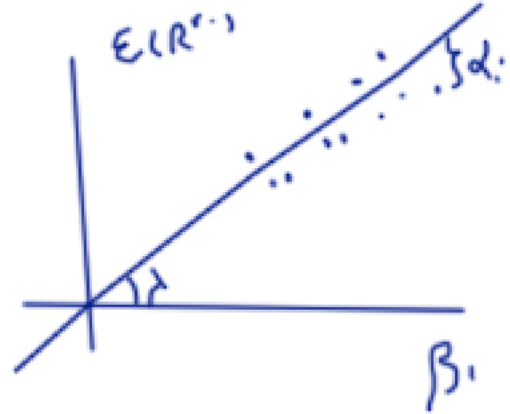
1. CLASSIC REGRESSION TESTS - MOTIVATION + OUTLINE



- MODEL $R^{ei}_t = a_i + \beta_i f_t + \varepsilon_{it}$

  $E(R^{ei}) = \beta_i \lambda + \alpha_i$

- ESTIMATE $\hat{\alpha}\ \hat{\beta}\ \hat{\lambda}$ ..
- STD ERRORS $\sigma(\hat{\alpha})\ \sigma(\hat{\beta})\ \sigma(\hat{\lambda})$
- TEST $\hat{\alpha}' V^{-1} \hat{\alpha}$ "TOO BIG"
- EVALUATE, DIAGNOSE SML "TOO FLAT"?

  $E(R^{ei}) = \beta_i \lambda + \overset{?}{c_i} + \overset{?}{\sigma_i^2} + \dots$ β VS CHARACTERISTICS? σ^2 β^2 MATTER?

  $\hat{\lambda}$ REASONABLE? ONE MODEL VS ANOTHER?

  (STATISTICAL VS. INFORMAL "TEST")

E(R) vs β, not m

- HISTORY: iid N REGRESSION

  NON GMM: BOOTSTRAP, MONTE CARLO

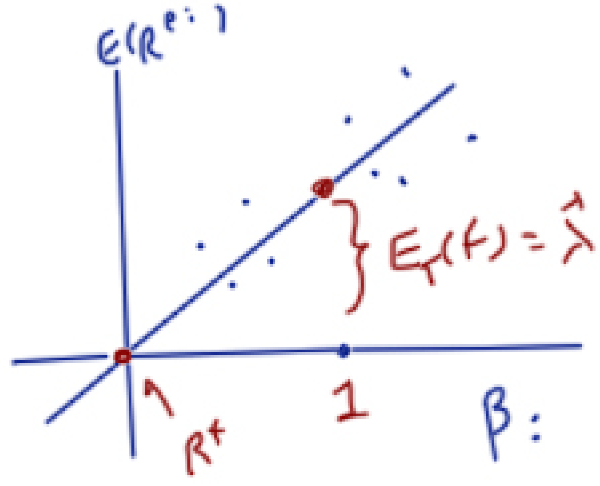
- ALL METHODS BASICALLY THE SAME

  1) TIME SERIES
  2) CROSS SECTION
  3) FAMA-MACBETH
  4) SDF-GMM

2. TIME SERIES / GRS [FAMA-FRENCH MULTIFACTOR ANOMALIES]

- f is a R^e, so $\lambda = E(f)$ [MODEL, POPULATION]

- $R^{ei}_t = \alpha_i + \beta_i f_t + \varepsilon_{it}$ $\quad t = 1, 2, \ldots T \quad \forall i$



IMPLIED $E(R^{ei}) = \alpha_i + \beta_i E(f)$

- ESTIMATES OLS $\hat{\alpha}$ $\hat{\beta}$; $\hat{\lambda} = E_T(f)$
- S.E. OLS [IID W OR GMM]
- TEST ALL α JOINTLY ZERO $\quad \hat{\alpha} = [\hat{\alpha}_1 \; \hat{\alpha}_2 \ldots \hat{\alpha}_N]'$

$$\hat{\alpha}' \operatorname{cov}(\hat{\alpha}, \hat{\alpha}')^{-1} \hat{\alpha} \sim \chi^2 \text{ OR } F$$

INTERCEPTS OF N REGRESSIONS?

a) ε^i IID N $\rightarrow$ $\dfrac{T-N-K}{N} \left[1 + \bar{f}' \hat{\Sigma}_f^{-1} \bar{f}\right]^{-1} \hat{\alpha}' \hat{\Sigma}^{-1} \hat{\alpha} \sim F_{N, T-N-K}$

$\operatorname{cov}(\varepsilon, \varepsilon')$

b) GMM: $g_T(b) = \begin{bmatrix} E_T(\varepsilon) \\ E_T(f_t \varepsilon_t) \end{bmatrix} = \begin{bmatrix} E_T(R^e_t - \alpha - \beta f_t) \\ E_T[(R^e_t - \alpha - \beta f_t) f_t] \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ $\quad \begin{bmatrix} \updownarrow N \\ \updownarrow N \end{bmatrix}$

STACK N REGRESSIONS IS EASY!

$\rightarrow \sigma(\hat{\alpha})\ \sigma(\hat{\beta})\ \ \hat{\alpha}' \operatorname{cov}(\hat{\alpha})^{-1} \hat{\alpha} \sim \chi^2$ ALLOWING A.C. + CHS.

c) MONTE CARLO | BOOTSTRAP FINITE SAMPLE

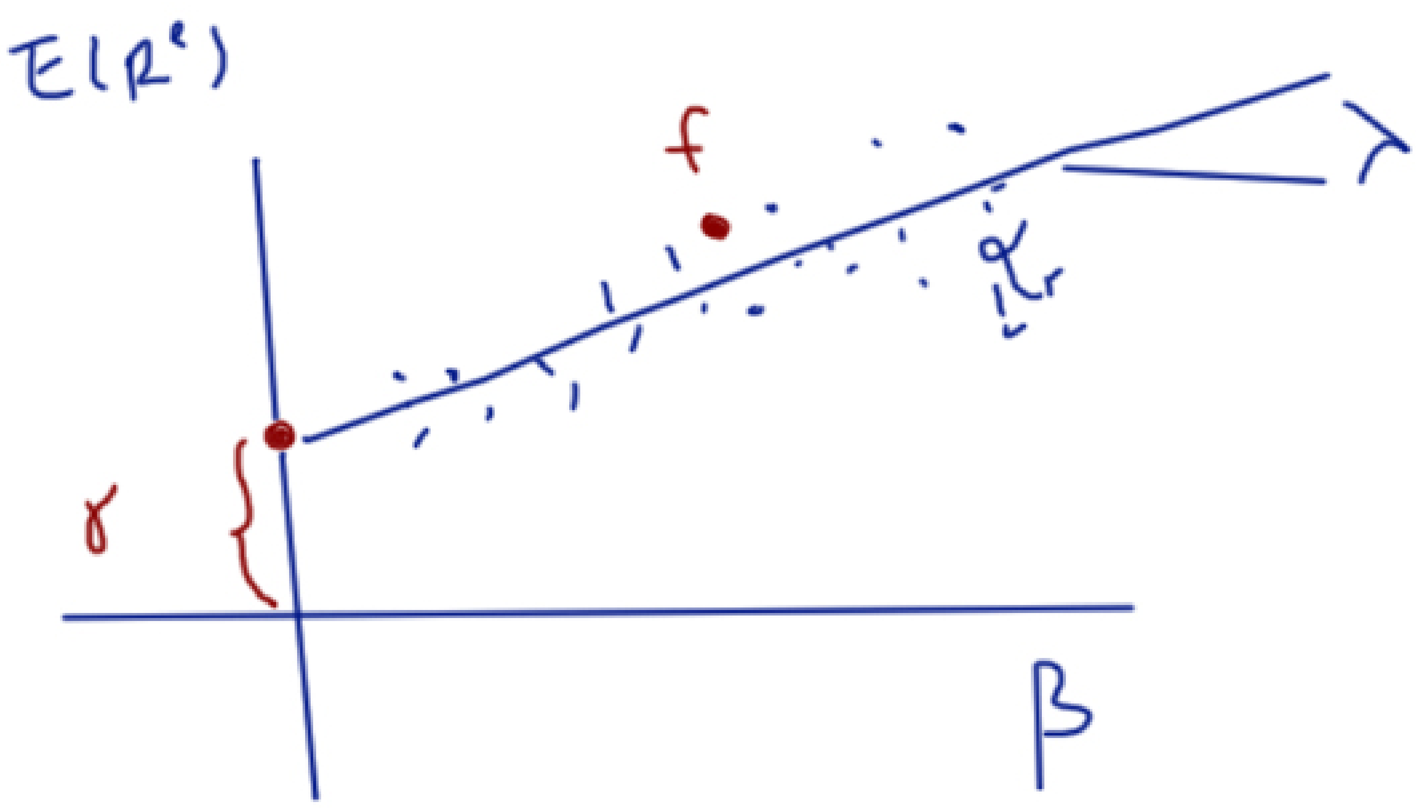
3. CROSS SECTION

a NOT α → β!

1 TS $\quad R^{ei}_t = a_i + \beta_i f_t + \varepsilon^i_t \quad t = 1,2 .. T \ \forall i$

2 CS $\quad E(R^{ei}) = (\gamma) + \beta_i \lambda + \alpha_i \quad i = 1..N$

$$y = a + x b + \varepsilon$$



- WHY? · "USE ALL ASSETS TO ESTIMATE γ"
  - · f NOT A R^e $\quad f_t = \Delta c_t$
- · ESTIMATES . $\hat\beta$: OLS
  - $\hat\gamma, \hat\lambda, \hat\alpha$ – OLS OR GLS CS $\qquad \Sigma / T$
  - $\hat\lambda = (\hat\beta'\hat\beta)^{-1}\hat\beta' E_T(R^e)$ OR $(\hat\beta' cov(\hat\alpha)^{-1}\hat\beta)^{-1}\hat\beta' cov(\hat\alpha)^{-1}\hat\beta$
  - $\hat\alpha = E_T(R^e) - \hat\beta \cdot \hat\lambda$

3. STD ERRORS / TEST $\cancel{\sigma(\hat{\lambda}) = (\beta'\beta)^{-1} \hat{\sigma}(\alpha)}$?

- β "GENERATED REGRESSORS". $\hat{\alpha}$: CORRELATED ACROSS i $\quad \hat{\alpha}_i - \alpha_i = \frac{1}{T}\sum_{t=1}^{T} \varepsilon_t^i$

$$\text{cov}(\hat{\alpha}) = \frac{1}{T}\Sigma \cdot \frac{1}{T}\text{cov}(5\,5')$$

- WAS: HARD. GMM: EASY

$$- \begin{bmatrix} I_N & & \\ & I_N & \\ & & \beta'\Sigma^{-1} \end{bmatrix} \begin{bmatrix} E_T(\overbrace{R_t^e - a - \beta f_t}^{\varepsilon_t}) \\ E_T[(R_t^e - a - \beta f_t) f_t] \\ E_T(\underbrace{R_t^e - \beta\lambda}_{\alpha}) \end{bmatrix}$$

$a \qquad g_T'(a, \beta, \lambda)$

$\beta' E_T(R^e) = \beta'\beta \cdot \lambda$

$\rightarrow \lambda = (\beta'\beta)^{-1}\beta' E_T(R^e)$

- GMM REDUCES TO CLASSIC

$(X'X)^{-1}X'\Omega X(X'X)^{-1}$ $\quad$ • SHANKEN CORRECTION" $\quad \hat{\sigma}/T$

$$\sigma^2(\hat{\lambda}_{OLS}) = \frac{1}{T}\left[(\beta'\beta)^{-1}\beta'\Sigma\beta(\beta'\beta)^{-1}[1+\lambda'\Sigma_f^{-1}\lambda] + \Sigma_f\right]$$

$$\text{cov}(\hat{\alpha}_{OLS}) = \frac{1}{T}\left[I - \beta(\beta'\beta)^{-1}\beta'\right]\Sigma[\cdot]'(1+\lambda'\Sigma_f^{-1}\lambda)$$

COV OF RESIDUALS!

- TEST $\quad \hat{\alpha}'\,\text{cov}^{-1}(\hat{\alpha})\,\hat{\alpha} \sim \chi^2$; IID N F TEST.

4. COMMENTS

TS

A $R^{ei}_t = \alpha_i + \beta_i f_t + \varepsilon^i_t \quad t = 1 \dots T \ \forall i$

$\Rightarrow$ B $E(R^{ei}) = \beta_i \underset{E(f)}{\lambda} + \alpha_i \quad i = 1 \dots N$

$=$

A $R^{ei} = a_i + \beta_i f_t + \varepsilon^i_t \quad t = 1 \dots T \ \forall i$

CS B $E(R^{ei}) = (\gamma) + \beta_i \lambda + \alpha_i \quad i = 1 \dots N$

$\hat{\lambda} = (\beta'\beta)^{-1}\beta' E_T(R^e)$ or $\hat{\lambda} = (\beta'\Sigma^{-1}\beta)^{-1}\beta'\Sigma^{-1}E_T(R^e)$

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1) TS A IS "MODEL OF VARIANCE"

B IS "MODEL OF MEAN" IMPLIED,

$t(\hat{\beta})$, $t(\hat{\lambda})$, R^2 AREN'T 'TEST'

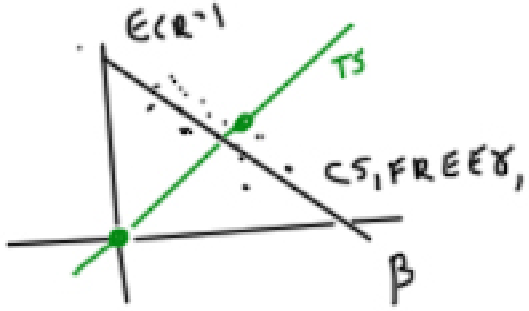
$\hat{\alpha}' V^{-1} \hat{\alpha}$ IS ONLY "TEST"

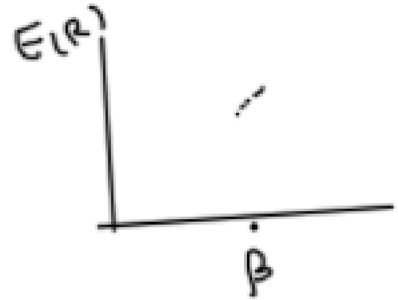
MODEL OF VARIANCE, DIAGNOSTICS IMPORTANT!

BUT NOT "TEST".

2) TS VS CS? SHOULD NOT MATTER UNLESS MODEL IS AWFUL

ASYMPTOTICALLY SAME





TEST ALL β's = TO EACH OTHER

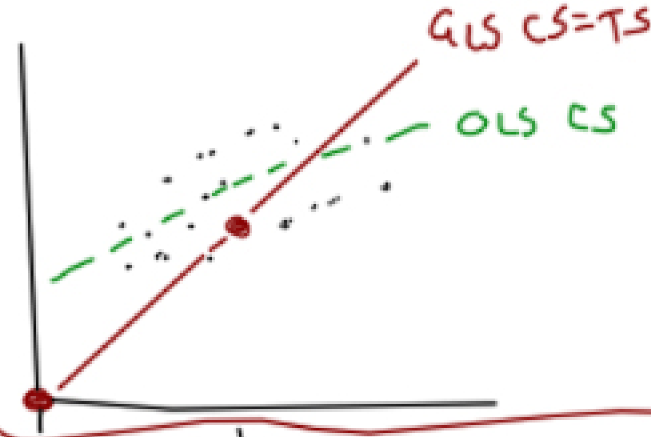
- CS: f IS NOT R^e. DON'T TRUST f, WANT TO USE OTHER ASSET INFO.

- GLS CS = TS = IID N MLE IF f IS A R^e

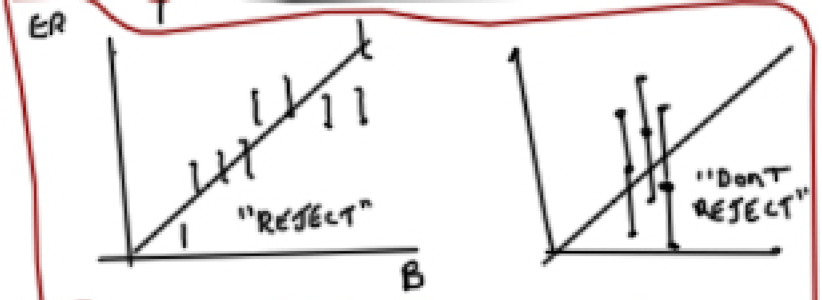
$$R^{ei}_t = a_i + \beta_i f_t + \varepsilon^i_t \rightarrow \Sigma = \begin{array}{c} f_t \\ \begin{bmatrix} \cdot & \cdot & \cdot & 0 \\ \cdot & \cdot & \cdot & 0 \\ \cdot & \cdot & \cdot & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

$$f_t = 0 + 1 \cdot f_t + 0$$

$E(R^{ei})$ ADDS NO INFO NOT IN f ABOUT $E(f) = \lambda$



⇒ AT LEAST INCLUDE f, R^f AS TEST ASSETS IN CS;



BUT REMEMBER FF-TESTS BORING

- $\hat{\alpha}' V^{-1} \hat{\alpha}$ "GRS" IS EASY USING CS! DO IT!

- WHY STILL USING N IID TESTS?? GMM/SIMULATION EASY! TRADITION/RHETORIC

A-MACBETH

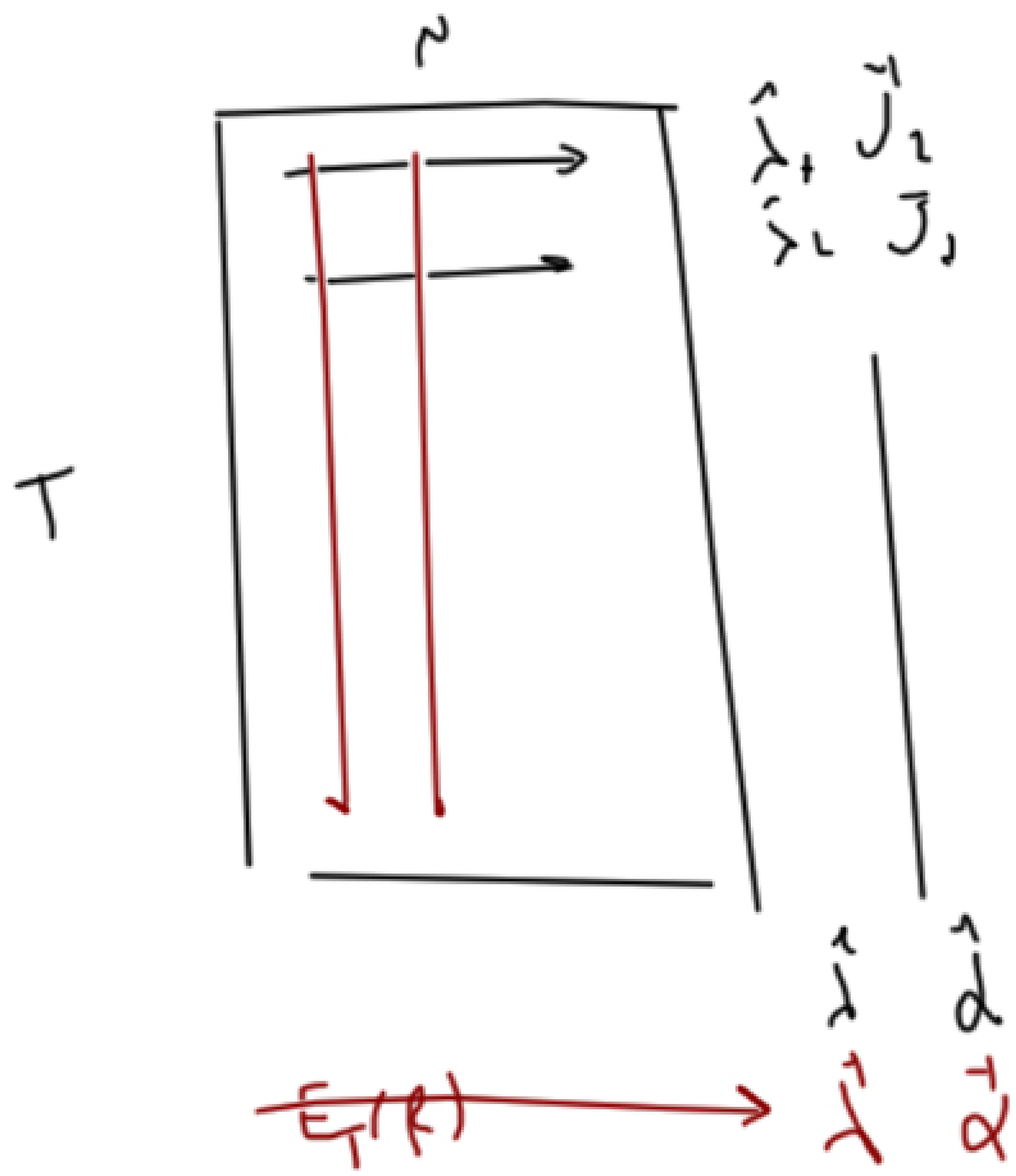
1. $TS \rightarrow \beta \qquad R^{ei}_t = a_i + \beta_i f_t + \varepsilon^i_t$

2. CS EACH t $\qquad R^{ei}_t = \beta_i' \lambda_t + \alpha_{it} \qquad i = 1 \ldots N \ \forall t$!

3. $\hat{\lambda} = E_T(\hat{\lambda}_t) \qquad \hat{\alpha} = E_T(\hat{\alpha}_t)$

4. $\sigma(\hat{\lambda}) = \dfrac{\sigma(\hat{\lambda}_t)}{\sqrt{T}}$ (OR GMM!) , $\operatorname{cov}(\hat{\alpha}, \hat{\alpha}) = \dfrac{\operatorname{cov}(\hat{\alpha}_t \hat{\alpha}_t')}{T}$ (OR GMM!)

[DO $\hat{\alpha}' \operatorname{cov}(\hat{\alpha}\hat{\alpha}')^{-1} \hat{\alpha} \sim \chi^2$!]



COMMENTS

1) IF β's CONSTANT OVERTIME

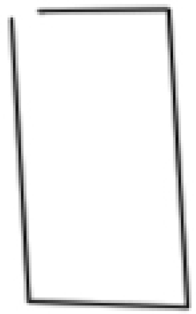
FMB = CS ESTIMATES.

FMB σ = CS σ − "SHANKEN CORRECTION"

2) FMB A WAY TO COMPUTE σ FOR CS

3) FMB FOR POOLED REGRESSION PANEL

$$y_{it} = a + bx_{it} + \varepsilon_{it}$$



WHAT IF $cov(\varepsilon_t \varepsilon_i) \neq I$? BIG IN FINANCE

→ OLS CONSISTENT, STD ERRORS WRONG.

FMB FOR STD ERRORS. IN 1970 NO $(x'x)^{-1}x'\Omega x(x'x)^{-1}$, "CLUSTER"

FMB EASY! MANY PAPERS IGNORE $cov(\varepsilon_i \varepsilon_j)$. t OFF BY 10!

42% — PETERSEN

3) FMB IGNORES VARIATION OVER TIME. $\hat{b}$ ONLY GETS CS VARIATION

6. GMM ISDF

$$0 = E(mR^e)$$

$$m = 1 - b'(f - Ef) = 1 - b'\tilde{f}$$

$$[\ 1 = E(mR^i)\ ]$$

$$g_T = E_T[\ R^e(1 - b\tilde{f})]$$

$$d = -E(R^e \tilde{f}') = -\text{cov}(R^e f') = -C$$

$$\min\ g_T' W g_T \rightarrow d' W g_T = 0$$

$$d' W\ E_T(R^e(1 - \tilde{f}'b)) = 0$$

$$c' W\ E_T(R^e) = c' W c b$$

$$b = (c' W c)^{-1} c' W E_T(R^e)$$

GMM IS A CS REGRESSION OF $E(R^e)$ ON $\text{cov}(R^e, f')$

BUT $E_T(1)$? "SHANKEN CORRECTION" Ef = PARAMETER

$$\underbrace{\begin{bmatrix} E_T(f R^e) W & 0 \\ 0 & I_n \end{bmatrix}}_{a} \underbrace{\begin{bmatrix} E_T[R^e - R^e(f' - Ef')]b \\ E_T[f - Ef] \end{bmatrix}}_{g_T} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$\vdots$

$$m = 1 - b\tilde{f},\quad 0 = E(mR^e)$$

$$\updownarrow$$

$$E(R^e) = \text{cov}(R^e f')b$$

GMM: SAMPLE EXTENSION

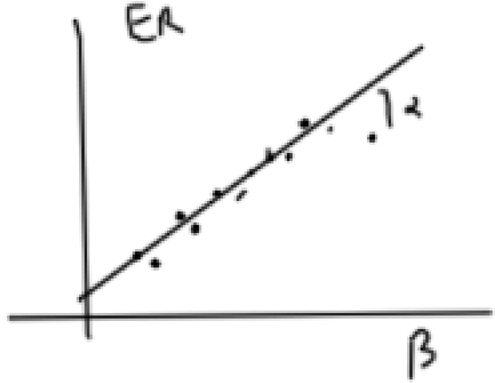
7. COMMENTS II

TIME SERIES

CROSS SECTION OLS GLS

FAMA MACBETH

GMM FIXED OPTIMAL S^{-1}



1) ALL ROUGHLY THE SAME!

2) ALL $\hat{\alpha}$ $\hat{\beta}$ $\hat{\lambda}$ ESTIMATE

$\hat{g}_T$ $\hat{b}$

$\sigma(\cdot)$ STD ERROR

$\hat{\alpha}' V^{-1} \hat{\alpha}$ $g_T' \operatorname{cov}_g g_T$ TEST

3) SHOULD GIVE ~ SAME ANSWER / UNDERSTAND =

4) UPDATE DISTRIBUTION THEORY!

5) "NOT REJECTED" ≠ "GOOD MODEL"!

→ LOTS OF PLOTS: ER vs β ETC.

8. TEST ONE MODEL VS ANOTHER.

"DIAGNOSTIC" "ALL MODELS FALSE" $\alpha' V^{-1} \alpha$ REJECTS IF $T \to \infty$

Q : $E(R^{ei}) = \alpha_i + \beta_i E(rmrf) + h_i E(hml) + s_i E(smb)$

$\lambda_m \qquad \lambda_h \qquad \lambda_s$

CAN WE DROP SMB?

WRONG ANSWERS

① $R^{ei}_t = \alpha_i + \beta_i\, rmrf_t + h_i\, hml_t + s_i\, smb_t + \varepsilon_{it}$

$E(smb) = \lambda$

$t(s_i) > 2$? R^2 DECLINE?

② $\lambda_s = 0$ $\quad t(\lambda_s) < 2(?)$ or $E(SMB) = 0$ (TS)

"IS THE FACTOR PRICED"

③ MODEL A GRS $\alpha' V^{-1} \alpha$ IS BETTER THAN B

RIGHT ANSWERS

① DOES $\alpha' V^{-1} \alpha$ RISE IF WE DROP SMB [SAME V!]

+ ALLOW β_i, h_i TO CHANGE?

② $m = 1 + b_m\, rmrf + b_h\, hml + b_s\, smb \qquad b_s = 0$

$\leftrightarrow E(R^e) = cov(R^e, rmrf) \cdot b_m \cdots cov(R^e, smb)\, b_s$

"DOES THE FACTOR HELP TO PRICE ASSETS"

ISSUE FACTORS MAY BE CORRELATED $\Longleftrightarrow$ SINGLE $\neq$ MULTIPLE β $\Leftrightarrow$ $b \neq \lambda$

MORE ANSWERS

(1) $smb_t = \alpha_s + \beta_s \, rmrf_t + h_s \, hml_t + \varepsilon_t^s$ $\quad \alpha_s = 0 \Rightarrow$ CAN DROP SMB

(2) $smb^T = \alpha_s + \varepsilon_t^s$ $\quad \lambda_s = E(smb^T) = 0 \Longleftrightarrow$ CAN DROP SMB

EXAMPLE

CAPM HOLDS

$$R_t^{ei} = \alpha_i + \beta_i R_t^{em} + m_i R_t^{e\,MSFT} + \varepsilon_{it}$$

$E(R^{e\,MSFT}) = \lambda_{MSFT} > 0$! $\quad t(m_i) > 0$! $\quad R^2 \uparrow$!

ANSWER $\tilde{R}^{e\,MSFT} = R^{e\,MSFT} - \beta^{MSFT} \cdot R^{em}$ $\quad$ IF CAPM $E(\tilde{R}^{e\,MSFT}) = 0$