MOST IMPORTANT RULE

$E(R^i) = a + b(\text{SIZE}_i) + c(B/M_i)$?

"FAMA FRENCH MODEL SAYS SMALL / VALUE STOCKS GET HIGHER RETURNS!

- DESCRIPTION VS EXPLANATION
- CHARACTERISTIC VS BETA
- WHO YOU ARE VS HOW YOU BEHAVE
- IF SO FORTUNE TO BE MADE!
- "EXPLANATION" MUST HAVE β's

$E(R^i) = \underline{\underline{b}}_i E(RMRF) + \underline{\underline{h}}_i E(HML) \ldots$

OPTIONS 1 INTRO AND PAYOFFS

- OBJECT: OPTION PRICE ← STOCK + BOND; "RELATIVE PRICING" NOT "EXPLANATION"
- METHOD: $P = E(mx)$ LEARN ABOUT m FROM S, B.
- WAS "ARBITRAGE" NOW LESS SO

PAYOFFS X

VALUE AT T

$C_T = X_T^C$
$= \max(S_T - X, 0)$

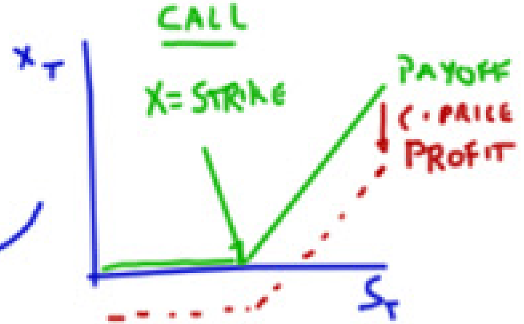
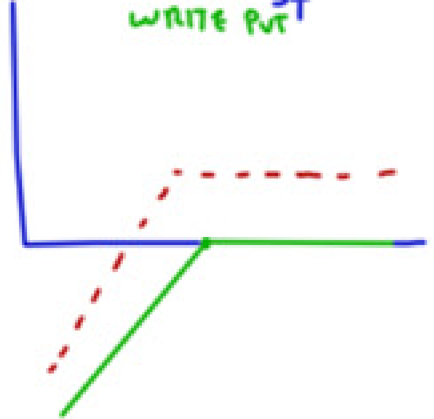
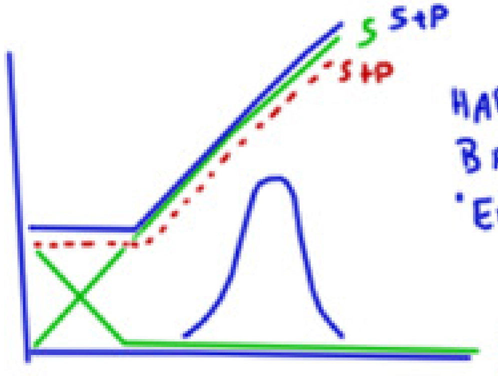


FIG. 17.1 ASSET PRICING



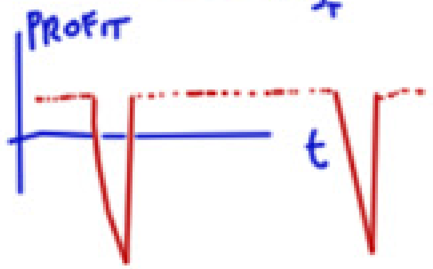
"BUY DISASTER INSURANCE"

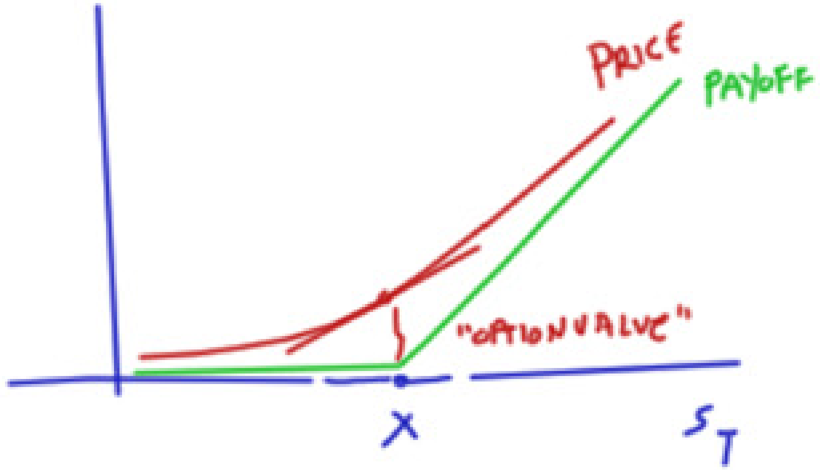
STOCK + OTM PUT



HABITS?
BANK
"EXPENSIVE"?

WRITE OTM PUT



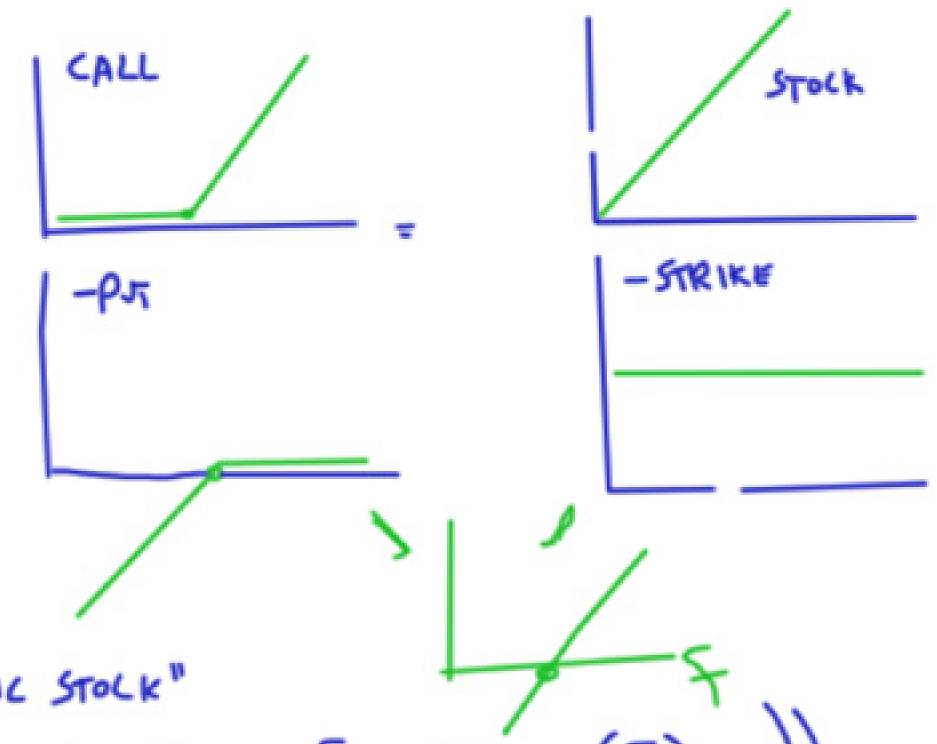


- OBJECT: PRICE, "δ"
- WHY: A) IDEAL TRADING VEHICLE
  - $S=100 \quad C=10 \quad \delta=\frac{1}{2} \rightarrow \beta=5!$
  - CAN'T DEFAULT!

  B) HEDGING, BET ON VOLATILITY, ETC

OPTIONS 2: ARBITRAGE BOUNDS

PRICE $\leftarrow S, B$? AVOID $V'(.)$



"SYNTHETIC STOCK"

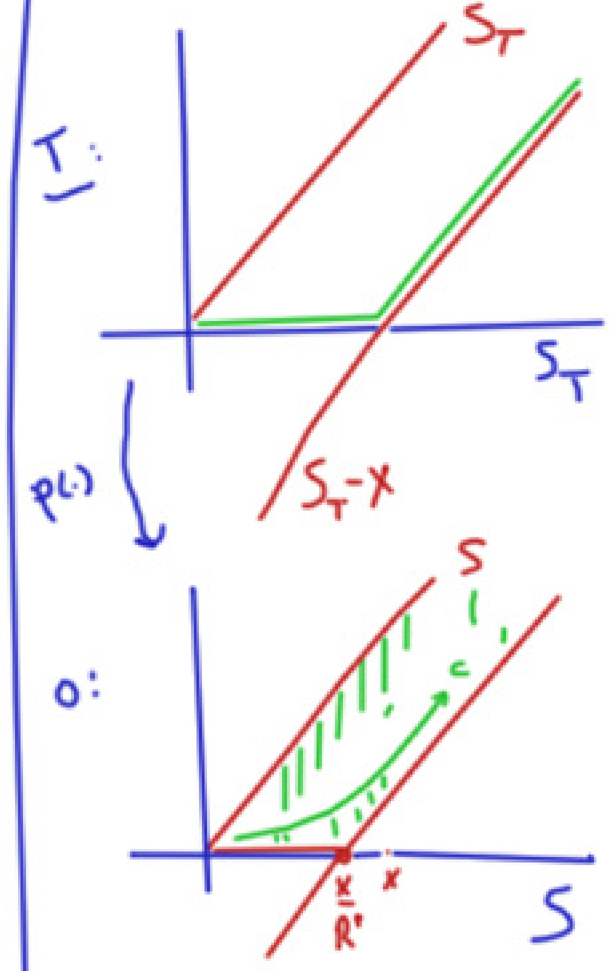
X: $\quad C_T - P_T = S_T - X \quad (T)$ $\quad \Downarrow P(.)$

$P=E(mx)$: $\quad C - P = S - X/R^f \quad (0 \text{ or } t)$

PUT-CALL PARITY $\quad$ (m: LOOP)

$E(mX^A) > E(mX^B)$

$X^A \geq X^B \quad (s.) \quad m>0 \Rightarrow P(X^A) \geq P(X^B)$



$\rightarrow$ $C > 0$
$C < S$
$C > S - X/R^f$

$\Rightarrow$ MORE FREQUENT TRADING!

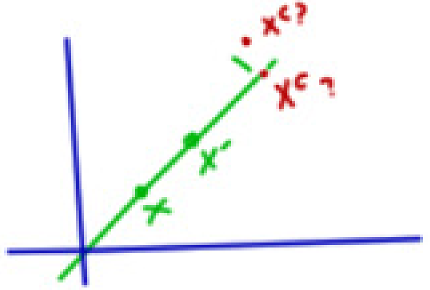
OPTIONS 3: BLACK-SCHOLES

- CONTINUOUS TRADING

$$\frac{dS}{S} = \mu\,dt + \sigma\,dz \quad ; \quad \frac{dB}{B} = r\,dt$$

- FIND x^* THAT PRICES S, B AND ALL PAYOFFS FORMED BY DYNAMIC TRADING.

$C = E(x^* x^c)$



$$x^* = \frac{d\Lambda}{\Lambda} = -r\,dt - \frac{\mu - r}{\sigma}\,dz$$

$$\left(\begin{array}{l} \text{CHECK}: E\left(\frac{d\Lambda}{\Lambda}\right) = -r\,dt \\ E\left(\frac{dS}{S}\right) - r\,dt = -E\left(\frac{d\Lambda}{\Lambda}\frac{dS}{S}\right) \end{array} \right)$$

$$\left[\begin{array}{l} \frac{S_T(\varepsilon)}{S_0} = e^{(\mu - \frac{1}{2}\sigma^2)T + \sigma\sqrt{T}\,\varepsilon} \\ \frac{\Lambda_T(\varepsilon)}{\Lambda_0} = e^{-\left[r + \frac{1}{2}\left(\frac{\mu - r}{\sigma}\right)^2\right]T - \frac{\mu - r}{\sigma}\sqrt{T}\,\varepsilon} \end{array} \right.$$

$\varepsilon \sim N(0,1)$ SAME ε

$$C_0 = E_0\left(\frac{\Lambda_T}{\Lambda_0}\max(S_T - X, 0)\right)$$

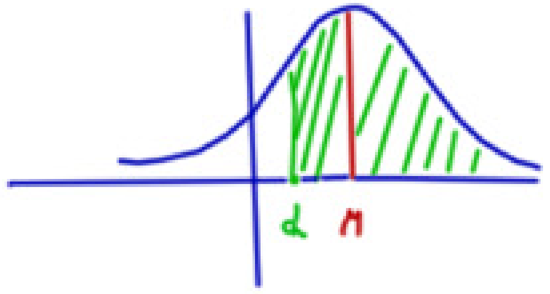
$$C_0 = S_0 \int_{S_T = X}^{\infty} \frac{\Lambda_T(\varepsilon)}{\Lambda_0} \cdot \frac{S_T(\varepsilon)}{S_0} \frac{e^{-\frac{1}{2}\varepsilon^2}}{\sqrt{2\pi}}\,d\varepsilon - X \int_{S_T = X}^{\infty} \frac{\Lambda_T(\varepsilon)}{\Lambda_0} e^{-\frac{1}{2}\varepsilon^2}\,d\varepsilon$$

...

$$C_0 = \frac{S_0}{\sqrt{2\pi}} \int_{S_T = X}^{\infty} e^{-\frac{1}{2}\left[\varepsilon - \left(\sigma - \frac{\mu - r}{\sigma}\right)\sqrt{T}\right]^2} d\varepsilon - \frac{X e^{-rT}}{\sqrt{2\pi}} \int_{S_T = X}^{\infty} e^{-\frac{1}{2}\left(\varepsilon - \frac{\mu - r}{\sigma}\sqrt{T}\right)^2} d\varepsilon$$

$$S_T = X \rightarrow \frac{X}{S_0} = e^{(\mu - \frac{1}{2}\sigma^2)T + \sigma\sqrt{T}\,\varepsilon} \rightarrow \varepsilon = -\frac{1}{\sigma\sqrt{T}}\left[\log\left(\frac{S_0}{X}\right) + \left(\mu - \frac{1}{2}\sigma^2\right)T\right]$$

$$\Phi(\mu - d) = \frac{1}{\sqrt{2\pi}} \int_{\varepsilon = d}^{\infty} e^{-\frac{1}{2}(\varepsilon - \mu)^2}\,d\varepsilon$$



$$\Rightarrow C_0 = S_0\,\Phi\left(\frac{\ln(S_0/X) + (r+\frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}}\right) - Xe^{-rT}\,\Phi\left(\frac{\ln(S/X) + (r-\frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}}\right)$$

- NOTE: . μ DISAPPEARED! "ARBITRAGE". μ INS!
  - . σ HERE! MORE $\sigma \rightarrow C\uparrow$
  - . "IMPLIED VOL" $\rightarrow \sigma$ TO JUSTIFY C, STANDARDIZES ACROSS X, T LIKE YIELD
- $\rightarrow$ DERIVATIVES, "GREEKS"

OPTIONS 4: OTHER METHODS

$\rightarrow$ DIFFERENTIAL EQUATION. SOLVE PRICE NOT DISCOUNT FACTOR

GUESS $C(S,t)$

ITO $dC = C_t\,dt + C_s\,dS + \frac{1}{2}C_{ss}\,dS^2$

$$dC = \left[C_t + C_s S\mu + \frac{1}{2}C_{ss}S^2\sigma^2\right]dt + C_s S\sigma\,dz$$

PRICE BY Λ

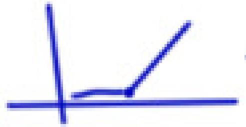
$$E\left(\frac{dC}{C}\right) - r\,dt = -E\left(\frac{dC}{C}\frac{d\Lambda}{\Lambda}\right)$$

$\times C$ $\quad C_t + C_s S\mu + \frac{1}{2}C_{ss}S^2\sigma^2 - rC = C_s S\sigma\left(\frac{\mu - r}{\sigma}\right)$

$$-rC + S r C_s + C_t + \frac{1}{2}C_{ss}S^2\sigma^2 = 0$$

"BLACK-SCHOLES DIFFERENTIAL EQ"

ANY ASSET WHOSE PRICE (S,t) SOLVES

BOUNDARY $C(S_T, T) =$  $= \max(S_T - X, 0)$

SOLVE BACK

$$C(S, t-\Delta t) = C(S,t) - \frac{\partial C(S)}{\partial t}\Delta t =$$

$$= C(S,t) + \left[rS\frac{\partial C(S,t)}{\partial S} + \frac{1}{2}\sigma^2 S^2\frac{\partial^2 C(S,t)}{\partial S^2} - rC(S,t)\right]\Delta t$$

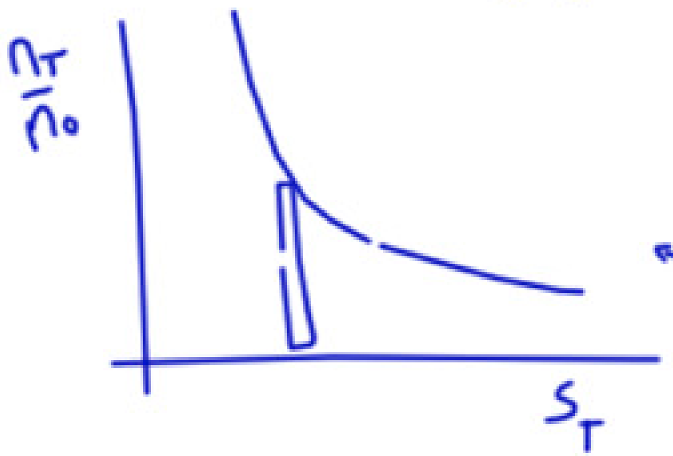
- GUESS + CHECK
- TRANSFORM TO INTEGRAL $\quad\rightarrow$ ARBITRAGE
- NUMERICAL.

OPTIONS 5

SPANNING, STATE PRICES + CURRENT MODELS

NOW

$$\frac{m_T}{m_0} = e^{\frac{1}{2}\left[\frac{\mu^2 - r^2}{\sigma^2} - (\mu + r)\right]T}\left(\frac{S_T}{S_0}\right)^{-\frac{\mu - r}{\sigma^2}}$$



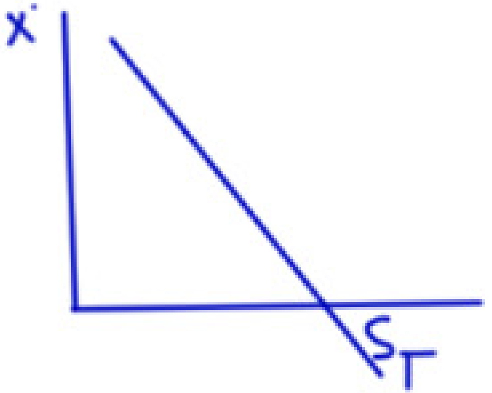
DISCOUNT FACTOR
STATE PRICE
MARGINAL UTILITY

FOR ALL MGD PORTFOLIOS

FOR R, R^f ONLY

WAS $\quad x^* = \frac{1}{R^f} - \frac{E(R - R^f)}{R^f \sigma^2}(R - R^f)$

$x^* = a - b\left(S_T / S_0\right)$



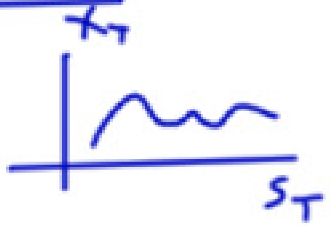
$$\frac{dV}{V} = r\,dt + w(t)\left(\frac{dS}{S} - r\,dt\right)$$

$V_T =$

EVEN IF B-S DOES NOT HOLD

SPANNING

"ANY"



← DYNAMIC TRADING S, B

← FULL SET OF C, P OPTIONS

ALL X

"COMPLETE MARKET"

FINDING Λ (EVEN WHEN B-S FAILS)

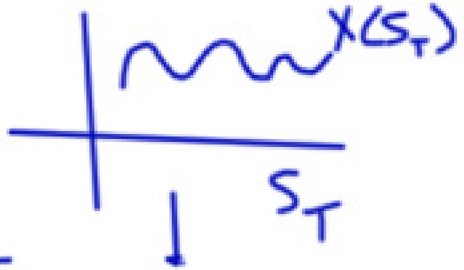
$$x^* = p' E(xx')^{-1} x$$

FACT YOU CAN FIND Λ FROM 2ND DERIVATIVE OF C WRT X

PAYOFF ε^2

PRICE $= \cdot\left[C(X - \varepsilon) - 2C(X) + C(X + \varepsilon)\right] \rightarrow \varepsilon^2 \frac{\partial^2 C(X)}{\partial X}$

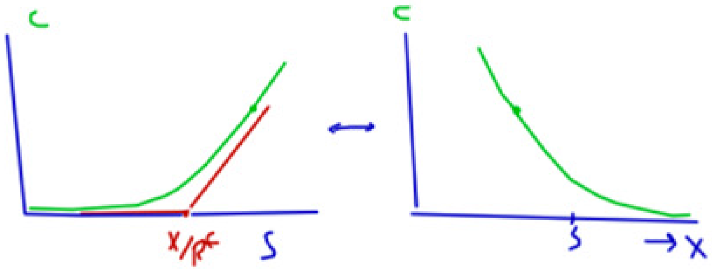
BUY $\frac{1}{\varepsilon^2} \rightarrow \frac{\partial^2 C}{\partial X^2} =$ C. CLAIM PRICE



$$\Rightarrow \quad P = \int \left.\frac{\partial^2 C(S, X)}{\partial X^2}\right|_{X = S_T} X(S_T)\, dS_T$$

$P(X(S_T))$?

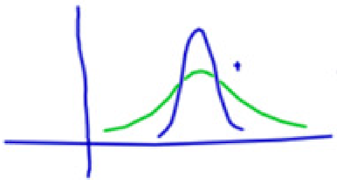
OPTIONS. DATA, SMILE, MODELS

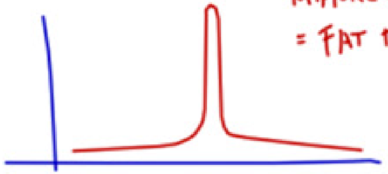


- OTM PUTS "CHEAP"?
- OTM PUTS "EXPENSIVE"!
  "WRITING DISASTER INSURANCE"
- FIX BLACK-SCHOLES?
  - RISK AVERSION?
  - PROBABILITY TOO!

1) PAST $dS_t/S_t = \mu dt + \sigma dz_t$; r^f

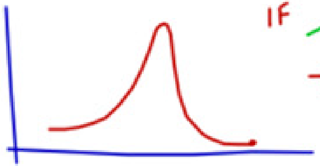
A) "STOCHASTIC VOLATILITY"





MIXTURE OF NORMALS
= FAT TAILS

VOLATILITY CHANGES,
HIGHER IN DOWNTURNS
⇒ FAT LEFT TAIL



IF 

→ FAT LEFT TAIL

$$\frac{dS_t}{S_t} = \mu_s dt + \sqrt{V_t}\, dz_t$$

— CORRELATION

$$dV_t = -\phi(\bar{V} - V_t) + \sigma_V \sqrt{V_t}\, dw_t$$

$$\frac{d\Lambda_t}{\Lambda_t} = -r\,dt - \frac{\mu_s - r}{V_t} dz_t - \gamma_w dw_t$$

"MARKET PRICE OF VOLATILITY RISK"

$\Rightarrow C^i(X^i, S, V, t;\ \gamma_w, \sigma_V)$

"FACTOR" MODEL → FIND V (LIKE σ)?

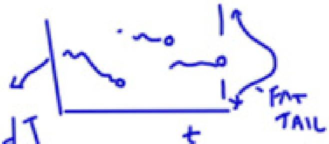
γ_w, σ_V FROM A FEW C^i

"SPAN" dw RISK

1) $C_t = E_t\left(\frac{\Lambda_T}{\Lambda_t} \cdot x_T^c\right)$ ⇐ YOU CAN DO THIS!

2) $\partial C/\partial t = \dots\ \partial C/\partial S,\ \partial C/\partial V$

B) JUMPS



$$\frac{dS_t}{S_t} = \mu\, dt + \sigma\, dz_t + dJ_t$$

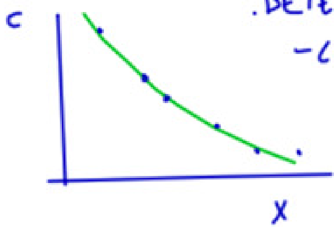
NO ITO

$$\frac{d\Lambda}{\Lambda} = -r\, dt - \frac{\mu - r}{\sigma} dz_t - \gamma_J\, dJ$$

MARKET PRICE OF JUMP RISK

- PRICING BY MOSTLY ARBITRAGE,

- A DISCIPLINED WAY TO PRICE "NEARBY" SECURITIES



. DETECT "MISPRICING"

- CONSTRUCT HEDGE

· ISOLATE PREMIUMS