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# Market microstructure and asset pricing: On the compensation for illiquidity in stock returns

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## Abstract

Models of price formation in securities markets suggest that privately informed investors create significant illiquidity costs for uninformed investors, implying that the required rates of return should be higher for securities that are relatively illiquid. We investigate the empirical relation between monthly stock returns and measures of illiquidity obtained from intraday data. We find a significant relation between required rates of return and these measures after adjusting for the Fama and French risk factors, and also after accounting for the effects of the stock price level.

*Key words:* Asset pricing; Market microstructure

*JEL classification:* G12; G14

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## 1. Introduction

At least since Bagehot (1971), it has been recognized that a primary cause of illiquidity in financial markets is the adverse selection which arises from the presence of privately informed traders. The theoretical implications of the

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adverse selection paradigm for financial market equilibrium have been analyzed extensively, and considerable effort has been expended on developing empirical techniques to measure adverse selection costs. In view of the considerable effort directed to the adverse selection paradigm, it is important to ask whether illiquidity due to information asymmetry significantly affects a firm's required rate of return.

Previous research on the return–illiquidity relation has focused on the quoted bid–ask spread as a measure of illiquidity. Thus, Amihud and Mendelson (1986) find evidence that asset returns include a significant premium for the quoted spread, while Eleswarapu and Reinganum (1993) question this result by showing that the return premium associated with the spread is primarily a seasonal phenomenon. However, the quoted bid–ask spread is a noisy measure of illiquidity because many large trades occur outside the spread and many small trades occur within the spread (see, e.g., Lee, 1993). Further, the theoretical models of Glosten (1989), Kyle (1985), and Easley and O'Hara (1987) and the empirical analysis of Glosten and Harris (1988) suggest that the liquidity effects of asymmetric information are most likely to be captured in the price impact of a trade, or the variable component of trading costs.

In this paper, we bring together diverse empirical techniques from asset pricing and market microstructure research to examine the return–illiquidity relation. Specifically, we estimate measures of illiquidity from intraday transactions data and use the Fama and French (1993) factors to adjust for risk. The use of transactions data enables us to estimate both the variable (trade-size-dependent) and the fixed costs of transacting. By empirically examining the effects of both variable and fixed components of illiquidity on asset returns we are able to shed light on the importance of the empirical measures of adverse selection in influencing asset returns. Moreover, since there is evidence that the activities of brokerage house analysts increase liquidity (Brennan and Subrahmanyam, 1995a), our findings have implications for the social value of security analysis.

We use intraday data from the Institute for the Study of Securities Markets for the years 1984 and 1988 and the methods of Glosten and Harris (1988) and Hasbrouck (1991) to decompose estimated trading costs into variable and fixed components. The basic data consist of the monthly returns on portfolios sorted by the estimated Kyle (inverse) measure of market depth,  $\lambda$  (estimated using the Glosten–Harris method), and firm size for the period 1984–1991. As mentioned earlier, unlike Amihud and Mendelson (1986) and Eleswarapu and Reinganum (1993), who use the simple capital asset pricing model to adjust returns for risk, we take as our null hypothesis the three-factor model developed by Fama and French (1993). These factors are the excess market return, the return on a portfolio which is long in small stocks and short in large stocks, and the return on a portfolio which is long in high book-to-market stocks and short in low book-to-market stocks.

We first estimate the intercepts from the time-series regressions of the excess returns on our  $\lambda$ -sorted portfolios on the Fama–French factors. We are able to

reject the null hypothesis that these intercepts are jointly zero. Our next step is to investigate directly the relation between the portfolio returns and our measures of market illiquidity. To accomplish this, we pool the time-series and cross-sectional data and perform generalized least squares (GLS) regressions of the portfolio returns simultaneously on measures of trading costs and the three Fama–French risk factors. This allows us to jointly estimate the factor coefficients and the coefficients of the illiquidity variables. We thus avoid the errors-in-variables problems associated with more traditional two-step Fama and MacBeth (1973) procedures.

We first perform regressions using indicator variables that correspond to the  $\lambda$  groups of the portfolios, in addition to the Fama–French risk factors, as our independent variables. We find that the coefficients on the indicators increase monotonically as we move from low- $\lambda$  to high- $\lambda$  portfolios. In subsequent regressions, we find that estimates of both the variable and the fixed components of the proportional cost of transacting are also significantly positively related to excess returns. The coefficient of the proportional spread, however, is negative, both when it is the only trading cost variable in the regression and when it is included along with our transaction cost variables. The sign of the spread coefficient is inconsistent with the role of this variable as a measure of the cost of transacting. We hypothesize that the spread is proxying for a risk variable associated with price level or firm size that is not captured by the Fama–French three-factor model. Our findings indicate that the explanatory power of the bid–ask spread appears largely to be due to the effect of (the reciprocal of) the price level. Indeed, the coefficient of the spread is not significant in the presence of the price level variable and our cost of illiquidity variables.

We also address the issue of seasonality raised by Eleswarapu and Reinganum (1993). A likelihood ratio test of seasonality leads us to conclude that there are no significant monthly seasonal components in the compensation for our transaction cost measures, the bid–ask spread, or the inverse price level variable, after allowing for the effect of the Fama–French risk factors.

The paper is organized as follows. Section 2 describes the estimation of the market illiquidity parameters and the portfolio formation procedure. Sections 3 and 4 report results obtained from performing Fama–French and GLS regressions, respectively, while Section 5 concludes.

## **2. Estimation of the market illiquidity parameters**

We refer to a trading cost that is a constant proportion of the value of the transaction as a fixed (proportional) cost; if the cost varies with the value of the transaction, it is referred to as a variable (proportional) cost. We estimate the parameters associated with fixed and variable proportional trading costs, denoted by  $\psi$  and  $\lambda$ , respectively, using Institute for the Study of Security

Markets (ISSM) data for the calendar years 1984 and 1988. This data set consists of all bid–ask quotations and time-stamped transaction prices and quantities for each NYSE/AMEX stock. We use two different empirical models of price formation, which we label the *Glosten–Harris (GH)* and *Hasbrouck–Foster–Viswanathan (HFV)* models and describe below.

### 2.1. The Glosten–Harris model

Let  $m_t$  denote the expected value of the security, conditional on the information set at time  $t$ , of a market maker who observes only the order flow,  $q_t$ , and a public information signal,  $y_t$ . Models of price formation such as Kyle (1985) and Admati and Pfleiderer (1988) imply that  $m_t$  will evolve according to

$$m_t = m_{t-1} + \lambda q_t + y_t, \quad (1)$$

where  $\lambda$  is the (inverse) market depth parameter. While theorists have attributed the costs of transacting in securities markets to adverse selection, inventory holding costs, and fixed costs, empirical studies have found that inventory holding costs appear to be small in an intraday setting (see, for example, Stoll, 1989; George, Kaul, and Nimalendran, 1991; Madhavan and Smidt, 1991). In order to allow for a fixed cost component of the price response to a transaction, we proceed as follows.

Let  $D_t$  denote the sign of the incoming order at time  $t$  (+1 for a buyer-initiated trade and  $-1$  for a seller-initiated trade). Since not all trades occur at the bid or ask quotes, we follow the convention of Lee and Ready (1991) and Madhavan and Smidt (1991) to assign values to  $D_t$ , i.e., to classify trades as buyer- or seller-initiated trades: if a transaction occurs above the prevailing quote mid-point, it is regarded as a purchase and vice versa. If a transaction occurs exactly at the quote mid-point, it is signed using the previous transaction price according to the tick test (i.e., a purchase if the sign of the last nonzero price change is positive and vice versa).

Given the order sign  $D_t$ , denoting the fixed cost component by  $\psi$ , and assuming competitive risk-neutral market makers, the transaction price,  $p_t$ , can be written as

$$p_t = m_t + \psi D_t. \quad (2)$$

Substituting out  $m_t$  using (1), we have

$$p_t = m_{t-1} + \lambda q_t + \psi D_t + y_t. \quad (3)$$

However, since  $p_{t-1} = m_{t-1} + \psi D_{t-1}$ , the price change,  $\Delta p_t$ , is given by

$$\Delta p_t = \lambda q_t + \psi [D_t - D_{t-1}] + y_t, \quad (4)$$

where  $y_t$  is the unobservable error term. Eq. (4) is used to estimate the Glosten–Harris  $\lambda$  as described below.

## 2.2. The Hasbrouck–Foster–Viswanathan model

Our second model closely parallels the framework used by Foster and Viswanathan (1993, pp. 202–206) to analyze intraday and interday variations in trade informativeness. Their framework, in turn, is based on Hasbrouck (1991). The advantage of using this approach is that it is valid for a relatively broad range of theoretical specifications. The model focuses on the price response to *unexpected* volume as the measure of the adverse selection component of the price change. The rationale is that if trades are autocorrelated or predictable from past price changes, then part of the contemporaneous order flow is predictable and should not be included in measuring the information content of a trade. In contrast to Hasbrouck (1991), and following Foster and Viswanathan (1993), we apply the model to transaction prices, rather than to bid–ask quotes.

Let  $\Delta p_t$  be the transaction price change for transaction  $t$ , let  $q_t$  be the signed trade quantity corresponding to the price change, and let  $D_t$  be the indicator corresponding to the direction of a trade, assigned a value following the procedure described in the previous subsection. The following model with five lags is considered for estimation:

$$q_t = \alpha_q + \sum_{j=1}^5 \beta_j \Delta p_{t-j} + \sum_{j=1}^5 \gamma_j q_{t-j} + \tau_t, \quad (5)$$

$$\Delta p_t = \alpha_p + \psi [D_t - D_{t-1}] + \lambda \tau_t + v_t. \quad (6)$$

The informativeness of trades in Eq. (6) is measured by the coefficient of  $\tau_t$ , the residual from the regression in (5). Thus, it is the response to the unexpected portion of the order flow in Eq. (5) (measured by  $\tau_t$ ) that measures trade informativeness. The coefficient on  $D_t - D_{t-1}$ , as before, measures the fixed cost component of the trading cost.

## 2.3. Empirical procedure

The estimation of the GH and HFV liquidity parameters proceeds as follows. First, we discard quotations and transactions which were reported out of sequence. Second, we omit the overnight price change in order to avoid contamination of the price change series by dividends, overnight news arrival, and special features associated with the opening procedure. Thus, the Hasbrouck–Foster–Viswanathan specification, which involves lagged values of price changes, uses missing values for the lags that involve the overnight price change. Third, to correct for reporting errors in the sequence of trades and quotations, we follow the convention employed in Madhavan and Smidt (1991) (and elsewhere) of delaying all quotations by five seconds. Finally, we use an error filter to screen out typographical errors. The error filter discards a trade if the trade price is too far outside the price range defined as the minimum range

that includes both the preceding bid and ask quotations and the immediately following trade price or bid and ask quotations. If the price falls outside this range by more than four times the width of the range, the trade is discarded. This filter is conservative and discards fewer than one in 40,000 observations in the sample considered.

Noting that ISSM identifies firms by their ticker symbols, we first identify the NYSE-listed firms on the 1984 (1988) ISSM tape whose ticker symbols were listed as active at any time in 1984 (1988) in the 1992 Center for Research in Security Prices (CRSP) name matrix (our study is restricted to common stock). There were 1,629 and 1,784 such firms in 1984 and 1988, respectively. The Glosten–Harris model represented by Eqs. (4) and the two-equation model of Foster–Viswanathan represented by Eqs. (5) and (6) are then estimated by ordinary least squares (OLS) for each of the 1,629 (1,784) firms separately for 1984 (1988), retaining the resulting estimates of  $\lambda$  and  $\psi$ . These estimates are used to form portfolios and to compute the cost of illiquidity measures described below.

Note that our estimation procedure ignores the discreteness of the price quotes. While Glosten and Harris (1988) find that this does not have a material effect on the estimated value of the market depth parameter,  $\lambda$ , they also conclude that the estimates of the fixed cost parameter,  $\psi$ , 'are quite sensitive to whether or not discreteness is modeled in the estimation process' (Glosten and Harris, 1988, p. 135). Glosten and Harris model discreteness for only 20 stocks, finding the econometric modeling of discreteness to be too costly for their full sample of 250 stocks. This exercise is prohibitive for our much larger sample of stocks. Therefore, our results concerning the influence of fixed costs of transacting must be interpreted with this limitation in mind.

In a Kyle-type model, the cost of trading  $x$  shares is  $\lambda x^2$ , so that, given the share price  $P$ , the marginal cost per dollar transacted when  $x$  shares are traded is  $2\lambda x/P$  and varies with the trade size. Thus, to take account of the variable proportional cost of transacting in different securities it is necessary to make an assumption about the sizes of the transactions in the securities. A natural approach is to use the average measured trade size. Thus, our first measure of the variable proportional cost of transacting is  $C_q \equiv \lambda q/P$ , where  $q$  is the average size of a transaction in the security. Since the marginal cost of transacting is linear in trade size, this procedure yields the average of the marginal costs realized by all transactors. This is the measure used by Glosten and Harris (1988) in their cross-sectional analysis of the  $\lambda$ 's.

A limitation of this measure of the variable proportional cost of transacting is that it takes no account of the distortionary effect of trading costs on the average transaction size.<sup>1</sup> Thus, if transaction sizes in extremely illiquid securities were

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<sup>1</sup>Both Glosten and Harris (1988) and Brennan and Subrahmanyam (1995b) find a significant negative effect of  $\lambda$  on average transaction size.

sufficiently small, this approach might yield a *lower* estimated variable cost for illiquid securities than for the relatively liquid ones. Thus, a second way to proxy for  $x$  is to assume that, in the absence of differential liquidity, the average transaction would be proportional to the total number of shares of a firm's stock outstanding. The relevant measure of the variable cost then becomes  $C_n \equiv \lambda n/P$ , where  $n$  is the number of shares outstanding. We recognize that both  $C_q$  and  $C_n$  are imperfect proxies for the variable cost. Therefore, in our empirical analysis, we use indicator variables based on  $\lambda$ , as well as transformations of  $C_q$  and  $C_n$ .

#### 2.4. Portfolio formation procedure

The portfolio formation proceeds as follows. First, all NYSE-listed securities on the CRSP tape at the beginning of each year from 1984 to 1987 are assigned to one of 30 portfolios according to their size and  $\lambda$  estimate based on 1984 transaction data. Thus, the securities are divided first into size quintiles according to the market value of the equity at the end of the previous year and then, within each size quintile, they are assigned to one of five portfolios with equal numbers of securities, based on their estimated GH  $\lambda$ . Securities for which no estimate of  $\lambda$  is available (for lack of transactions data on the ISSM tape) are assigned to a sixth portfolio for each size quintile. To facilitate usage of log-linear specifications involving  $\lambda$  (see Table 5 to be discussed later), we omit firms with negative estimates of  $\lambda$  from our portfolios. However, this procedure leads to the omission of no more than one firm per portfolio per year. The same portfolio formation procedure is repeated for CRSP data from 1988 to 1991 using estimates of  $\lambda$  derived from 1988 transaction data. This procedure yields a total of 30 portfolios, for five of which there is no estimate of  $\lambda$ .

Equally weighted monthly excess returns are calculated for each of the 30 portfolios from January 1984 to December 1991, using the one-month risk-free rate from the CRSP bond files. The values of  $\lambda$  and  $\psi$  for each portfolio are taken to be the equally weighted average  $\lambda$  and  $\psi$ , which are based on the 1984 estimates for the years 1984 to 1987 and on the 1988 estimates for the years 1988 to 1991. The fixed proportional cost component,  $\psi/P$ , is calculated by first dividing the estimated  $\psi$  by the average closing price in 1984 (1988) for each stock and then taking an equally weighted average across stocks within a portfolio. The *proportional spread* for each security in 1984 and 1988 is computed by averaging the proportional quoted spread (i.e., the quoted spread divided by the average of the bid and ask prices) across all quotations during the year. The proportional spread for each portfolio,  $SP$ , is the equally weighted average of the security spreads using the 1984 calculation for 1984–1987 and the 1988 calculation for 1988–1991.

For each calendar year, the equally weighted average size of the firms in each portfolio is calculated using the logarithm of the market value of equity

outstanding at the end of the previous year as the measure of size. (A firm is eliminated from the sample if the relevant market value of equity is missing from the CRSP tape.) Finally, for each portfolio the equally weighted values of our variable cost of transacting variables,  $C_q = \lambda q/P$  and  $C_n = \lambda n/P$ , are calculated for each year; for portfolios formed in 1984 through 1987 (1988 through 1991)  $\lambda$  is the value estimated from the 1984 (1988) transactions data,  $q$  is the average transaction size in 1984 (1988),  $n$  is the (monthly) average number of shares outstanding in 1984 (1988), and  $P$  is the (monthly) average price of the relevant stock in 1984 (1988). For certain firms and certain months, data on the number of shares outstanding and/or the price level in the relevant year are missing from the CRSP tape. We use a firm in our portfolio formation procedure if there is data on both these variables for at least one month, otherwise the firm is eliminated from the sample. We do not update  $n$  and  $P$  each year since we expect  $\lambda$  to be affected by changes in  $n$  and  $P$  and, due to lack of data availability, we are unable to update the  $\lambda$  or  $q$  component of the cost. Our procedure amounts to assuming that for a given security  $\lambda q/P$  and  $\lambda n/P$  are intertemporal constants. To the extent that this assumption is not valid, the power of our tests will be reduced. The time-series mean values of the monthly returns, sizes, illiquidity, and cost of illiquidity measures (based on the GH  $\lambda$ ), and proportional spreads for each of the 30 portfolios are presented in Table 1. Panel A of Table 1 shows that for a given size group the average return tends to increase with  $\lambda$ , and for a given  $\lambda$  group the average return tends to increase with firm size. As can be seen from panel B,  $\lambda$  and the fixed and variable cost of transacting measures show considerable variation within each size group and are positively related. Also, the average proportional fixed cost component  $\psi/P$  is smaller than the average proportional spread, reflecting the notion that a large proportion of transactions take place within the quoted bid–ask spread.

Table 2 reports the correlations between the variables. Average return is positively correlated with size and negatively correlated with both the spread and the proportional fixed cost,  $\psi/P$ . The proportional spread and  $\psi/P$  have a correlation of 0.78, which is to be expected since they are both measures of the fixed proportional component of trading costs. The two measures of the variable proportional cost of transacting,  $C_n$  and  $C_q$ , have a correlation of 0.93, but their correlations with the spread are only 0.16 and 0.38, respectively. An interesting feature of the data is that while  $\lambda$  has only a low correlation with the spread and  $\psi/P$  in the overall sample (Table 2), within each size group  $\lambda$  is almost perfectly negatively correlated with these variables (panel B of Table 1).<sup>2</sup> Thus, within a size group the estimated fixed proportional cost of transacting is negatively related to the variable cost. We are not aware of any model of market making that would predict such a relation. The pattern may be related to market

<sup>2</sup>We are grateful to the referee for bringing this point to our attention.



Table 1

Summary statistics: Average values of monthly return, size, and liquidity variables for 30 portfolios of NYSE stocks sorted by size and the Glosten–Harris measure of illiquidity,  $\lambda$ , for the period 1984–1991

$\lambda$  estimates the derivative of transaction price (\$/share) with respect to signed trade size (shares, positive for trades initiated by buyers). Portfolios are formed annually from all NYSE firms active at the beginning of the year. Within each calendar year, size is measured as market value of equity at the end of the preceding year.  $C_q$  equals  $\lambda$  times the average trade size divided by the monthly average closing price.  $C_n$  equals  $\lambda$  times the monthly average number of shares outstanding divided by monthly average closing price, and  $\psi/P$  denotes the fixed component of trading costs as a proportion of the monthly average closing price. The proportional spread is calculated by averaging the proportional quoted spread (i.e., the quoted spread divided by the average of the bid and ask prices) across all quotations during the year. For the 1984–1987 period,  $\lambda$  and all other liquidity variables are estimated using 1984 data. For the 1988–1991 period, they are estimated from 1988 data. The portfolio labeled 0 in the  $\lambda$  group column denotes the portfolio for which data on the above liquidity variables are not available. The variable and fixed components of liquidity,  $\lambda$  and  $\psi$ , respectively, are estimated using the Glosten–Harris (1988) method.

## (A) Average monthly returns (percentages) and firm size (millions of dollars)

Size group	$\lambda$ group					
	0	1	2	3	4	5
<i>Average return</i>						
1	− 0.87	− 1.35	− 0.42	− 0.40	− 0.40	0.34
2	0.18	0.77	0.93	1.19	1.42	1.62
3	0.54	1.20	1.13	1.41	1.52	1.68
4	1.30	1.43	1.39	1.35	1.74	1.87
5	1.30	1.69	1.78	1.82	1.86	2.03
<i>Average firm size</i>						
1	38.40	46.18	48.66	49.97	49.99	49.32
2	134.58	166.71	166.46	158.10	159.94	149.42
3	341.75	464.21	470.75	457.30	461.83	434.58
4	2,525.33	1,234.82	1,270.71	1,265.69	1,227.04	1,146.93
5	5,107.72	7,107.38	7,546.87	8,535.76	4,837.67	5,164.03

## (B) Liquidity variables

Size group	$\lambda$ group				
	1	2	3	4	5
$\lambda * 100$					
1	0.0074	0.0244	0.0472	0.0899	0.2784
2	0.0058	0.0175	0.0334	0.0640	0.2347
3	0.0035	0.0106	0.0201	0.0347	0.1304
4	0.0027	0.0079	0.0136	0.0227	0.0559
5	0.0019	0.0058	0.0102	0.0151	0.0395

Table 1 (continued)

Size group	$\lambda$ group				
	1	2	3	4	5
<i>Proportional spread</i>					
1	0.0262	0.0179	0.0142	0.0131	0.0124
2	0.0131	0.0100	0.0095	0.0088	0.0075
3	0.0114	0.0086	0.0076	0.0068	0.0062
4	0.0079	0.0081	0.0065	0.0059	0.0054
5	0.0053	0.0049	0.0049	0.0044	0.0040
<i><math>C_q * 100</math></i>					
1	0.0262	0.0418	0.0536	0.0834	0.1267
2	0.0080	0.0166	0.0234	0.0318	0.0566
3	0.0043	0.0098	0.0132	0.0168	0.0285
4	0.0026	0.0062	0.0089	0.0121	0.0192
5	0.0016	0.0037	0.0052	0.0069	0.0089
<i><math>C_n</math></i>					
1	2.42	3.54	3.67	6.84	8.49
2	1.02	1.60	2.15	3.02	6.08
3	0.92	1.67	1.70	2.15	4.09
4	0.82	1.36	1.73	2.19	3.28
5	1.08	2.15	2.74	2.51	3.27
<i><math>\psi/P</math></i>					
1	0.0173	0.0094	0.0073	0.0070	0.0050
2	0.0061	0.0044	0.0041	0.0037	0.0032
3	0.0051	0.0034	0.0030	0.0028	0.0024
4	0.0036	0.0026	0.0022	0.0022	0.0021
5	0.0023	0.0018	0.0015	0.0015	0.0014

makers' tradeoffs between the fixed and variable components across stocks that differ in trading volume and transaction frequency.

### 3. Fama–French intercepts

Fama and French (1993) propose a three-factor model of common stock returns. These factors are the market excess return, a size factor, and a book-to-market factor. The size factor is the return on a portfolio that is long in small stocks and short in large stocks. Similarly, the book-to-market factor is the

Table 2

Cross-sectional correlation matrix: Matrix provides correlations between monthly return, size, and liquidity variables for 25 portfolios of NYSE stocks sorted by size and the Glosten–Harris measure of illiquidity,  $\lambda$ , for the period 1984–1991

Five portfolios for which data on  $\lambda$  are not available are omitted from the sample.  $\lambda$  estimates the derivative of transaction price (\$/share) with respect to signed trade size (shares, positive for trades initiated by buyers). Portfolios are formed annually from all NYSE firms active at the beginning of the year. Within each calendar year, size is measured as market value of equity at the end of the preceding year.  $C_q$  equals  $\lambda$  times the average trade size divided by the monthly average closing price,  $C_n$  equals  $\lambda$  times the monthly average number of shares outstanding divided by monthly average closing price, and  $\psi/P$  denotes the fixed component of trading costs as a proportion of the monthly average closing price. The proportional spread is calculated by averaging the proportional quoted spread (i.e., the quoted spread divided by the average of the bid and ask prices) across all quotations during the year. For the 1984–1987 period,  $\lambda$  and all other liquidity variables are estimated using 1984 data. For the 1988–1991 period, they are estimated from 1988 data. The variable and fixed components of liquidity,  $\lambda$  and  $\psi$ , respectively, are estimated using the Glosten–Harris (1988) method.

	Average return	Average size	$\lambda$	Prop. spread	$C_q$	$C_n$	$\psi/P$
Avg. return	1.00	0.49	-0.08	-0.95	-0.52	-0.30	-0.92
Avg. size	0.49	1.00	-0.29	-0.52	-0.41	-0.21	-0.44
$\lambda$	-0.08	-0.29	1.00	0.03	0.82	0.88	-0.03
Prop. spread	-0.95	-0.52	0.03	1.00	0.38	0.16	0.78
$C_q$	-0.52	-0.41	0.83	0.38	1.00	0.93	0.31
$C_n$	-0.30	-0.21	0.88	0.16	0.93	1.00	0.13
$\psi/P$	-0.92	-0.44	-0.03	0.78	0.31	0.13	1.00

return on a portfolio that is long in stocks with high book-to-market ratios and short in stocks with low book-to-market ratios. We take the Fama–French model as our null hypothesis and test whether variables related to the cost of transacting have additional explanatory power for the cross-section of returns. First, following Fama and French (1993), we perform OLS time-series regressions of the excess returns on our 30 portfolios on the factors:<sup>3</sup>

$$R_{it} = \alpha_i + \beta_i R_{mt} + \delta_i SMB_t + \kappa_i HML_t + e_{it}, \quad (7)$$

where  $R_{mt}$  is the excess return on the market portfolio in month  $t$ ,  $SMB_t$  and  $HML_t$  are the returns in month  $t$  on the Fama–French size and book-to-market factors, respectively, and  $R_{it}$  is the excess return on portfolio  $i$ . Under the null hypothesis that the costs of transacting have no effect on expected returns the intercepts in these regressions are equal to zero. Following Fama and French,

<sup>3</sup>We thank Gene Fama and Ken French for making the returns on their factor portfolios available to us.

we test the null hypothesis using the Gibbons, Ross, and Shanken (1989) statistic, which can be defined as follows. Let there be  $N$  time-series observations,  $L$  portfolios, and  $K - 1$  explanatory variables (excluding the intercept). Further, let  $X$  denote the matrix of regressors. Then, the test statistic is given by

$$(A' \Sigma^{-1} A) \frac{N - K - L + 1}{L * (N - K) * \omega_{1,1}},$$

where  $A$  is the column vector of the regression intercepts,  $\Sigma$  is the variance-covariance matrix of the residuals from the regressions, and  $\omega_{1,1}$  is the diagonal element of  $(X' X)^{-1}$  corresponding to the intercept. Under the null hypothesis this statistic has an  $F$ -distribution with  $L$  and  $N - K - L + 1$  degrees of freedom. The estimated coefficients of Eq. (7) for each of the 30 portfolios are reported in Table 3. The  $F$ -test strongly rejects at the 5% and 1% levels the null hypothesis that the intercepts are jointly equal to zero, both when the intercepts for the missing- $\lambda$  portfolios are included and when they are excluded from the test. This suggests that the size and book-to-market effects documented by Fama and French do not proxy for transaction cost effects. However, the three-factor model, by accounting for variation in returns that is unrelated to trading costs, makes the relationship between trading costs and returns more transparent. There also appears to be a pattern associated with firm size (unrelated to  $\lambda$ ) in the intercepts, in that they generally increase as one moves from the smallest to largest firm quintiles. To account for this finding, we include firm size in addition to the illiquidity variables in the GLS regressions whose results appear in the next section.

#### 4. GLS regressions

In this section, we report the results of pooled cross-section time-series regressions of the portfolio returns on various trading cost measures and the three Fama–French risk factors. By estimating simultaneously the factor coefficients and the coefficients of the trading cost variables, we avoid the errors-in-variables problems associated with more traditional Fama and MacBeth (1973) procedures. Throughout the analysis, we restrict our attention to the 25 portfolios for which estimates of  $\lambda$  are available. The estimation proceeds as follows. Define  $R$  as the  $(25T \times 1)$  vector of portfolio excess returns, where  $T$  is the total number of time-series observations, and the vector is ordered by month so that the first 25 observations correspond to the portfolio returns in month 1. Define  $X$  as the partitioned matrix

$$X = [WZ], \tag{8}$$

Table 3

Intercepts from Fama–French OLS regressions for 30 portfolios of NYSE stocks sorted by size and the Glisten–Harris measure of illiquidity,  $\lambda$ , for the period 1984–1991

$\lambda$  estimates the derivative transaction price (\$/share) with respect to signed trade size (shares, positive for trades initiated by buyers). Portfolios are formed annually from all NYSE firms active at the beginning of the year. Within each calendar year, size is measured as market value of equity at the end of the preceding year. For the 1984–1987 period,  $\lambda$  and all other liquidity variables are estimated using 1984 data. For the 1988–1991 period, they are estimated from 1988 data. The portfolio labeled 0 in the  $\lambda$  group column denotes the portfolio for which data on  $\lambda$  are not available. The table presents intercepts from the following time-series regressions:

$$R_{it} = \alpha_i + \beta_i MKT_t + \delta_i SMB_t + \kappa_i HML_t + u_{it},$$

where  $R_{it}$  is the excess return on portfolio  $i$  in month  $t$ , and  $MKT_t$ ,  $SMB_t$ , and  $HML_t$  denote the returns on the Fama and French (1993) factors related to the market, firm size, and the book-to-market ratio in month  $t$ . The bottom of the table presents the Gibbons, Ross, and Shanken (1989) test of the hypothesis that the intercepts jointly equal zero. Intercepts are reported in percentage terms ( $t$ -statistics are in parentheses).

Size group	$\lambda$ group					
	0	1	2	3	4	5
1	-1.21 (-2.50)	-2.14 (-3.62)	-1.51 (-3.17)	-1.34 (-3.34)	-1.25 (-3.78)	-0.45 (-1.76)
2	-0.39 (-1.18)	-0.42 (-1.76)	-0.19 (-0.99)	0.13 (0.72)	0.37 (1.99)	0.80 (4.63)
3	0.03 (0.11)	0.02 (0.09)	-0.17 (-1.04)	0.26 (1.39)	0.41 (2.57)	0.66 (4.55)
4	0.86 (2.24)	0.27 (1.54)	0.07 (0.51)	-0.07 (-0.47)	0.41 (2.42)	0.63 (4.16)
5	0.71 (1.47)	0.29 (2.05)	0.33 (3.03)	0.38 (3.96)	0.38 (3.11)	0.60 (4.74)

(1)  $F$ -value for the Gibbons, Ross, and Shanken test that the intercepts jointly equal zero is 4.77 ( $p$ -value =  $9.08 \times 10^{-8}$ ). (2)  $F$ -value for the Gibbons, Ross, and Shanken test that the intercepts equal zero (excluding portfolios with missing liquidity parameters) is 5.70 ( $p$ -value =  $5.50 \times 10^{-9}$ ).

where  $Z$  is a  $(25T \times 75)$  matrix of the Fama–French factors. The first 25 columns of  $Z$  consist of  $T$  stacked  $(25 \times 25)$  diagonal matrices with identical elements  $R_{mt}$ , the excess return on the market in month  $t$ ,  $t = 1, \dots, T$ ; the second 25 columns consist similarly of the size factor,  $SML_t$ , and the last 25 columns contain the book-to-market factor,  $HML_t$ .  $W$  is a  $(25T \times (k + 1))$  matrix, whose first column is a vector of units and whose remaining  $k$  columns are the vectors of the  $k$  portfolio attributes (trading cost measures) whose influence on returns we wish to assess.

We first perform the OLS pooled cross-section time-series regression

$$R = X\beta + \varepsilon, \tag{9}$$

where  $\beta$  is a  $k + 76$  vector of coefficients, the first element of which is the constant term of the regression, the next  $k$  elements of which are the coefficients of the  $k$  trading cost measures included in the regression, and the last 75 elements of which are the coefficients of the three Fama–French factors for the 25 portfolios ordered by portfolio.  $\varepsilon$  is a  $25T \times 1$  vector of errors. In our application the sample consists of monthly returns from January 1984 to December 1991 so that  $T = 96$ . In order to obtain the GLS estimator of  $\beta$ ,  $\Omega$ , the variance–covariance matrix of errors in (9), is estimated assuming that the portfolio return errors are serially independent, but allowing for cross-sectional dependence. Then  $\Omega$  is a  $(25T \times 25T)$  block-diagonal matrix, whose typical element is the  $25 \times 25$  covariance matrix of portfolio return errors: it is estimated using the residuals from (9). The GLS estimate of  $\beta$  is given by

$$\hat{\beta} = (X' \hat{\Omega}^{-1} X)^{-1} X' \hat{\Omega}^{-1} Y,$$

where  $\hat{\Omega}$  is the estimate of  $\Omega$  from the first-stage regressions.

#### 4.1. GLS regressions using indicator variables for the $\lambda$ groups

Our first set of regressions uses dummy variables for the  $\lambda$  quintiles, treating the smallest  $\lambda$  group as the base case. Then  $k = 4$  and the last four columns of  $W$  contain the dummy variables for group membership. The GLS estimates of the constant term and the dummy variable coefficients are presented in Table 4 along with the average characteristics of the five portfolios within each  $\lambda$  quintile. (The coefficients of the Fama–French factors are omitted to save space.) The dummy variable coefficients increase monotonically as we move from low to high  $\lambda$  quintiles, and the coefficients are statistically significant for all the four nonbase  $\lambda$  quintiles. Note that the variation in average firm size across the quintiles is not monotonic, while the average value of  $\lambda$  increases monotonically across the quintiles. This suggests that the pattern in the indicator variable coefficients is unrelated to firm size.<sup>4</sup>

#### 4.2. The functional form of the cost of illiquidity

While our results thus far confirm the hypothesis that portfolios with a higher  $\lambda$  have a higher risk-adjusted return, they do not distinguish between the influence of the  $\lambda$  variable and the transaction cost variables  $C_q$  and  $C_n$ . To explore this issue, as well as to analyze the influence of the proportional fixed

<sup>4</sup>Using the actual  $\lambda$  values for the portfolios in the GLS regression, as opposed to indicator variables, leads to essentially similar results. Further, including firm size as an independent variable in the regression does not materially alter the coefficients of the indicator variables. For brevity, these results are not reported.

Table 4

Dummy variable GLS regressions, using the Fama–French factors, for the 25 portfolios of NYSE stocks sorted by size and the Glosten–Harris measure of illiquidity,  $\lambda$ , for the period 1984–1991

Five portfolios for which data on  $\lambda$  are not available are omitted from the sample.  $\lambda$  estimates the derivative of transaction price (\$/share) with respect to signed trade size (shares, positive for trades initiated by buyers). Portfolios are formed annually from all NYSE firms active at the beginning of the year. Within each calendar year, size is measured as market value of equity at the end of the preceding year. For the 1984–1987 period,  $\lambda$  and all other liquidity variables are estimated using 1984 data. For the 1988–1991 period, they are estimated from 1988 data. Variables  $L2$  through  $L5$  denote indicators corresponding to the second through fifth  $\lambda$  groups, arranged in increasing order of  $\lambda$ . The group with the smallest  $\lambda$  forms the base case. The regression equation is

$$R_{it} = \alpha_i + \sum_{k=2}^5 \gamma_k L_{ik} + \beta_i MKT_t + \delta_i SMB_t + \kappa_i HML_t + \varepsilon_{it},$$

where  $R_{it}$  denotes the excess return on portfolio  $i$  in month  $t$ , the  $L$ 's denote the dummy variables, and  $MKT$ ,  $SMB$ , and  $HML$  denote the Fama–French factors related to the market, firm size, and the book-to-market ratio, respectively. The second through fourth columns report the average values of firm size,  $\lambda$ ,  $C_q$ , which equals  $\lambda$  times the average trade size divided by monthly average closing price, and  $C_n$ , which equals  $\lambda$  times the number of shares outstanding divided by monthly average closing price for each of the five  $\lambda$  groups ( $t$ -statistics are in parentheses).

	Estimate*10 <sup>3</sup> ( $t$ -statistic)	Average size (\$ millions)	$\lambda$ *100	$C_q$ *10 <sup>3</sup>	$C_n$
Constant	0.01 (0.01)				
$L1$	Base case	1,735.79	0.0042	0.085	1.25
$L2$	1.63 (2.05)	1,773.08	0.0132	0.156	2.06
$L3$	2.82 (2.88)	1,955.04	0.0249	0.208	2.40
$L4$	4.37 (3.86)	1,298.49	0.0452	0.302	3.34
$L5$	5.52 (5.26)	1,324.33	0.1478	0.479	5.04

component  $\psi/P$ , we repeat the generalized least squares regressions of the type (9), with different regressors for the last  $k$  columns of the matrix  $W$ . We use, in turn, both the GH and the HFV measures of  $\lambda$  and  $\psi$  to compute  $C_q$ ,  $C_n$ , and  $\psi/P$ .

First, for direct comparability with Amihud and Mendelson (1986), we perform the GLS regression including the time-series average of the bid–ask spread as a regressor; the results appear in the first column of panel A in Table 5. (Again, we omit the estimated portfolio factor loadings which are close to those reported in Table 3.) The spread enters the regression with a strongly significant

Table 5

Pooled time-series cross-sectional GLS regressions using the cost of illiquidity variables for 25 portfolios of NYSE stocks sorted by size and the Glosten–Harris measure of illiquidity,  $\lambda$ , for the period 1984–1991

Five portfolios for which data on  $\lambda$  are not available are omitted from the sample.  $\lambda$  estimates the derivative of transaction price (\$/share) with respect to signed trade size (shares, positive for trades initiated by buyers). Portfolios are formed annually from all NYSE firms active at the beginning of the year. Within each calendar year, size is measured as market value of equity at the end of the preceding year.  $C_q$  equals  $\lambda$  times the average trade size divided by the monthly average closing price,  $C_n$  equals  $\lambda$  times the monthly average number of shares outstanding divided by monthly average closing price, and  $\psi/P$  denotes the fixed component of trading costs as a proportion of the monthly average closing price. The proportional spread is calculated by averaging the proportional quoted spread (i.e., the quoted spread divided by the average of the bid and ask prices) across all quotations during the year.  $\log(C_q)$  and  $C_q^2$  are defined as the average of the logarithm of  $C_q$  and the average of the squared value of  $C_q$  for the relevant portfolio respectively, and  $C_n$ ,  $C_n^2$ , and  $(\psi/P)^2$  are defined similarly. For the 1984–1987 period,  $\lambda$  and all other liquidity variables are estimated using 1984 data. For the 1988–1991 period, they are estimated from 1988 data.  $(1/P)$  is the mean of the inverse monthly average closing price for each portfolio. The regression equation is

$$R_{it} = \alpha + \sum_{k=1}^N \gamma_k L_{ik} + \beta_i MKT_t + \delta_i SMB_t + \kappa_i HML_t + \varepsilon_{it},$$

where  $R_{it}$  denotes the excess return on portfolio  $i$  in month  $t$ , the  $L$ 's denote the illiquidity variables, and the inverse price variable (if included), and  $MKT$ ,  $SMB$ , and  $HML$  denote the Fama–French factors related to the market, firm size, and the book-to-market ratio, respectively.  $N$  is the total number of illiquidity variables plus one if the inverse price level variable is included in the regression ( $t$ -statistics are in parentheses).

(A)  $C_q$  as measure of the cost of illiquidity

	(1)	(2)	(3)	(4)	(5)
Constant*1000	9.04 (8.42)	10.78 (8.94)	27.41 (1.99)	9.02 (7.06)	− 26.90 (− 0.94)
		<b>10.41</b> <b>(8.54)</b>	<b>25.59</b> <b>(1.91)</b>	<b>9.10</b> <b>(7.15)</b>	<b>− 31.54</b> <b>(− 1.16)</b>
$C_q$		3.21 (2.03)		1.55 (4.90)	2.18 (3.24)
		<b>3.48</b> <b>(2.07)</b>		<b>1.37</b> <b>(4.27)</b>	<b>2.23</b> <b>(3.28)</b>
$\psi/P$		1.52 (2.45)		0.82 (0.86)	7.64 (2.85)
		<b>1.38</b> <b>(2.20)</b>		<b>0.79</b> <b>(0.84)</b>	<b>9.32</b> <b>(3.41)</b>
$\log(C_q)/1000$			1.08 (3.14)		
			<b>1.13</b> <b>(3.18)</b>		



Table 5 (continued)

	(1)	(2)	(3)	(4)	(5)
log( $\psi/P$ )/1000			0.96 (0.49) <b>0.67</b> <b>(0.35)</b>		
$C_q^2 * 1000$				- 6.44 (- 4.39) <b>- 5.22</b> <b>(- 3.65)</b>	- 8.19 (- 3.76) <b>- 7.75</b> <b>(- 3.63)</b>
$(\psi/P)^2 * 100$				0.31 (0.36) <b>0.38</b> <b>(0.46)</b>	0.79 (3.53) <b>0.95</b> <b>(4.37)</b>
(1/P)					- 0.45 (- 3.26) <b>- 0.57</b> <b>(- 3.99)</b>
Prop. spread	- 0.93 (- 6.00)	- 1.88 (- 4.89) <b>- 1.75</b> <b>(- 4.49)</b>	- 1.21 (- 3.94) <b>- 1.12</b> <b>(- 3.73)</b>	- 1.48 (- 3.04) <b>- 1.45</b> <b>(- 3.01)</b>	- 0.57 (- 0.99) <b>- 0.37</b> <b>(- 0.66)</b>
Avg. log (size)/10 <sup>3</sup>					1.43 (1.22) <b>1.60</b> <b>(1.44)</b>
<b>(B) <math>C_n</math> as measure of market illiquidity</b>					
	(1)	(2)	(3)	(4)	(5)
Constant* 1000	9.04 (8.42)	8.94 (7.36) <b>8.28</b> <b>(6.65)</b>	26.86 (1.95) <b>24.95</b> <b>(1.87)</b>	8.22 (6.62) <b>7.65</b> <b>(6.01)</b>	- 3.02 (- 0.15) <b>- 6.52</b> <b>(- 0.35)</b>
$C_n/1000$		0.64 (3.24) <b>0.70</b> <b>(3.30)</b>		1.01 (4.30) <b>0.98</b> <b>(4.01)</b>	1.18 (3.19) <b>1.25</b> <b>(3.40)</b>
$(\psi/P)$		1.56 (2.65) <b>1.41</b> <b>(2.39)</b>		2.52 (2.54) <b>2.06</b> <b>(2.09)</b>	11.23 (4.42) <b>12.04</b> <b>(4.75)</b>

Table 5 (continued)

	(1)	(2)	(3)	(4)	(5)
$\log(C_n)/100$			0.15 (2.90) <b>0.16</b> <b>(2.90)</b>		
$\log(\psi/P)/100$			0.27 (1.40) <b>0.25</b> <b>(1.33)</b>		
$C_n^2/1000$				- 0.01 (- 2.74) <b>- 0.01</b> <b>(- 2.15)</b>	- 0.02 (- 4.81) <b>- 0.02</b> <b>(- 4.67)</b>
$(\psi/P)^2 * 100$				- 0.88 (- 0.98) <b>- 0.61</b> <b>(- 0.70)</b>	1.06 (4.42) <b>1.14</b> <b>(4.92)</b>
$(1/P)$					- 0.63 (- 4.56) <b>- 0.70</b> <b>(- 5.05)</b>
Prop. spread	- 0.93 (- 6.00)	- 1.79 (- 5.14) <b>- 1.65</b> <b>(- 4.65)</b>	- 1.18 (- 3.83) <b>- 1.09</b> <b>(- 3.59)</b>	- 2.14 (- 4.56) <b>- 1.88</b> <b>(- 3.99)</b>	- 1.19 (- 2.25) <b>- 0.85</b> <b>(- 1.61)</b>
Avg. log (size)/10 <sup>3</sup>					0.36 (0.45) <b>0.46</b> <b>(0.62)</b>

but negative sign. This finding is contrary to the hypothesized role of the spread as a proxy for market illiquidity, but is consistent with the results in Eleswarapu and Reinganum (1993) for the 1981-90 period (see panel B of their Table 4).

In order to investigate whether the negative sign on the spread can be explained by the inclusion of our transaction cost variables, we include the time-series averages of  $C_q$  and  $\psi/P$  in the GLS regression. The coefficients are reported in the second column of panel A in Table 5.<sup>5</sup> The coefficients of the

<sup>5</sup>Note that there is time-series variation in  $C_q$  only on account of changing portfolio membership and the switch from the 1984 to 1988 estimates of  $C$ .

variable cost measure  $C_q$  and the fixed cost measure  $\psi/P$  are both positive and significant for both the GH and HFV measures of  $\lambda$ , but the coefficient of the spread remains negative and strongly significant.

Amihud and Mendelson (1986) suggest that investors with long horizons will require a smaller premium for illiquidity and, in equilibrium, will tend to hold the relatively illiquid stocks. This clientele effect will give rise to a concave relation between returns and the costs of transacting. However, the consideration of both the fixed and variable components of transaction costs gives rise to the possibility of a second clientele, this time in trade sizes. Thus, investors who trade small orders will be at a comparative advantage relative to large traders in trading high  $\lambda/P$  stocks. We should therefore expect the stocks with high values of  $\lambda/P$  to be held by investors who have long horizons *and* small trade sizes. Stocks with high values of the proportional fixed component  $\psi/P$ , however, would be held by investors with long horizons, *regardless* of the investors' preferred trade sizes. This suggests that the variable element of the proportional cost admits two clientele effects (associated with both trade size and horizon), while the fixed element admits only one (associated only with horizon). We would therefore expect the concavity between the relation between returns and the proportional cost of transacting to be greater for the variable component than for the fixed component.

To take account of potential nonlinearities in the relation between return and the cost of illiquidity, we replace  $C_q$  and  $\psi/P$  with the time-series average of the portfolio value of the logarithms of  $C_q$  and  $\psi/P$ . The results are reported in the third column of panel A in Table 5. With this modified specification, the variable cost of transacting variable becomes more strongly significant for both measures of  $\lambda$ , consistent with the trade size clientele effect discussed above. However, the fixed cost of transacting variable,  $\psi/P$ , is now less significant, suggesting that the horizon clientele effects discussed by Amihud and Mendelson may not be so important.

As an alternative empirical specification, we also include quadratic functions of  $C_q$  and  $\psi/P$  in the GLS regression, with squared terms computed by taking the time-series average of the average values of the squared measures within each portfolio. The relevant coefficients are reported in the fourth column of the table. For both measures of  $\lambda$ , the coefficient of the linear term  $C_q$  is positive, and that of the squared term  $C_q^2$  is negative, confirming the concave nature of the relation; both coefficients are strongly significant. This finding supports the notion that clientele effects cause the effect of our illiquidity measures on returns to be overstated for portfolios with large values of these measures. Note, however, that the coefficients of the linear and squared terms involving  $\psi/P$ , the fixed cost component, are not significant, though they are both positive.

The negative and significant coefficient on the bid–ask spread in all the regressions discussed above is puzzling and points to some misspecification in our regressions. One possibility is that the spread proxies for risk variables

related to firm size and the price level (see Miller and Scholes, 1982, pp. 1132–1133) that are omitted from the Fama–French model. To test this hypothesis, we include the average of the inverse price level and the average of the portfolio value of (the logarithm of) firm size in the regression represented by column (5) of panel A in Table 5. (The inverse price level for each security is computed as the inverse of the closing price at the end of the previous month.) The price variable is strongly significant, while in the presence of this variable, the spread variable becomes insignificant; the logarithm of firm size is not significant either. This suggests that the spread effect observed in the first four regressions is due to the spread proxying for a risk variable that is associated with (the reciprocal of) the price variable. This would explain why Amihud and Mendelson find a positive premium associated with the spread in one sample period while Eleswarapu and Reinganum find a negative premium in a different sample period. The coefficients of both  $\psi/P$  and  $(\psi/P)^2$  are now positive and strongly significant, while there is not much alteration in the magnitude or significance of the coefficients associated with  $C_q$  for either measure of  $\lambda$ .

Thus the results of the extended regression are consistent with a risk model in which the Fama–French risk factors are complemented with a price factor. Further, there are premiums associated with both the fixed and variable proportional costs of trading. The relation between the premium and the variable element is increasing and concave, consistent with a clientele effect in trade sizes. The relation between the premium and the fixed element is increasing but convex which is inconsistent with a clientele effect in horizons of the type suggested by Amihud and Mendelson (1986). One possible reason for this convex relation is that (as we noted earlier) the techniques used by Glosten and Harris (1988), Hasbrouck (1991), and Foster and Viswanathan (1993) (and thus, by us) ignore discreteness on the grounds that it is prohibitively expensive to address. This may cause the fixed component to be underestimated when it is large. A second possibility is that even with the inclusion of the inverse price variable we have been unable to capture completely the structure of risk premiums.

Panel B of Table 5 repeats the same regressions as those in panel A using  $C_n$  as the variable component of illiquidity. The results are qualitatively very similar, and indeed somewhat stronger, using this measure of illiquidity. In particular, the coefficients of the linear terms of both the variable and fixed components of transacting are now significant in the quadratic specification. Further, the results using (log) firm size and the inverse of the price level mimic those reported in panel A.

The results reported in this section are consistent with our fixed and variable transaction cost variables having a significant positive effect on equilibrium rates of return, particularly when the  $C_n$  measure of variable trading costs is used.

### 4.3. Seasonality in the compensation for market illiquidity

Eleswarapu and Reinganum (1993) report that the association between the bid–ask spread and returns is seasonal, and is mainly confined to the month of January. In the context of the results in Table 5, it is of interest to examine two issues. First, is there a seasonal component to the compensation for our transaction cost measures? Second, does the seasonal component in the coefficient of the bid–ask spread found by Eleswarapu and Reinganum survive in the expanded Fama–French model setting?

We assess the presence of seasonal effects using a likelihood ratio test. Thus, we estimate regressions of the form of Eq. (9) by GLS, allowing for seasonal effects. First, the regression is estimated including the Fama–French factors and allowing for seasonal effects in the coefficients of all the six variables in column (5) of Table 5, i.e., the inverse price level and the five illiquidity variables ( $C_q$ ,  $C_q^2$ ,  $\psi/P$ ,  $[\psi/P]^2$ , and the proportional bid–ask spread). We do this by expanding the column corresponding to a given variable in (9) to a 12-column matrix whose columns consist of the variable interacted with 12 monthly dummy variables. Let the estimated variance–covariance matrix from this unconstrained regression be denoted by  $\hat{\Omega}_u$  and the vector of regression residuals by  $\varepsilon_u$ .

Consider a test of whether the coefficients of a subset consisting of  $N \leq 6$  variable(s)  $X_i$ ,  $i = 1, \dots, N$ , have seasonal components associated with them. For this, we perform a constrained GLS regression, which includes all of the five illiquidity variables and the price level, but in which the variables  $X_1, \dots, X_N$  are constrained to have the same coefficient each month. Let  $\varepsilon_c$  denote the vector of residuals from this constrained regression. Then, under the null hypothesis of no seasonals associated with the coefficients of  $X_1, \dots, X_N$ ,

$$\varepsilon_c' \hat{\Omega}_u^{-1} \varepsilon_c - \varepsilon_u' \hat{\Omega}_u^{-1} \varepsilon_u$$

is asymptotically distributed  $\chi^2$  with degrees of freedom equal to the number of constraints, which, in turn, is equal to  $11 \times N$ .

Our first test is of the null hypothesis of no seasonal in the coefficients of the linear and quadratic measures of the fixed and variable elements of the proportional cost of trading,  $C_q$ ,  $C_q^2$ ,  $\psi/P$ , and  $(\psi/P)^2$ . As shown in the first panel of Table 6, we cannot reject the null of no seasonal in the premium associated with these transaction cost measures. Nor can we reject the null of no seasonal in the risk premium associated with the inverse price variable. Finally, unlike Eleswarapu and Reinganum, we are also unable to reject the null of no seasonal in the coefficient associated with the bid–ask spread. Thus we find no evidence of seasonality in the premium associated with transaction costs.

It is natural to ask why the seasonal in the spread premium documented by Eleswarapu and Reinganum disappears in our regressions. We are unable to provide a definitive answer because our analysis uses a different sample period. However, a second major difference between our analysis and that of

Table 6

Seasonality in the compensation for illiquidity: This table presents the results of testing for seasonality in the coefficients of the various cost of illiquidity variables, using pooled time-series cross-sectional GLS regressions for 25 portfolios of NYSE stocks sorted by size and the Glosten–Harris measure of illiquidity,  $\lambda$ , for the period of 1984–1991

Five portfolios for which data on  $\lambda$  are not available are omitted from the sample.  $\lambda$  estimates the derivative of transaction price (\$/share) with respect to signed trade size (shares, positive for trades initiated by buyers). Portfolios are formed annually from all NYSE firms active at the beginning of the year. Within each calendar year, size is measured as market value of equity at the end of the preceding year.  $C_q$  equals  $\lambda$  times the average trade size divided by the monthly average closing price,  $C_n$  equals  $\lambda$  times the monthly average number of shares outstanding divided by monthly average closing price, and  $\psi/P$  denotes the fixed component of trading costs as a proportion of the monthly average closing price. The proportional spread is calculated by averaging the proportional quoted spread (i.e., the quoted spread divided by the average of the bid and ask prices) across all quotations during the year.  $C_q^2$  is defined as the average of the squared value of  $C_q$  for the relevant portfolio;  $(\psi/P)^2$  is defined similarly. For the 1984–1987 period,  $\lambda$  and all other liquidity variables are estimated using 1984 data. For the 1988–1991 period, they are estimated from 1988 data.  $(1/P)$  is the mean of the inverse monthly average closing price for each portfolio.

The quantity  $-2\ln(LLR)$  denotes twice the natural logarithm of the likelihood ratio;  $-2\ln(LLR)$  is calculated as  $\varepsilon_c' \hat{\Omega}_c^{-1} \varepsilon_c - \varepsilon_u' \hat{\Omega}_u^{-1} \varepsilon_u$ , where  $\varepsilon_c$  is the vector of residuals from a constrained GLS regression in which some variable(s) is(are) constrained to have the same coefficient each month,  $\varepsilon_u$  is the vector of residuals from the unconstrained regression, and  $\hat{\Omega}_u$  is the estimated residual variance–covariance matrix from the unconstrained regression in which the coefficients of all the variables are allowed to differ across months. Each of the constrained and unconstrained GLS regressions contains the Fama and French factors, the five illiquidity variables, i.e.,  $C_q$ ,  $C_q^2$ ,  $\psi/P$ ,  $(\psi/P)^2$ , and the proportional spread, together with the inverse price level variable  $1/P$ .

Under the null hypothesis,  $-2\ln(LLR)$  is distributed  $\chi^2$  with degrees of freedom equal to the number of constraints, which in turn, is equal to 11 times the number of variables whose coefficients are constrained to be the same each month.

Null hypothesis	No. of constraints ( $k$ )	$2\ln(LLR)$	$p$ -value
No seasonal in the coefficient of $C_q$ , $C_q^2$ , $\psi/P$ , $(\psi/P)^2$	44	19.07	0.9996
No seasonal in the coefficient of $(1/\text{price})$	11	1.77	0.9991
No seasonal in the coefficient of the proportional spread	11	9.23	0.6011

Eleswarapu and Reinganum is the use of the Fama–French risk factor model which absorbs any seasonality associated with firm size or book-to-market ratio. Further, the January coefficient of the spread is *negative* and significant (and indeed is the only significant coefficient among the 12) in the unconstrained GLS regression. This appears to support the notion that the spread proxies for some omitted risk factor, rather than for illiquidity, in our sample period.

## 5. Conclusion

In this paper, we unify recent techniques from the asset pricing and market microstructure literatures to examine the association between average risk-adjusted rates of return and both the variable and the fixed components of the cost of transacting. As the variable component is derived from models that take account of the adverse selection caused by privately informed traders, we are able to shed light on the significance of the empirical measures of adverse selection costs in determining required rates of return on equity. This exercise takes on particular significance given the voluminous work, both theoretical and empirical, on the adverse selection paradigm in recent years.

Our main findings are that there is a significant return premium associated with both the fixed and variable elements of the cost of transacting. The relation between the premium and the variable cost is concave, which is consistent with clientele effects caused by small traders concentrating in the less liquid stocks. However, the relation between the premium and the estimated fixed cost component is convex. This is inconsistent with the horizon clientele effect proposed by Amihud and Mendelson (1986), and may be the result of our inability to estimate this parameter accurately on account of price discreteness. Alternatively, it may be due to incomplete risk adjustment by the three-factor Fama–French model we use. We also find that even after risk adjustment using this model there is an additional risk premium associated with an inverse price factor. There is no evidence of seasonality in the premiums associated with our cost of transacting variables. Finally, an interesting byproduct of our analysis is the finding that controlling for firm size, there appears to be a negative relation between the variable and fixed costs of transacting. Theoretical and empirical understanding of this phenomenon appears to be a fruitful area for future research.

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