

10.7 Fama and French Mutual Funds notes

Why the Fama-French simulation works to detect skill, even without knowing the characteristics of skill.

The genius of the Fama-French simulation is that it lets us see if there are some “skilled” managers *without* trying to identify which funds are skilled – without trying to identify a characteristic (past returns, past alphas, morningstar rating, MBA of manager, etc.) that we think is associated with skill, form portfolios, and watch returns going forward. Of course that also means we don’t learn how to identify those skilled managers either.

How does it work? Consider a very simple example – a world of Bernie Medoffs; well, of the guy Bernie was pretending to be. Suppose there are two managers, one with positive alpha who always generates 5% returns, and one with negative alpha, who always generates -5% returns. Now, look at a sample. Subtract from each fund its sample mean — 5% in the first case, -5% in the second case – and simulate what’s left over. There’s nothing left over, so in this “no skill” (true alpha = 0) simulation, the distribution of estimated alphas should always stick at 0. The fact that we see “fat tails” in the distribution of estimated alphas tells us that there is some skill and some “negative skill.”

More generally, the amount of return variability *over time* tells you how much return could possibly be generated by luck. This observation is really just the standard error of the mean. If returns vary by $\sigma(R)$ *over time*, then the mean return should vary by only $\sigma(R)/\sqrt{T}$ *across funds*. This time we get to see lots of draws, unlike the usual application of standard error of the mean where you only see one draw. If mean returns vary across funds by more than luck can account for, then we know there is some skill. By looking at *how much* fatter the actual distribution of mean returns is than would be generated by luck, we can find the distribution of true underlying skill.

Fama and French don’t look at mean returns. First, they adjust for factor exposures; for each fund they run the usual regression $R_t^{ei} = \alpha_i + b_{irmr}f_t + h_{ihml}h_t + s_{ismb}b_t + \varepsilon_t^i$, and they look at the estimated $\hat{\alpha}_i$, and save the residuals $\hat{\varepsilon}_t^i$ (hats mean estimates). Then they do a bootstrap: They keep the ε_t^i of all the funds at each moment in time together and re-sample months. (We’ll do a bootstrap later if this is confusing.) This controls for the fact that returns might not be normally distributed, and by resampling the entire month of returns for all funds together they capture the fact that returns might be correlated across funds, so each fund is not really a separate experiment. Finally, they evaluate each fund by its t statistic $\hat{\alpha}_i / [\sigma(\varepsilon^i)/\sqrt{T}]$. This is a good idea. If we just looked at the distribution of alphas, some funds that are alive for shorter times or have larger tracking error $\sigma(\varepsilon)$ are more likely than others to achieve big alphas by chance, and to figure out “what is the chance of seeing so many big alphas” we’d have to keep track of all that. But these are refinements on the basic idea.

Here’s a simple example. Ignore factors and suppose returns are normal. Fund returns are generated by fund-specific skill α_i (which is constant over time – this is the key to measuring it) and luck,

$$R_t^i = \alpha_i + \varepsilon_t^i$$

Assume the ε^i have the same variance for each fund and are uncorrelated across funds, to make the example simple.

Now, let’s think about the distribution of average returns = estimated alpha. The estimated alphas for a given fund will be

$$\hat{\alpha}_i = E(R_t^i) = \alpha_i + \frac{1}{T} \sum_{t=1}^T \varepsilon_t^i$$

Thus, the estimated alpha has a distribution

$$\hat{\alpha}_i \sim \alpha_i + N(0, \sigma_\varepsilon^2/T)$$

These estimated alphas capture some skill as well as some luck. This is our standard error of the mean formula. It means “if you ran this fund over and over again, the mean would fill out this distribution.” But since the funds are the same in this example, it also means “this is how much variation we expect *across funds*.”

If there is no skill, then, we expect to see a distribution of estimated alphas across funds that has variance σ_ε^2/T .

Now, what if there is some skill? Skill is also distributed *across funds*. To keep it simple, I’ll write that the distribution of alphas across funds is also normal,

$$\alpha_i \sim N(\mu_\alpha, \sigma_\alpha^2).$$

This is the distribution of true skill across funds; I assume that skill is picked once and for all at the start of the sample and stays constant over time once picked. Since the sum of normals is normal, and the subsequent luck is independent of the true alphas, the distribution of estimated alphas across funds will now be

$$\hat{\alpha}_i \sim \left[N(\mu_\alpha, \sigma_\alpha^2) + N(0, \sigma_\varepsilon^2/T) \right] = N(\mu_\alpha, \sigma_\alpha^2 + \sigma_\varepsilon^2/T).$$

We have a normal distribution of estimated alphas. Some of the variation across funds comes from luck (σ_ε) and some from skill (σ_α). You can see how adding some distribution of true skill fattens up the distribution of estimated alphas across funds. If we see too many good funds and too many bad funds, we can infer that there is some underlying skill, though we still cannot tell whether a *particular* fund was skilled or lucky.

Let’s see how the FF procedure separates these two components. They take out the sample mean, so they form

$$e_t^i = R_t^i - \frac{1}{T} \sum_{t=1}^T R_t^i = \alpha_i + \varepsilon_t^i - \frac{1}{T} \sum_{t=1}^T (\alpha_i + \varepsilon_t^i) = \varepsilon_t^i - \frac{1}{T} \sum_{t=1}^T \varepsilon_t^i$$

Notice that this always removes the true alpha, and leaves us a series that is just demeaned luck. Reshuffling (simulating) the e_t^i , their simulation delivers the predicted cross-sectional distribution of estimated alphas, under the assumption that true alpha is zero for everyone,

$$\hat{\alpha}^{sim} = \frac{1}{T} \sum_{t=1}^T e_t^i$$

Using the standard error of the mean logic, the variance across funds of this quantity is

$$\sigma^2(\hat{\alpha}^{sim}) = \sigma^2 \left(\frac{1}{T} \sum_{t=1}^T e_t^i \right) = \sigma^2 \left(\varepsilon_t^i - \frac{1}{T} \sum_{t=1}^T \varepsilon_t^i \right) = \frac{(T-1)}{T} \frac{\sigma_\varepsilon^2}{T}$$

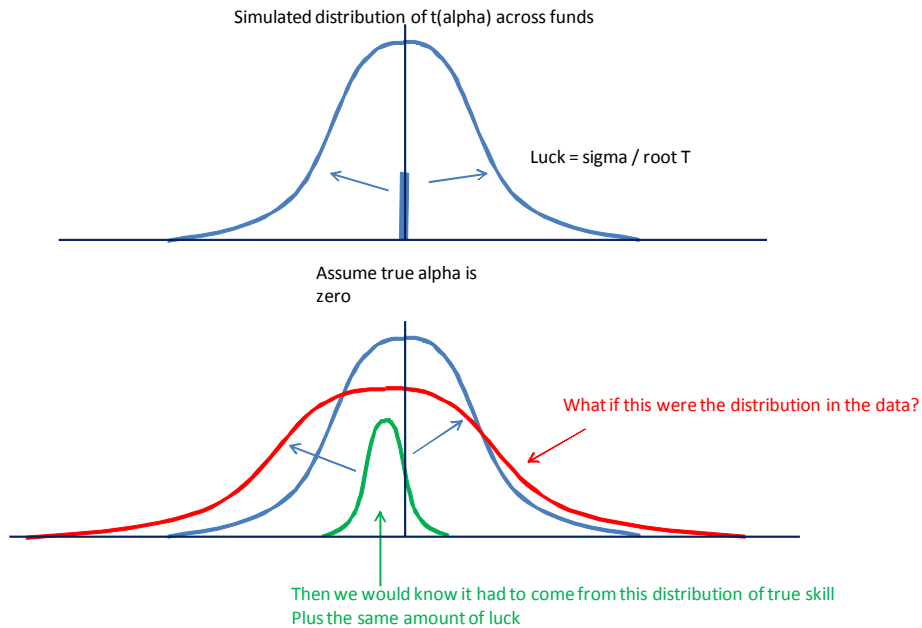
Compare this to the actual distribution of estimated alphas, from above, with variance

$$\sigma^2(\hat{\alpha}_i) = \sigma_\alpha^2 + \frac{\sigma_\varepsilon^2}{T}.$$

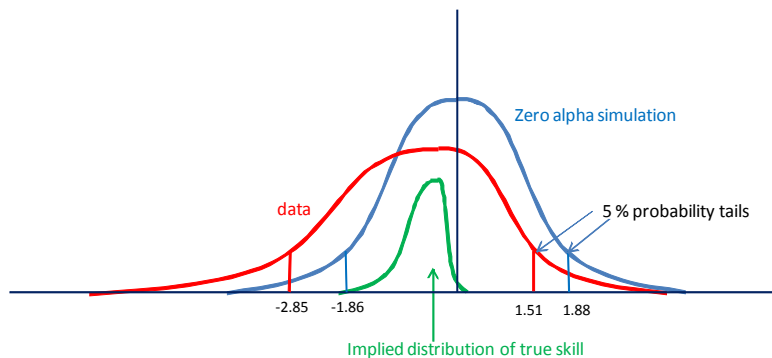
If there were no real alpha, then the simulation would give the variance (and hence distribution) of the actual estimated alphas. The simulation would be a little narrower, because of the $(T-1)/T$ term. This should be small, (and FF could correct for it.) But if there is substantial true alpha, then the distribution of the simulated alphas $\sigma^2(\hat{\alpha}^{sim})$ will be much narrower than the distribution of the true alphas, with variance $\sigma^2(\hat{\alpha}_i) = \sigma_\alpha^2 + \sigma_\varepsilon^2/T$. We would see a lot more large alphas than there should be by chance.

Here is a picture that will make it all clear. The top graph shows the simulation. Assume all funds have zero alpha, and by simulation (or standard error of the mean), we see what the distribution of estimated alphas

should be. Now suppose we saw a wider distribution as in the second graph. There is only so much luck one can have (that was the Bernie Medoff tipoff – if returns are incredibly stable over time, it’s impossible to have that much good luck.) Thus, we know that the distribution of observed fund alpha t statistics had to come from a distribution of skill (green) plus the luck.



For example, look at Table 4, top left block. The 5 Pct and 95 Pct numbers mean this on the graph, and as I eyeball the results it looks something like the following



Rather depressing overall.

Beyond Fama and French

This discussion says we can go beyond what FF did, and *directly estimate the distribution of alphas across funds*. If the distributions are normal, *the variance of the actual distribution of estimated alphas minus the variance of the simulated distribution of estimated alphas is a direct estimate of the variance across funds in true alpha*. Of course, *the mean of the distribution of estimated alphas is the mean of the distribution of true alphas* which we knew all along – “the average fund underperforms the market.”

It's also possible to directly estimate the distribution of true alphas. The distribution of alpha t statistics comes from luck and skill as follows.

$$f(t_\alpha) = \int_{-\infty}^{\infty} f(t_\alpha|\alpha) \times f(\alpha) d\alpha$$

Finding the unknown $f(\alpha)$ that “fattens up” $f(t_\alpha|\alpha)$ just enough is easy if you do the probabilities at discrete points

$$f(t_{\alpha i})\Delta t_{\alpha i} = \sum_j f(t_{\alpha i}|\alpha_j)\Delta t_{\alpha i} \times f(\alpha_j)\Delta \alpha_j$$

This is a matrix multiplication so we can find $f(\alpha_j)$ as a simple matrix inversion. (Well, keeping it numerically stable is hard, but conceptually, this is just a matrix inversion.)

The simulation tells us $f(t_\alpha|\alpha = 0)$. In general, we would have to do a lot of simulations to find $f(t_\alpha|\alpha)$ —assume a new value of α , then simulate again, and so forth. However since luck is independent of skill, $f(t_\alpha|\alpha)$ is the same as $f(t_\alpha|\alpha = 0)$ but just shifted over to the right. (This procedure uses the same assumption as in the simulation, that tracking error and fund life are independent of alpha.)

Alas, from the numbers in Fama and French’s paper we can only recover the distribution of the “true t_α ” meaning the true alpha divided by the standard error of estimated alphas. To go back from true t_α to “true alpha” we need to know $\sigma(\varepsilon)$ and T . Fama and French can easily calculate it, but we don’t have those numbers. I make some calculations below.

In sum, if we write the sample as $t_{\hat{\alpha}} = \hat{\alpha} / (\sigma_\varepsilon / \sqrt{T})$ and the true alpha divided by standard error as $t_\alpha = \alpha / (\sigma / \sqrt{T})$ then we have

$$f(t_{\hat{\alpha} i})\Delta t_{\hat{\alpha} i} = \sum_j f(t_{\hat{\alpha} i}|t_{\alpha j})\Delta t_{\hat{\alpha} i} \times f(t_{\alpha j})\Delta t_{\alpha j}$$

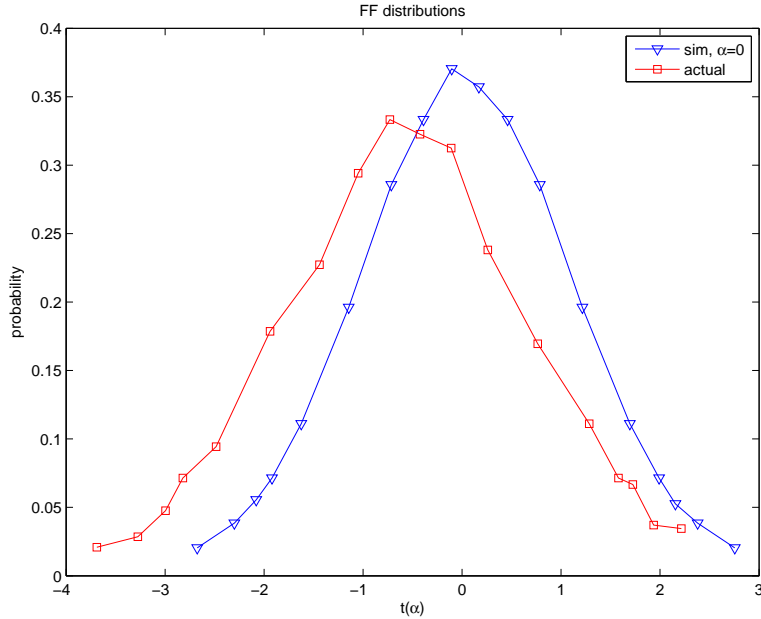
We can figure out the $f(t_{\hat{\alpha} i}|t_{\alpha j})$ from the simulation

$$\begin{aligned} f(t_{\hat{\alpha} i}|t_{\alpha j}) &= f(t_{\hat{\alpha} i} - t_{\alpha j}|t_{\alpha j} = 0) \\ f(t_{\hat{\alpha} i}|t_{\alpha j})\Delta t_{\hat{\alpha} i} &= [f(t_{\hat{\alpha} i} - t_{\alpha j}|t_{\alpha j} = 0)\Delta t_{\alpha j}] \frac{\Delta t_{\hat{\alpha} i}}{\Delta t_{\alpha j}} \end{aligned}$$

and now we can invert to find $f(t_{\alpha j})$.

To illustrate, I will make graphs corresponding to FF’s Table 4, bottom left panel. These are four-factor alphas.

Fama and French tabulate the *cumulative* distribution. To plot it as a density, I made histograms using the midpoints. For example, if the 95% value happened at $t_\alpha = 1.5$ and the 96% value happened at $t_\alpha = 1.6$, I drew a histogram with 1% probability at $t_\alpha = 1.55$. Of course, FF could easily calculate histograms as well. The first figure presents that result.

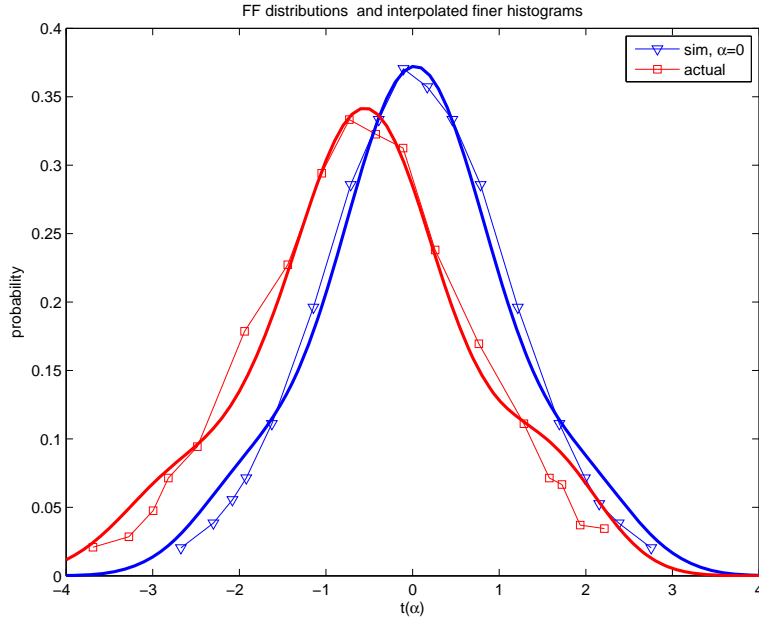


You can see that the actual distribution of t_α is shifted to the left and somewhat spread out compared to the distribution we expect if all true alphas are zero.

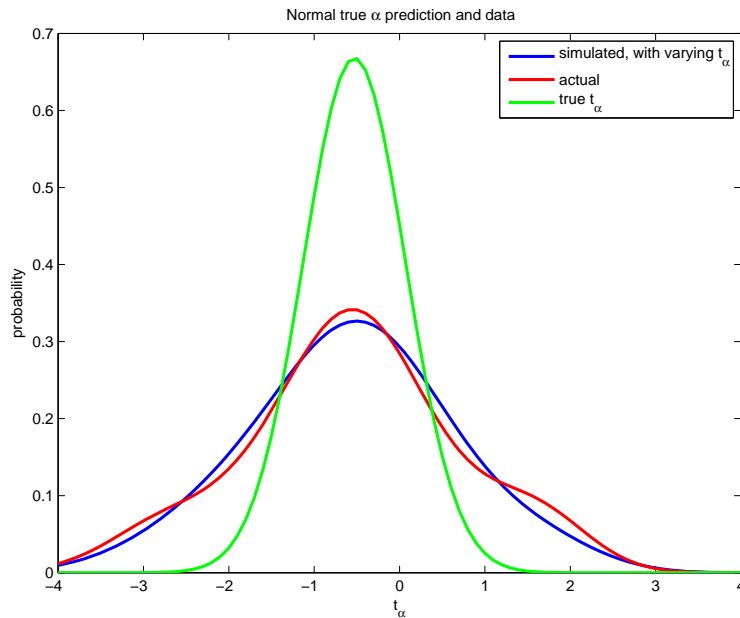
The table below gives the moments. Under the simulation, the mean alpha t-stats are zero (up to simulation error) and the standard deviation across funds is 1.18. This is what we would observe if there were no skill at all and all alphas were zero. The actual distribution is shifted to the left, with a mean -0.53, and a wider distribution with standard deviation of 1.32. Assuming normality, then, we can immediately infer that the distribution of the “true alpha” t-statistic (true alpha/ $(\sigma(\varepsilon)/\sqrt{T})$) has a mean of -0.53 and a standard deviation across funds of 0.60.

	sim. $\alpha = 0$	actual	true α
$E(t_\alpha)$	0.03	-0.53	-0.53
$\sigma(t_\alpha)$	1.18	1.32	0.60

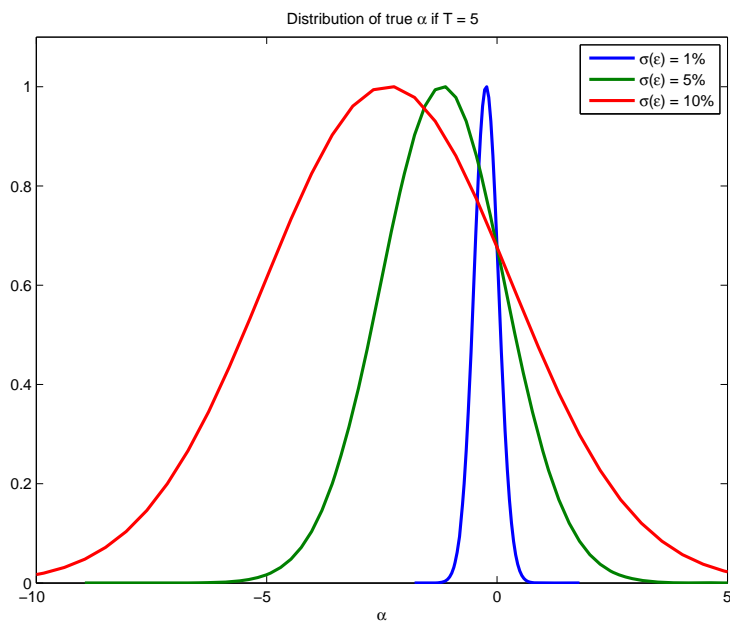
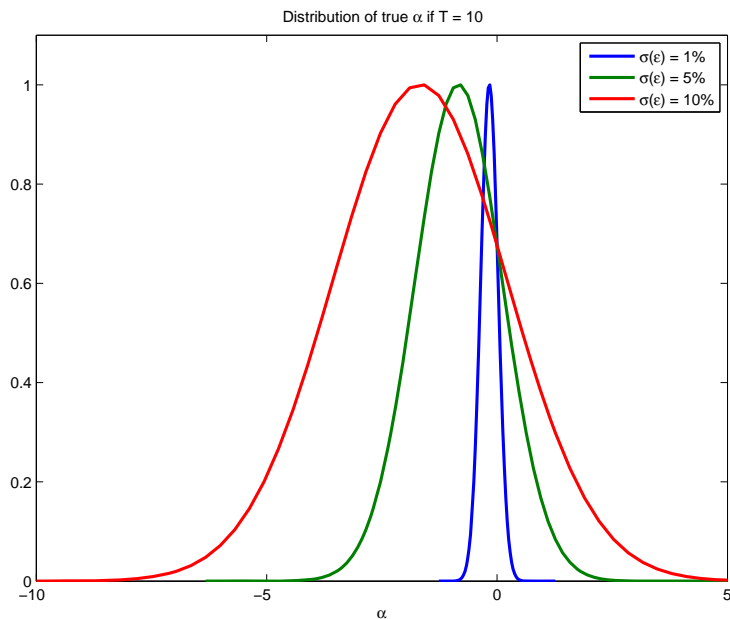
For the other calculations, it’s easier for me to infer the histogram on evenly spaced values on the x axis. (This isn’t really necessary, but it really helped the programming for this quick example. Also, FF can easily calculate these if they want to.) I did this by smoothing across the histogram. The next figure shows you the FF “histogram” and the histograms I use with a finer and even set of breakpoints. (This is a “kernel density estimate.”) As you can see, it’s a decent approximation.



Before doing the “nonparametric” (i.e. histogram) of the true alpha distribution, I plot here the normal value. The “true t_α here has normal distribution as calculated above with $\mu = -0.53, \sigma = 0.60$. The “simulated with varying t_α ” line then is formed from the simulation multiplied by this “true τ_α ” $\int_{-\infty}^{\infty} f(t_{\hat{\alpha}}|t_\alpha) \times f(t_\alpha) dt_\alpha$. This calculation comes quite close to the “actual” which is the actual distribution of $t_{\hat{\alpha}}$ across funds. The fact that blue and red lines are so close means that the *normal* distribution of true alphas is an excellent approximation. (I used these as starting points for a more accurate calculation that finds $f(t_\alpha)$ that will *exactly* match the actual distribution, but that calculation isn’t working yet.)



OK, now we have the implied distribution of true alpha t-statistics. What does this mean about true alphas? I don't know average fund ages and tracking errors, but we can make some simple assumptions. Here are the implied distributions of true alphas for $T = 10$ years and $T = 5$ years, and various tracking errors.



These distributions wider than you might have thought, given that the distribution of fund alpha t statistics didn't look much wider than the simulated distribution. However, we add squares of standard deviations. It took a good deal of alpha t statistic to widen up the fund distribution; $0.60^2 + 1.18^2 = 1.32^2$.