Problem Set 3

Due in class, Mon Nov 11

1. We wondered a bit in class about, "isn't a tautology to 'explain' 25 size and b/m portfolios by size and b/m factors?" "What if you formed 26 portfolios by sorting on the first letter of the stock? Wouldn't the AL minus MZ portfolio "explain" everything? This problem explores that question. Along the way it builds an understanding of the relationship between factors for *means* and factors for *variances.*

   So, suppose we have 25 random portfolios. What happens if we do the Fama French exercise on these random portfolios? Rather than work through a lot of algebra, we'll answer this question by simulation.

   So, create 25 monthly "portfolio returns" using the random number generator. Make them have different means, from 0 to 1% per month as you go from portfolio 1 to portfolio 25, with annualized 20% standard deviation, and uncorrelated with each other. Then, create a "market" portfolio which is just the average of your 25 portfolios. Create a "hml" portfolio composed of the average return the top 8 (roughly top 1/3, portfolios 1-8) portfolios minus the average return of the bottom 8 (portfolios 17-25). Run regressions of each portfolio return on the "market" and "hml" with a constant

   $$R_t^{ei} = \alpha_i + b_i rmrf_t + h_i hml_t + \varepsilon_t^i$$

   (a) Make a plot or tables of average return $E(R^{ei})$, α_i, b_i, h_i and R^2 versus $i = 1..25$.

   (b) Explain the patterns in b_i, h_i and R^2, and the pattern in α_i relative to $E(R^{ei})$. Why aren't the betas all zero? To what extent do each of these look like FF's results, and where do they differ? (Working out the coefficient and R^2 of R^{ei} regressed on $\sum_{j=1}^{N} R^{ei}$ will provide insight here)

   (c) Now, what part of the FF Table 1 results do look like these results and might be spurious? What parts of FF's results do not look like these results and so are not spurious? How might FF have run things a bit differently to clear up the small problems you do see here?

   (d) I decided not to give you the following problem: Rerun both the simulation and the Fama French model, but this time *exclude each portfolio i from the factors on the right hand side.* I.e. for each portfolio i, recreate a market return, a hml return, and a smb return from the other portfolios, but excluding portfolio i. So, instead of doing it, given what you've learned so far, what do you think would happen? I.e. how would the b, h in the simulation and b, h, s in FF, be affected by this change? How would the R^2 in the simulation and the R^2 in FF be affected by this change? How would the alphas be affected?

   Note: Create a very long sample. We're not interested in standard errors here or variation about true values due to short samples. Experiment a bit and choose a sample long enough that the numbers seem to have settled close to their population values, but not so long that it takes your computer forever to run the program

2. Please turn in the plots and tables you made for the coursera Fama French replication problem.

3. As you can see, the observation that *value returns are correlated with each other* is crucial to the whole FF exercise. Maybe the content of Table 1 really is just the *covariance* information – all the value returns are correlated, so if you think you'll make a ton by buying value companies and diversifying the risk you're wrong. Maybe in fact it is "just" an APT. But maybe its central

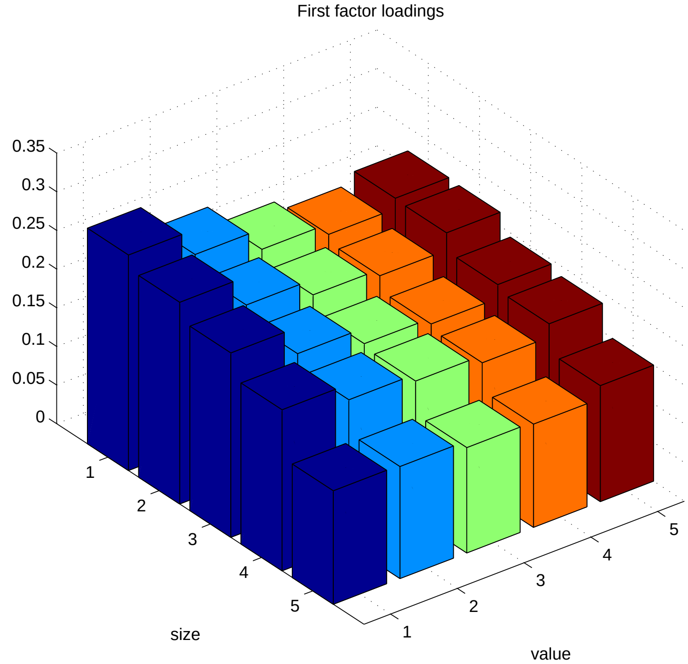
message to "behavioral" explanations is, "if the value effect is just sentiment, why do all the value firms *rise and fall together* next year as if they are exposed to some common factor. WIth all that in mind, let's approach the FF data as an APT. My solutions use the 1963-today sample.

(a) Form the covariance matrix of the FF 25 portfolios, and take the eigenvalue decomposition to form factors that explain the variance of the 25 returns.

(Digression on factor analysis. If you form $Q\Lambda Q' = cov(R^e R^{e\prime}) = \Sigma$, with $QQ' = Q'Q = I$ and Λ diagonal, $\Lambda = Q'\Sigma Q$. then the columns of Q tell you both the "weights", which linear combination of returns forms each factor, $f_t = Q'R^e_t$, and they also give you the "loadings", the OLS regression coefficients of data on factors, $R^e_t = Qf_t = q_1 f_{1t} + q_2 f_{2t} ...$ where q_i are the columns of q. The factors so constructed are orthogonal, $cov(f_t, f_t') = Q'\Sigma Q = \Lambda$, and ordered from largest to smallest variance. Since they are orthogonal, a factor model formed by leaving out some factors retains the property that columns of Q are OLS regressions of returns on factors, $R^e_t = q_1 f_{1t} + q_2 f_{2t} + \varepsilon_t$ will estimate the same q_1 and q_2. Each factor explains the largest possible amount of variance of the returns, while staying orthogonal to the others.)

(b) Plot the standard deviation of the factors (square root of Λ diagonals), then look at numbers. How many factors seem important? (If you make a plot or bar plot of the factor standard deviations, it jumps out at you where to start throwing out factors)

(c) Now, let's look at the four factors with largest standard deviations. First we want to look at the corresponding columns of Q (look again at the interpretation of these numbers). I found the plots most revealing by using the same size-by-book/market display that I used to display the FF model before. For example, here's the first factor loadings. Since they are all positive, it is a sort of level factor – you form the factor by roughly an equal weighted portfolio of the 25 returns, and these are the OLS regressions of returns on the factor. Interestingly, it weights small firms even more than large firms (rmrf is *value* weighted, so would put lots of weight on large.) So, do like this, present the plots, and answer, *How can you interpret the remaining three of the first four factors?* (Yes 4 – let's go one past FF and see what happens.)



Note: in matching what comes out of this procedure with the Fama-French model, keep in mind that you can always use linear combinations of factors. Mathematically, if you have a factor model

$$R_t = q_1 x_t^{(1)} + q_2 x_t^{(2)} + q_3 x_t^{(3)}$$

then you could just as easily rewrite this as a factor model

$$\begin{aligned} R_t &= q_1 x_t^{(1)} + [q_2 + q_3] \left[x_t^{(2)} + x_t^{(3)}\right] + [q_2 - q_3] \left[x_t^{(2)} - x_t^{(3)}\right] \\ R_t &= q_1 x_t^{(1)} + \tilde{q}_2 \tilde{x}_t^{(2)} + \tilde{q}_3 \tilde{x}_t^{(3)} \end{aligned}$$

These factors will no longer have the special property that each in turn captures as much variance as possible, but once we've settled on how many factors we want, that's not so important anymore. The upshot is, this procedure may well produce factors that are *combinations* of hml and smb, especially since hml and smb have very similar variance. Factor models are always only identified up to such "rotations." Eigenvalue decompositions identify rotation by maximizing variance explained, but that isn't always interesting.

(d) Take the first three and then the first four factors, construct a time series of the factors from the underlying 25 FF excess returns,($x_t = Q'y_t$) and then run time series regressions of the 25 excess returns against these factors. ($R_t^{ei} = \alpha_i + b_{1i}x_{1t} + b_{2i}x_{2t} + ... + \varepsilon_{it}$, i.e. do just like Fama and French but using the first three and then four x factors rather than rmf, hml, smb. You can use tsregress2 or your program.)

   i. How do the regression coefficients compare with the columns of Q?
   ii. What are the R^2 in these regressions, and how do they compare to the FF case? How do they work as factor models of *returns*, i.e. of return *variance.*
   iii. How well do these models work as *expected return* factor models? Are the alphas small? (No need to repeat GRS tests, etc. Just plot or make a table so you can see how big they are.)
   iv. How do these models compare to the Fama French regressions?

Note: It is *not* true that factor models will attack average returns or alphas one by one. The factor model finds in order the largest common movements in the *covariance* matrix of returns. There is no mathematical reason that this should relate at all to the *mean* returns or alphas left over from the last factor model. In fact, you will see a pattern that each factor model attacks patterns seen in mean returns of the last factor model. This is *finance*, not math, it's a sign of the basic idea in finance working. The central idea in finance is that risk premia will attach only to common, undiversifiable movements in asset returns. And, lo, it does.

4. We discussed the question, "Did Fama and French need the size factor?" Let's answer it.

(a) Tabulate the mean of rmrf, hml, smb, their sharpe ratios and their t statistics. Are they economically and statistically significant?

(b) As we discussed, a factor "is priced" if its mean is zero. A factor "is needed on the margin to price other assets' 'if its alpha is zero or, equivalently, if the mean of an orthogonalized version of the factor is zero. $E(smb - b_{smb}rmrf - h_{smb}hml)$ is both the mean of the orthogonalized smb factor and the alpha of smb given the other factors. Tabulate the alpha and its t statistic of

   i. smb vs rmrf and hml
   ii. smb vs. rmrf
   iii. hml vs. rmrf and smb

iv. hml vs. rmrf

Which factors do we need by this test?

(c) Let's try a FF 2 factor model. What would have happened if they left out smb? How would it have looked different from the three factor model? Tabulate α, b, h, R^2 $t(\alpha)$ and $\sqrt{\frac{1}{N}\sum \alpha_i^2}$. Make a plot of actual $E(R^{ei})$ vs predicted $b_i E(rmrf) + h_i E(hml)$. Compare to the three factor model. Is it useful to include smb even if the evidence is marginal on whether smb is "priced" or is useful in pricing other assets?