

2014 Final Exam

Name (Print clearly): _____

Section: _____

Mailfolder location or address: _____

Directions

DO NOT START UNTIL WE TELL YOU TO DO SO. *Read these directions* in the meantime.

Please *do not tear your exam apart*. Answer the questions in the space provided. There are some extra pages at the end if you run out of space (but if you do, it means you're writing too much.)

You can rip off the formula sheet and blank pages at the back for throw-away scrap paper if you wish.

Show your work. An answer that comes without justification the right answer but coming miraculously from the wrong logic will be graded as wrong. Also, by showing your work you will get partial credit.

Keep your answers *short*. We are only looking for the right answer; we will grade off for a memory dump of unrelated stuff as it reveals you don't know what's relevant to the question.

Put your answers in a *box* or *underline* to make sure we find them. Make sure you answer each direct question. The questions are not clever or subtle. In each case, we just want to know the one obvious point.

For fact questions, *quote the author and paper*, or *state that the fact comes from a problem set* if such a source is relevant.

This is an *closed-book, closed-note* exam. The number questions are all easy. If not, your answer is wrong. You may use a calculator. You may not use a laptop computer, tablet/ ipad, cell phone, Google glasses, etc.

Each question has a suggested time, which is also the number of points it will count in grading. Small times (5 min) require shorter answers. The total time is 3:00. Keep moving, the questions are easy but there are a lot of them.

Thursday section: Do not discuss the contents of this exam with anyone until Sat 12 PM. Any information passed to the other section is not only a serious honor code violation, it lowers your grade directly.

Booth honor code required statement: I pledge my honor that I have not violated the honor code during this examination.

Signature: _____

1. (15) Consider our standard return forecast VAR

$$\begin{aligned}r_{t+1} &= b_r dp_t + \varepsilon_{t+1}^r; \quad b_r = 0.1, \quad \sigma(\varepsilon^r) = 0.194 \\ dp_{t+1} &= \phi dp_t + \varepsilon_{t+1}^{dp}; \quad \phi = 0.94, \quad \sigma(\varepsilon^{dp}) = 0.17\end{aligned}$$

These parameters imply that

$$\sigma^2(dp_t) = \frac{\sigma^2(\varepsilon^{dp})}{1 - \phi^2} = 0.5^2$$

$$\sigma^2(r) = b_r^2 \sigma^2(dp) + \sigma^2(\varepsilon^r) = 0.1^2 \times 0.5^2 + 0.194^2 = 0.20^2$$

(I reverse-engineered $\sigma(\varepsilon^r)$ and $\sigma(\varepsilon^{dp})$ to give these answers) and that if you regress returns on past returns only, you get a zero coefficient,

$$r_{t+1} = \beta_r r_t + v_{t+1}^r; \quad \beta_r = 0, \quad \sigma(v_{t+1}^r) = 0.20.$$

All regressions include constants, which are not important for this problem. In each case, give a formula, a number to two decimals, and short explanation:

- (a) (3) Find the implied coefficient for forecasting 5 year returns ($r_{t+1} + r_{t+2} + \dots + r_{t+5}$) from dp_t .
- (b) (3) Find the R^2 for forecasting one-year returns from dividend yields.
- (c) (3) Find the R^2 for forecasting five-year returns from dividend yields.
- (d) (3) Find the standard deviation of one-year expected returns, $\sigma[E_t(r_{t+1})]$.
- (e) (3) What message do you draw from these calculations?

More space for #1

2. (10) High prices relative to current dividends must signal higher future dividends, or lower future returns. This statement means that a combination of regression coefficients must add up to one. Given our return-forecasting regression

$$r_{t+1} = b_r^{(1)} dp_t + \varepsilon_{t+1}^r; \quad b_r = 0.1$$

- (a) (5) *What are the regression coefficients that must add up to 0.9 to make the first sentence precise?*

Use this notation,

$$\begin{aligned} r_{t+k} &= b_r^{(k)} dp_t + \varepsilon_{t+k}^r; & \Delta d_{t+k} &= b_d^{(k)} dp_t + \varepsilon_{t+k}^d \\ dp_{t+k} &= b_{dp}^{(k)} dp_t + \varepsilon_{t+k}^{dp}; & \sum_{j=k}^m \rho^{j-1} r_{t+j} &= b_r^{(k,m)} dp_t + v_{t+m}^r \end{aligned}$$

and write out the regressions for any other notation you wish to use. The return and present value identities may be useful:

$$r_{t+1} = -\rho dp_{t+1} + dp_t + \Delta d_{t+1}; \quad dp_t = \sum_{j=1}^{\infty} \rho^{j-1} r_{t+j} - \sum_{j=1}^{\infty} \rho^{j-1} \Delta d_{t+j}$$

- (b) (5) Now, suppose you run multiple regressions,

$$r_{t+1} = b_r dp_t + c_r z_t + \varepsilon_{t+1}^r;$$

- i. Are there z variables that help to forecast returns in such a regression? (This is a fact from your readings.)
- ii. If $c > 0$, can you find other things that z must also forecast?

More space for #2

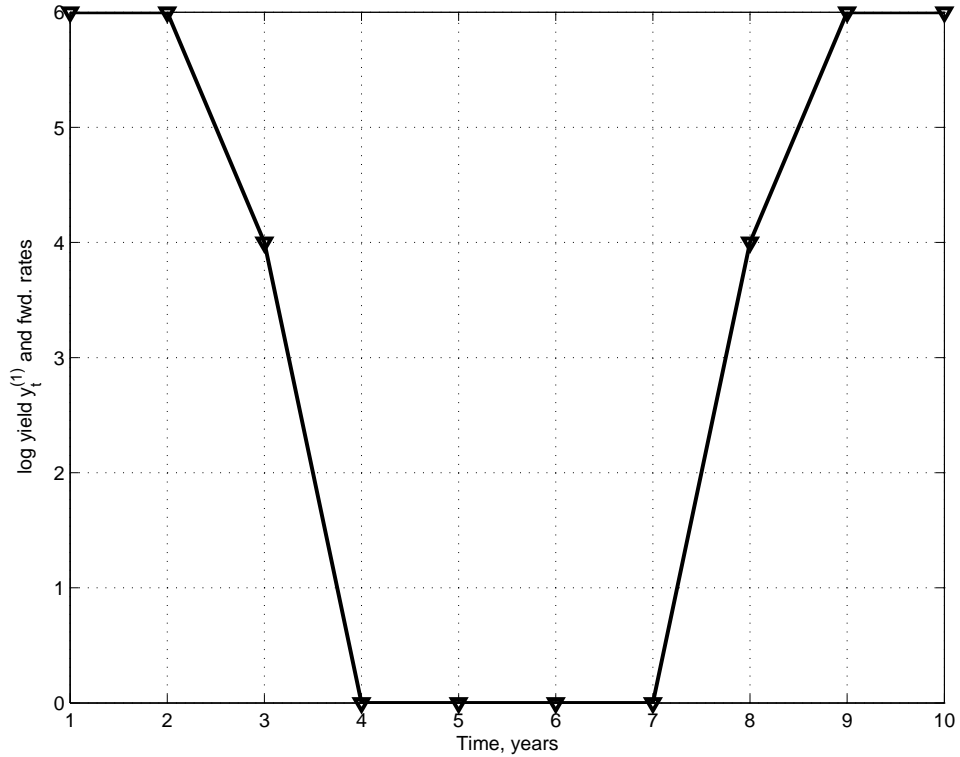
3. (20) Let's construct a simple term structure model. Suppose the one year rate follows an AR(1)

$$y_{t+1}^{(1)} - \delta = \rho \left(y_t^{(1)} - \delta \right) + \varepsilon_{t+1}$$

and suppose the pure expectations hypothesis holds with no risk premiums.

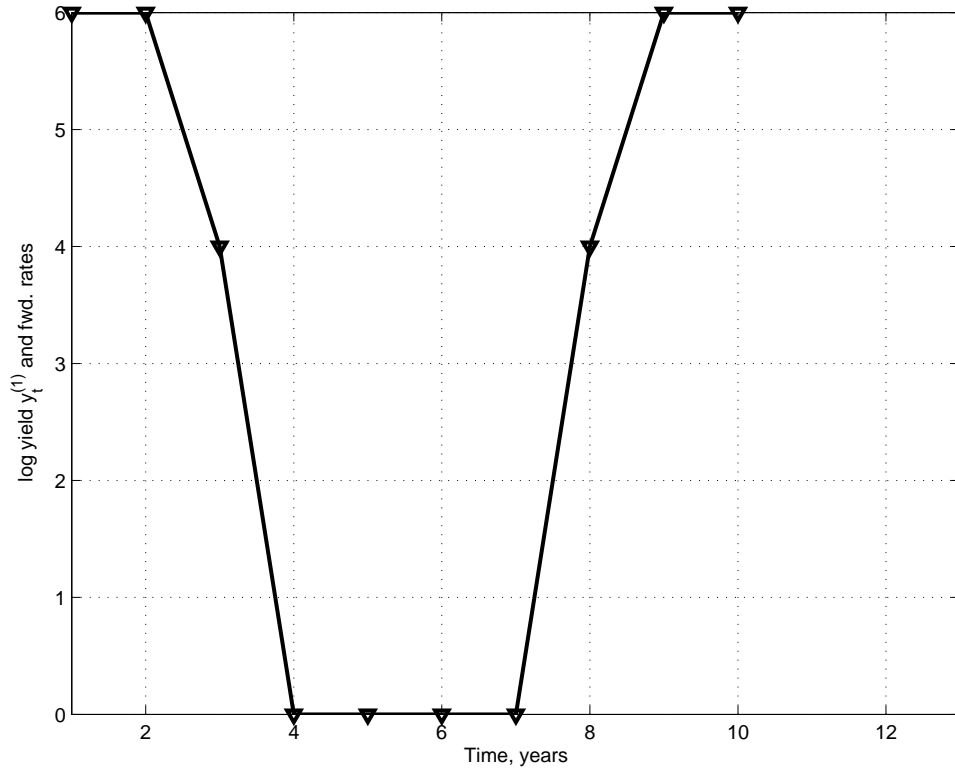
- (a) (5) Find a formula for forward rates $f_t^{(2)}, f_t^{(3)}, f_t^{(4)}$. Express your result as a one-factor model for forward rates. ($f_t^{(n)} = \text{constant} + \text{loading}_n \times (y_t^{(1)} - \delta)$)
- (b) (5) Using $\delta = 0.04$ (i.e. 4%) and $\rho = 0.5$, fill in the following table and sketch the path of $f_t^{(2)}, f_t^{(3)}, f_t^{(4)}$ on the plot below. (I plotted $y_t^{(1)}$ for you.)

$100 \times y_t^{(1)}$:	6%	4%	0%
$100 \times f_t^{(2)}$:			
$100 \times f_t^{(3)}$:			
$100 \times f_t^{(4)}$:			



More space for #3a,b

- (c) (5) Now, calculate at each date t , the path of future interest rates $E_t(y_{t+j}^{(1)})$ implied by the expectations hypothesis. Put these expectations on the following plot, i.e. at each t make a line connecting $y_t^{(1)}$, $E_t(y_{t+1}^{(1)})$, $E_t(y_{t+2}^{(1)})$, etc. (You did this in a problem set.)



- (d) (5) The path of forward rates you sketched ought to look like a reasonable representation of the yield and forward-rate data over the course of a business cycle. And we assumed the expectations hypothesis to make it. Yet Fama and Bliss show that the expectations hypothesis is seriously wrong. What feature which you can see in the two plots shows the failure of the expectations hypothesis?

More space for #3

4. (20) Fama and French “Multifactor anomalies”

- (a) (5) Table 1 runs a regression. Write down the regression. Be careful with the i 's and t 's vs. $t+1$. If you are unsure of notation, define symbols.
- (b) (5) Characterize FF's results for each of the following statistics, and explain which ones support Fama and French's claim that their regression is a good model of average returns? i) Size of α , ii) α t statistics, iii) pattern of b, h, s , iv) b, h, s t statistics v) R^2 , vi) GRS test that alphas are jointly zero.

Multifactor anomalies, continued.

- (c) (5) Are subsequent returns of companies that have, for 5 years had strong growth in sales higher or lower than subsequent returns of companies that have had 5 years of declining sales, including the companies that go out of business so stockholders lose everything?
- (d) (5) What do Fama and French's results suggest about a strategy that combines value and momentum? Will it do better or worse than strategies that focus on each one alone? (Hint: the ability of the FF model to explain momentum is the key here.)

5. (20) More anomalies.

(a) (5) The following is Table IV From Fama and French “Dissecting Anomalies.”

	<i>Int</i>	<i>MC</i>	<i>B/M</i>	<i>Mom</i>	<i>Zero</i> <i>NS</i>	<i>NS</i>	<i>Neg</i> <i>Ac/B</i>	<i>Pos</i> <i>Ac/B</i>	<i>dA/A</i>	<i>Neg Y</i>	<i>Pos</i> <i>Y/B</i>	<i>R</i> ²
Market												
Average	1.81	-0.18	0.26	0.50	-0.11	-1.90	0.03	-0.34	-0.81	0.06	0.92	0.04
<i>t</i> -statistic	5.36	-4.36	3.77	3.24	-2.41	-8.59	0.20	-2.72	-7.37	0.55	3.19	
Micro												
Average	2.63	-0.46	0.23	0.41	-0.16	-1.94	0.00	-0.28	-0.83	-0.01	0.55	0.03
<i>t</i> -statistic	7.41	-6.95	3.19	2.51	-2.83	-6.74	0.03	-2.02	-6.82	-0.11	1.50	
Small												
Average	1.01	-0.03	0.30	0.82	-0.04	-1.49	-0.09	-0.45	-0.57	0.01	1.19	0.05
<i>t</i> -statistic	2.02	-0.37	3.41	4.65	-0.55	-4.42	-0.28	-2.24	-3.10	0.03	2.36	
Big												
Average	1.06	-0.08	0.17	0.78	-0.12	-1.71	0.12	-0.38	-0.17	0.11	0.75	0.08
<i>t</i> -statistic	2.61	-1.96	1.79	3.92	-1.59	-5.28	0.32	-1.49	-0.86	0.46	1.56	

Explain what the numbers in the first row of this table are. Writing an equation or two is good. You can stop after MC, B/M and Mom.

(b) (3) “Stock issues should forecast returns, but their effect should be subsumed by book/market. When companies have lower cost of capital, they have higher market values and invest more. Issuing equity and B/M are the same signal.” What does the table say about the power to issue equities (NS = net stock issues) in a *multiple* regression sense, holding B/M effects constant?

#5 continues

- (c) (3) The coefficients on B/M are different for different size categories. How might you capture the B/M effects across size categories in one regression? (Hint: you did this in a problem set.)
- (d) (3) The last columns of the above table show that profitability is not useful for forecasting returns. Yet Novy-Marx and the latest Fama French find that profitability is very useful for forecasting returns. How do you resolve this apparent contradiction?

#5 continues

- (e) (3) On that topic, the old “good company vs. good stock” fallacy also says to avoid stocks of highly profitable companies, because the market knows they are profitable and has already included that fact in the stock price. How do Novy Marx and FF avoid this fallacy? (An equation will help here.)
- (f) (3) You investigate a trading strategy and alas the 1-10 portfolios have exactly the same average returns. Is it possible you have a good strategy anyway? Give an example that shows yes or proves no from our readings

6. (10) Carhart

- (a) (5) A fund enthusiast would say that higher fees pay for better research which delivers more alpha to the investor. A total Chicago cynic would say that investors are paid no more than what they can get elsewhere, so that higher fees have no effect at all on returns to investors. Where do the facts lie in this spectrum, according to Carhart?
- (b) (5) A mutual fund manager complains, "Carhart's results are bogus. He sorted mutual funds by their one-year past returns. Everyone knows that's mostly luck. He should have looked at funds based on 5 year performance averages, like Morningstar does. Then he would have seen some alphas!" How would Carhart respond?

7. (10) Below, find an excerpt from Fama and French's Table 3 from "Skill vs. Luck"

- (a) (4) What is the key assumption under "Simulated?"
- (b) (3) What does 1.30 mean in "simulated"? What does the relative position of 1.30 in "Actual" vs. "Simulated" mean?
- (c) (3) What do -1.71 and -2.84 mean? Is this normal, or a puzzle?

Percentile	Simulated	Actual
5	-1.71	-2.84
50	-0.01	-0.62
90	1.30	1.01
93	1.60	1.30
95	1.68	1.54

Table 3 - Percentiles of $t(\alpha)$ estimates for actual and simulated fund returns...[3-factor adjusted] net fund returns...

8. (10) Berk and Green

- (a) (5) You are a fund manager, and you can make 5% alpha on the first \$20 million dollars under management. If you get any more money, all you can do is stuff it into the index at 0% alpha. You charge 1% AUM fees. In Berk and Green's world what is the equilibrium of this situation?
- (b) (5) FF make tables of gross and net alpha. Would Berk and Green measure your skill by gross alpha or net alpha? (Warning, this is a slightly tricky question.)

9. (20) Theory. The log utility CAPM. Suppose investors have log utility,

$$E \sum_{t=0}^{\infty} \beta^t u(C_t); \quad u(C_t) = \log(C_t); \quad \beta = \frac{1}{1 + \delta}$$

The second equality just defines a useful rate δ . Assume that consumption growth follows a given process. If you need it, assume that consumption growth is independent over time,

$$\begin{aligned} \frac{C_{t+1}}{C_t} &= e^{\Delta c_{t+1}}; \quad \Delta c_{t+1} = g + \varepsilon_{t+1}, \\ E_t \left(\frac{C_{t+1}}{C_t} \right) &= e^{g + \frac{1}{2}\sigma^2}; \quad E_t \left(\frac{C_t}{C_{t+1}} \right) = e^{-g + \frac{1}{2}\sigma^2}. \end{aligned}$$

This problem does not take a lot of algebra, but it helps a lot if you think about where you're going.

- (a) (5) Find the price/dividend ratio P_t/C_t of a security that pays consumption itself as the “dividend,” i.e. if you pay P_t you get $\{C_{t+1}, C_{t+2}, C_{t+3}, \dots\}$.
- (b) (5) Find the rate of return of the consumption claim

$$R_{t+1}^c = \frac{P_{t+1} + C_{t+1}}{P_t}$$

in terms of consumption itself, i.e. use the part a result to get rid of the P_{t+1} and P_t to find R_{t+1} in terms of consumption only.

#9 continues

- (c) (5) Now, recall the standard consumption-based model specifies

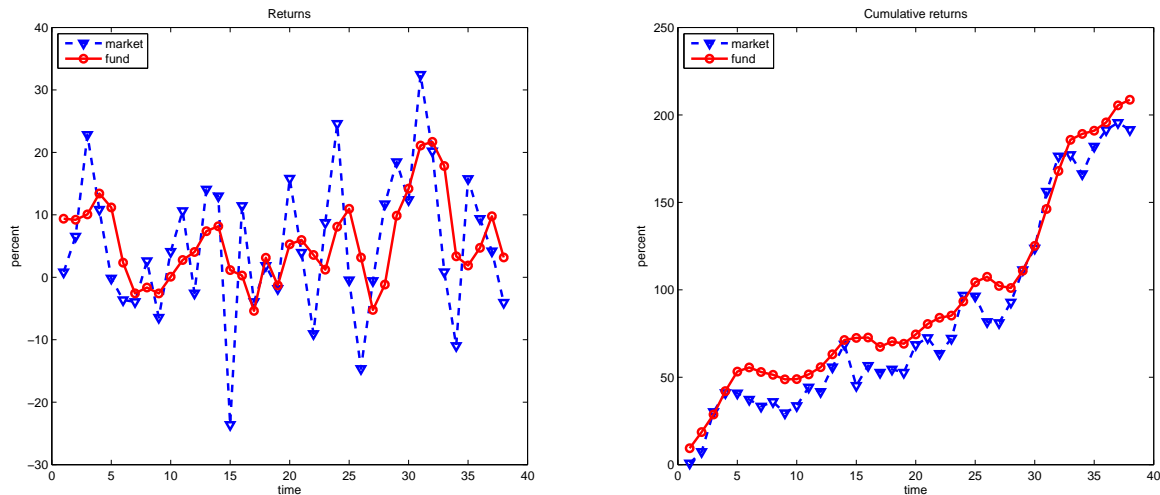
$$E(R_{t+1}^{ei}) \approx \gamma \text{cov}(R_{t+1}^{ei}, \Delta c_{t+1}) = \beta_{i, \Delta c} \lambda$$

where $\Delta c_{t+1} = \log(C_{t+1}/C_t)$. Use your result in part b to get rid of consumption and express expected returns as a factor model using the consumption claim return R^c .

- (d) (5) Find the interest rate R^f in this model in terms of β or δ , g , σ^2 or other properties of consumption .

10. (10)

(a) (5) A hedge fund's performance is graphed below. (This is simulated data. I have a model in mind, your job is to figure it out.)

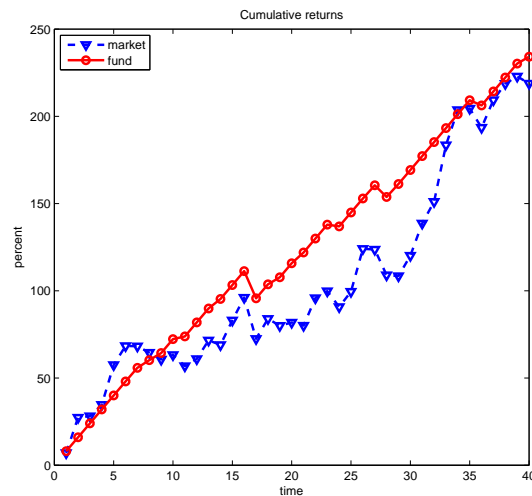
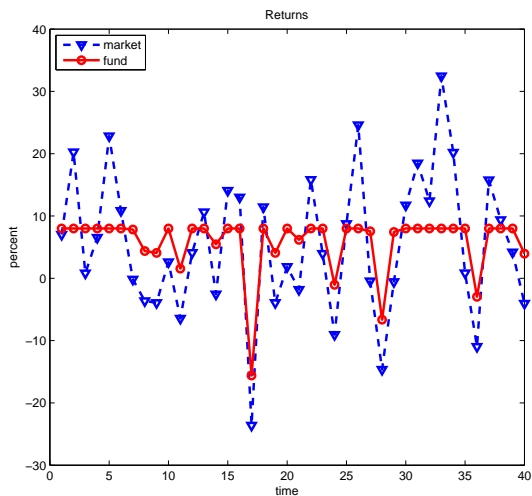


Here are the standard statistics estimated in this simulated data.

	$E(R)$	$\sigma(R)$	E/σ	α	β	R^2	$\sigma(\varepsilon)$
fund	5.5	6.6	0.83	4.0	0.29	0.26	5.7
market	5.0	11.4	0.44	0	1	1	0

This looks like a great fund. Looking hard at both graphs, suggest an alternative evaluation procedure that might not get such good results, and guess how it would come out. Writing an equation is a good idea, as is referencing a procedure you read about or did in the class. Hint: The fact that the fund and market cumulative returns start together and also end together is significant. How does this happen with $\beta = 0.29$?

(b) (5) Same question, different fund:



	$E(R)$	$\sigma(R)$	E/σ	α	β	R^2	$\sigma(\varepsilon)$
fund	5.9	4.8	1.21	4.2	0.3	0.53	3.3
market	5.5	11.4	0.48	0	1	1	0

11. (10) 3Com-Palm and stocks as money.

- (a) (3) The 3Com-Palm event is evidence of short sales constraints. At its peak, how much of the available Palm shares were sold short?
- (b) (3) If you can't short Palm, you can sell the synthetic Palm in options markets, by buying a put and writing a call. Why not?
- (c) (4) "Money as stock" points out that every "bubble" has featured one additional interesting characteristic. What is it?

12. (5) It turns out that signed orderflow – volume signed by who initiated the trade – is strongly correlated with price changes. Prices do rise on “buy” volume. But this could be “price pressure,” or “inventory management,” selling volume pushes price down temporarily, or it could be “price discovery,” people who know the price will go down anyway sell and make some money.

State three pieces of Brandt and Kavajecz’s evidence for the “price discovery” view?

13. (10) You are examining de-meaned sovereign credit default swaps spreads on Freedonia, Tomania, and Grand Fenwick, $[FD_t, TO_t, GF_t]$. You notice a strong correlation structure: the covariance matrix is

$$\Sigma = \begin{bmatrix} 1 & 0.9 & 0.9 \\ 0.9 & 1 & 0.9 \\ 0.9 & 0.9 & 1 \end{bmatrix}.$$

Since each variance is exactly one, this is also the correlation matrix. You want to express these CDS spreads as a factor model. There seems to be a common “fictional country” factor driving them. Applying the eigenvalue decomposition $\Sigma = Q\Lambda Q'$ you obtain

$$Q = \begin{bmatrix} 0.58 & 0 & 0.82 \\ 0.58 & 0.71 & -0.41 \\ 0.58 & -0.71 & -0.41 \end{bmatrix}$$

$$\Lambda = \begin{bmatrix} 2.8 & 0 & 0 \\ 0 & 0.1 & 0 \\ 0 & 0 & 0.1 \end{bmatrix} = \begin{bmatrix} 1.67^2 & 0 & 0 \\ 0 & 0.32^2 & 0 \\ 0 & 0 & 0.32^2 \end{bmatrix}$$

Write the components of the one-factor representation that captures the greatest variance, i.e., with the objective of producing the largest possible R^2 : i.e.

- (a) (3) What are the β in

$$CDS_t = \begin{bmatrix} FD_t \\ TO_t \\ GF_t \end{bmatrix} = \begin{bmatrix} \beta_{FD} \\ \beta_{TO} \\ \beta_{GF} \end{bmatrix} F_t + \begin{bmatrix} \varepsilon_t^{FD} \\ \varepsilon_t^{TO} \\ \varepsilon_t^{GF} \end{bmatrix}?,$$

- (b) (3) How do you construct the factor F_t from the vector of CDS spreads CDS_t ? and
- (c) (1) What name would you give the resulting factor?
- (d) (3) What is the R^2 of each line of the single-factor model? (Note: If $y = xb + \varepsilon$, $R^2 = \sigma^2(xb)/\sigma^2(y) = 1 - \sigma^2(\varepsilon)/\sigma^2(y)$; and $\sigma^2(y) = \sigma^2(xb) + \sigma^2(\varepsilon)$.)

Note: two decimal points are enough for numerical answers. If you get stuck on numbers, give a formula. Explain your approach in any case, don't just write down a number.

More space for #13

14. (10) You are considering investing in two managers, and of course the market index. You have a mean-variance objective with risk aversion $\gamma = 2$. Your assessment of the market portfolio is a mean $E(R^{em}) = 8\%$, volatility $\sigma(R^{em}) = 20\%$. You run CAPM regressions for the two managers

$$R_t^{ei} = \alpha_i + \beta_i R_t^{em} + \varepsilon_t^i$$

with result $\beta_1 = 2, \beta_2 = 2; \sigma(\varepsilon) = 10\%$ for both managers, and the residuals ε have correlation -0.5 . You believe $\alpha_1 = -0.3\%, \alpha_2 = 1.2\%$.

- (a) (7) Find the optimal allocation to the market index and to the two managers. Express the answer in terms of a weight w_m on the excess market return R_t^{em} and weights w_α on “portable alpha” or beta-hedged portfolios for each of the two managers, i.e., write the optimal portfolio return in the form, $R^{ep} = w_m R^{em} + w_1 (R^{e1} - \beta_1 R_t^{em}) + w_2 (R^{e2} - \beta_2 R_t^{em})$ and find the weights w_m, w_1, w_2 .

(Hints:

$$\begin{bmatrix} 1 & -0.5 \\ -0.5 & 1 \end{bmatrix}^{-1} = \frac{1}{3} \begin{bmatrix} 4 & 2 \\ 2 & 4 \end{bmatrix}$$

You should not need a calculator for this problem.)

- (b) (3) Is it possible that in an example like this, that you end up investing a positive amount with a manager who has negative alpha? (Get ready to market your negative alpha hedge fund!) Explain if it is, or prove it is not.

More space for any question

More space for any question

More space for any question

More space for any question

Formulas

Prediction and present value

$$\text{If } x_t = \phi x_{t-1} + \varepsilon_t \text{ then } E_t(x_{t+j}) = \phi^j x_t$$

$$r_{t+1} \approx -\rho dp_{t+1} + \Delta d_{t+1} + dp_t; \quad dp_t \equiv d_t - p_t$$

$$p_t - d_t \approx \sum_{j=1}^{\infty} \rho^{j-1} \Delta d_{t+j} - \sum_{j=1}^{\infty} \rho^{j-1} r_{t+j}; \quad \rho = \frac{1}{1 + D/P} \approx 0.96$$

$$M_t = E_t \sum_{\tau=1}^{\infty} \frac{Y_{t+\tau} - dB_{t+\tau}}{(1+r)^\tau} \quad (\text{Novy-Marx})$$

VAR

$$r_{t+1} = b_r \times dp_t + \varepsilon_{t+1}^r; \quad b_r \approx 0.1$$

$$\Delta d_{t+1} = b_d \times dp_t + \varepsilon_{t+1}^d; \quad b_d \approx 0$$

$$dp_{t+1} = b_{dp} \times dp_t + \varepsilon_{t+1}^{dp}; \quad b_{dp} \approx 0.94$$

Geometric sums

$$\sum_{j=0}^{\infty} z^j = \frac{1}{1-z} \text{ if } \|z\| < 1; \quad \sum_{j=1}^{\infty} \beta^j = \frac{\beta}{1-\beta} = \frac{\frac{1}{1+\delta}}{1 - \frac{1}{1+\delta}} = \frac{1}{\delta}$$

Discount factors, consumption and models

$$p_t = E_t(m_{t+1} x_{t+1}) = E_t \left(\beta \frac{u'(c_{t+1})}{u'(c_t)} x_{t+1} \right)$$

$$p_t = E_t \sum_{j=0}^{\infty} \beta^j \frac{u'(c_{t+j})}{u'(c_t)} x_{t+j}$$

$$m_{t+1} = \beta \left(\frac{c_{t+1}}{c_t} \right)^{-\gamma} \approx 1 - \delta - \gamma \Delta c_{t+1}$$

$$0 = E_t(m_{t+1} R_{t+1}^e); \quad 1 = E_t(m_{t+1} R_{t+1})$$

$$R^f = 1/E(m_{t+1}) = 1/E_t \left(\beta \frac{u'(c_{t+1})}{u'(c_t)} \right) \approx 1 + \delta + \gamma E_t(\Delta c_{t+1})$$

$$E(R_{t+1}^e) = -R_t^f \text{cov}(m_{t+1}, R_{t+1}^e) \approx \gamma \text{cov}(\Delta c_{t+1}, R_{t+1}^e)$$

Empirical methods GRS test:

$$T \left[1 + E(f)' \Sigma_f^{-1} E(f) \right]^{-1} \hat{\alpha}' \Sigma^{-1} \hat{\alpha} \sim \chi_N^2$$

$$\frac{T - N - K}{N} \left[1 + E(f)' \hat{\Sigma}_f^{-1} E(f) \right]^{-1} \hat{\alpha}' \hat{\Sigma}^{-1} \hat{\alpha} \sim F_{N, T-N-K}$$

Term structure

$$p_t^{(n)} = \text{log price at } t \text{ of bond that comes due at } t+n, \text{ e.g. } -0.20$$

$$y_t^{(n)} \equiv -\frac{1}{n} p_t^{(n)}; \quad f_t^{(n)} \equiv p_t^{(n-1)} - p_t^{(n)};$$

$$r_{t+1}^{(n)} \equiv p_{t+1}^{(n-1)} - p_t^{(n)}; \quad r x_{t+1}^{(n)} = r_{t+1}^{(n)} - y_t^{(1)}$$

Expectations:

$$\begin{aligned} y_0^{(n)} &= \frac{1}{n} E \left(y_t^{(1)} + y_{t+1}^{(1)} + y_{t+2}^{(1)} + \dots y_{t+n-1}^{(1)} \right) + (\text{risk premium}) \\ f_t^{(n)} &= E_t(y_{t+n-1}^{(1)}) + (\text{risk premium}) \\ E_t \left[r_{t+1}^{(n)} \right] &= y_t^{(1)} + (\text{risk premium}) \end{aligned}$$

Fama-Bliss regression

$$\begin{aligned} rx_{t+1}^{(n)} &= r_{t+1}^{(n)} - y_t^{(1)} = a + b \left(f_t^{(n)} - y_t^{(1)} \right) + \varepsilon_{t+1} \\ y_{t+n-1}^{(1)} - y_t^{(1)} &= a + b \left(f_t^{(n)} - y_t^{(1)} \right) + \varepsilon_{t+1} \end{aligned}$$

Cochrane-Piazzesi regression

$$\begin{aligned} \overline{rx}_{t+1} &= \frac{1}{4} \sum_{n=2}^5 rx_{t+1}^{(n)} = \gamma' f_t + \varepsilon_{t+1} \\ rx_{t+1}^{(n)} &= b_n (\gamma' f_t) + \varepsilon_{t+1}^{(n)} \end{aligned}$$

Portfolios

Classic mean-variance

$$w_0 = \frac{1}{\gamma} \Sigma^{-1} E(R^e); \quad \Sigma = \text{cov}(R^e)$$

With a factor model

$$\begin{aligned} R_{t+1}^p &= R^f + w_m R_{t+1}^{em} + w'_\alpha (\alpha + \varepsilon). \\ w_m &= \frac{1}{\gamma} \frac{E(R^{em})}{\sigma^2(R^{em})}; \quad w_\alpha = \frac{1}{\gamma} \Sigma^{-1} \alpha; \quad \Sigma \equiv E(\varepsilon_{t+1} \varepsilon'_{t+1}) \end{aligned}$$

Multifactor, y = state variable, and relative to the market if everyone is like this

$$\begin{aligned} w &= \frac{1}{\gamma} \Sigma^{-1} E(R^e) + \beta_{R,y'} \frac{\eta}{\gamma} \\ R^i &= R^f + \frac{\gamma^m}{\gamma^i} R^{em} + \frac{1}{\gamma^i} (\eta^{i'} - \eta^{m'}) R^{ez}; \quad R^{ez} \equiv \beta_{y,R'} R^e \\ E(R^e) &= \text{cov}(R^e, R^m) \gamma^m - \text{cov}(R^e, y') \eta^m \end{aligned}$$

Bayesian portfolios

$$w = \frac{1}{\gamma} \frac{E(R^e)}{\sigma^2(R^e) + \sigma^2[E(R^e)]}$$