Formulas

Prediction and present value

$$\text{If } x_t = \phi x_{t-1} + \varepsilon_t \;\text{ then } E_t(x_{t+j}) = \phi^j x_t$$

$$r_{t+1} \approx -\rho dp_{t+1} + \Delta d_{t+1} + dp_t;\; dp_t \equiv d_t - p_t$$

$$p_t - d_t \approx \sum_{j=1}^{\infty} \rho^{j-1} \Delta d_{t+j} - \sum_{j=1}^{\infty} \rho^{j-1} r_{t+j}; \quad \rho = \frac{1}{1+D/P} \approx 0.96$$

$$M_t = E_t \sum_{\tau=1}^{\infty} \frac{Y_{t+\tau} - dB_{t+\tau}}{(1+r)^{\tau}} \text{ (Novy-Marx)}$$

VAR

$$\begin{aligned} r_{t+1} &= b_r \times dp_t + \varepsilon^r_{t+1};\; b_r \approx 0.1 \\ \Delta d_{t+1} &= b_d \times dp_t + \varepsilon^d_{t+1}; b_d \approx 0 \\ dp_{t+1} &= b_{dp} \times dp_t + \varepsilon^{dp}_{t+1}; b_{dp} \approx 0.94 \end{aligned}$$

Geometric sums

$$\sum_{j=0}^{\infty} z^j = \frac{1}{1-z} \text{ if } \|z\| < 1;\; \sum_{j=1}^{\infty} \beta^j = \frac{\beta}{1-\beta} = \frac{\frac{1}{1+\delta}}{1-\frac{1}{1+\delta}} = \frac{1}{\delta}$$

Discount factors, consumption and models

$$p_t = E_t(m_{t+1} x_{t+1}) = E_t\left(\beta \frac{u'(c_{t+1})}{u'(c_t)} x_{t+1}\right)$$

$$p_t = E_t \sum_{j=0}^{\infty} \beta^j \frac{u'(c_{t+j})}{u'(c_t)} x_{t+j}$$

$$m_{t+1} = \beta\left(\frac{c_{t+1}}{c_t}\right)^{-\gamma} \approx 1 - \delta - \gamma \Delta c_{t+1}$$

$$0 = E_t(m_{t+1} R^e_{t+1});\; 1 = E_t(m_{t+1} R_{t+1})$$

$$R^f = 1/E(m_{t+1}) = 1/E_t\left(\beta \frac{u'(c_{t+1})}{u'(c_t)}\right) \approx 1 + \delta + \gamma E_t(\Delta c_{t+1})$$

$$E(R^e_{t+1}) = -R^f_t cov(m_{t+1}, R^e_{t+1}) \approx \gamma cov(\Delta c_{t+1}, R^e_{t+1})$$

Empirical methods GRS test:

$$T\left[1 + E(f)' \Sigma_f^{-1} E(f)\right]^{-1} \hat{\alpha}' \Sigma^{-1} \hat{\alpha} \,\tilde{}\, \chi^2_N$$

$$\frac{T-N-K}{N}\left[1 + E(f)' \hat{\Sigma}_f^{-1} E(f)\right]^{-1} \hat{\alpha}' \hat{\Sigma}^{-1} \hat{\alpha} \,\tilde{}\, F_{N,T-N-K}$$

Term structure

$$\begin{aligned} p_t^{(n)} &= \text{log price at } t \text{ of bond that comes due at } t+n\text{, e.g. -0.20} \\ y_t^{(n)} &\equiv -\frac{1}{n} p_t^{(n)}; \quad f_t^{(n)} \equiv p_t^{(n-1)} - p_t^{(n)}; \\ r_{t+1}^{(n)} &\equiv p_{t+1}^{(n-1)} - p_t^{(n)}; rx_{t+1}^{(n)} = r_{t+1}^{(n)} - y_t^{(1)} \end{aligned}$$

Expectations:

$$\begin{aligned} y_0^{(n)} &= \frac{1}{n}E\left(y_t^{(1)}+y_{t+1}^{(1)}+y_{t+2}^{(1)}+...y_{t+n-1}^{(1)}\right)+\text{(risk premium)} \\ f_t^{(n)} &= E_t(y_{t+n-1}^{(1)})+\text{(risk premium)} \\ E_t\left[r_{t+1}^{(n)}\right] &= y_t^{(1)}+\text{(risk premium)} \end{aligned}$$

Fama-Bliss regression

$$\begin{aligned} rx_{t+1}^{(n)} &= r_{t+1}^{(n)}-y_t^{(1)}=a+b\left(f_t^{(n)}-y_t^{(1)}\right)+\varepsilon_{t+1} \\ y_{t+n-1}^{(1)}-y_t^{(1)} &= a+b\left(f_t^{(n)}-y_t^{(1)}\right)+\varepsilon_{t+1} \end{aligned}$$

Cochrane-Piazzesi regression

$$\begin{aligned} \overline{rx}_{t+1} &= \frac{1}{4}\sum_{n=2}^{5}rx_{t+1}^{(n)}=\gamma' f_t+\varepsilon_{t+1} \\ rx_{t+1}^{(n)} &= b_n\left(\gamma\prime f_t\right)+\varepsilon_{t+1}^{(n)} \end{aligned}$$

Portfolios
Classic mean-variance

$$w_0=\frac{1}{\gamma}\Sigma^{-1}E(R^e);\ \Sigma=cov(R^e)$$

With a factor model

$$\begin{aligned} R_{t+1}^p &= R^f+w_mR_{t+1}^{em}+w_\alpha'(\alpha+\varepsilon). \\ w_m &= \frac{1}{\gamma}\frac{E(R^{em})}{\sigma^2(R^{em})};\ w_\alpha=\frac{1}{\gamma}\Sigma^{-1}\alpha;\quad \Sigma\equiv E(\varepsilon_{t+1}\varepsilon_{t+1}') \end{aligned}$$

Multifactor, y = state variable, and relative to the market if everyone is like this

$$w=\frac{1}{\gamma}\Sigma^{-1}E(R^e)+\beta_{R,y'}\frac{\eta}{\gamma}$$

$$R^i=R^f+\frac{\gamma^m}{\gamma^i}R^{em}+\frac{1}{\gamma^i}\left(\eta^{i\prime}-\eta^{m\prime}\right)R^{ez};\ R^{ez}\equiv\beta_{y,R'}R^e$$

$$E(R^e)=cov(R^e,R^m)\gamma^m-cov(R^e,y')\eta^m$$

Bayesian portfolios

$$w=\frac{1}{\gamma}\frac{E(R^e)}{\sigma^2(R^e)+\sigma^2\left[E(R^e)\right]}$$