac{P_{-M}^*}{P_{-M}}.$$

Here, π_{-M} denotes the solution with $\delta = 0$, so actual inflation after the shock is announced is $\pi_{-M}^* = \pi_{-M} + \delta_{-M}$. The basic solution for inflation (88) includes a jump in inflation when the shock is announced, and I have defined δ as additional unexpected changes in inflation. Surpluses rise to $se^{\Delta s}$, giving

$$\sum_{j=0}^{\infty} \left(\prod_{k=0}^{M-1} \frac{1}{1+f^{(k)}} \right) \left(\prod_{k=M}^{j-1} \frac{1}{1+f^{(k)}+i} \right) \frac{B^{(j)}}{P_{-M}^*} = \sum_{t=-M}^{\infty} \beta^{(t-M)} \frac{u'(C_t)}{u'(C_{-M})} se^{\Delta s} \tag{93}$$

To easily calculate the multiperiod discount factor on the right hand side, I use

$$\frac{u'(C_t)}{u'(C_\tau)} = \frac{e^{-\gamma(c+x_t)}}{e^{-\gamma(c+x_\tau)}} = e^{-\frac{1}{\sigma}(x_t-x_\tau)}$$

Dividing pre and post shock values of (93), s cancels and

$$e^{\pi_{-M}+\delta_{-M}} = \frac{\sum_{j=0}^{\infty} \left(\prod_{k=0}^{M-1} \frac{1}{1+f^{(k)}} \right) \left(\prod_{k=M}^{j-1} \frac{1}{1+f^{(k)}+i} \right) B^{(j)}}{\sum_{j=0}^{\infty} \left(\prod_{k=0}^{j-1} \frac{1}{1+f^{(k)}} \right) B^{(j)}} \frac{\sum_{t=-M}^{\infty} \beta^{(t-M)}}{\sum_{t=-M}^{\infty} \beta^{(t-M)} e^{-\frac{1}{\sigma}(x_t-x_{-M})}} e^{-\Delta s}.$$

Conversely, then, we can find the surplus required to support a given time -M shock δ_{-M} – whether that surplus comes from active or from passive fiscal policy – by solving for Δs,

$$e^{\Delta s} = \frac{\sum_{j=0}^{\infty}\left(\prod_{k=0}^{M-1}\frac{1}{1+f^{(k)}}\right)\left(\prod_{k=M}^{j-1}\frac{1}{1+f^{(k)}+i}\right)B^{(j)}}{\sum_{j=0}^{\infty}\left(\prod_{k=0}^{j-1}\frac{1}{1+f^{(k)}}\right)B^{(j)}}\frac{\sum_{t=-M}^{\infty}\beta^{(t-M)}}{\sum_{t=-M}^{\infty}\beta^{(t-M)}e^{-\frac{1}{\sigma}(x_t-x_{-M})}}e^{-(\pi_{-M}+\delta_{-M})}.$$

(94)

For each choice of δ_{-M}, then, I find the solution for inflation and interest rates by (90)-(92); I compute the product of real rates in the bottom right term of (94), and I compute the required percentage change in surplus Δs. To find the fiscal-theory / long-term debt solution, I search for the δ_{-M} that produces $\Delta s = 0$. It is important to treat the numerator and denominator of the last term of (94) equally. If one truncates the denominator, truncate the numerator at the same point.

9.6   Linearized valuation equation

To linearly approximate (94), write

$$e^{\Delta s} \approx V\frac{\sum_{t=-M}^{\infty}\beta^{(t-M)}}{\sum_{t=-M}^{\infty}\beta^{(t-M)}e^{-\frac{1}{\sigma}(x_t-x_{-M})}}e^{-(\pi_{-M}+\delta_{-M})}.$$

(95)

$$1+\Delta s \approx (1+v)\frac{\sum_{t=-M}^{\infty}\beta^{(t-M)}}{\sum_{t=-M}^{\infty}\beta^{(t-M)}\left(1-\frac{1}{\sigma}(x_t-x_{-M})\right)}(1-(\pi_{-M}+\delta_{-M})).$$

(96)

$$1+\Delta s \approx (1+v)\frac{1}{1-\frac{\sum_{t=-M}^{\infty}\beta^{(t-M)}\frac{1}{\sigma}(x_t-x_{-M})}{\sum_{t=-M}^{\infty}\beta^{(t-M)}}}(1-(\pi_{-M}+\delta_{-M})).$$

(97)

$$\Delta s \approx v + \frac{\sum_{t=-M}^{\infty}\beta^{(t-M)}\frac{1}{\sigma}(x_t-x_{-M})}{\sum_{t=-M}^{\infty}\beta^{(t-M)}}-(\pi_{-M}+\delta_{-M}).$$

(98)

$$\Delta s \approx v + (1-\beta)\sum_{t=-M}^{\infty}\beta^{(t-M)}\frac{1}{\sigma}(x_t-x_{-M})-(\pi_{-M}+\delta_{-M}).$$

(99)

In numerical experimentation, it turns out that the exact and linearized approach produce almost exactly the same answer to the first few decimals. So, the nonlinearity of long-term present values is not an issue for this magnitude – a few percent at most – of interest rate variation.

For one-period debt, v is unchanged so we have

$$\Delta s \approx -\Delta E_t\left(\pi_t\right) + \frac{1-\beta}{\sigma}\sum_{j=0}^{\infty}\beta^j \Delta E_t\left(x_{t+j} - x_t\right) \tag{100}$$

where $\Delta E_t \equiv E_t - E_{t-1}$ and t is the date of the announcement of a new policy.

The first term of (100) captures the fact that unexpected inflation devalues outstanding government debt. In the second term, $(x_{t+j} - x_t)/\sigma$ is the real interest rate between time t and time $t+j$. So this term captures the fact that if real rates rise, the government must pay more interest on the debt.

9.7 The Model with Money

This section derives the model with money (73). The utility function is

$$\max E \int_{t=0}^{\infty} e^{-\delta t} u(c_t, M_t/P_t) dt.$$

The present-value budget constraint is

$$\frac{B_0 + M_0}{P_0} = \int_{t=0}^{\infty} e^{-\int_{s=0}^{t} r_s ds}\left[c_t - y_t + s_t + (i_t - i_t^m)\frac{M_t}{P_t}\right] dt$$

where

$$r_t = i_t - \frac{dP_t}{P_t}$$

and s denotes real net taxes paid, and thus the real government primary surplus. This budget constraint is the present value form of

$$d(B_t + M_t) = i_t B_t + i_t^m M_t + P_t(y_t - c_t - s_t).$$

Introducing a multiplier λ on the present value budget constraint, we have

$$\frac{\partial}{\partial c_t} : e^{-\delta t} u_c(t) = \lambda e^{-\int_{s=0}^{t} r_s ds},$$

where (t) means $(c_t, M_t/P_t)$. Differentiating with respect to time,

$$-\delta e^{-\delta t} u_c(t) + e^{-\delta t} u_{cc}(t)\frac{dc_t}{dt} + e^{-\delta t} u_{cm}(t)\frac{dm_t}{dt} = -\lambda r_t\, e^{-\int_{s=0}^{t} r_s ds}$$

where $m_t \equiv M_t / P_t$. Dividing by $e^{-\delta t} u_c(t)$, we obtain the intertemporal first order condition:

$$- \frac{c_t u_{cc}(t)}{u_c(t)} \frac{dc_t}{c_t} - \frac{m_t u_{cm}(t)}{u_c(t)} \frac{dm_t}{m_t} = (r_t - \delta) \, dt. \tag{101}$$

The first-order condition with respect to M is

$$\frac{\partial}{\partial M_t} : e^{-\delta t} u_m(t) \frac{1}{P_t} = \lambda e^{-\int_{s=0}^{t} r_s ds} (i_t - i_t^m) \frac{1}{P_t}$$

$$e^{-\delta t} u_m(t) = e^{-\delta t} u_c(t) (i_t - i_t^m)$$

$$\frac{u_m(t)}{u_c(t)} = i_t - i_t^m. \tag{102}$$

The last equation is the usual money demand curve.

Thus, an equilibrium $c_t = y_t$ satisfies

$$\frac{-c_t u_{cc}(t)}{u_c(t)} \frac{dc_t}{c_t} - \frac{m_t u_{cm}(t)}{u_c(t)} \frac{dm_t}{m_t} = -\delta dt + \left(i_t - \frac{dP_t}{P_t} \right) dt \tag{103}$$

$$\frac{u_m(t)}{u_c(t)} = i_t - i_t^m \tag{104}$$

$$\frac{B_0 + M_0}{P_0} = \int_{t=0}^{\infty} e^{-\int_{s=0}^{t} r_s ds} \left[s_t + (i_t - i_t^m) \frac{M_t}{P_t} \right] dt \tag{105}$$

The last equation combines the consumer's budget constraint and equilibrium $c = y$. I call it the government debt valuation formula.

9.7.1   CES functional form

I use a standard money in the utility function specification with a CES functional form,

$$u(c_t, m_t) = \frac{1}{1 - \gamma} \left[c_t^{1-\theta} + \alpha m_t^{1-\theta} \right]^{\frac{1-\gamma}{1-\theta}}.$$

I use the notation $m = M/P$, with capital letters for nominal and lowercase letters for real quantities.

This CES functional form nests three important special cases. Perfect substitutes is the case $\theta = 0$:

$$u(c_t, m_t) = \frac{1}{1 - \gamma} \left[c_t + \alpha m_t \right]^{1-\gamma}.$$

The Cobb-Douglas case is $\theta \longrightarrow 1$:

$$u(c_t, m_t) \longrightarrow \frac{1}{1-\gamma} \left[c_t^{\frac{1}{1+\alpha}} m_t^{\frac{\alpha}{1+\alpha}} \right]^{1-\gamma}. \tag{106}$$

The monetarist limit is $\theta \to \infty$:

$$u(c_t, m_t) \to \frac{1}{1-\gamma} \left[\min\left(c_t, \alpha m_t \right) \right]^{1-\gamma}.$$

I call it the monetarist limit because money demand is then $M_t/P_t = c_t/\alpha$, i.e. $\alpha = 1/V$ is constant, and the interest elasticity is zero. The separable case is $\theta = \gamma$:

$$u(c_t, m_t) = \frac{1}{1-\gamma} \left[c_t^{1-\gamma} + \alpha m_t^{1-\gamma} \right].$$

In the separable case, u_c is independent of m, so money has no effect on the intertemporal substitution relation, and hence on inflation and output dynamics in a new-Keynesian model under an interest rate target. Terms in $(\theta - \gamma)$ or $(\sigma - \xi)$ with $\sigma = 1/\gamma$ and $\xi = 1/\theta$ will characterize deviations from the separable case, how much the marginal utility of consumption is affected by money.

With this functional form, the derivatives are

$$u_c = \left[c_t^{1-\theta} + \alpha m_t^{1-\theta} \right]^{\frac{\theta-\gamma}{1-\theta}} c_t^{-\theta}$$

$$u_m = \left[c_t^{1-\theta} + \alpha m_t^{1-\theta} \right]^{\frac{\theta-\gamma}{1-\theta}} \alpha m_t^{-\theta}.$$

Equilibrium condition (104) becomes

$$\frac{u_m(t)}{u_c(t)} = \alpha \left(\frac{m_t}{c_t} \right)^{-\theta} = i_t - i_t^m. \tag{107}$$

The second derivative with respect to consumption is

$$\frac{u_{cc}}{u_c} = (\theta - \gamma) \frac{1}{\left[c_t^{1-\theta} + \alpha m_t^{1-\theta} \right]} c_t^{-\theta} - \theta c_t^{-1}$$

$$-\frac{c u_{cc}}{u_c} = -\frac{(\theta - \gamma) c_t^{1-\theta} - \theta \left[c_t^{1-\theta} + \alpha m_t^{1-\theta} \right]}{\left[c_t^{1-\theta} + \alpha m_t^{1-\theta} \right]}$$

$$-\frac{cu_{cc}}{u_c} = \frac{\gamma c_t^{1-\theta} + \theta\alpha m_t^{1-\theta}}{c_t^{1-\theta} + \alpha m_t^{1-\theta}}$$

$$-\frac{cu_{cc}}{u_c} = \gamma\frac{\left[1 + \frac{\theta}{\gamma}\alpha\left(\frac{m_t}{c_t}\right)^{1-\theta}\right]}{\left[1 + \alpha\left(\frac{m_t}{c_t}\right)^{1-\theta}\right]}.$$

The cross derivative is

$$\frac{mu_{cm}}{u_c} = (\theta - \gamma)\frac{\alpha m_t^{1-\theta}}{c_t^{1-\theta} + \alpha m_t^{1-\theta}}$$

$$= (\theta - \gamma)\frac{\alpha\left(\frac{m_t}{c_t}\right)^{1-\theta}}{1 + \alpha\left(\frac{m_t}{c_t}\right)^{1-\theta}}.$$

or, using (107)

$$\frac{mu_{cm}}{u_c} = (\theta - \gamma)\frac{\left(\frac{m_t}{c_t}\right)(i_t - i_t^m)}{1 + \left(\frac{m_t}{c_t}\right)(i_t - i_t^m)}.$$

9.7.2   Money demand

Money demand (107) can be written

$$\frac{m_t}{c_t} = \left(\frac{1}{\alpha}\right)^{-\xi}(i_t - i_t^m)^{-\xi}. \tag{108}$$

where $\xi = 1/\theta$ becomes the interest elasticity of money demand, in log form, and α governs the overall level of money demand.

The steady state obeys

$$\frac{m}{c} = \left(\frac{1}{\alpha}\right)^{-\xi}(i - i^m)^{-\xi}. \tag{109}$$

so we can write money demand (108) in terms of steady state real money as

$$\frac{m_t}{c_t} = \left(\frac{m}{c}\right)\left(\frac{i_t - i_t^m}{i - i^m}\right)^{-\xi}, \tag{110}$$

avoiding the parameter α. (Throughout, numbers without time subscripts denote steady state values.)

The product $\frac{m}{c}(i - i^m)$, the interest cost of holding money, appears in many subsequent

expressions. It is

$$\frac{m}{c}\left(i - i^m\right) = \left(\frac{1}{\alpha}\right)^{-\xi}\left(i - i^m\right)^{1-\xi}.$$

With $\xi < 1$, as interest rates go to zero this interest cost goes to zero as well.

9.7.3   Intertemporal Substitution

The first order condition for the intertemporal allocation of consumption (103) is

$$-\frac{c_t u_{cc}(t)}{u_c(t)}\frac{dc_t}{c_t} - \frac{m_t u_{cm}(t)}{u_c(t)}\frac{dm_t}{m_t} = -\delta dt + \left(i_t - \pi_t\right)dt$$

where $\pi_t = dP_t/P_t$ is inflation. This equation shows us how, with nonseparable utility, monetary policy can distort the allocation of consumption over time, in a way not captured by the usual interest rate effect. That is the central goal here. In the case of complements, $u_{cm} > 0$ (more money raises the marginal utility of consumption), larger money growth makes it easier to consume in the future relative to the present, and acts like a higher interest rate, inducing higher consumption growth.

Substituting in the CES derivatives,

$$\gamma\frac{1 + \frac{\theta}{\gamma}\alpha\left(\frac{m_t}{c_t}\right)^{1-\theta}}{1 + \alpha\left(\frac{m_t}{c_t}\right)^{1-\theta}}\frac{dc_t}{c_t} - (\theta - \gamma)\frac{\alpha\left(\frac{m_t}{c_t}\right)^{1-\theta}}{1 + \alpha\left(\frac{m_t}{c_t}\right)^{1-\theta}}\frac{dm_t}{m_t} = -\delta dt + \left(i_t - \pi_t\right)dt$$

and using (107) to eliminate α

$$\gamma\frac{1 + \frac{\theta}{\gamma}\left(\frac{m_t}{c_t}\right)\left(i_t - i_t^m\right)}{1 + \left(\frac{m_t}{c_t}\right)\left(i_t - i_t^m\right)}\frac{dc_t}{c_t} - (\theta - \gamma)\frac{\left(\frac{m_t}{c_t}\right)\left(i_t - i_t^m\right)}{1 + \left(\frac{m_t}{c_t}\right)\left(i_t - i_t^m\right)}\frac{dm_t}{m_t} = -\delta dt + \left(i_t - \pi_t\right)dt \qquad (111)$$

We can make this expression prettier as

$$\gamma\frac{dc_t}{c_t} + (\theta - \gamma)\frac{\left(\frac{m_t}{c_t}\right)\left(i_t - i_t^m\right)}{1 + \left(\frac{m_t}{c_t}\right)\left(i_t - i_t^m\right)}\left(\frac{dc_t}{c_t} - \frac{dm_t}{m_t}\right) = -\delta dt + \left(i_t - \pi_t\right)dt$$

Rexpressing in terms of the intertemporal substitution elasticity $\sigma = 1/\gamma$ and interest elasticity

of money demand $\xi = 1/\theta$, and multiplying by σ,

$$\frac{dc_t}{c_t} + \left(\frac{\sigma - \xi}{\xi}\right) \frac{\left(\frac{m_t}{c_t}\right)(i_t - i_t^m)}{1 + \left(\frac{m_t}{c_t}\right)(i_t - i_t^m)} \left(\frac{dc_t}{c_t} - \frac{dm_t}{m_t}\right) = -\delta\sigma dt + \sigma(i_t - \pi_t)\, dt. \qquad (112)$$

We want to substitute interest rates for money. To that end, differentiate the money demand curve

$$\frac{m_t}{c_t} = \left(\frac{m}{c}\right)\left(\frac{i_t - i_t^m}{i - i^m}\right)^{-\xi}$$

$$\frac{m_t}{c_t}\left(\frac{dm_t}{m_t} - \frac{dc_t}{c_t}\right) = -\xi\left(\frac{m}{c}\right)\left(\frac{i_t - i_t^m}{i - i^m}\right)^{-\xi}\frac{d(i_t - i_t^m)}{i_t - i_t^m}$$

$$\left(\frac{dc_t}{c_t} - \frac{dm_t}{m_t}\right) = \xi\frac{\frac{m}{c}}{\frac{m_t}{c_t}}\left(\frac{i_t - i_t^m}{i - i^m}\right)^{-\xi}\frac{d(i_t - i_t^m)}{i_t - i_t^m}$$

Substituting,

$$\frac{dc_t}{c_t} + \left(\frac{\sigma - \xi}{\xi}\right) \frac{\left(\frac{m_t}{c_t}\right)(i_t - i_t^m)}{1 + \left(\frac{m_t}{c_t}\right)(i_t - i_t^m)} \left(\xi\frac{\frac{m}{c}}{\frac{m_t}{c_t}}\left(\frac{i_t - i_t^m}{i - i^m}\right)^{-\xi}\frac{d(i_t - i_t^m)}{i_t - i_t^m}\right) = -\delta\sigma dt + \sigma(i_t - \pi_t)\, dt.$$

$$\frac{dc_t}{c_t} + (\sigma - \xi)\frac{m}{c}\frac{1}{1 + \left(\frac{m_t}{c_t}\right)(i_t - i_t^m)}\left(\frac{i_t - i_t^m}{i - i^m}\right)^{-\xi}d(i_t - i_t^m) = -\delta\sigma dt + \sigma(i_t - \pi_t)\, dt.$$

With $x_t = \log c_t$, $dx_t = dc_t/c_t m$, approximating around a steady state, and approximating that the interest cost of holding money is small, $\left(\frac{m}{c}\right)(i - i^m) \ll 1$, we obtain the intertemporal substitution condition modified by interest costs,

$$\frac{dx_t}{dt} + (\sigma - \xi)\frac{m}{c}\frac{d(i_t - i_t^m)}{dt} = \sigma(i_t - \pi_t). \qquad (113)$$

In discrete time,

$$E_t x_{t+1} - x_t + (\sigma - \xi)\left(\frac{m}{c}\right)\left[E_t\left(i_{t+1} - i_{t+1}^m\right) - (i_t - i_t^m)\right] = \sigma(i_t - E_t\pi_{t+1}).$$

For models with monetary control, one wants an IS curve expressed in terms of the monetary aggregate. From (112), with the same approximations and $\tilde{m} = \log(m)$,

$$\frac{dx_t}{dt} + \left(\frac{\sigma - \xi}{\xi}\right)\left(\frac{m}{c}\right)(i - i^m)\left(\frac{dx_t}{dt} - \frac{d\tilde{m}_t}{dt}\right) = \sigma(i_t - \pi_t)\, dt. \qquad (114)$$

In discrete time,

$$(E_t x_{t+1} - x_t) + \left(\frac{\sigma - \xi}{\xi}\right) \left(\frac{m}{c}\right) (i - i^m) \left[(E_t x_{t+1} - x_t) - E_t \left(\tilde{m}_{t+1} - \tilde{m}_t\right)\right] = \sigma \left(i_t - \pi_t\right). \quad (115)$$