2014 Final Exam Answers

1. Note there was a typo. I wrote $\sigma^2(v_{t+1}^r) = 0.20$ but I meant $\sigma(v_{t+1}^r) = 0.20$ of course. Either way gets credit.

    (a)

$$
\begin{aligned}
b_r^{(5)} &= b_r(1 + \phi + \phi^2 + \ldots + \phi^4) = b_r\frac{1 - \phi^5}{1 - \phi} \\
&= 0.10\frac{1 - 0.94^5}{1 - 0.94} = 0.44349 \text{ or } 0.44
\end{aligned}
$$

    (b)

$$
R^2 = \frac{b_r^2\sigma^2(dp)}{\sigma^2(r)} = \frac{0.1^2 \times 0.5^2}{0.2^2} = 0.0625 \text{ or } 0.06
$$

    (c) Here you have to put together use of VAR with the univariate regression, i.e. the fact that with uncorrelated returns the variance grows with horizon. To do that you have to have figured out that it really is ok to believe returns are uncorrelated while also predictable from dp.

$$
R^2 = \frac{\left[b_r^{(5)}\right]^2\sigma^2(dp)}{\sigma^2(r_{t+1} + \ldots + r_{t+5})} = \frac{0.44349^2 \times 0.5^2}{5 \times 0.2^2} = 0.24585 \text{ or } 0.25
$$

    (d)

$$
\sigma(E_t(R_{t+1})) = b_r\sigma(dp) = 0.1 \times 0.5 = 0.05 = 5\%
$$

    (e) Though the one-year R^2, which is the variance of expected returns / variance of returns is small, the standard deviation of expected returns is large relative to the level of expected returns, and the long-horizon R^2 is large. These are measures of "economic importance."

2.

    (a)

$$
\begin{aligned}
r_{t+1} &= -\rho dp_{t+1} + dp_t + \Delta d_{t+1} \\
dp_t &= \sum_{j=1}^{\infty}\rho^{j-1}r_{t+j} - \sum_{j=1}^{\infty}\rho^{j-1}\Delta d_{t+j}
\end{aligned}
$$

The trick is to run both sides of the present value identity on to dp_t. You can then write things several ways.

$$
1 = b_r^{(1)} + \sum_{j=2}^{\infty}\rho^{j-1}b_r^{(j)} - \sum_{j=1}^{\infty}\rho^{j-1}b_d^{(j)}
$$

and hence

$$
0.9 = \sum_{j=2}^{\infty}\rho^{j-1}b_r^{(j)} - \sum_{j=1}^{\infty}\rho^{j-1}b_d^{(j)}
$$

You can also write

$$
1 = b_r + b_r^{(2,\infty)} - b_d^{(1,\infty)}
$$

and hence

$$
0.9 = b_r^{(2,\infty)} - b_d^{(1,\infty)}.
$$

You could regress the return identity on dp, and get

$$b_r = -\rho b_{dp} + 1 + b_d$$

and thus

$$0.9 = \rho b_{dp} - b_d$$

That's good for partial credit, but it doesn't really fulfill the first sentence, which does not include future dp.

(b)

    i. cay the consumption-wealth ratio helps to forecast. "Discount rates."

    ii. Words are ok here – z must also forecast future dividend growth or returns past t=1. Equations are better, the corresponding identity is

$$dp_t = \sum_{j=1}^{\infty} \rho^{j-1} r_{t+j} - \sum_{j=1}^{\infty} \rho^{j-1} \Delta d_{t+j}$$

$$0 = c_r^{(1)} + \sum_{j=2}^{\infty} \rho^{j-1} c_r^{(j)} - \sum_{j=1}^{\infty} \rho^{j-1} c_d^{(j)}$$

so if $c_r^{(1)} > 0$, z_t must also forecast future returns, in the opposite direction, or dividends in the same direction.

3.

(a)

$$\begin{aligned}
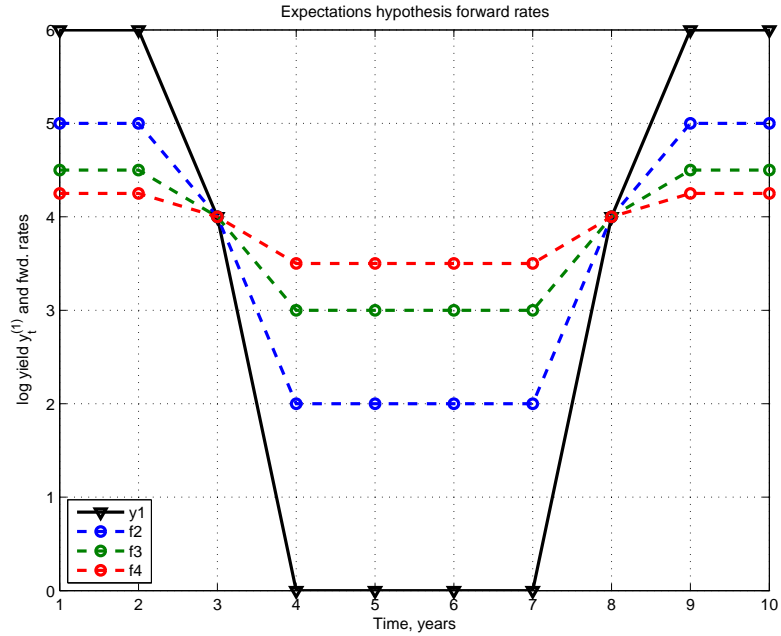f_t^{(2)} &= E_t y_{t+1}^{(1)} = \delta + \rho(y_t^{(1)} - \delta) \\
f_t^{(3)} &= E_t y_{t+2}^{(1)} = \delta + \rho^2(y_t^{(1)} - \delta) \\
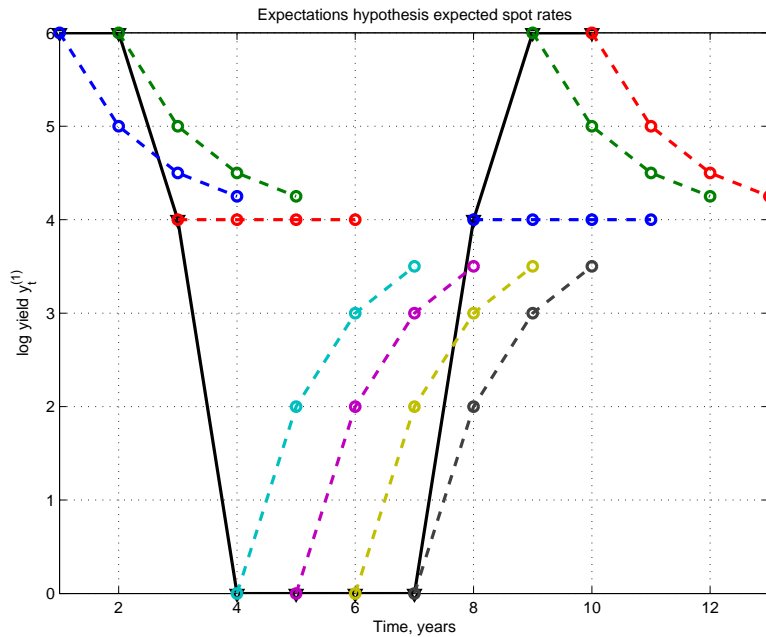f_t^{(4)} &= E_t y_{t+3}^{(1)} = \delta + \rho^3(y_t^{(1)} - \delta) \\
f_t^{(n)} &= E_t y_{t+n-1}^{(1)} = \delta + \rho^{(n-1)}\left(y_t^{(1)} - \delta\right)
\end{aligned}$$

(b) $\rho^2 = 0.25$, $\rho^3 = 0.125$ so

| $y_t^{(1)}$	6%	4%	0%
$f_t^{(2)}$	5	4	2
$f_t^{(3)}$	4.5	4	3
$f_t^{(4)}$	4.25	4	3.5

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Expectations hypothesis forward rates

(c) The numbers are the same, we just move the forward rates dots 1,2,3, periods forward and connect the dots differently.

Expectations hypothesis expected spot rates

(d) If rates follow the assumed AR(1), then events like the graphed one should be very rare. As shown in the second plot, You don't see the *time series* forecast error in the first plot. Put another way, the $y_t^{(1)}$ process graphed is much more persistent than an AR(1) with $\rho = 0.5$. People are generating bond prices as if there is a quickly mean reverting AR(1), but the actual process doesn't revert so fast, so you make

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money. You could assume $\rho = 1$ to generate the slow mean reversion, but then the forward rates would not be upward sloping. At $\rho = 1$, with the expectations hypothesis, all the forward rates collapse to the spot rate. So, *to make a graph that looks like the forward rate data I have to assume people expect interest rates to revert back a lot faster than interest rates actually do revert back.*

As another way to see the point, (far beyond what I expect on an exam) here is a plot of excess returns $rx_t^{(n)}$ through the episode, and the mean excess returns in the episode are as given in the table. The investor makes money through the episode. The market is "expecting" yields to rise, but it doesn't happen fast enough.
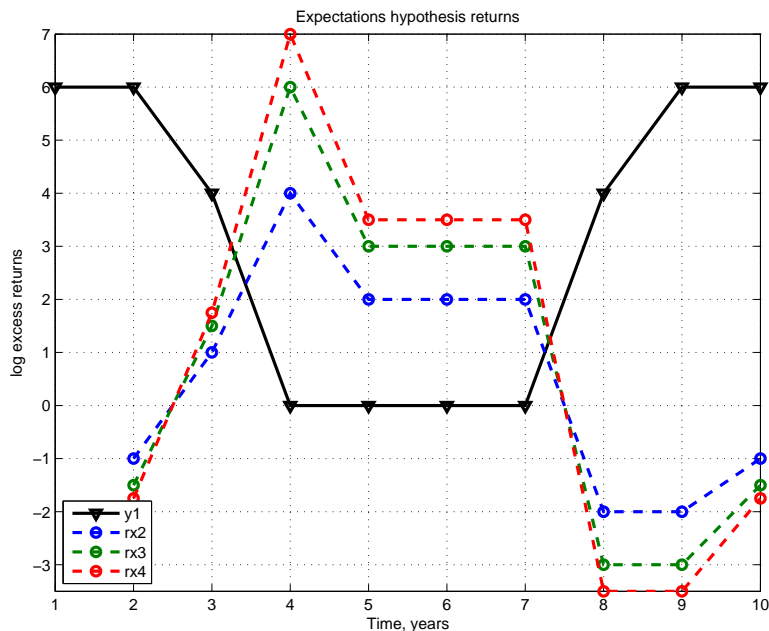
4.

    (a)
$$R_t^{ei} = \alpha_i + b_i rmrf_t + h_i hml_t + s_i smb_t + \varepsilon_t^i \quad T = 1..T \text{ for each } i$$

    R^{ei} = excess returns on 25 size and b/m sorted portfolios, $rmrf_t$ = market excess return hml_t = long value short growth factor return smb_t = long small short big factor return.

    (b) i) alphas are mostly small (0.10) relative to the expected return spread (0.50 or more), except small growth. Supports. ii) alpha t statistics are mostly below 2, except small growth. Mostly supports. iii) b are all about 1, h rise for value firms, s rise for small firms. This is key support. iv) the t statistics are huge, but these tell us if the b,h,s are different from zero, not different from each other. They measure a bit the contribution of each factor to the "model of returns" but say nothing about the "model of expected returns" v) R^2 is 0.90-0.95. Great for "model of returns" but really tangential to the "model of expected returns." vi) GRS rejects, bad news for the FF model. But models are models, and statistical rejection does not mean the glass is not 95% full. Σ is small, so $\hat{\alpha}' \hat{\Sigma}^{-1} \hat{\alpha}$ can be big even with small α.

    (c) High sales growth have low returns, with a spread of about 0.5% / month for the 1-10 portfolios. This includes out of business companies.

    (d) We saw that the momentum portfolios have negative hml betas. This means value and momentum are negatively correlated, so a portfolio of the two has higher Sharpe ratio than either alone.

5.

(a) This is a table of Fama MacBeth regression coefficients. The first row is

$$R_{t+1}^{ei} = a_t + b_t MC_{it} + c_t B/M_{it} + d_t Mom_{it} + \ldots + \varepsilon_{i,t+1} \; i = 1, 2, \ldots N \; \forall t$$

then

$$b = \frac{1}{T} \sum_{t=1}^{t} b_t; c = \frac{1}{T} \sum_{t=1}^{t} c_t; \; d = \frac{1}{T} \sum_{t=1}^{t} d_t;$$

etc. If you write that it's a cross sectional regression

$$E\left(R_{t+1}^{ei}\right) = a + bMC_i + cB/M_i + dMom_i + \ldots + \varepsilon_i; \; i = 1, 2, \ldots N \; \forall t$$

that's almost right but not quite. These are not averages of portfolio returns!

(b) This is a multiple regression, not a single regression or portfolio sort. So the interpretation of the NS coefficient is exactly a measure of expected returns if we see more issues, holding B/M constant.

(c) In the problem set you handled this with an interaction term.

$$R_{t+1}^{ei} = a_t + b_t MC_{it} + c_t B/M_{it} + d_t \left(MC_{it} \times B/M_{it}\right) + \ldots + \varepsilon_{i,t+1} \; i = 1, 2, \ldots N \; \forall t$$

which is the same thing as

$$R_{t+1}^{ei} = a_t + b_t MC_{it} + \left(c_t + d_t MC_{it}\right) B/M_{it} + \ldots + \varepsilon_{i,t+1} \; i = 1, 2, \ldots N \; \forall t$$

(d) Novy Marx and FF are using *gross* profits to assets, not *net*. Though net in the end matters to investors, NM claims (and finds) that gross profitability is a better forecaster of future profitability.

(e)
$$\frac{M_t}{B_t} = E_t \sum_{\tau=1}^{\infty} \frac{Y_{t+\tau} - dB_{t+\tau}}{B_t \left(1+r\right)^\tau} \; \text{(Novy-Marx)}$$

Higher Y by itself generates higher M with no change in r. But higher Y controlling for M must mean higher r.

(f) Varying betas with no change in average return is just as much a puzzle as varying expected returns with no betas. Betting against beta by Frazzini et al.

6.

(a) 1% higher fees lead to 1.54% lower return to investors. The evidence is a FmB cross sectional regression of alphas on fees.

(b) Carhart did sort on 5 year averages, and found weaker results – almost no expected return spread based on 5 year return averages.

7.

(a) The key assumption under simulated is that no funds have any true alpha, positive or negative.

(b) 1.30 means that if all funds really have exactly zero alpha, then we expect to see that 10% of the funds in a sample will have an alpha t statistic greater than 1.30 just due to chance. In fact, 7% had a t stat greater than 1.30. Thus there are actually 3% too few funds with alpha greater than 1.30 than there should be.

(c) 5% of funds should have performance below -1.71. In fact, 5% of funds have performance below -2.84. This is a bit puzzling – why have negative alpha when you can just buy the index? But we have not chalked up all the costs here.

8. Berk and Green

(a) You grow to $100M AUM, you still make 5% alpha x $20M = $1 million; your fees are 1% = $1 million, and your investors get the index.

(b) Neither. They measure skill by gross alpha times AUM, or \$1 million. alpha to investors is always zero in their world.

9.

(a)

$$P_t = E_t \sum_{j=1}^{\infty} \beta^j \left(\frac{C_{t+j}}{C_t} \right)^{-1} C_{t+j}$$

$$\frac{P_t}{C_t} = E_t \sum_{j=1}^{\infty} \beta^j = \frac{\beta}{1-\beta} = \frac{\frac{1}{1+\delta}}{1 - \frac{1}{1+\delta}} = \frac{1}{\delta}$$

(b)

$$
\begin{aligned}
R^c_{t+1} &= \frac{P_{t+1} + C_{t+1}}{P_t} = \frac{C_{t+1}/\delta + C_{t+1}}{C_t/\delta} = \frac{C_{t+1}/\delta + C_{t+1}}{C_t/\delta} \\
&= \frac{C_{t+1}}{C_t} \frac{1/\delta + 1}{1/\delta} = \frac{C_{t+1}}{C_t} \frac{1+\delta}{1} = \frac{1}{\beta} \frac{C_{t+1}}{C_t}
\end{aligned}
$$

(c)

$$\log R^c_{t+1} = r^c_{t+1} = -\log \beta + \Delta c_{t+1}$$

so

$$E(R^{ei}_{t+1}) \approx cov(R^{ei}_{t+1}, r^c_{t+1}) = \beta_{i,r^c} \lambda$$

(d)

$$R^f = \frac{1}{E(m)} = \frac{1}{E\left[\beta \frac{u'(C_{t+1})}{u'(C_t)}\right]} = \frac{1}{\beta E\left[\left(\frac{C_{t+1}}{C_t}\right)^{-1}\right]} = \frac{1}{\beta e^{-g+\frac{1}{2}\sigma^2}} = (1+\delta) e^{g-\frac{1}{2}\sigma^2}$$

10.

(a) You can see in the right graph that the long run beta is one. And you can see in the left graph that fund returns tend to lag. Hmm, How about,

$$R_t = 0 + \frac{1}{3} R^m_t + \frac{1}{3} R^m_{t-1} + \frac{1}{3} R^m_{t-2} + \varepsilon_t$$

for a total beta including lags of 1.0. This is the Asness et al stale pricing issue. The data is generated this way with $\varepsilon = 0$.

(b) This is screaming for up and down betas. I generated the data with

$$R_t = 9 + 1.0 \ (R^m < 0)$$

Any expression that the up beta is zero and down beta is one is ok. We can't infer anything about alphas without a real rate of return on the right hand side.

11.

(a) 147%. Any answer between 100 and 200% is fine. The constraint was an overwhelming demand to short and only 5% of the shares outstanding.

(b) Put call parity failed. The synthetic Palm traded substantially below the actual Palm.

(c) Lots of trading volume.

12.

(a) The change in yield of each bond depends on the 2-5 year on the run order flow in a multiple regression, not its own order flow.

(b) The off the run bonds are more affected by on the run order flow than their own order flow.

(c) There is no bounce-back, where yesterday's order flow moves today's price in the opposite direction.

13.

(a) You have to know that the answer is "columns of Q", so $\begin{bmatrix} \beta_{FD} \\ \beta_{TO} \\ \beta_{GF} \end{bmatrix} = \begin{bmatrix} 0.58 \\ 0.58 \\ 0.58 \end{bmatrix}$

(b) and $F_t = \begin{bmatrix} 0.58 \\ 0.58 \\ 0.58 \end{bmatrix}' \begin{bmatrix} FD_t \\ TO_t \\ GF_t \end{bmatrix}$

(c) This is a "level" or "average" factor if I've ever seen one.

(d) $0.58^2 \times 1.67^2/1 = 0.938\,19 = 0.94$ This requires you to know that the variance of the factor is 2.8. Alternatively, the residual $1 - 0.82^2 \times 0.1 = 0.93$, the same thing up to numerical differences.

14. This is your reward for doing the last problem set.

(a) The key here is to separate the "active" and "passive" portfolios.

$$ w^m = \frac{1}{\gamma}\frac{E(R^{em})}{\sigma^2(R^{em})} = \frac{1}{2}\frac{0.08}{0.20^2} = \frac{1}{2}\frac{0.08}{0.04} = 1 $$

$$
\begin{aligned}
w_\alpha &= \frac{1}{\gamma}\Sigma^{-1}\alpha = \frac{1}{2}\frac{1}{0.01}\begin{bmatrix} 1 & -0.5 \\ -0.5 & 1 \end{bmatrix}^{-1}\begin{bmatrix} \alpha_1 \\ \alpha_2 \end{bmatrix} \\
&= \frac{1}{2}\frac{1}{0.01}\frac{1}{3}\begin{bmatrix} 4 & 2 \\ 2 & 4 \end{bmatrix}\begin{bmatrix} -0.003 \\ 0.0012 \end{bmatrix} \\
&= \frac{1}{10}\frac{1}{3}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} -3 \\ 12 \end{bmatrix} \\
&= \frac{1}{10}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} -1 \\ 4 \end{bmatrix} = \begin{bmatrix} 0.2 \\ 0.7 \end{bmatrix}
\end{aligned}
$$

(b) This example gives positive weight. The negative correlation means that the negative alpha manager helps to reduce risk so much, it's worth keeping him. The reason is his negative correlation – the portfolio of the two managers exploits the good manager's alpha and uses the bad manager's ε to diversify. If the ε were uncorrelated, then

$$ \Sigma^{-1}\alpha = \begin{bmatrix} \sigma_1^2 & 0 \\ 0 & \sigma_2^2 \end{bmatrix}^{-1}\begin{bmatrix} \alpha_1 \\ \alpha_2 \end{bmatrix} = \begin{bmatrix} \alpha_1/\sigma_1^2 \\ \alpha_2/\sigma_2^2 \end{bmatrix} $$

and positive α must correspond to positive weight.