Final Exam

YOUR NAME: _______________________________________.

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(Economics, Booth PhD/MBA mailfolders, elsewhere)

INSTRUCTIONS

DO NOT TURN OVER THIS PAGE UNTIL YOU ARE TOLD TO DO SO.

Please sit in alternate seats. This is a closed-book, closed-note exam. Please get rid of everything but pen/pencil. Please don't rip the exam apart–keep it stapled.

Put your answers in the spaces provided. There is extra paper stapled to the end of the exam. You can also use the paper at the end of the exam for scratch. *Show your work* – quote what equation you start with, and explain the logic you use to get your answer. When the main answer is an equation or a number, please put a box around it so we can find it. Keep answers short; I'm only looking for the really obvious big point, I don't give more credit for long winded answers, and I can take points off if you add things that are wrong or irrelevant. Make sure you try every question – I can't give partial credit for a blank answer!

The suggested times add up to 2:10, so you have plenty of extra time. Think before you write!

Booth rules require that the following statement is placed on all exams, and that you sign it.

I pledge my honor that I have not violated the Honor Code during this examination.

Signature: ____________________________

Formula Sheet

Discount factors

$$
\begin{aligned}
p &= E(mx); \quad E(mR)=1; \quad E(mR^e)=0 \\
E(R^e) &= -\frac{1}{E(m)}E(m,R^e); \quad E(dR)-r^f dt = -E\left(\frac{d\Lambda}{\Lambda}dR\right)
\end{aligned}
$$

$$
\begin{aligned}
x^* &= p'E(xx')^{-1}x \\
x^* &= \frac{1}{R^f}-\frac{1}{R^f}\left[E(R)-R^f\right]\Sigma^{-1}\left[R-E(R)\right] \\
\frac{d\Lambda^*}{\Lambda^*} &= -r^f dt - \left[\mu - r^f dt\right]\Sigma^{-1}\sigma dz \text{ if } dR = r^f dt+\mu dt+\sigma dz
\end{aligned}
$$

$$
\begin{aligned}
R^* &= x^*/p(x^*); \; E(R^{*2}) = E(R^*R) \;\; \forall R \in \underline{R} \\
R^{e*} &= proj(1|\underline{R}^e), \; E(R^{e*}R^e) = E(R^e) \;\; \forall R^e \in \underline{R}^e \\
E(R^{*2}) &= 1/E(x^{*2}); x^* = R^*/E(R^{*2}) \\
\text{If } R^f &\in \underline{X}, R^f = 1/E(x^*) = E(R^{*2})/E(R^*); \; R^f = R^* + R^f R^{e*}
\end{aligned}
$$

GMM

$$
g_T(b) = E_T\left[u_t(b)\right] = E_T\left[f(x_t,b)\right]; \; a_T g_T(\hat{b}) = 0
$$

$$
\begin{aligned}
\sqrt{T}(\hat{b}-b) &\rightarrow N(0, \left(ad\right)^{-1}aSa'(ad)^{-1\prime}) \\
\sqrt{T}g_T(\hat{b}) &\rightarrow N(0, \left[I-d(ad)^{-1}a\right]S\left[I-d(ad)^{-1}a\right]')
\end{aligned}
$$

$$
d = \frac{\partial g(b)}{\partial b'}; \; S = \sum_{j=-\infty}^{\infty} E\left[f(x_t,b)f(x_{t-j},b)'\right] = \sum_{j=-\infty}^{\infty} E_t\left[u_t(b)u_{t-j}(b)\right]
$$

Efficient GMM

$$
a = d'S^{-1}
$$

$$
\begin{aligned}
var(\tilde{b}_2) &= \frac{1}{T}(d'S^{-1}d)^{-1} \\
cov\left[g_T(\hat{b}_2)\right] &= \frac{1}{T}\left(S-d(d'S^{-1}d\right)^{-1}d') \\
g_T' cov(g_T)^+ g_T &= T g_T' S^{-1} g_T \,\tilde{}\, \chi^2_{\#mom-\#par}
\end{aligned}
$$

Fun facts

$$
\begin{aligned}
\sigma^2(x_{t+1}) &= E(\sigma_t^2(x_{t+1})) + \sigma^2(E_t(x_{t+1})) \\
Ee^x &= E^{E(x)+\frac{1}{2}\sigma^2(x)} \text{ if } x \text{ is normal} \\
\int_0^\infty e^{-\rho t}dt &= \frac{1}{\rho} \text{ if } \rho > 0 \\
\text{Ito's lemma} &: df(x,t) = f_t dt + f_x dx + \frac{1}{2}f_{xx}dx^2 \\
&= f_t dt + \left(f_x\mu_x + \frac{1}{2}f_{xx}\sigma_x^2\right)dt + f_x\sigma_x dz
\end{aligned}
$$

1. (30) Let's think about how our asset pricing formulas would change if we recognize that the consumption series we're using is durable. Assume a single durable good, so the representative investor objective is

$$E\sum_{j=0}^{\infty}\beta^j u(k_{t+j})\text{s.t. } k_t = (1-\delta)\,k_{t-1} + c_t$$

c_t now represents durable good purchases. General hint: This problem does *not* require lots of algebra. I used no more than 3 lines for each part. I strongly advise you to work it out on the scratch paper at the end before answering it here!

a) (5) State the investor's first order conditions for buying an asset with price p_t and payoff x_{t+1}. How is this equation different from the standard nondurable case $pu'(c_t) = E_t\left[\beta u'(c_{t+1})x_{t+1}\right]$?

b) (10) Now assume a constant riskfree rate $R^f = 1/\beta$. Use the equation for pricing the risk free rate to collapse the new terms, so you have an asset pricing equation $p = E(mx)$ expressed in terms of $u'(k_t)$ and $u'(k_{t+1})$.

(Hints: 1) Do the $\delta = 1$ case first, then show this solution works for the $\delta < 1$ case. You do not have to prove this is the only solution. 2) If you're having trouble, start with quadratic utility, and then generalize to arbitrary $u(k)$.)

c) (7.5) In the case of power utility, express the discount factor in terms of c_{t+1}/c_t and a purchases/stock c_t/k_t ratio. Suppose as in the Campbell/Cochrane model that the variance of purchases growth $\sigma(c_{t+1}/c_t)$is constant over time, When does this model generate high risk premia – in booms when purchases are high relative to the stock of durables or in recessions when purchases are low relative to the stock?

d) (7.5) Express the model in continuous time,

$$\begin{aligned} &\max E_0\int_0^{\infty} e^{-\rho t}u(k_t)dt \\ dk_t &= -\delta k_t dt + c_t dt \end{aligned}$$

assume c follows a diffusion process and power utility $u'(k) = k^{-\gamma}$. Assume your results from part a, b go through so $\Lambda_t = e^{-\rho t}u'(k_t)$. By characterizing this discount factor, do risk premia increase or decrease in this model relative to the nondurable model? How might you modify this continuous-time setup to generate the opposite result (one sentence)?

2) (15) You're analyzing small value stocks. When you run a simple time-series regression, you find a large alpha,

$$R^{ei}_{t+1} = 0.20 + 1.0 \times R^{em}_{t+1} + \varepsilon_{t+1}$$

However, the returns are log-normal – gross returns can never be negative – so you try it in logs, obtaining a much better-behaved regression (annual units)

$$\log R^{i}_{t+1} - R^{f} = 0 + 1.0 \times \left(\log R^{m}_{t+1} - R^{f}\right) + \varepsilon_{t+1};\ \sigma^2(\varepsilon) = 0.50$$

(R^f is constant in your sample.)

a) (5) Does this observation rescue the discrete-time CAPM?

b) (10) Does this observation rescue the continuous-time CAPM? If the continuous-time CAPM holds with $\alpha = 0$, $\beta = 1$, what should you observe?

3. An economy consists of two consumers A and B. They have power utility with the same discount rate δ but different risk aversion γ_A, γ_B. Thus, A maximizes

$$U(\{c_A\}) = E\int_{t=0}^{\infty} e^{-\delta t}\frac{c_{At}^{1-\gamma_A}}{1-\gamma_A}dt$$

and similarly for B. They have access to complete markets. They live in an endowment economy with total consumption c_{At} at each date

a) Write and solve the planner's problem max $U(\{c_A\}) + \lambda U(\{c_B\})$ to find c_{At} and c_{Bt} as a function of c_t. (You can leave this expression with $f(c_{At}) = c_t$ form; solving it for $c_{At} = g(c_t)$ is not pretty.)

b) Sketch c_{At} and c_{Bt} as a function of c_t. Use $\lambda = 1$ and $\gamma_A = 2$,$\gamma_B = 1$ This tells us how consumption is optimally split between a more risk averse and less risk averse investor. (Since its' hard to solve $c_{At} = g(c_t)$, what you're really sketching is $c_t = f(c_{At})$, just put c_A on the y axis) (For you to think about: Who gets consumption in bad times? Who gets consumption as $c \to \infty$? How is this distribution of results fair given that $\lambda = 1$ and the planner loves everyone equally? The graphs should tell you something about the fortunes of the "high beta rich" in the recent market downturns.)

c) Now, how do asset prices behave. I want to use this model to go after the intuition that in a downturn, less risk averse people lose more. Then the "average investor" is more risk averse, so aggregate risk aversion and the equity premium rises. This is a potential explanation for time-varying risk aversion and expected returns.

To get there, remind yourself that the discount factor is equal to either investor's marginal rate of substitution. Now, assume aggregate consumption is a random walk

$$\frac{dc}{c} = \mu dt + \sigma dz$$

Find the process for either individual's consumption dc_A/c_A etc. Apply the continuous-time Hansen-Jagannathan bound to show that the risk premium rises after aggregate consumtpion declines.

(Hint: you should get the familiar expression, with a weighted average of individual risk aversions in place of aggregate risk aversion. The weigts depend on consumption shares, so when c_A/c rises, the aggregate risk aversion rises. Hint 2: differentiate your sharing rule $f(c_{At}) = c_t$, and find values for μ_A, σ_A in $\frac{dc_A}{c_A} = \mu_A dt + \sigma_A dz$ to make it work. Express answers in terms of ratios like c_A/c, c/c You can ignore the difficult μ_A term and only look for σ_A, since μ does not appear in the continuous-time Hansen-Jagannathan bound. Find a final answer that is symmetric in A and B.)

4) (30) You're evaluating asset pricing models empirically.

a) (15) You are studying the moment $0 = E((C_{t+1}/C_t)^{-\gamma} R^e_{t+1})$ with two assets. (Say, rmrf and hml.) In a first-stage $\min g_T'(\gamma) g_T(\gamma)$,you find very large (but finite) standard errors for γ but the J_T test rejects with very large precision. It seems weird that we can't estimate the model parameters well, but we can nonetheless reject it with seemingly great confidence.

i) Can this finding reflect some pathology; i.e. is the GMM distribution theory potentially misleading? Explain how things might have gone wrong.

ii) *Must* this finding reflect some pathology, or is it possible that this is just a good characterization of the model's behavior? Explain yes/no (Hint: plots of $g_T(\gamma)$ may help in both cases.)

b) (5) A paper estimates a linearized model $m = 1 - \gamma \Delta c_{t+1}$ on the Fama-French 25 portfolios. It finds $\gamma \approx 100$, $\sigma(\hat{\gamma}) = 0.10$, the J_T test does not reject, and pricing errors $E(mR^e)$ are less than 1%, compared with an expected return spread $E(R^e)$ of 15%. What could be wrong? What diagnostic would you like to see?

c) (5) A paper estimates the same linear model by a cross-sectional regression $E(R^{ei}) = \gamma + \beta_{i\Delta c} \lambda_{\Delta c} + \alpha_i$, it plots "actual" $E(R^{ei})$ vs. "predicted" expected returns, which have a nice fit and a 90% cross-sectional R^2. What could be wrong? What diagnostic would you like to see?

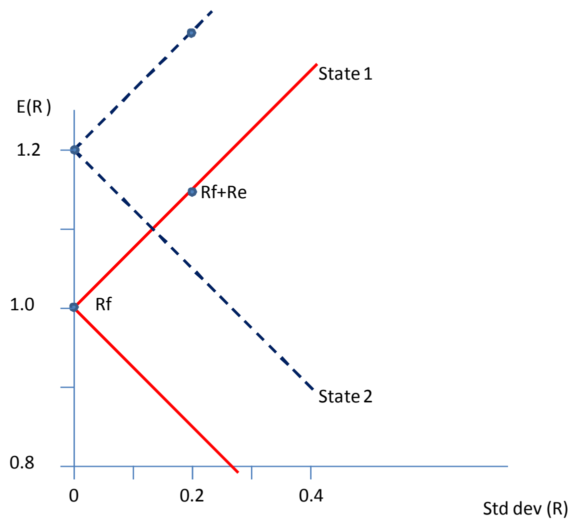
d) (5) A paper estimates the same model by Fama-MacBeth cross-sectional regression, and concludes that the model is good because the t on $\lambda_{\Delta c}$ is greater than two. It "does not do the GRS test because that test requires a time-series regression." Can the author do a GRS - style test here, or must he/she stop with the λ test? (2-3 equations and a few lines are enough.)

5) (20) There are two states of the world $i = 1, 2$, with different values of the riskfree rate R_i^f, as illustrated below There is also a single risky excess return R^e with iid mean $\mu = E_t(R_{t+1}^e)$ and $\sigma^2 = \sigma_t^2(R_{t+1}^e)$. Suppose $R_1^f = 1$ (i.e. 0%), and $R_2^f = 1.2$ (i.e. 20%), $\mu/\sigma = 1$ and $\sigma = 0.20$ as graphed.

a) (10) Find R^* and R^{e*}, expressed as combinations of R^f and R^e. Give equations and numbers.

b) (5) Check that R^f has the correct Hansen-Richard representation.

c) (5) Find the return in state 2 that must be paired with the state-1 riskfree return to form an unconditionally mean-variance efficient return. Place it on the graph; explain why it is not the risk free rate $R_2^f = 1.2$.



8. (10)

a) (5) We looked at the Merton portfolio problem in class, giving portfolio weights

$$\alpha = \frac{1}{\gamma}\Sigma^{-1}(\mu - r) + \beta_{dR,dy'}\frac{\eta}{\gamma}$$

or, expressed relative to the market,

$$\begin{aligned} dR^i &= rdt + \frac{\gamma^m}{\gamma^i}\left(dR^m - rdt\right) + \frac{1}{\gamma^i}\left(\eta^{i\prime} - \eta^{m\prime}\right) dR^z \\ dR^z &\equiv \beta_{dy,dR'}\left(dR - rdt\right) \end{aligned}$$

How do we modify this answer if, instead of power utility, the investor has a subsistence level, $u(c_t) = (c_t - x)^{1-\gamma}?$. (You do not have to re-solve the model, just indicate how the answer would change.)

b) (5) We derived in class that with power utility and lognormal returns, the risky asset weight above reduces to

$$\alpha = \frac{\mu - r}{\gamma\sigma^2}.$$

Using $\mu - r = 8\%, \sigma = 16\%$, $\gamma = 3.125$ leads to 100% equities. This looks like a reasonable risk aversion number. Yet the equity premium literature complains that at $\mu - r = 8\%, \sigma = 16\%$, investors with these kinds of risk aversion should want to buy much more stock. How do we reconcile these two calculations?

Answer Sketch

1. a)

$$\frac{\partial U}{\partial c_t} = E_t \sum_{j=0}^{\infty} \beta^j (1-\delta)^j u'(k_{t+j})$$

$$p_t E_t \left[\sum_{j=0}^{\infty} \beta^j (1-\delta)^j u'(k_{t+j}) \right] = E_t \left[x_{t+1} \beta \sum_{j=0}^{\infty} \beta^j (1-\delta)^j u'(k_{t+1+j}) \right]$$

Difference: There are all these forward-looking terms.

b)

$$E_t \sum_{j=0}^{\infty} \beta^j (1-\delta)^j u'\left(k_{t+j}\right) = E_t \sum_{j=0}^{\infty} \beta^j (1-\delta)^j u'(k_{t+1+j})$$

$$\begin{aligned} & u'(k_t) + \beta(1-\delta) E_t u'(k_{t+1}) + \beta^2 (1-\delta)^2 E_t u'(k_{t+2}) \\ = \; & E_t u'(k_{t+1}) + \beta(1-\delta) E_t u'(k_{t+2}) + \beta^2 (1-\delta)^2 E_t u'(k_{t+3}) + \ldots \end{aligned}$$

If $\delta = 1$, we have $E_t u'(k_{t+j}) = u'(k_t)$. Try that again... it still works for $\delta < 1$.

$$p_t E_t \sum_{j=0}^{\infty} \beta^j (1-\delta)^j u'(k_{t+j}) = E_t \left[x_{t+1} \beta \sum_{j=0}^{\infty} \beta^j (1-\delta)^j u'(k_{t+j+1}) \right]$$

$$p_t u'(k_t) \frac{1}{1-\beta(1-\delta)} = E_t \left[x_{t+1} \beta u'(k_{t+1}) \frac{1}{1-\beta(1-\delta)} \right]$$

$$p_t u'(k_t) = \beta E_t \left[x_{t+1} u'(k_{t+1}) \right]$$

There is no difference at all. Intuition? The relative price of k and c are always one, so you can buy and sell durable goods as freely as you want. That hinges on the $R^f \beta = 1$ assumption. To increase k_t by one, not changing anything else, you buy one more unit of c_t, but then decrease c_{t+1} by $(1-\delta)$ units for sure. To decrease k_{t+1} by one, you do the opposite with c_{t+1} and c_{t+2}. The relative prices here are always the same – there is no distortion between t and $t+1$ because you have to go through these steps.

c)

$$\begin{aligned} p_t &= \beta E_t \left[x_{t+1} \frac{u'(c_{t+1} + (1-\delta) k_t)}{u'(k_t)} \right] \\ & \left(\frac{c_{t+1} + (1-\delta) k_t}{k_t} \right)^{-\gamma} \\ & \left(\frac{c_{t+1}}{k_t} + (1-\delta) \right)^{-\gamma} \\ & \left(\frac{c_{t+1}}{c_t} \frac{c_t}{k_t} + (1-\delta) \right)^{-\gamma} \end{aligned}$$

when c_t/k_t is high, m is more sensitive to consumption shocks, so you get more $\sigma(m)$ and more risk premia in booms.

d) You are given $\Lambda_t = e^{-\rho t}u'(k_t)$ Then just apply ito's lemma:

$$\begin{aligned}
\Lambda &= e^{-\rho t}u'(k_t) = e^{-\rho t}k_t^{-\gamma} \\
dk_t &= -\delta k_t + c_t dt \\
d\Lambda &= -\rho\Lambda dt - \gamma\Lambda\frac{dk_t}{k_t} + \frac{1}{2}(\gamma)(\gamma+1)\Lambda\frac{dk_t^2}{k_t} \\
\frac{d\Lambda}{\Lambda} &= -\rho dt - \gamma\left(-\delta + \frac{c_t}{k_t}\right)dt
\end{aligned}$$

this is deterministic, so there are no risk premia.$E\left(\frac{d\Lambda^2}{\Lambda^2}\right) = 0$ We need a consumption process that is "noisier" than a diffusion, so that the services process can be as noisy as a diffusion.

You did not have to resolve the problem – you were invited to start at $\Lambda_t = e^{-\rho t}u'(k_t)$. FYI here is the whole derivation:

$$\begin{aligned}
p_t E_t \int_{s=0}^{\infty} e^{-\rho s}u'(k_{t+s})e^{-\delta s}ds &= E_t\int_0^{\infty} e^{-\rho s}x_{t+s}dsE_{t+s}\int_{\tau=0}^{\infty} e^{-\rho\tau}u'(k_{t+s+\tau})e^{-\delta\tau}d\tau \\
p_t E_t \int_{s=0}^{\infty} e^{-(\rho+\delta)s}u'(k_{t+s})ds &= E_t\int_{s=0}^{\infty}\int_{\tau=0}^{\infty} e^{-\rho s}x_{t+s}e^{-(\rho+\delta)\tau}u'(k_{t+s+\tau})dsd\tau
\end{aligned}$$

if there is a constant risk free rate $= \rho$, then $p = 1$ corresponds to $x = \rho$

$$E_t\int_{s=0}^{\infty} e^{-(\rho+\delta)s}u'(k_{t+s})ds = E_t\int_{s=0}^{\infty}\int_{\tau=0}^{\infty} e^{-\rho s}\rho e^{-(\rho+\delta)\tau}u'(k_{t+s+\tau})dsd\tau$$

Guess $E_t u'(k_{t+s}) = u'(k_t)$

$$\begin{aligned}
\int_{s=0}^{\infty} e^{-(\rho+\delta)s}ds &= \rho\int_{s=0}^{\infty}\int_{\tau=0}^{\infty} e^{-\rho s}e^{-(\rho+\delta)\tau}dsd\tau \\
\frac{1}{\rho+\delta} &= \int_{s=0}^{\infty} e^{-\rho s}ds\int_{\tau=0}^{\infty} e^{-(\rho+\delta)\tau}d\tau
\end{aligned}$$

yes, so first order conditions are

$$p_t u'(k_t) = E_t\int_0^{\infty} e^{-\rho s}u'(k_{t+s})x_{t+s}ds.$$

so $\Lambda_t = e^{-\rho t}u'(k_t)$.

2)

a) No, investors care about arithmetic returns, because those are the ones you put into portoflios

b) You should see a -50% intercept.

$$\begin{aligned}
dR^{ei} &= dR^{em} + dz_i \\
\frac{dP^i}{P^i} - r^f dt &= \left(\frac{dP^m}{P^m} - r^f dt\right) + dz_i \\
d\log P^i &= \frac{dP^i}{P^i} - \frac{1}{2}\sigma^2\left(\frac{dP^i}{P^i}\right) = \frac{dP^i}{P^i} - \frac{1}{2}\left(\sigma_m^2 + \sigma_i^2\right) \\
d\log P^i + \left(\frac{1}{2}\left(\sigma_m^2 + \sigma_i^2\right) - r^f\right)dt &= \left(d\log P^m + \left(\frac{1}{2}\sigma_m^2 - r^f\right)dt\right) + dz_i \\
\log\frac{P_T^i}{P_0^i} + \left(\frac{1}{2}\left(\sigma_m^2 + \sigma_i^2\right) - r^f\right)T &= \log\frac{P_T^m}{P_0^m} + \left(\frac{1}{2}\sigma_m^2 - r^f\right)T + \int_0^T dz_i \\
\log\frac{P_T^i}{P_0^i} - r^f T &= \left(-\frac{1}{2}\sigma_i^2 T\right) + \left(\log\frac{P_T^m}{P_0^m} - r^f T\right) + \varepsilon_T
\end{aligned}$$

3

$$\max E \int e^{-\delta t} \frac{c_{At}^{1-\gamma_A}}{1-\gamma_A} dt + \lambda \int e^{-\delta t} \frac{c_{Bt}^{1-\gamma_B}}{1-\gamma_B} \quad s.t. \;\; c_{At} + c_{Bt} = c_t$$

$$\begin{aligned} c_{At}^{-\gamma_A} &= \lambda c_{Bt}^{-\gamma_B} \\ \left(\lambda c_{Bt}^{-\gamma_B}\right)^{-\frac{1}{\gamma_A}} + c_{Bt} &= c_t \\ \lambda^{-\frac{1}{\gamma_A}} c_{Bt}^{\frac{\gamma_B}{\gamma_A}} + c_{Bt} &= c_t \end{aligned}$$

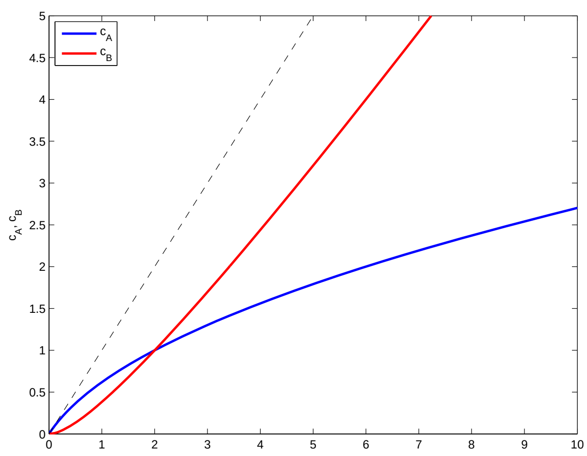
similarly

$$\begin{aligned} \frac{1}{\lambda} c_{At}^{-\gamma_A} &= c_{Bt}^{-\gamma_B} \\ \left(\frac{1}{\lambda} c_{At}^{-\gamma_A}\right)^{-\frac{1}{\gamma_B}} + c_{At} &= c_t \\ \lambda^{\frac{1}{\gamma_B}} c_{At}^{\frac{\gamma_A}{\gamma_B}} + c_{At} &= c_t \end{aligned}$$

$$\lambda^{\frac{1}{\gamma_B}} c_{At}^{\frac{\gamma_A}{\gamma_B}} + c_{At} = c_t$$

For $\lambda = 1$, $\gamma_A/\gamma_B = 1$ you verify $c_A = c_B$ For $\gamma_A/\gamma_B = 2$,

$$\begin{aligned} c_{Bt}^{\frac{1}{2}} + c_{Bt} &= c_t \\ c_{At}^2 + c_{At} &= c_t \end{aligned}$$



As you see, the risk averse get everything in bad times, but the risk tolerant get greater and greater shares in good times. As c rises, the risk tolerant get all the additional gains – dc_B/dc approaches 1! That compensates them for their willingness to really suffer in bad times.

$$
\begin{aligned}
\lambda^{\frac{1}{\gamma_B}} c_{At}^{\frac{\gamma_A}{\gamma_B}} + c_{At} &= c_t \\
\left[\lambda^{\frac{1}{\gamma_B}} \frac{\gamma_A}{\gamma_B} c_{At}^{\frac{\gamma_A}{\gamma_B}-1} + 1\right] dc_A + \frac{1}{2}\lambda^{\frac{1}{\gamma_B}} \left(\frac{\gamma_A}{\gamma_B}\right)\left(\frac{\gamma_A}{\gamma_B} - 1\right) c_{At}^{\frac{\gamma_A}{\gamma_B}-2} dc_{At}^2 &= dc_t \\
\left[\lambda^{\frac{1}{\gamma_B}} \frac{\gamma_A}{\gamma_B} \frac{c_{At}^{\frac{\gamma_A}{\gamma_B}}}{c} + \frac{c_A}{c}\right] \frac{dc_A}{c_A} + \frac{1}{2}\lambda^{\frac{1}{\gamma_B}} \left(\frac{\gamma_A}{\gamma_B}\right)\left(\frac{\gamma_A}{\gamma_B} - 1\right) \frac{c_{At}^{\frac{\gamma_A}{\gamma_B}}}{c} \frac{dc_{At}^2}{c_A^2} &= \frac{dc_t}{c_t} \\
\left[\lambda^{\frac{1}{\gamma_B}} \frac{\gamma_A}{\gamma_B} \frac{c_{At}^{\frac{\gamma_A}{\gamma_B}}}{c} + \frac{c_A}{c}\right] \sigma_A &= \sigma
\end{aligned}
$$

Using

$$
\begin{aligned}
c_A^{-\gamma_A} &= \lambda c_B^{-\gamma_B} \\
\lambda^{\frac{1}{\gamma_B}} c_A^{\frac{\gamma_A}{\gamma_B}} &= c_B
\end{aligned}
$$

we then have

$$
\left[\frac{\gamma_A}{\gamma_B}\frac{c_{Bt}}{c} + \frac{c_A}{c}\right]\sigma_A = \sigma
$$

$$
\sigma_A = \frac{\sigma}{\frac{\gamma_A}{\gamma_B}\left(\frac{c_{Bt}}{c_t}\right) + \frac{c_{At}}{c_t}} = \frac{1}{\gamma_A}\frac{\sigma}{\frac{1}{\gamma_B}\frac{c_{Bt}}{c_t} + \frac{1}{\gamma_A}\frac{c_{At}}{c_t}}
$$

Now, the HJ bound,

$$
E_t(dR) - r dt = -E_t\left(dR\frac{d\Lambda}{\Lambda}\right) = \gamma_i E_t\left(dR\frac{dc_i}{c_i}\right)
$$

$$
\frac{E_t(dR) - r dt}{\sigma_t(dR)} \le \gamma_i \sigma_t\left(\frac{dc_i}{c_i}\right) = \left(\frac{1}{\gamma_B}\frac{c_{Bt}}{c_t} + \frac{1}{\gamma_A}\frac{c_{At}}{c_t}\right)^{-1}\sigma
$$

Thus asset returns are priced by an "aggregate risk aversion coefficient"

$$
\frac{1}{\gamma_m} = \left(\frac{1}{\gamma_B}\frac{c_{Bt}}{c_t} + \frac{1}{\gamma_A}\frac{c_{At}}{c_t}\right)
$$

$$
\frac{E_t(dR) - r dt}{\sigma_t(dR)} \le \gamma_m \sigma\left(\frac{dc}{c}\right)
$$

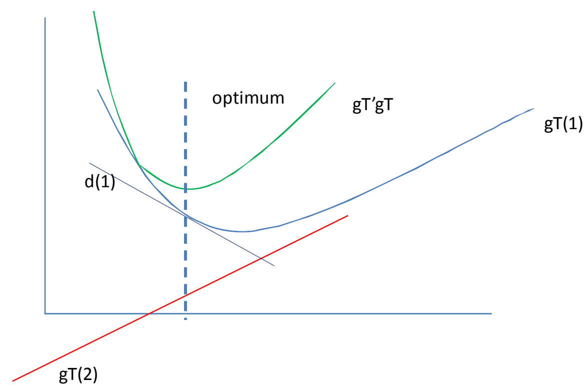
But, as $\sigma(dc_A/c_A)$ varies over time this risk aversion varies. We can either think of asset prices as priced with time-varying volaltilites of individual consumption, despite constant aggregate consumption volatility, or we can think of assets as priced by a "representative consumer" with time-varying risk aversion that depends on the consumption shares.

Finding μ_A is straightforward but a mess. It's implication is that interest rates also vary over time, despite constant aggregate consumption growth.

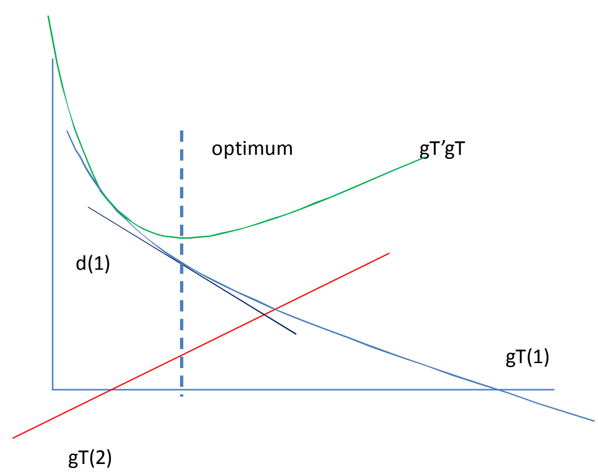
This problem is inspired by "The Young, the Old, The Conservative, the Bold" by Stavros Panageas and Nicolae Garleanu.

4)

a) i) Yes, as in the homework it is possible that one or both of the moments has no value of γ for which $g_T(\gamma) = 0$. So long as the minimum does not happen at the same point γ, the minimizer will settle on a place at which $dg_T/d\gamma \neq 0$ for both of them, so you will have a nonzero d matrix and finite standard errors. My picture has an example in which one moment does not intersect zero. The d matrix is $[1 - 1]$ here, so $d'd = 2$ and the standard errors will be fine. If this represents the *asymptotic* behavior of the system, the GMM distribution theory is inconsistent.



ii) Alas, it is also possible that this situation occurs when the model is just fine. GMM is local, so can't really tell what's happening away from the minimum. If the d matrix is small – the curves are flat, but high up, at the minimum, we'll see large standard errors but a big JT stat rejection. Alas, "right" or "wrong" here depends on what happens in bigger samples, something we don't know much about.



b) with $E(\Delta c) \approx 1\%$ this paper has $E(m) \approx 0$, so $E(mR^e) = E(m)E(R^e) + cov(m, R^e)$ can be small while the true $\alpha = E(mR^e)/E(m) = E(R^e) - cov(m, R^e)/E(m)$ is huge. For example, if $cov(m, R^e) = 0$, you can still get 0 pricing error with $\alpha = E(R^e)$. Let's see $E(m)$, or better $1/E(m)$ translated to annual percent units.

c) Huge γ can hide very bad performance, as in the problem set example with a negative slope! Let's see γ at a minimum, estimate the model without a constant, include the riskfree rate in the analysis, or call "predicted" only $\beta_{i\Delta c}\lambda$ not $\gamma + \beta_{i\Delta c}\lambda$.

d) It's easy. From the α_t ($N \times 1$) estimates (residual in time t cross-sectional regression), form $cov(\alpha_t, \alpha_t')$, then form $E(\alpha_t)' \left[cov(\alpha_t, \alpha_t')/T\right] E(\alpha_t)$ which has a χ^2 distribution.

5) a) Lots of ways to do this. Chooose any defining property of R^* and R^{e*} and work out what the weights must be.

i) You know that R^* is the conditionally or unconditinally minimum second moment return. Thus,

$$\begin{aligned} R^* &= R_i^f + w_i R^e \\ E_t(R^{*2}) &= \left(R_i^f + w_i\mu\right)^2 + w_i^2\sigma^2 \end{aligned}$$

d/dw_i :

$$\begin{aligned} 2R^f\mu + 2w_i\mu^2 + 2w_i\sigma^2 &= 0 \\ w_i &= -R^f\frac{\mu}{\mu^2+\sigma^2} \\ R^* &= R_i^f - R_i^f\frac{\mu}{\mu^2+\sigma^2}R^e \end{aligned}$$

Numerically

$$\begin{aligned} R^* &= R_i^f - R_i^f\frac{\mu/\sigma}{(\mu^2/\sigma^2+1)}\frac{1}{\sigma}R^e \\ R^* &= R_i^f - R_i^f\frac{1}{2}\frac{1}{0.2}R^e \\ R_1^* &= 1 - \frac{1}{0.4}R^e = 1 - 2.5\times R^e \\ R_2^* &= 1.2 - 1.2\frac{1}{0.4}R^e = 1.2 - 3R^e \end{aligned}$$

ii) Another way to do it. You know

$$E(R^*R^e) = 0$$

$$\begin{aligned} 0 &= E(R^*R^e) \\ 0 &= E(\left(R^f + w_iR^e\right)R^e) \\ 0 &= R^f\mu + w_i\left(\mu^2+\sigma^2\right) \\ -\frac{R^f\mu}{(\mu^2+\sigma^2)} &= w_i \\ R^* &= R^f - R^f\frac{\mu}{\mu^2+\sigma^2}R^e \end{aligned}$$

iii) starting with $E(R^{*2}) = E(R^*R^f)$ also works.

For R^{e*} we have

$$R^{e*} = w_iR^e$$

I use the defining property

$$\begin{aligned} E(R^{e*}R^e) &= E(R^e) \\ E(w_i R^{e2}) &= E(R^e) \\ w_i &= \frac{\mu}{\mu^2+\sigma^2} \\ R^{e*} &= \frac{\mu}{\mu^2+\sigma^2}R^e \end{aligned}$$

Numerically,

$$R^{e*} = \frac{\mu/\sigma}{(\mu^2/\sigma^2+1)\,\sigma}R^e = \frac{1}{2}\frac{1}{0.2}R^e = 2.5R^e$$

b) The Check:

$$\begin{aligned} R^* &= R^f - R^f\frac{\mu}{\mu^2+\sigma^2}R^e \\ R^* &= R^f - R^f R^{e*} \\ R^* + R^f R^{e*} &= R^f \end{aligned}$$

c) In state 1, of course,

$$R_1^f = R^* + R_1^f R^{e*}$$
$$1 = R^* + 1R^{e*}$$

you can check this holds. The UCMVF is

$$R^{mv} = R^* + wR^{e*}$$

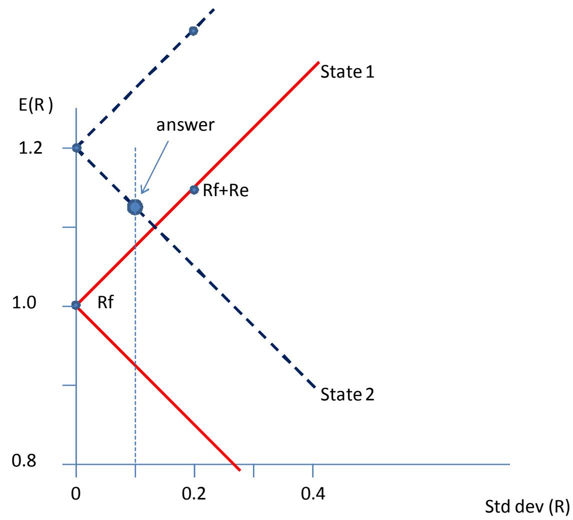
for fixed w, which must be 1 in this case. So, in state2, we must be looking for

$$R^{mv} = R^* + 1 \times R^{e*}$$

The key is you must use the new R^f in R^* and R^{e*} for this state,

$$R^{mv} = R^* + 1 \times R^{e*}$$
$$R^{mv} = R^f - R^f\frac{\mu}{\mu^2+\sigma^2}R^e + \frac{\mu}{\mu^2+\sigma^2}R^e$$
$$R^{mv} = R^f + \left(1-R^f\right)\frac{\mu}{\mu^2+\sigma^2}R^e$$
$$R^{mv} = R^f + \left(1-R^f\right)\frac{\mu/\sigma}{(\mu^2/\sigma^2+1)\,\sigma}R^e$$
$$R^{mv} = 1.20 - 0.2\frac{1}{2\times 0.2}R^e$$
$$R^{mv} = 1.20 - \frac{1}{2}R^e$$

so it's just halfway down. By choosing less mean you get less unconditional variance by getting less variance of conditional mean than you would have with Rf in both states



8. a) no change, γ is WV_W/V_{WW}. It will affect the value of γ and η.once we get around to solving the Bellman equation. b) consumption advice is $c = kW$ so $\sigma(\Delta c) = \sigma(\Delta w)$. If we saw this, the EP puzzle would agree.