Business 35150 **John H. Cochrane**

2013 Final Exam

Name (Print clearly):

Section:

Mailfolder location:

Directions

DO NOT START UNTIL WE TELL YOU TO DO SO. *Read these directions* in the meantime.

Please *do not tear your exam apart.* Answer the questions in the space provided. There are some extra pages at the end if you run out of space (but if you do, it means you're writing too much.)

You can rip off the formula sheet and blank pages at the back for throw-away scrap paper if you wish.

Show your work. An answer that comes without justification the right answer but coming miraculously from the wrong logic will be graded as wrong. Also, by showing your work you will get partial credit.

Keep your answers *short.* We are only looking for the right answer; we will grade off for a memory dump of unrelated stuff as it reveals you don't know what's relevant to the question.

Put your answers in a *box* or *underline* to make sure we find them. Make sure you answer each direct question. The questions are not clever or subtle. In each case, we just want to know the one obvious point.

For fact questions, *quote the author and paper*, or *state that the fact comes from a problem set* if such a source is relevant.

This is an *closed-book, closed-note* exam. Not even calculators are allowed. The number questions are all easy. If not, your answer is wrong. You may not use a laptop computer, PDA, ipad, cell phone, etc.

Each question has a suggested time, which is also the number of points it will count in grading. Small times (5 min) require shorter answers. The total time is 2:50.

Thursday section: Do not discuss the contents of this exam with anyone until Sat 12 PM. There are two sections of this class, and any information passed to the other section is not only a serious honor code violation, it lowers your grade directly.

Booth honor code required statement: I pledge my honor that I have not violated the honor code during this examination.

Signature:

1. (30) Consider our standard return forecast VAR

$$\begin{aligned} r_{t+1} &= b_r dp_t + \varepsilon^r_{t+1} \\ dp_{t+1} &= \phi dp_t + \varepsilon^{dp}_{t+1} \\ \Delta d_{t+1} &= b_d dp_t + \varepsilon^d_{t+1} \end{aligned}$$

and use the return identity

$$r_{t+1} = -\rho dp_{t+1} + dp_t + \Delta d_{t+1}; \;\; \rho = 0.96.$$

(a) (5) Using the return identity, suppose $b_d = 0$ (which is close to true). Find b_r in terms of ρ and ϕ. Keep $b_d = 0$ for the rest of the problem.

(b) (5) Suppose $cov(\varepsilon^d_{t+1}, \varepsilon^{dp}_{t+1}) = 0$ (which is close to true). Using the return identity,

i. Find $cov(\varepsilon^r_{t+1}, \varepsilon^{dp}_{t+1})$ in terms of variances of the shocks $\sigma^2(\varepsilon)$ and other model parameters. Is the covariance positive or negative? (Hint: Find the restriction on the shocks $\varepsilon^r_{t+1}, \varepsilon^d_{t+1}, \varepsilon^{dp}_{t+1}$ that follows from the return identity. Then, with $\varepsilon^r_{t+1} = ...$ you can multiply both sides by $\varepsilon^{dp}_{t+1} \times ..$ and then take expectations)

ii. The issue: are shocks to *returns* ε^r_{t+1} positively, negatively or un-correlated with shocks to *expected returns* ε^{dp}_{t+1}? Does this correlation have the same or opposite sign as the same correlation (returns and expected returns) for bonds?
Keep the assumption $cov(\varepsilon^d_{t+1}, \varepsilon^{dp}_{t+1}) = 0$ and its implications for the rest of the problem as well.

(c) (5) One question on our minds all quarter has been whether stocks are riskier for long-run investors. Applying the portfolio weight formula $w = \frac{1}{\gamma}\frac{E(r)}{\sigma^2(r)}$ to log returns, r_{t+1} for a one-year investor and $r_{t+1} + r_{t+2}$ for a two-year investor, (with $r^f = 0$, and ignoring the logs vs levels issue)

i. Show that stocks are riskier for long-run investors if $cov(r_{t+1}, r_{t+2}) > 0$ and safer for long-run investors if $cov(r_{t+1}, r_{t+2}) < 0$.

ii. Show that if returns are independent over time, so $cov(r_{t+1}, r_{t+2}) = 0$, the portfolio allocation is the same for one year and two year horizons.

(d) (15) But our VAR says returns are not independent over time. Let's figure out if that means stocks are better or worse for long run investors. We will work out what the VAR implies for $cov(r_{t+1}, r_{t+2})$.

Intuition: There are two offsetting effects. In one effect, the slow-moving dp induces momentum or positive covariance – a high dp_t means high returns r_{t+1}, and it's likely dp_{t+1} and return r_{t+2} will be high again next year as well. In the other effect, stocks are a bit like bonds: a rise in expected returns $E_{t+1}r_{t+2}$ means prices p_{t+1} and returns r_{t+1} decline, so there is a negative correlation between current return r_{t+1} and subsequent returns r_{t+2}. Your job is to quantify these two effects and see which one dominates.

i. To get started, write each of r_{t+1} and r_{t+2} in terms of dp_t and subsequent shocks.

ii. Suppose $\varepsilon^r = 0$ always, i.e. $\sigma^2(\varepsilon^r) = 0$. Find the covariance $cov(r_{t+1}, r_{t+2})$. Is it positive or negative? Which intuition does this case represent? (Hint: There are several equivalent ways to write the covariance. Any correct expression that allows you to figure out the sign is sufficient. You may leave the answer in terms of $\sigma^2(dp_t)$, since we know that's positive.)

iii. Now, allow $\sigma^2(\varepsilon^r) > 0$, but hold dp_t fixed. Find the covariance $cov(r_{t+1}, r_{t+2}|dp_t)$. ($|dp_t$ just means "holding dp_t fixed" or the "conditional covariance cov_t.") Is this covariance positive or negative? Which intuition does this case represent? (Hint: You treat dp_t and any other variables dated t or earlier as constants. The part b result will come in handy.)

iv. Now find $cov(r_{t+1}, r_{t+2})$ in general, with both dp_t variation and with all shocks ε turned on. To simplify the answer use $\rho = \phi$. Which effect wins? I.e., *with stock return predictability as given in our VAR, are stocks safer or riskier in the long run?*

(Hint:$\sigma^2(dp_t) = \frac{1}{1-\phi^2}\sigma^2(\varepsilon^{dp})$, and the result from part a will help. Generic hint: Remember that the covariance of ε_{t+1} with any variable dated t, $t-1, t-2$,.. or earlier is zero.)

(More space for #1)

(More space for #1)

2. (10) Consider again the standard VAR with values

$$
\begin{aligned}
r_{t+1} &= 0.1 \times dp_t + \varepsilon^r_{t+1} \\
dp_{t+1} &= 0.94 \times dp_t + \varepsilon^{dp}_{t+1} \\
\Delta d_{t+1} &= 0 \times dp_t + \varepsilon^d_{t+1}
\end{aligned}
$$

and use the return identity

$$r_{t+1} = -0.96 \times dp_{t+1} + dp_t + \Delta d_{t+1}$$

Calculate the response to

(a) a dividend growth $\varepsilon^d_1 = 1$ shock with no movement in dividend yield $\varepsilon^{dp}_1 = 0$.

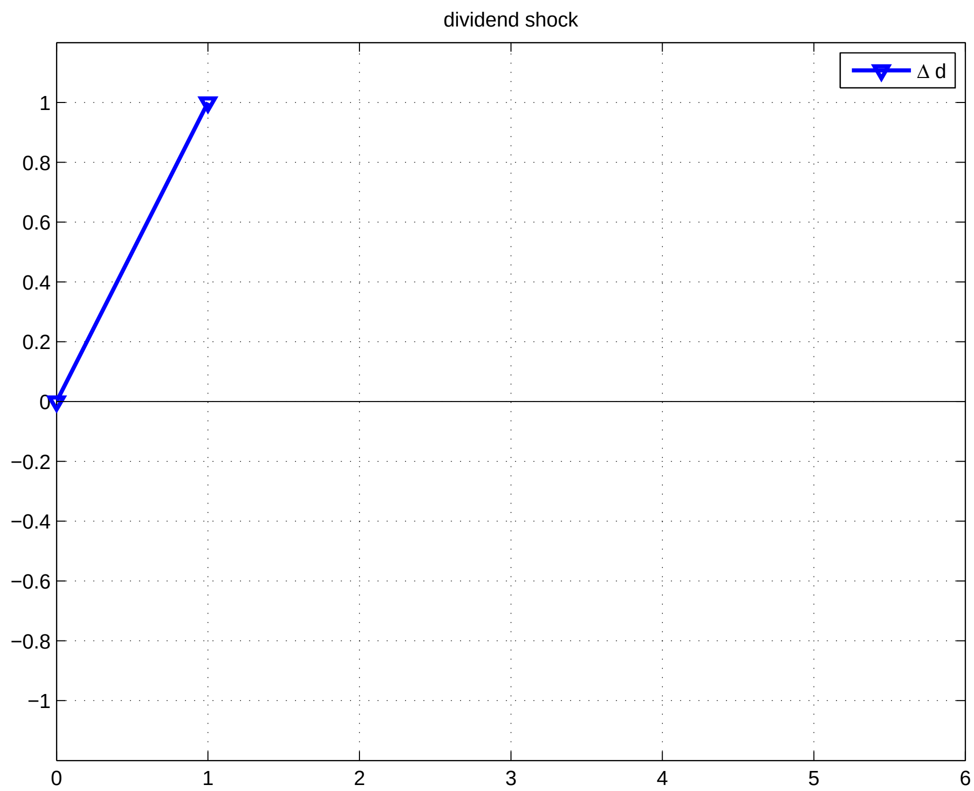
(b) a dividend yield $\varepsilon^{dp}_1 = 1$ shock with no movement in dividend growth $\varepsilon^d_1 = 0$, and

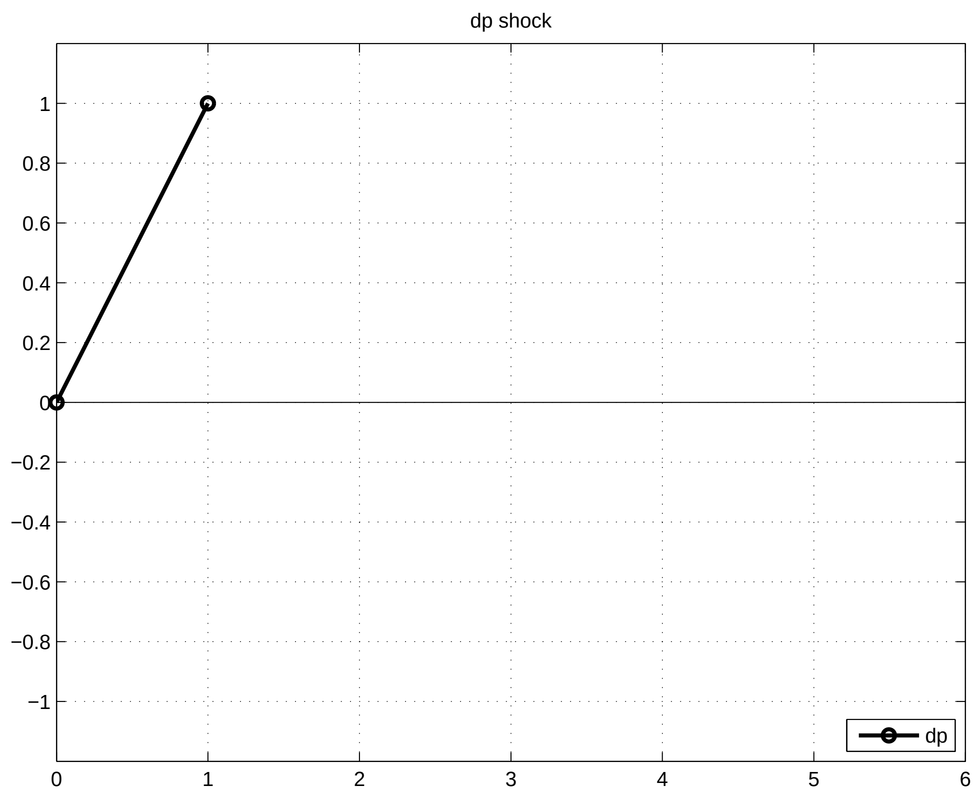
Put the three responses for each case in the table (or a larger version) and on the graph on the next page. FYI: $0.94^2 = 0.88$; $0.94^3 = 0.83$; $0.94^4 = 0.78$.

t	0	1	2	3	4
ε^d_t	0	1	0	0	0
dp_t	0	0			
r_t	0				
Δd_t	0	1			

t	0	1	2	3	4
ε^{dp}_t	0	1	0	0	0
dp_t	0	1			
r_t	0				
Δd_t	0	0			

Graphs for #2 (fill in all three lines, r,Δd, dp.)





3. (15) Let's construct a simple term structure model. Suppose the one year rate follows an AR(1)

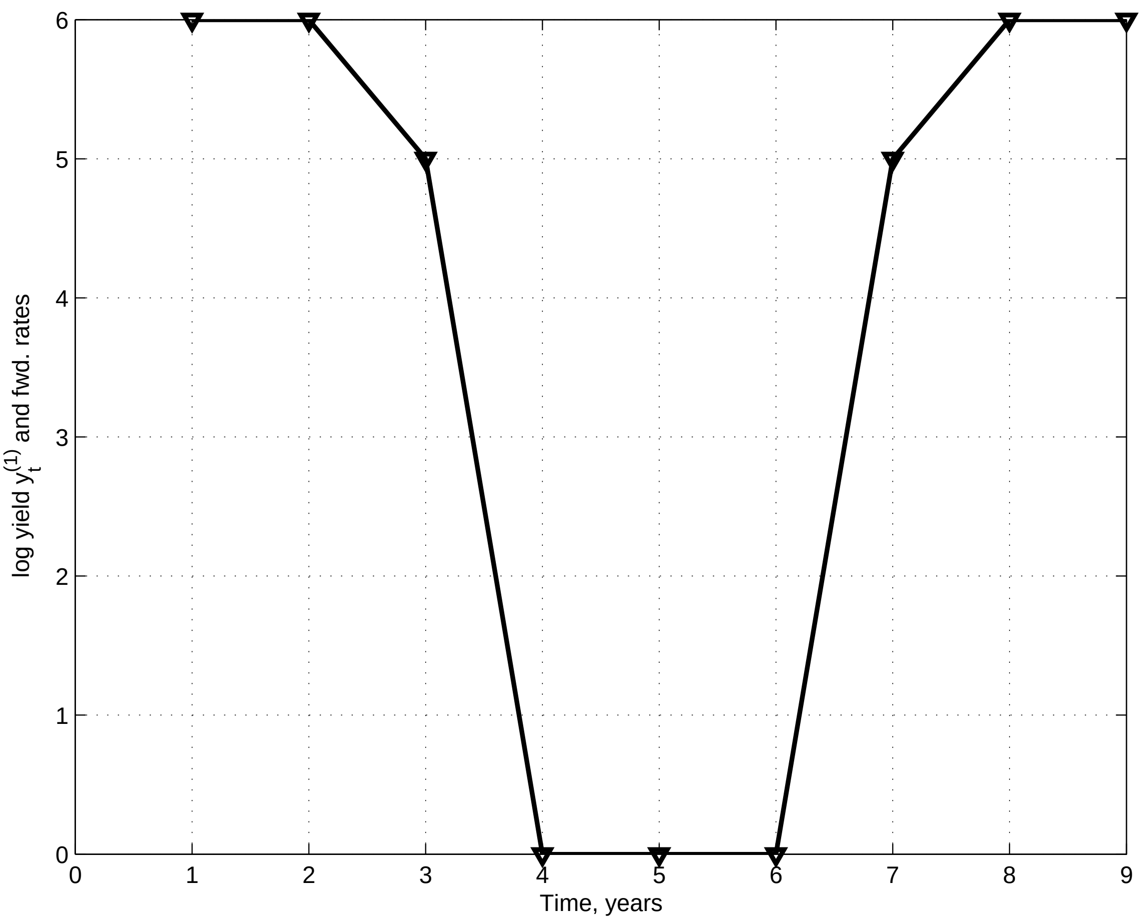
$$y_{t+1}^{(1)} - \delta = \rho\left(y_t^{(1)} - \delta\right) + \varepsilon_{t+1}$$

and suppose the pure expectations hypothesis holds with no risk premiums.

(a) (5) Find a formula for forward rates $f_t^{(2)}, f_t^{(3)}, f_t^{(4)} f_t^{(n)}$. Express your result as a one-factor model for forward rates. ($f_t^{(n)}$ = constant + loading$_n$×factor$_t$)

(b) (5) Using $\delta = 0.05$ (i.e. 5%) and $\rho = 0.5$, fill in the following table (or a larger version) and sketch the path of $f_t^{(2)}$, $f_t^{(3)}$, $f_t^{(4)}$ on the plot below. (We have plotted $y_t^{(1)}$ for you over time t.)

$100 \times y_t^{(1)}$:	6%	5%	0%
$100 \times f_t^{(2)}$:			
$100 \times f_t^{(3)}$:			
$100 \times f_t^{(4)}$:			



(a) (5) The path of forward rates you sketched ought to look like a reasonable representation of the yield and forward-rate data over the course of a business cycle. And we assumed the expectations hypothesis to make it. Yet Fama and Bliss show that the expectations hypothesis is seriously wrong. What can you see in the data plot which shows the failure of the expectations hypothesis?

(Hint: If one year yields follow an AR(1) with $\rho = 0.5$, should we see a plot like this very often? If we tried $\rho = 1$ to make the one-year yield unforecastable, what would the plot look like? There is a tension between the model that generates the cross section of yields and the time series of this plot. What is it? You don't have to answer these questions, but they should help you to answer the main one.)

4. (10) Suppose the current yield curve is as shown in the second row. (The numbers are 100 × log yield, i.e. 1 means 0.01)

   (a) (5) Fill in the f and p rows. (If this is too small, make a larger version in the blank space.)

n	1	2	3	4	5
$y_t^{(n)}$	0	1	2	3	4
$f_t^{(n)}$	0				
$p_t^{(n)}$					
$E_t(y_{t+n-1}^{(1)})$	0				

   (b) (5) Use the table below of simplified Fama-Bliss regression coefficients to fill in the final $E_t(y_{t+n-1}^{(1)})$ row. I.e. *Use the Fama Bliss coefficients, not the expectations hypothesis to find the path of expected future short rates.* (Use the coefficients in the table not 0 and 1)

	$rx_{t+1}^{(n)} = a + b\left(f_t^{(n)} - y_t^{(1)}\right) + \varepsilon_{t+1}$				$y_{t+n-1}^{(1)} - y_t^{(1)} = a + b\left(f_t^{(n)} - y_t^{(1)}\right) + \varepsilon_{t+1}$			
n	a	b	$\sigma(b)$	R^2	a	b	$\sigma(b)$	R^2
2	0	0.80	0.27	0.11	0	0.20	0.27	0.01
3	0	1.10	0.35	0.13	0	0.50	0.33	0.05
4	0	1.40	0.43	0.15	0	0.80	0.26	0.14
5	0	1.05	0.49	0.07	0	0.90	0.17	0.17

5. (20) Fama and French "Multifactor anomalies"

   (a) (5) Table 1 runs a regression. Write down the regression. Be careful with the i's and t's. Define the variables that you use.

   (b) (5) Fama and French describe their paper as a "parsimonious description of returns and average returns." Of $E(R^e)$, $E(rmrf)$, $E(hml)$, $E(smb)$, α, $t(\alpha)$, b, h, s, $t(b), t(h), t(s)$, R^2, GRS, which statistics are important to document this claim for i) Returns and ii) Average returns.

   (Note: you may not use all the suggested statistics, and you don't have to comment on each statistic. Just say which statistics are important to each point, also showing you understand the difference between the two points.)

(c) (5) Fama and French report that the GRS F test rejects their model at the 0.004 level. Yet, they still think it's a pretty good model. Why? (Note: The GRS statistic is $\frac{T-N-K}{N}\left[1+E(f)'\hat{\Sigma}_f^{-1}E(f)\right]^{-1}\hat{\alpha}'\hat{\Sigma}^{-1}\hat{\alpha}\tilde{}F_{N,T-N-K}$)

(d) (5) Suppose you find

$$smb_t = 0 + b_{smb} \times rmrf_t + h_{smb} \times hml_t + \varepsilon_t^s$$

What effect would dropping smb have on Fama French's description of i) returns and ii) average returns?

6. (10) Fama and French "Dissecting Anomalies" present Table II of average monthly returns of stocks, sorted into portfolios based on whether the company is issuing or repurchasing shares:

	Negatives			Positives					Pos H –
	Low	High	Zeros	Low	2	3	4	High	Neg L
Sorting on Net Stock Issues, *NS*									
			Average Value-Weight Returns						
Market	0.26	0.11	−0.11	−0.03	0.08	0.11	−0.08	−0.29	−0.54
Micro	0.26	0.10	−0.08	0.10	0.02	0.28	−0.13	−0.45	−0.71
Small	0.26	0.20	−0.08	0.11	0.15	0.12	0.11	−0.36	−0.62
Big	0.26	0.11	−0.13	−0.04	0.08	0.11	−0.09	−0.27	−0.54

They also present Table III of how big (relative to assets) the stock issues were in each portfolio:

Average of Annual EW Average Values								
	Negatives			Positives				
	Low	High	Zeros	Low	2	3	4	High
Sorting on Net Stock Issues, *NS*								
Market	−5.73	−0.48	0.00	0.14	0.57	1.48	4.50	24.04
Micro	−6.20	−0.46	0.00	0.14	0.58	1.48	4.47	26.03
Small	−5.48	−0.48	0.00	0.15	0.58	1.48	4.50	22.18
Big	−4.82	−0.51	0.00	0.15	0.57	1.49	4.56	20.29

(a) (3) In Table II, perhaps this pattern of average returns is just a reflection of the value effect: Stocks with a low cost of capital have high market values relative to book values (earnings are discounted at a low rate), they respond by issuing more shares, and we see the lower average returns on average. How would FF respond?

(b) (7) In Table II average returns do not decrease linearly across portfolios, with the change from Low, 2 , 3 much less than the change from 3, 4, 5. In Table III the difference across portfolios is even more striking, with a huge increase from 3 to high. Is the non-uniformity in Table II a problem, and the huge difference between Table II and Table III a problem? If not, why does this happen? (Pictures may help here. Hint1: The portfolios are formed with equal numbers of stocks in them. Hint 2: Expected returns should be linear in betas. Need they be linear in anomaly variables?)

(More space for #6)

7. (10)

   (a) (5) A mutual fund manager complains, "Carhart's results are bogus. He sorted mutual funds by their one-year past returns. Everyone knows that's mostly luck. He should have looked at funds based on 5 year performance averages, like Morningstar does. Then he would have seen some alphas!" How would Carhart respond?

(b) (5) What point was Carhart making with this picture?

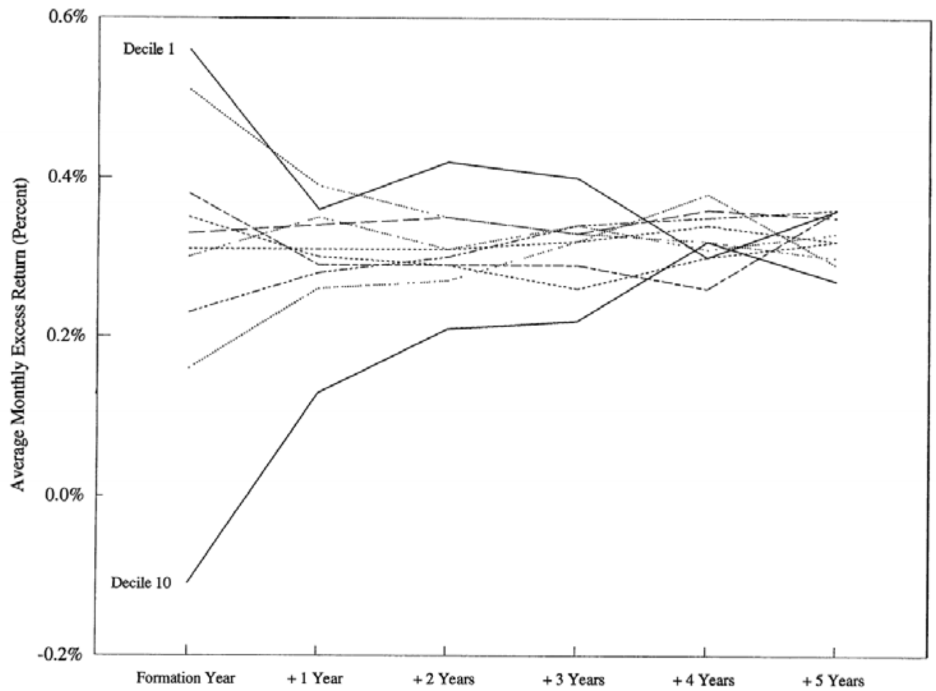


Figure 2. Post-formation returns on portfolios of mutual funds sorted on lagged one-year return. In each calendar year from 1962 to 1987, funds are ranked into equal-weight decile portfolios based on one-year return. The lines in the graph represent the excess returns on the decile portfolios in the year subsequent to initial ranking (the "formation" year) and in each of the next five years after formation. Funds with the highest one-year return comprise decile 1 and funds with the lowest comprise decile 10. The portfolios are equally weighted each month, so the weights are readjusted whenever a fund disappears from the sample.

8. (10) Below, find an excerpt from Fama and French's Table 3 from "Skill vs. Luck"

   (a) What is the key assumption under "simulated?"
   (b) What does 1.30 mean in "simulated"? What does the relative position of 1.30 in "Actual" vs. "Simulated" mean?
   (c) What do -1.71 and -2.84 mean? Is this normal, or a puzzle?
   (d) Fama and French blast Berk with the 50% row, the alpha delivered to investors by the average fund is -0.62. How would Berk measure skill instead? How does Berk answer the -0.62% alpha charge?

Percentile	Simulated	Actual
5	-1.71	-2.84
50	-0.01	-0.62
90	1.30	1.01
93	1.60	1.30
95	1.68	1.54

Table 3 - Percentiles of t(α) estimates for actual and simulated fund returns...[3-factor adjusted] net fund returns...

9. (15) The "binomial model." Suppose that there are two states tomorrow, "up" and "down," and each can happen with probability 1/2. Consumption is $c_t = 1$ today, and $c_{t+1} = 2$ in the "up" state and $c_{t+1} = 1/2$ in the "down" state. Assume $\gamma = 1$ ($u(c) = \log(c)$), $\beta = 1$, and calculate the following

   (a) Find the price of a bond – an asset that pays 1 in each state

   (b) Find the price of an asset that pays $x = 1$ in the up state and $x = -1$ in the down state.

   (c) Find the price of an asset that pays $x = -1$ in the up state and $x = 1$ in the down state.

   (d) Compare b and c. Which of the assets has greater mean payoff $E(x)$? Greater variance of payoff $\sigma(x)$? Explain why they differ in price.

   (e) Find the price of an asset that pays off one unit in the up state, and zero units in the down state, and the price of an asset that pays of zero units in the up state and one unit in the down state. These are "contingent claims." Which is more valuable? Why?

   (f) Now, rather than value the asset in part b directly, let's value it by arbitrage. Find the number of contingent claims from part e that replicate the asset of part b. Find the price of the replicating portfolio. Do you get the same answer?

More space for #9

10. (10)

(a) (5) Perhaps Palm's price was higher because it was more liquid than 3com. Was Palm more or less liquid than 3Com?

(b) (5) "Money as stock" presented the following table. The coefficient is about 1.0. So what's the point of the table?

	b	R^2	$\sigma(y)$	$\sigma(\varepsilon)$
One day returns:				
$\text{Palm}_t = a + b3\text{Com}_t + \varepsilon_t$	0.96	0.60	7.2	4.5
$(\text{Palm}_t - \beta\text{Nasdaq}_t) = a + b(3\text{Com}_t - \beta\text{Nasdaq}_t) + \varepsilon_t$	0.93	0.53	6.9	4.6
Five day returns:				
$\text{Palm}_t = a + b3\text{Com}_t + \varepsilon_t$	1.03	0.69	15.0	8.3
$(\text{Palm}_t - \beta\text{Nasdaq}_t) = a + b(3\text{Com}_t - \beta\text{Nasdaq}_t) + \varepsilon_t$	0.95	0.54	13.4	10.0

Table 1. Regressions of Palm returns on 3Com returns. σ units are daily percent returns. Sample: 03 March 2000 - 27 July 2000.

11. (10) It turns out that signed orderflow – volume signed by who initiated the trade – is strongly correlated with price changes. Prices do rise on "buy" volume.

   (a) (5) What's the difference between the "price pressure" and "price discovery" views of this correlation?

   (b) (5) What is Brandt and Kavajecz's most important piece of evidence for the "price discovery" view?

12. (20)

(a) (5) There are two managers, with $\alpha = +1\%$ and -0.5% respectively. Each has $\sigma(\varepsilon) = 10\%$ tracking error. Finish constructing the example so that, using the standard power utility portfolio theory, you actually invest a positive amount in the -0.5% alpha manager. Hint:

$$\begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix}^{-1} = \frac{1}{1-\rho^2} \begin{bmatrix} 1 & -\rho \\ -\rho & 1 \end{bmatrix}$$

(b) (5) Suppose Fama and French are right, and value represents a state variable for recession risk; people are anxious to get rid of such stocks, driving down their prices and up their expected returns. This means the market is efficient though, so everyone should just hold the market portfolio anyway. Right?

(c) (5) Long term bonds have zero (close enough to it for this problem) average excess return but 10% volatility, and (suppose) that bond returns are uncorrelated with other asset returns (this isn't part a again). Thus, by $w = \frac{1}{\gamma}\frac{E(R^e)}{\sigma^2(R^e)}$ logic, you shouldn't hold long-term bonds. Right?

(d) (5) Even if they have zero beta, zero alpha on all known factors, industry portfolios are very important in portfolio theory. People should short the industry portfolio that is correlated with their human capital, their business, or their home prices (local businesses) so that their portfolios help insure human capital, business, or real estate risk. Where do we put this insight in to our portfolio equation?

Hints for 12:

$$w = \frac{1}{\gamma}\Sigma^{-1}E(R^e) + \beta_{R,y'}\frac{\eta}{\gamma}$$

$$R^i = R^f + \frac{\gamma^m}{\gamma^i}R^{em} + \frac{1}{\gamma^i}\left(\eta^{i\prime} - \eta^{m\prime}\right)R^{ez};\ R^{ez} \equiv \beta_{y,R'}R^e$$

$$E(R^e) = cov(R^e, R^m)\gamma^m - cov(R^e, y')\eta^m$$

Formulas

Prediction and present value

$$\text{If } x_t = \phi x_{t-1} + \varepsilon_t \;\text{ then } E_t(x_{t+j}) = \phi^j x_t$$

$$r_{t+1} \approx -\rho dp_{t+1} + \Delta d_{t+1} + dp_t;\; dp_t \equiv d_t - p_t$$
$$p_t - d_t \approx E_t \sum_{j=1}^{\infty} \rho^{j-1} \Delta d_{t+j} - E_t \sum_{j=1}^{\infty} \rho^{j-1} r_{t+j}; \quad \rho = \frac{1}{1+D/P} \approx 0.96$$

VAR

$$\begin{aligned} r_{t+1} &= b_r \times dp_t + \varepsilon^r_{t+1};\; b_r \approx 0.1 \\ \Delta d_{t+1} &= b_d \times dp_t + \varepsilon^d_{t+1}; b_d \approx 0 \\ dp_{t+1} &= b_{dp} \times dp_t + \varepsilon^{dp}_{t+1}; b_{dp} \approx 0.94 \end{aligned}$$

$$\sum_{j=0}^{\infty} z^j = \frac{1}{1-z} \text{ if } \|z\| < 1$$

Discount factors, consumption and models

$$p_t = E_t(m_{t+1}x_{t+1}) = E_t\left(\beta \frac{u'(c_{t+1})}{u'(c_t)} x_{t+1}\right)$$

$$m_{t+1} = \beta \left(\frac{c_{t+1}}{c_t}\right)^{-\gamma} \approx 1 - \delta - \gamma \Delta c_{t+1}$$
$$0 = E_t(m_{t+1}R^e_{t+1});\; 1 = E_t(m_{t+1}R_{t+1})$$
$$R^f = 1/E(m_{t+1}) = 1/E_t\left(\beta \frac{u'(c_{t+1})}{u'(c_t)}\right) \approx 1 + \delta + \gamma E_t(\Delta c_{t+1})$$
$$E(R^e_{t+1}) = -R^f_t cov(m_{t+1}, R^e_{t+1}) \approx \gamma cov(\Delta c_{t+1}, R^e_{t+1})$$

Empirical methods

GRS test:

$$\begin{aligned} &T\left[1 + E(f)'\Sigma_f^{-1}E(f)\right]^{-1} \hat{\alpha}'\Sigma^{-1}\hat{\alpha} \,\tilde{}\, \chi^2_N \\ &\frac{T-N-K}{N}\left[1 + E(f)'\hat{\Sigma}_f^{-1}E(f)\right]^{-1} \hat{\alpha}'\hat{\Sigma}^{-1}\hat{\alpha} \,\tilde{}\, F_{N,T-N-K} \end{aligned}$$

Term structure

$$\begin{aligned} p^{(n)}_t &= \text{ log price at } t \text{ of bond that comes due at } t+n, \text{ e.g. -0.20} \\ y^{(n)}_t &\equiv -\frac{1}{n}p^{(n)}_t; \quad f^{(n)}_t \equiv p^{(n-1)}_t - p^{(n)}_t; \\ r^{(n)}_{t+1} &\equiv p^{(n-1)}_{t+1} - p^{(n)}_t; rx^{(n)}_{t+1} = r^{(n)}_{t+1} - y^{(1)}_t \\ y^{(n)}_t &= \frac{1}{n}\left(y^{(1)}_t + f^{(2)}_t + f^{(3)}_t + .. + f^{(n)}_t\right) \end{aligned}$$

Expectations:

$$\begin{aligned}
y_0^{(n)} &= \frac{1}{n}E\left(y_t^{(1)} + y_{t+1}^{(1)} + y_{t+2}^{(1)} + ...y_{t+n-1}^{(1)}\right) + \text{(risk premium)} \\
f_t^{(n)} &= E_t(y_{t+n-1}^{(1)}) + \text{(risk premium)} \\
E_t\left[r_{t+1}^{(n)}\right] &= y_t^{(1)} + \text{(risk premium)} \\
r_t^{US} &= r_t^{Euro} + s_t - E_t s_{t+1} + \text{ (risk premium)}
\end{aligned}$$

Fama-Bliss regression

$$\begin{aligned}
rx_{t+1}^{(n)} &= r_{t+1}^{(n)} - y_t^{(1)} = a + b\left(f_t^{(n)} - y_t^{(1)}\right) + \varepsilon_{t+1} \\
y_{t+n-1}^{(1)} - y_t^{(1)} &= a + b\left(f_t^{(n)} - y_t^{(1)}\right) + \varepsilon_{t+1}
\end{aligned}$$

Cochrane-Piazzesi regression

$$\begin{aligned}
\overline{rx}_{t+1} &= \frac{1}{4}\sum_{n=2}^{5} rx_{t+1}^{(n)} = \gamma' f_t + \varepsilon_{t+1} \\
rx_{t+1}^{(n)} &= b_n\left(\gamma\prime f_t\right) + \varepsilon_{t+1}^{(n)}
\end{aligned}$$

Portfolios

Quadratic utility, independent returns, or mean-variance objective,

$$w_0 = \frac{1}{\gamma}\Sigma^{-1}E(R^e);\ \Sigma = cov(R^e)$$

With a factor model

$$\begin{aligned}
R_{t+1}^p &= R^f + w_m R_{t+1}^{em} + w_\alpha'(\alpha + \varepsilon). \\
w_m &= \frac{1}{\gamma}\frac{E(R^{em})}{\sigma^2(R^{em})};\ w_\alpha = \frac{1}{\gamma}\Sigma^{-1}\alpha;\quad \Sigma \equiv E(\varepsilon_{t+1}\varepsilon_{t+1}')
\end{aligned}$$

Multifactor, y = state variable, and relative to the market if everyone is like this

$$w = \frac{1}{\gamma}\Sigma^{-1}E(R^e) + \beta_{R,y'}\frac{\eta}{\gamma}$$

$$R^i = R^f + \frac{\gamma^m}{\gamma^i}R^{em} + \frac{1}{\gamma^i}\left(\eta^{i\prime} - \eta^{m\prime}\right)R^{ez};\ R^{ez} \equiv \beta_{y,R'}R^e$$

$$E(R^e) = cov(R^e, R^m)\gamma^m - cov(R^e, y')\eta^m$$

Bayesian portfolios

$$\begin{aligned}
f(R) &= \int f(R|\mu)f(\mu)d\mu. \\
&\quad R\tilde{\ }N(\mu,\sigma^2),\ \mu\tilde{\ }N(\bar{\mu},\sigma_\mu^2) \text{ then } f(R)\tilde{\ }N(\bar{\mu},\sigma^2+\sigma_\mu^2) \\
\text{i.e. predictive variance} &= \text{return variance + uncertainty about the mean}
\end{aligned}$$

2013 Final Exam Answers

1.

(a) Plugging the VAR into the identity, the terms multiplying dp_t must equate so

$$\begin{aligned} b_r dp_t + \varepsilon^r_{t+1} &= -\rho\left(\phi dp_t + \varepsilon^{dp}_{t+1}\right) + dp_t + \left(b_d dp_t + \varepsilon^d_{t+1}\right) \\ b_r &= 1 - \rho\phi + b_d \end{aligned}$$

that means $b_r = 1 - \rho\phi$.

(b) The shock terms must also equate,

$$\varepsilon^r_{t+1} = -\rho\varepsilon^{dp}_{t+1} + \varepsilon^d_{t+1}$$

so multiplying be ε^{dp} and taking expectation,

$$cov(\varepsilon^r_{t+1}, \varepsilon^{dp}_{t+1}) = -\rho\sigma^2\left(\varepsilon^{dp}_{t+1}\right) + cov(\varepsilon^{dp}_{t+1}, \varepsilon^d_{t+1}) = -\rho\sigma^2\left(\varepsilon^{dp}_{t+1}\right)$$

(c)

$$\begin{aligned} \sigma^2\left(r_{t+1} + r_{t+2}\right) &= 2\sigma^2(r) + 2cov(r_{t+1}, r_{t+2}) \\ E\left(r_{t+1} + r_{t+2}\right) &= 2E(r) \end{aligned}$$

$$w = \frac{1}{\gamma}\frac{E\left(r_{t+1} + r_{t+2}\right)}{\sigma^2\left(r_{t+1} + r_{t+2}\right)} = \frac{1}{\gamma}\frac{E(r)}{\sigma^2(r) + cov(r_{t+1}, r_{t+2})} > \text{ or } < \frac{1}{\gamma}\frac{E(r)}{\sigma^2(r)}. \text{ as } cov < 0 \text{ or } > 0.$$

(d) Using the VAR,

i.

$$\begin{aligned} r_{t+1} &= b_r dp_t + \varepsilon^r_{t+1} \\ r_{t+2} &= b_r\phi dp_t + b_r\varepsilon^{dp}_{t+1} + \varepsilon^r_{t+2} \end{aligned}$$

so

$$cov(r_{t+1}, r_{t+2}) = cov\left(b_r dp_t + \varepsilon^r_{t+1}, b_r\left(\phi dp_t + \varepsilon^{dp}_{t+1}\right) + \varepsilon^r_{t+2}\right)$$

ii. we turn off the ε^r_{t+1},

$$\begin{aligned} cov(r_{t+1}, r_{t+2}) &= cov\left(b_r dp_t, b_r\left(\phi dp_t + \varepsilon^{dp}_{t+1}\right)\right) \\ &= b_r^2\phi\sigma^2(dp_t) > 0 \end{aligned}$$

Intuition. dp is very slow moving. If $\varepsilon^r_{t+1} = 0$, returns have the same slow-moving and highly autocorrelated process as dividend yields themselves. This answer is good enough. You can also go one step further and write

$$= b_r^2\frac{\phi}{1-\phi^2}\sigma^2(\varepsilon^{dp}) > 0$$

or even

$$= (1-\rho\phi)^2\,\frac{\phi}{1-\phi^2}\sigma^2(\varepsilon^{dp}) > 0$$

iii. If dp_t is a number, then

$$\begin{aligned} cov_t(r_{t+1}, r_{t+2}) &= cov\left(\varepsilon^r_{t+1}, b_r\varepsilon^{dp}_{t+1} + \varepsilon^r_{t+2}\right) \\ &= cov\left(\varepsilon^r_{t+1}, b_r\varepsilon^{dp}_{t+1}\right) \\ &= -b_r\rho\sigma^2(\varepsilon^{dp}_{t+1}) < 0 \end{aligned}$$

Here is where stocks are like bonds. A positive shock to dp is a negative shock to returns which is a positive shock to expected returns.

(e) Now the whole thing.

$$\begin{aligned}
cov(r_{t+1}, r_{t+2}) &= cov\left[b_r dp_t + \varepsilon^r_{t+1}, b_r\left(\phi dp_t + \varepsilon^{dp}_{t+1}\right) + \varepsilon^r_{t+2}\right] \\
&= b_r^2 \phi \sigma^2(dp_t) + b_r cov(\varepsilon^{dp}_{t+1}, \varepsilon^r_{t+1}) \\
&= b_r^2 \phi \sigma^2(dp_t) - b_r \rho \sigma^2(\varepsilon^{dp}) \\
&= b_r\left(\frac{1-\rho\phi}{1-\phi^2}\phi - \rho\right)\sigma^2(\varepsilon^{dp})
\end{aligned}$$

with $\rho = \phi$ we have $cov(r_{t+1}, r_{t+2}) = 0$! *Predictability does not affect the safety of stocks in the long run!* This is an initially counterintuitive result. We see here the two countervaling effects. You probably thought stocks safer in the long run because you thought of the bond-like effect, a negative shock to prices is a positive shock to returns. But this leaves out the smooth dp effect: dp is a very smooth variable, giving a very slow moving component to returns, which induces positive serial correlation. The two effects exactly offset!

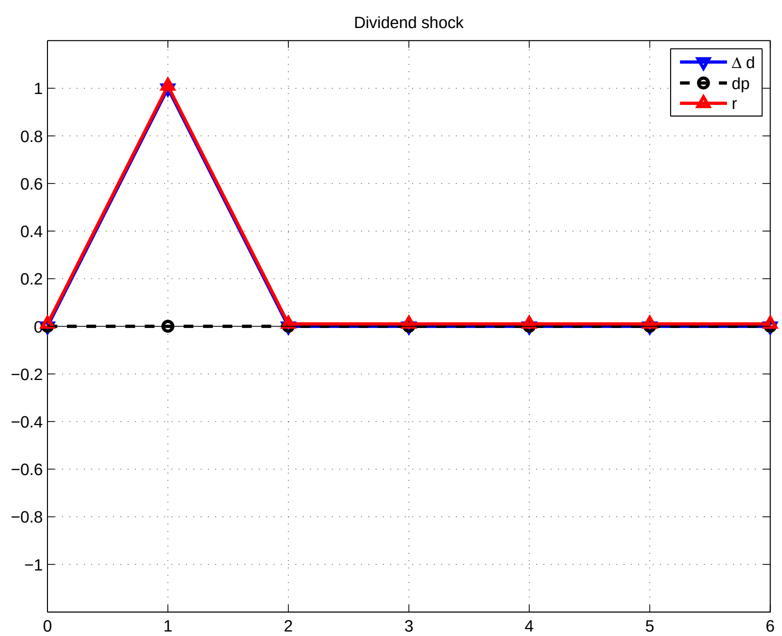
2. The key here is that you must use the identity to find the return shock,

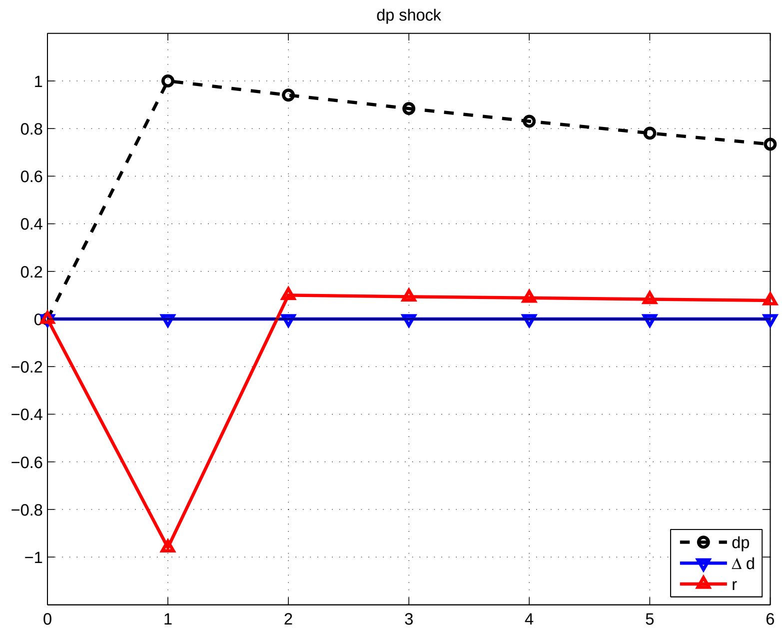
$$\varepsilon^r_{t+1} = -\rho\varepsilon^{dp}_{t+1} + \varepsilon^d_{t+1}$$

in the first case $\varepsilon^r = 1$. In the second case $\varepsilon^r = -\rho = -0.96$ Then, just substituting the VAR you have

ε^d :	0	1	2	3	4
dp_t	0	0	0	0	0
r_t	0	1	0	0	0
Δd_t	0	1	0	0	0

	0	1	2	3	4
dp_t	1	0.94	0.88	0.83	0.78
r_t	-0.96	0.1	0.094	0.088	0.083
Δd_t	0	0	0	0	0





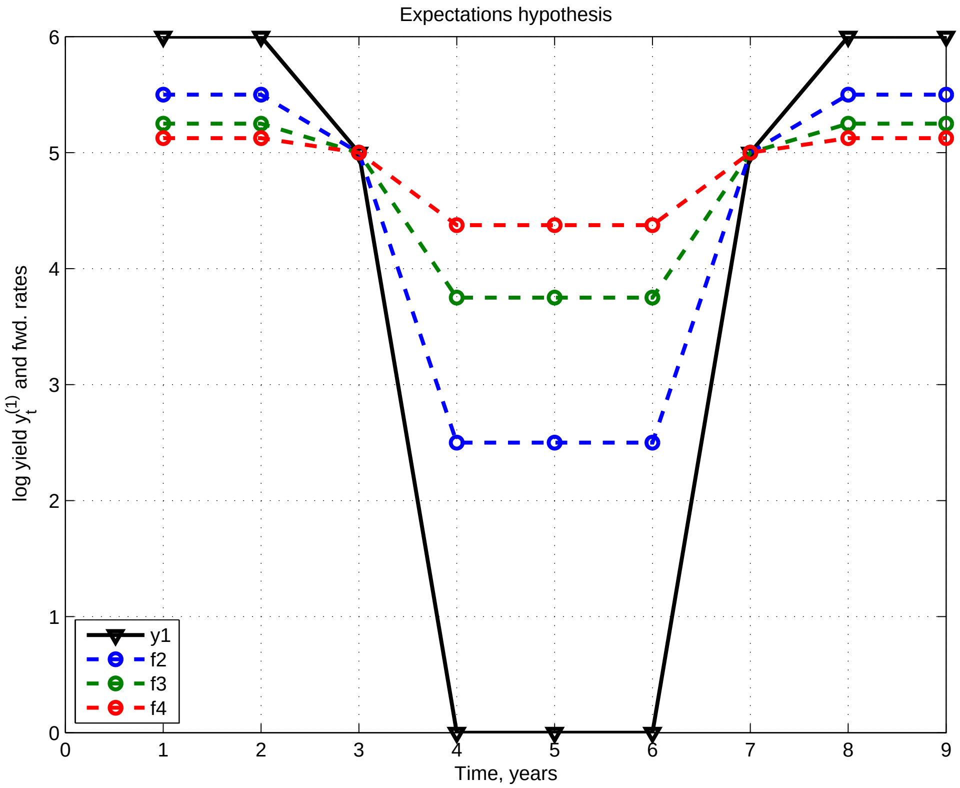
3.

(a)

$$
\begin{aligned}
f_t^{(2)} &= E_t y_{t+1}^{(1)} = \delta + \rho(y_t^{(1)} - \delta) \\
f_t^{(3)} &= E_t y_{t+2}^{(1)} = \delta + \rho^2(y_t^{(1)} - \delta) \\
f_t^{(4)} &= E_t y_{t+3}^{(1)} = \delta + \rho^3(y_t^{(1)} - \delta) \\
f_t^{(n)} &= E_t y_{t+n-1}^{(1)} = \delta + \rho^{(n-1)} \left(y_t^{(1)} - \delta \right)
\end{aligned}
$$

(b) $\rho^2 = 0.25$, $\rho^3 = 0.125$ so

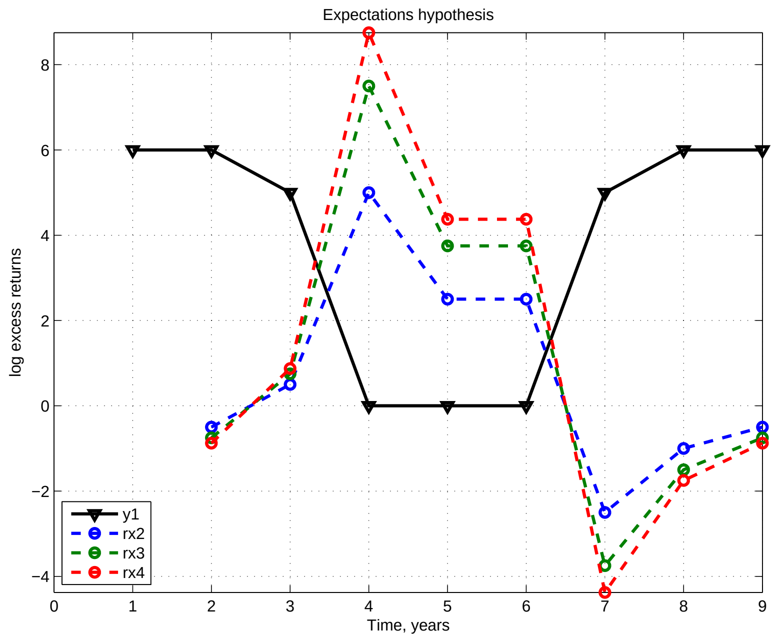
$y_t^{(1)}$	6%	5%	0%
$f_t^{(2)}$	5.5	5	2.5
$f_t^{(3)}$	5.25	5	3.75
$f_t^{(4)}$	5.125	5	4.375



(c) If rates follow the assumed AR(1), then events like the graphed one should be very rare. We are always expecting yields to bounce back up, but it takes them forever. You don't see that *time series* forecast error in this plot. Put another way, the $y_t^{(1)}$ process graphed is much more persistent than an AR(1) with $\rho = 0.5$. People are generating bond prices as if there is a quickly mean reverting AR(1), but the actual process doesn't revert so fast, so you make money. You could assume $\rho = 1$ to generate the slow mean reversion, but then the forward rates would not be upward sloping. At $\rho = 1$, with the expectations hypothesis, all the forward rates collapse to the spot rate. So, *to make a graph that looks like the forward rate data I have to assume people expect interest rates to revert back a lot faster than interest rates actually do revert back.*

As another way to see the point, (far beyond what I expect on an exam) here is a plot of excess returns $rx_t^{(n)}$ through the episode, and the mean excess returns in the episode are as given in the table. The investor makes money through the episode. The market is "expecting" yields to rise, but it doesn't happen fast enough.

$E(rx^{(2)})$	$E(rx^{(3)})$	$E(rx^{(4)})$
0.75%	1.12%	1.31%



4.

(a)

n	1	2	3	4	5
$y_t^{(n)}$	0	1	2	3	4
$f_t^{(n)}$	0	2	4	6	8
$p_t^{(n)}$	0	-2	-6	-12	-20
$E_t(y_{t+n-1}^{(1)})$	0	0.4	2	4.8	7.2

(b) $y_t^{(1)}$ is always zero, so $f_t^{(n)} - y_t^{(1)} = f_t^{(n)}$. Numbers, $0.20 \times 2 = 0.40.$; $0.50 \times 4 = 2$; $0.80 \times 6 = 4.8$; $0.90 \times 8 = 7.2$

5.

(a)

$$R_t^{ei} = \alpha_i + b_i rmrf_t + h_i hml_t + s_i smb_t + \varepsilon_t^i \quad T = 1..T \text{ for each } i$$

R^{ei} = excess returns on 25 size and b/m sorted portfolios, $rmrf_t$ = market excess return hml_t = long value short growth factor return $smb_t =$ long small short big factor return.

(b)

i. The large R^2 is the most important statistic to say this is a good model of "returns" i.e. a factor model. The size and pattern of the b, h, s along with their t statistics are good confirming evidence. The α don't matter to this point

ii. The economically small alphas (mostly) are the most important statistics. The fact that b, h, s vary in the same direction as $E(R^e)$ is good confirming evidence. The R^2 is irrelevant to this point.

(c) The large R^2 means Σ is small, so $\hat{\alpha}'\hat{\Sigma}^{-1}\hat{\alpha}$ can be big even with small α.

(d)

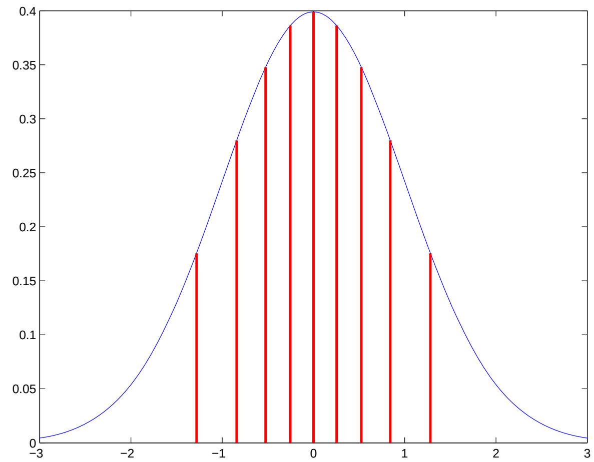
i. Dropping smb would lower R^2 and hence worsen the model of "returns." It would also lower t statistics.

ii. Dropping smb would have precisely no effect on the FF alphas, and hence its model of expected returns.

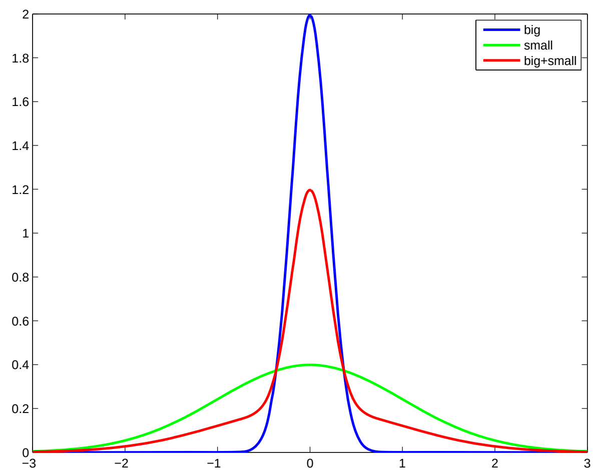
6.

(a) Fama and french's returns are net of a matching portfolio, with the same book/market and size. Thus, this represents a multiple regression, the effect of NS independent of value

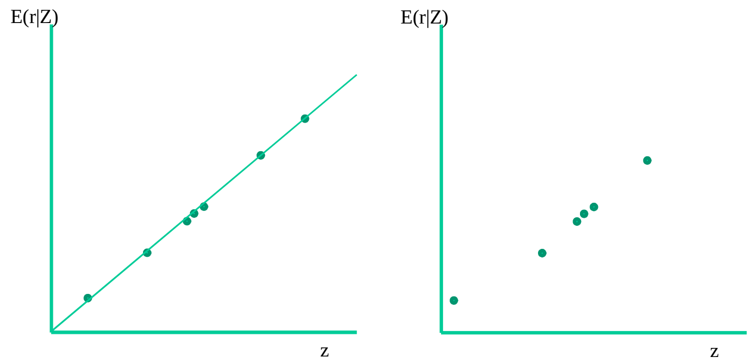
(b) No, S shaped patterns are perfectly normal. There are three points to make here. First, when a variable is spread out, portfolios with even numbers of firms always have more extreme values in the tails. Thus, we expect S shapes in the anomaly variables – middle of the x axis buckets here.



Second, when we mix small and large firms, we are in effect drawing from two different distributions. The tails are going to be fatter, and represent the smallest firms. This fat tailed distribution makes the extremes even more extreme. The extremes are the smallest and most volatile firms.



However, this phenomenon should only lead to extremes in the tails of both anomaly variable and returns, if returns are really linear in the anomaly variable (left). How can the z variable be more spread out than the returns? Well, nobody said the function had to be linear. It's fine if the function is S shaped. We're only describing returns here. Theory says expected returns must be linear in betas, but not in anomaly variables.



7.

(a) Carhart did sort on 5 year averages, and found weaker results – almost no expected return spread based on 5 year return averages

(b) If his one-year return continuation was really skill, then average returns should be higher in skilled portfolios for much longer times, as long as skill lasts. Let's hope that's more than a year.

8.

(a) The key assumption under simulated is that no funds have any true alpha, positive or negative.

(b) 1.30 means that if all funds really have exactly zero alpha, then we expect to see that 10% of the funds in a sample will have an alpha t statistic greater than 1.30 just due to chance. In fact, 7% had a t stat greater than 1.30. Thus there are actually 3% too few funds with alpha greater than 1.30 than there should be.

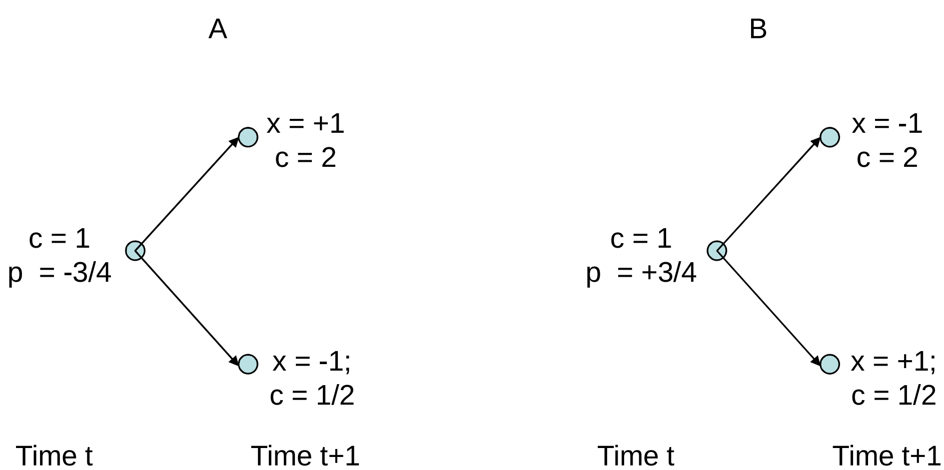
(c) 5% of funds should have performance below -1.71. In fact, 5% of funds have performance below -2.84. This is a bit puzzling – why have negative alpha when you can just buy the index? But we have not chalked up all the costs here.

(d) Berk says we should measure skill by i) gross alpha (before fees) ii) alpha times assets under management – gross fees really, but with the assumption that alpha to investors is zero. iii) he wants tradeable benchmarks, available at the time. No cost-free hml factors in 1967.

9. For any payoff $x = \{x_u, x_d\}$,

$$p = E(mx) = \pi_u \frac{1}{c_u} x_u + \pi_d \frac{1}{c_d} x_d$$

$$p = E(mx) = \frac{1}{2}\frac{1}{2} x_u + \frac{1}{2}\frac{2}{1} x_d = \frac{1}{4} x_u + x_d$$



(a) For the bond, $x_u = x_d = 1$

$$p = E(mx) = \frac{1}{4}1 + 1 = 1.25$$

A bond price can exceed one meaning a negative real interest rate. 1/2 is such a terrible outcome that the consumer would really like to save to prevent it.

(b)

$$p = \frac{1}{4}1 + (-1) = -0.75$$

The price is negative. Well, losing a dollar in the state of the world that consumption goes down by half is a terrible idea, and you would pay not to take that bet.

(c)

$$p = \frac{1}{4}(-1) + 1 = 0.75$$

The situation is exactly reversed

(d) The mean $E(x) = 0$ is the same and the variance is the same. They differ by *in which state of nature* you take losses. That matters *This is important. Risk is not standard deviation, its covariance with consumption.*

(e)

$$\begin{aligned} p &= \frac{1}{4}1 + 0 = \frac{1}{4} \\ p &= \frac{1}{4}0 + 1 = 1 \end{aligned}$$

The claim that pays in the bad state is much more valuable. You're hungrier in the bad state and willing to pay more

(f) Part b is +1 contingent claim to the good state and -1 contingent claim to the bad state. The value of this arbitrage portfolio is $1/4 - 1 = -3/4$, the same value.

10.

(a) Palm had more turnover, a usual sign of 'liquidity." But it had a higher bid/ask spread. More quantity at higher price...we diagnosed this in class as a large *demand* for trading *despite* illiquidity, not a movement on the demand curve for trading induced by a big supply of liquidity. The facts:

TECH STOCK CARVE-OUTS 261

TABLE 8
VOLUME, LIQUIDITY, AND INSTITUTIONAL OWNERSHIP

	Turnover		Bid/Ask Spread		Institutional Ownership	
	Parent (1)	Subsidiary (2)	Parent (3)	Subsidiary (4)	Parent (5)	Subsidiary (6)
Creative/UBID	23.98	106.47	.69	.93	17.71	10.38
HNC/Retek	3.68	22.19	.32	.26	96.38	72.28
Daisytek/PFSWeb	2.42	25.53	.62	.81	71.88	69.95
Metamor/Xpedior	2.13	11.79	.42	.49	53.06	35.96
3Com/Palm	4.54	19.18	.09	.14	52.22	46.01
Methode/Stratos	2.63	41.67	.42	.20	69.47	36.63
Average	6.56	37.80	.43	.47	60.12	45.20
Difference, parent vs. subsidiary	31.24		.04		−14.92	
t-statistic	2.83		.62		−3.06	

NOTE.—Turnover is daily volume as a percentage of parent shares outstanding or subsidiary shares trading. Subsidiary shares trading are shares sold to the public in the IPO. Volume is average daily volume from the first 20 trading days after the IPO date (not including the first day of trading). The shares outstanding of the parent are taken from CRSP, and the shares issued in the IPO are taken from company SEC filings. Bid/ask spread is the average percentage of prices from the first 20 trading days after the IPO date (not including the first day of trading). Institutional ownership, from 13F filings to the SEC (via Securities Data Corp.), pertains to the first quarterly filing after the IPO. Institutional ownership refers to a percentage of parent shares outstanding or subsidiary shares trading.

(b) The R^2 is surprisingly low, and the tracking error huge – $\sigma(\varepsilon)$ is half the size of $\sigma(y)$! This means you can't trade on palm news by buying 3 com. Palm is "special" for information trading, a key requirement for the monetary theory.

(a) Causality. Price pressure says that selling volume pushes prices down. Price discovery says there is a piece of news, which will depress prices eventually. The informed learn it first, and trade. Then the price goes down.

(b) The change in yield of each bond depends on the 2-5 year on the run order flow in a multiple regression, not its own order flow.

(a)

$$\begin{bmatrix} 1 & -\rho \\ -\rho & 1 \end{bmatrix}^{-1} = \frac{1}{1-\rho^2}\begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix}$$

$$\begin{aligned} w &= \frac{1}{\gamma} \times \frac{1}{0.1^2}\begin{bmatrix} 1 & -\rho \\ -\rho & 1 \end{bmatrix}^{-1}\begin{bmatrix} 1 \\ -0.5 \end{bmatrix} \\ &= \frac{1}{\gamma} \times \frac{1}{1-\rho^2}\begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix}\begin{bmatrix} 1 \\ -0.5 \end{bmatrix} \\ &= \frac{1}{\gamma} \times \frac{1}{1-\rho^2}\begin{bmatrix} 1-0.5\rho \\ \rho-0.5 \end{bmatrix} \end{aligned}$$

we just need the returns to be sufficiently negatively correlated, $\rho > 0.5$

(b) No. People who are not exposed should buy the value stocks.

(c) No. The hedging demand term is strong. Bond expected returns rise when bond prices fall.

Business 35150 **John H. Cochrane**

2012 Final Exam

Name (Print clearly):

Section:

Mailfolder location:

Directions

DO NOT START UNTIL WE TELL YOU TO DO SO. *Read these directions* in the meantime.

Please *do not tear your exam apart*. Answer the questions in the space provided. There are some extra pages at the end if you run out of space (but if you do, it means you're writing too much.)

You can rip off the formula sheet and blank pages at the back for throw-away scrap paper if you wish.

Show your work. An answer that comes out of the blue or is the right answer but coming from the wrong equation will be graded as wrong. Also, by showing your work you may get partial credit.

Keep your answers *short*. We are only looking for the right answer; we will grade off for a memory dump of unrelated stuff as it reveals you don't know what's relevant to the question. Put your answers in a *box* or *underline* to make sure we find them. Make sure you answer each direct question. The questions are not clever or subtle. In each case, we just want to know the one obvious point.

For fact questions, *quote the author and paper*, or *state that the fact comes from a problem set* if such a source is relevant.

This is an *closed-book, closed-note* exam. You may use a calculator, but you do not need one; all the answers come out to simple numbers. You may not use a laptop computer, PDA, ipad, cell phone, etc.

Each question has a suggested time, which is also the number of points it will count in grading. Small times (5 min) require shorter answers. The total time is 2:15 = 135 so you have plenty of time.

Friday section: Do not discuss the contents of this exam with anyone until Sat 12 PM. There are two sections of this class, and any information passed to the other section is not only a serious honor code violation, it lowers your grade directly.

Booth honor code required statement: I pledge my honor that I have not violated the honor code during this examination.

Signature:

1. (5) You see the stock market fall by 10%, $\Delta p_t = -0.10$. Does this fact imply that expected returns rise, fall, or stay the same relative to what you expected before the shock – i.e are prices are expected to "mean revert," continue with "momentum" or stay the same? The answer is "it depends," so explain what else you need to know, and say how much expected returns change in a few cases. Use the VAR we developed in class, see the formula sheet for a reminder.

2. (20) Suppose the regressions in logs had come out instead (ignoring constants) to

$$\begin{aligned} r_{t+1} &= 0.2 \times dp_t + \varepsilon^r_{t+1} \\ dp_{t+1} &= 0.64 \times dp_t + \varepsilon^{dp}_{t+1} \end{aligned}$$

a) What value do you expect for b_d in

$$\Delta d_{t+1} = b_d \times dp_t + \varepsilon^d_{t+1}?$$

Hint: Use the return identity $r_{t+1} \approx -\rho dp_{t+1} + \Delta d_{t+1} + dp_t$ (formula sheet) to connect coefficients. Give approximate answers, i.e. $0.96 \times 0.94 \approx 0.90$ is fine. Is your number the "right" sign – high prices mean higher future dividend growth?

b) What value do you expect for long-run return and dividend growth forecast coefficients,

$$\begin{aligned} \sum_{j=1}^{\infty} \rho^{j-1} r_{t+j} &= a + b_r^{lr} \times dp_t + \varepsilon^{r,lr}_{t+1}. \\ \sum_{j=1}^{\infty} \rho^{j-1} \Delta d_{t+j} &= a + b_d^{lr} \times dp_t + \varepsilon^{d,lr}_{t+1}? \end{aligned}$$

c) Looking at the present value identity (formula sheet) We decided that all variation in price-dividend ratios corresponded to variation in expected returns and none to expected dividend growth. How is that conclusion altered by the fact that returns are even more predictable in this case? (Hint: think about running both sides of the present value identity on dp_t, and multiplying by $var(dp_t)$)

3. (5) Fama and French ("Multifactor explanations", "Dissecting anomalies") show that portfolios of smaller stocks (low market equity) earn higher average expected returns. This fact seems to offer an amazing profit opportunity: We'll form a holding company ("Booth Hathaway"). We'll buy lots of small stocks and earn their high expected returns. We'll issue stock as a single company. Our total market equity will be so large that we'll have to pay only a small expected return to our investors. We can pay ourselves huge salaries off the difference.

How would Fama and French respond? This seems like an awfully "inefficient" conclusion!

4) a(5) Which gets better returns going forward, stocks that had great past sales growth, or stocks of companies whose sales are going down? Are the high expected return stocks riskier, in the sense that they are more affected by market downturns? Cite evidence from a paper you read.

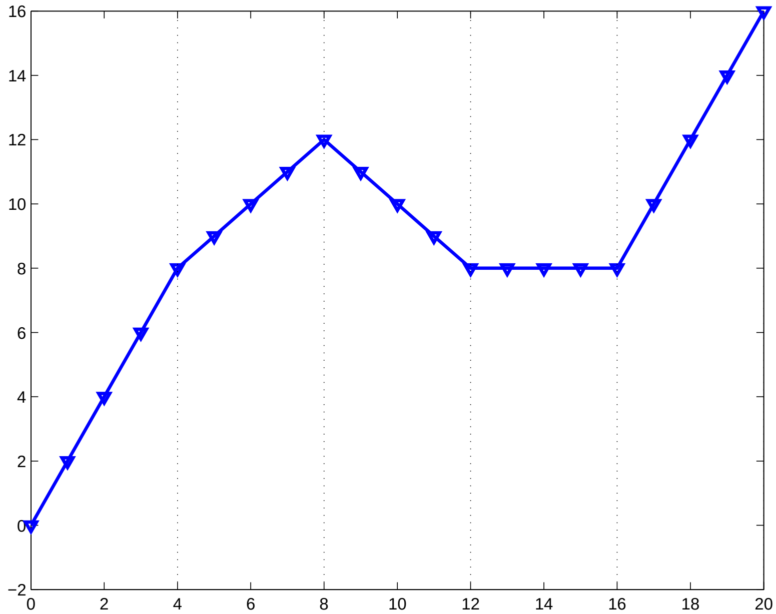
b(5) If we form a momentum portfolio, from stocks that did well last year, are the returns on that portfolio correlated with the returns on value stocks over the next year? I.e. if value stocks go up, do momentum stocks tend to go up, down, or remain the same? Cite evidence from a paper you read.

5. (5) a) Some behavioral researchers claim that managers exploit "bubbles," issuing stock when their stock is "overpriced," and repurchasing when it's "underpriced." As a result, they say that high stock issues should forecast low returns. Leaving aside the explanations, is the *fact* right, or does the sign go the other way (high stock issues forecast high returns)?

b) Whatever the sign, do net stock issues add additional information about returns along with all the other forecasters?

In both cases, be specific, alluding to regression or portfolio evidence.

6. (10)The graph represents consumption over time, in percent (100 x log). Use the consumption-based model to find and plot the interest rate over time, also in percent, assuming people know ahead of time where consumption is going. Use discount rate $\delta = 2\%$, and risk aversion $\gamma = 2$, and approximate as necessary to get round (integer) answers. Hint: Start by making a table of interest rates for consumption growth -1,0,1,and 2%. Make sure you put the interest rate at the right moment in time. t vs. $t+1$ is vital here! What do you learn about how interest rates should move over the business cycle?



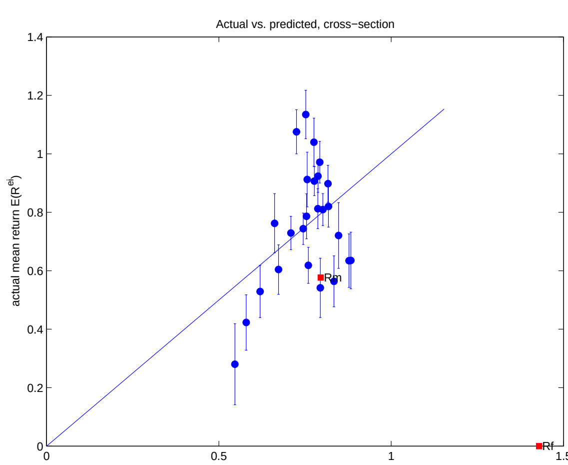
7a) (5) "The CAPM doesn't work. You get much higher returns on small stocks than on big stocks." Is this correct?

7b) (5) A friend brings in the following table of results

$\hat{\gamma}$	$\hat{\lambda}$	$\sigma(\hat{\gamma})$	$\sigma(\hat{\lambda})$	$\sqrt{\frac{1}{25}\sum \alpha_i^2}$
1.38	-0.57	0.40	0.19	0.15

CAPM, 1947-2010, FF 25 size and B/M portfolios. Estimate of $E(R^{ei}) = \gamma + \lambda\beta_i + \alpha_i, i = 1, 2, ..25$ by cross-sectional regression.

You ask for a graph and he produces the following graph of $E(R^{ei})$ (vertical axis) vs. predicted mean return, $\gamma + \lambda\beta_i$ (horizontal axis). Ok, he says, it's not perfect, but it's not a total disaster either.



Did something go wrong here? Can you suggest a better procedure?

8. a) (5) A mutual fund manager complains, "Carhart's results are bogus. He sorted mutual funds by their one-year past returns. Everyone knows that's mostly luck. He should have looked at funds based on 5 year performance averages, like Morningstar does. Then he would have seen some alphas!" How would Carhart respond?

8. b) (5) Berk argues that there can be alpha even if mutual fund returns to investors do not persist over time, and that flows following past returns are not irrational. Which of the following considerations is key to this argument, and explain why. (Focus on the right one, briefly comment on the wrong ones).

a) Managers can only achieve alpha up to a certain scale

b) Managers raise their fees (as a percent of assets under management) if they do well

c) Momentum in underlying stocks explains the appearance of persistent returns

9) (10) Below, find an excerpt from Fama and French's Table Mutual Funds 3

a) What is the key assumption under "simulated?"

b) What does 1.68 mean in "simulated"? What does 2.04 mean? What does the relative position of 1.68 in "Actual" vs. "Simulated" mean?

c) How does this table address the claim, "the only reason you see some funds with really good performance is that they got lucky?"

d) What do -1.71 and -2.19 mean? Is this normal, or a puzzle?

Percentile	Simulated	Actual
5	-1.71	-2.19
50	-0.01	-0.06
90	1.30	1.59
91	1.38	1.68
95	1.68	2.04

Table 3 - Percentiles of t(α) estimates for actual and simulated fund returns...[3-factor adjusted] gross fund returns....

10. (15)

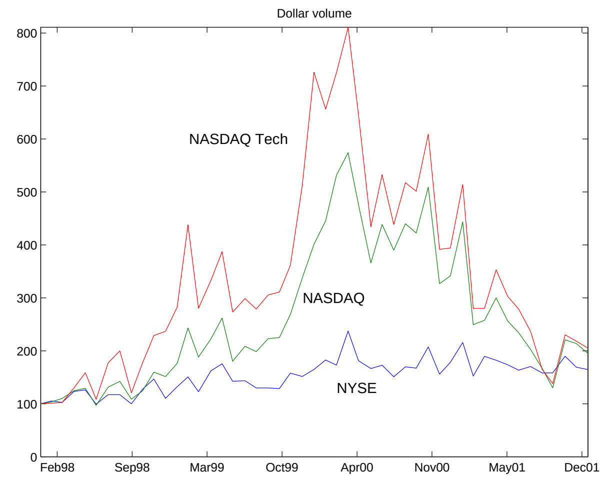
a) On the day that Palm went public, what happened to 3Com's price?

b) Did short sales constraints mean that nobody in fact was able to short Palm stock?

c) Was Palm more or less liquid than 3Com?

d) If you want to buy Palm because you think it will go up next week, why not buy 3Com instead? After all, 3Com owns 95% of Palm so it will go up too. (Be specific about facts.)

e) What implication did Cochrane draw from this graph?



Dollar volume on NYSE, NASDAQ and NASDAQ with SIC code 737. Series are normalized to 100 on Jan 1 1998.

11. (10)

a) A broker-dealer lost money and is running short of cash. Why does it not just issue more equity?

b) Derivatives are exempt from bankruptcy – they get paid first. Why does this make sense? Since the firm typically is running a matched book, with no overall derivative exposure, why does it case a problem in bankruptcy?

c) Why does it hurt the bank if you pull securities of your brokerage account? After all, they're just executing trades for a fee; the securities you own are yours. Missing a few weeks of fee income isn't going to make them bankrupt.

d) Why, according to Gorton and Metrick, did a run at Lehman spark a crisis, but a run at MF Global did not?

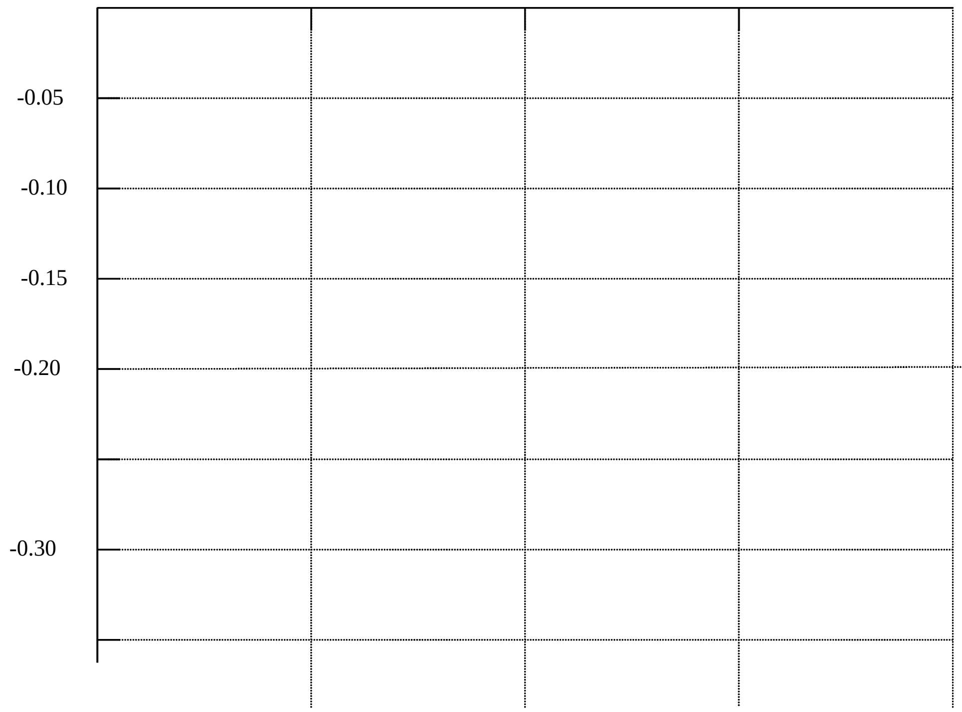
.

12. (15) The price of one, two and three year bonds is $p_0^{(1)} = -0.05$, $p_0^{(2)} = -0.15, p_0^{(3)} = -0.30$

a) Find today's yields and forward rates

b) Plot the expected evolution of these bonds' prices over time, according to the expectations hypothesis.

c) Plot the expected evolution of these bonds' prices for the first year, according to the Fama Bliss regressions, specializing all the coefficients to 1 and 0 as appropriate.


-0.05
-0.10
-0.15
-0.20
-0.30

12. (5) Cochrane and Piazzesi run regressions

$$rx_{t+1}^{(n)} = a_n + \beta_{n,1} y_t^{(1)} + \beta_{n,2} f_t^{(2)} + \beta_{n,3} f_t^{(3)} + \beta_{n,4} f_t^{(4)} + \beta_{n,5} f_t^{(5)} + \varepsilon_{t+1}^{n}$$

They find betas in a tent shape across the right hand variables. What pattern do they find in these betas *across maturity* n? Write an equation that captures this pattern.

13. (15) Suppose the one-year rate is an MA(1),

$$y_t^{(1)} = \varepsilon_t + \varepsilon_{t-1}.$$

$E_t(\varepsilon_{t+1}) = 0; E(\varepsilon_t) = 0$ as usual. Form a term-structure model, by supposing that the expectations hypothesis holds. (You're looking for yields and forward rates of all maturities as a function of two "factors" ε_t and ε_{t-1}.)

a) Find forward rates (at time t, for maturity 2,3,4,...n)

b) Find yields (at time t, for maturity 2,3,4,...n).

c) plot the yield and forward curves on a day in which $\varepsilon_t = 1; \varepsilon_{t-1} = 1$.

(Hint: You may think you got it wrong because the answer is too simple. Don't worry, it really is simple. This problem does NOT involve a lot of algebra.)

14(5) You form an optimal portfolio of the 25 Fama French size and b/m sorted returns. You use the mean-variance formula

$$\text{"optimal"}: w = \frac{1}{\gamma}\Sigma^{-1}E(R^e)$$

Here are the results, in percent. Did something go wrong, and if so what? Explain, using an equation or a graph.

	low	2	3	4	high
small	-149	51	69	96	52
2	-19	-57	190	-13	-60
3	29	-34	-31	-93	41
4	116	-39	-42	35	-2
large	87	-19	8	-22	2

rmrf	hml	smb
-94	77	-69

15. (5) You have risk aversion $\gamma = 1$, and returns are independent over time. Your best guess is that the mean annual premium μ is 4% with volatility σ =20%,

a) What should your allocation to stocks be?

b) In fact you don't really know what the mean return is. Reflecting on it, your uncertainty about the mean return $\sigma(\mu)$ is 10 percentage points, and both the actual return and your uncertainty about it are normally distributed. (Numbers are easy to calculate, not realistic.) How does this consideration change your optimal allocation to stocks?

Business 35150 **John H. Cochrane**

2012 Final Exam Answers

1. (5) You see the stock market fall by 10%, $\Delta p_t = -0.10$. Does this fact imply that expected returns rise, fall, or stay the same relative to what you expected before the shock – i.e are prices are expected to "mean revert," continue with "momentum" or stay the same? The answer is "it depends," so explain what else you need to know, and say

how much expected returns change in a few cases. Use the VAR we developed in class, see the formula sheet for a reminder.

ANSWER: $r_{t+1} = b_r \times (d_t - p_t) + \varepsilon_{t+1}$. It depends on what happened to d_t. If p changes with no change in d, it means expected returns rise by about $0.1 \times 10\% = 1\%$. If d changed 10% as well, then there is no change in expected returns.

2. (20) Suppose the regressions in logs had come out instead (ignoring constants) to

$$\begin{aligned} r_{t+1} &= 0.2 \times dp_t + \varepsilon^r_{t+1} \\ dp_{t+1} &= 0.64 \times dp_t + \varepsilon^{dp}_{t+1} \end{aligned}$$

a) What value do you expect for b_d in

$$\Delta d_{t+1} = b_d \times dp_t + \varepsilon^d_{t+1}?$$

Hint: Use the return identity $r_{t+1} \approx -\rho dp_{t+1} + \Delta d_{t+1} + dp_t$ (formula sheet) to connect coefficients. Give approximate answers, i.e. $0.96 \times 0.94 \approx 0.90$ is fine. Is your number the "right" sign – high prices mean higher future dividend growth?

b) What value do you expect for long-run return and dividend growth forecast coefficients,

$$\begin{aligned} \sum_{j=1}^{\infty} \rho^{j-1} r_{t+j} &= a + b^{lr}_r \times dp_t + \varepsilon^{r,lr}_{t+1}. \\ \sum_{j=1}^{\infty} \rho^{j-1} \Delta d_{t+j} &= a + b^{lr}_d \times dp_t + \varepsilon^{d,lr}_{t+1}? \end{aligned}$$

c) Looking at the present value identity (formula sheet) We decided that all variation in price-dividend ratios corresponded to variation in expected returns and none to expected dividend growth. How is that conclusion altered by the fact that returns are even more predictable in this case? (Hint: think about running both sides of the present value identity on dp_t, and multiplying by $var(dp_t)$)

ANSWER:

a) Regressing both sides of the return identity on dp_t, $b_r = 1 + b_d - \rho b_{dp}$ Hence $b_d = b_r + \rho b_{dp} - 1$. In the old regression $b_d = 0.1 + 0.96 \times 0.94 - 1 = 0$. In the new regression $b_d = 0.2 + 0.96 \times 0.64 - 1 = -0.2$. Negative is the "right" sign.

b) $b^{lr}_r = b_r(1 + \rho b_{dp} + \rho^2 b^2_{dp} + ...) = b_r/(1 - \rho b_{dp}) = 0.2/(1 - 0.96 \times 0.64) = 0.2/0.4 = 1/2$. Similarly, $b^{lr}_d = -1/2$.

c) Running both sides of the present value identity on dp, $1 = b^{lr}_r - b^{lr}_d ; 1 = 1/2 - (-1/2)$. We interpreted the two terms as fractions of var dp explained, so with these numbers the variance of prices comes half from expected returns and half from expected dividend growth. (If you state the formula

$$var\left(p_t - d_t\right) \approx cov\left(p_t - d_t, \sum_{j=1}^{\infty} \rho^{j-1} \Delta d_{t+j}\right) - cov\left(p_t - d_t, \sum_{j=1}^{\infty} \rho^{j-1} r_{t+j}\right)$$

that's even better, but just stating the answer in terms of regression coefficients is enough.

3. (5) Fama and French ("Multifactor explanations", "Dissecting anomalies") show that portfolios of smaller stocks (low market equity) earn higher average expected returns. This fact seems to offer an amazing profit opportunity: We'll form a holding company ("Booth Hathaway"). We'll buy lots of small stocks and earn their high expected returns. We'll fund the purchase by issuing stock as a single company, and our total market equity will be so large

that we'll have to pay only a small expected return to our investors. We can pay ourselves huge salaries off the difference.

How would Fama and French respond? This seems like an awfully "inefficient" conclusion!

ANSWER: The new company would inherit the beta of small stocks, and, since expected returns are really a function of beta, not of market cap, our company would have to pay the expected return of small stocks. (For this answer it really doesn't matter whether market beta is enough, or whether small firm beta gets a special premium. The point is that expected return is really a function of beta, not of size, and size is only coincidentally correlated with beta in the other firms.)

4a(5) Which gets better returns going forward, stocks that had great past sales growth, or stocks of companies whose sales are going down? Are the high expected return stocks riskier, in the sense that they are more affected by market downturns? Cite evidence from a paper you read.

ANSWER: the low sales growth stocks have higher expected returns. This does not correspond to higher market betas. It does correspond to larger hml betas. Fama and French "Multifactor anomalies"

4b)(5) If we form a momentum portfolio, from stocks that did well last year, are the returns on that portfolio correlated with the returns on value stocks over the next year? I.e. if value stocks go up, do momentum stocks tend to go up, down, or remain the same?

ANSWER

High momentum stocks have low h values. In "Multifactor anomalies" So momentum is negatively correlated with value.

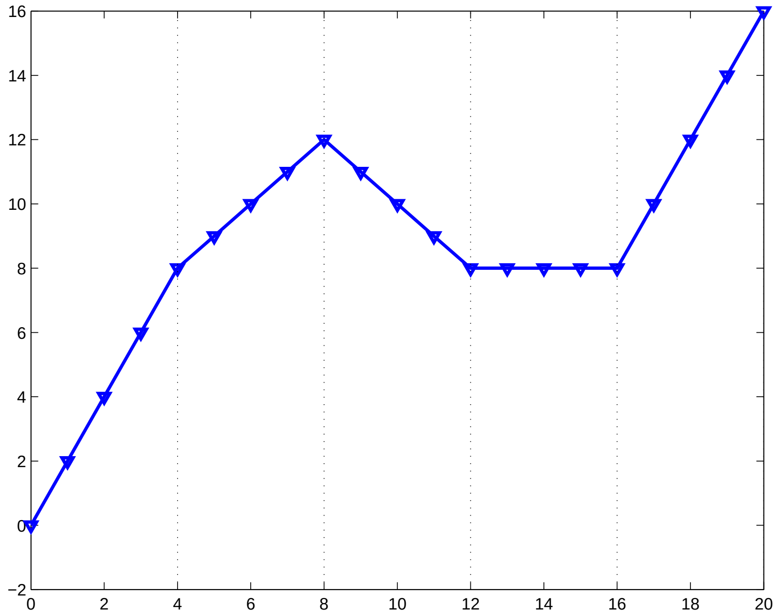
5. (10) 5. (5) a) Some behavioral researchers claim that managers exploit "bubbles," issuing stock when their stock is "overpriced," and repurchasing when it's "underpriced." As a result, they say that high stock issues should forecast low returns. Leaving aside the explanations, is the *fact* right, or does the sign go the other way (high stock issues forecast high returns)?

b) Whatever the sign, do net stock issues add additional information about returns along with all the other forecasters?

In both cases, be specific, alluding to regression or portfolio evidence.

ANSWER FF dissecting anomalies. Yes, net issues do correspond to low returns and vice versa. Portfolios sorted by low stock issuance or repurchase have high subsequent returns and vice versa. Regressions $R_{t+1} = a + bNS_t + \varepsilon_{t+1}$ work. The portfolios are net of matched size and BM stocks; the regressions include size, bm and lots of other variables, so NS is an independent forecaster.

6. (10)The graph represents consumption over time, in percent (100 x log). Use the consumption-based model to find and plot the interest rate over time, also in percent, assuming people know ahead of time where consumption is going. Use discount rate $\delta = 2\%$, and risk aversion $\gamma = 2$, and approximate as necessary to get round (integer) answers. Hint: Start by making a table of interest rates for consumption growth -1,0,1,and 2%. Make sure you put the interest rate at the right moment in time. t vs. $t+1$ is vital here! What do you learn about how interest rates should move over the business cycle?

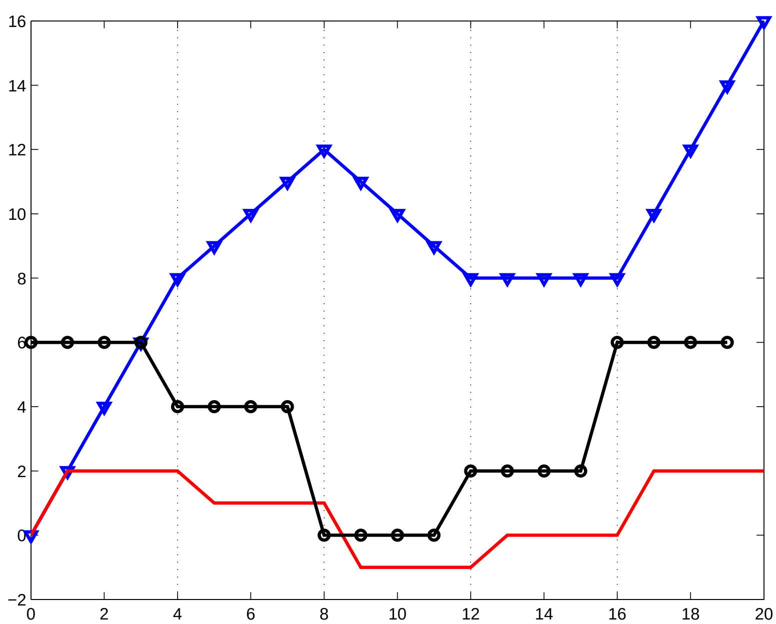


ANSWER This is from a problem set.

$$r_t^f = \delta + \gamma E_t \Delta c_{t+1} = 2 + 2 * E_t \Delta c_{t+1}$$

$E_t \Delta c_{t+1}$	-1	0	1	2
$2 + 2 * E \Delta c$	0	2	4	6

I graphed Δc_{t+1} in red and r_t^f in black. This is the interest rate quoted at time t for loans from t to t+1, and is conventionally dated as of time t. I graphed it that way. That's why the interest rate moves one period before the peaks of the consumption series. There's a bit of a lesson here. See the "recession" in the second part of the plot. Interest rates move pretty much contemporaneously with the *growth rate* of consumption, with only the one-period advance notice. Interest rates move ahead of recessions as defined by the *level* of consumption. Much popular discussion confuses the level and growth views of where we are in economic cycles.



The hard part is the t vs t+1. The interest rate at t reflects consumption growth over the *next* year.

7a) (5) "The CAPM doesn't work. You get much higher returns on small stocks than on big stocks." Is this correct?

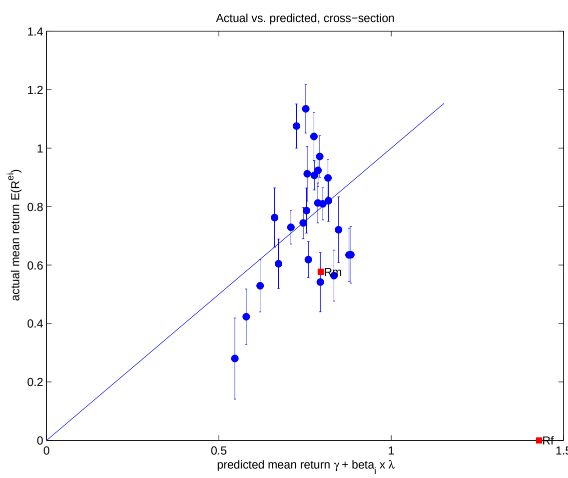
ANSWER: a) Two mistakes: i) higher average returns by themselves don't mean anything, the question is whether they are matched by higher betas. ii) Actually, small stock average returns are matched by higher CAPM betas, as we saw in class

7b) (5) A friend brings in the following table of results

$\hat{\gamma}$	$\hat{\lambda}$	$\sigma(\hat{\gamma})$	$\sigma(\hat{\lambda})$	$\sqrt{\frac{1}{25}\sum \alpha_i^2}$
1.38	-0.57	0.40	0.19	0.15

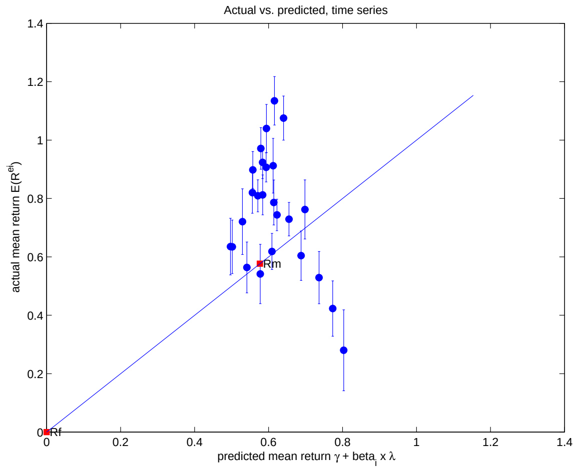
CAPM, 1947-2010, FF 25 size and B/M portfolios. Estimate of $E(R^{ei}) = \gamma + \lambda\beta_i + \alpha_i, i = 1, 2, ..25$ by cross-sectional regression.

You ask for a graph and he produces the following graph of $E(R^{ei})$ (vertical axis) vs. predicted mean return, $\gamma + \lambda\beta_i$ (horizontal axis). Ok, he says, it's not perfect, but it's not a total disaster either.



Did something go wrong here? Can you suggest a better procedure?

ANSWER: This is from a problem set. This is the cross sectional regression with a free constant. Note the constant is huge and the market premium is negative. The actual performance of this model is awful A graph like the following is an ideal answer, average returns vs betas,



A time series regression or including the factor portfolios (including rf) as test assets are ways to fix this.

8. a) (5) A mutual fund manager complains, "Carhart's results are bogus. He sorted mutual funds by their one-year past returns. Everyone knows that's mostly luck. He should have looked at funds based on 5 year performance averages, like Morningstar does. Then he would have seen some alphas!" How would Carhart respond?

ANSWER: Carhart also sorted funds on 5 year formation, and found even less result there than with sorts based on one-year performance.

8. b) (5) Berk argues that there can be alpha even if mutual fund returns to investors do not persist over time, and that flows following past returns are not irrational. Which of the following considerations is key to this argument, and explain why. (Focus on the right one, briefly comment on the wrong ones).

a) Managers can only achieve alpha up to a certain scale

b) Managers raise their fees (as a percent of assets under management) if they do well

c) Momentum in underlying stocks explains the appearance of persistent returns

persistent returns

ANSWER:

a is the right answer. As funds rush in, returns to investors decline. It's important that b does not happen, otherwise we wouldn't need new funds to give more money to the managers. c is irrelevant, that was Carhart's point not Berk and Green's.

9) (10) Below, find an excerpt from Fama and French's Table Mutual Funds 3

a) What is the key assumption under "simulated?"

b) What does 1.68 mean in "simulated"? What does 2.04 mean? What does the relative position of 1.68 in "Actual" vs. "Simulated" mean?

c) How does this table address the claim, "the only reason you see some funds with really good performance is that they got lucky?"

d) What do -1.71 and -2.19 mean? Is this normal, or a puzzle?

Percentile	Simulated	Actual
5	-1.71	-2.19
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90	1.30	1.59
91	1.38	1.68
95	1.68	2.04

Table 3 - Percentiles of t(α) estimates for actual and simulated fund returns...[3-factor adjusted] gross fund returns....

ANSWER

a) The key assumption under simulated is that no funds have any alpha, positive or negative.

b) 1.68 means that if all funds really have exactly zero alpha, then we expect to see that 5% of the funds in a sample will have an alpha t statistic greater than 1.68 just due to chance. In fact, 5% of funds had a t stat greater than 2.04, and 9% had a t stat greater than 1.68. Thus there are 4% too many funds with alpha greater than 1.68 than there should be.

c) Actually, there are more funds with very large alpha than there should be just due to luck. Not many, but a small number. (4%, above)

d) 5% of funds should have performance below -1.71. In fact, 5% of funds have performance below -2.19. This is a bit puzzling – why have negative alpha when you can just buy the index? But maybe they're just on the way out.

10. (15)

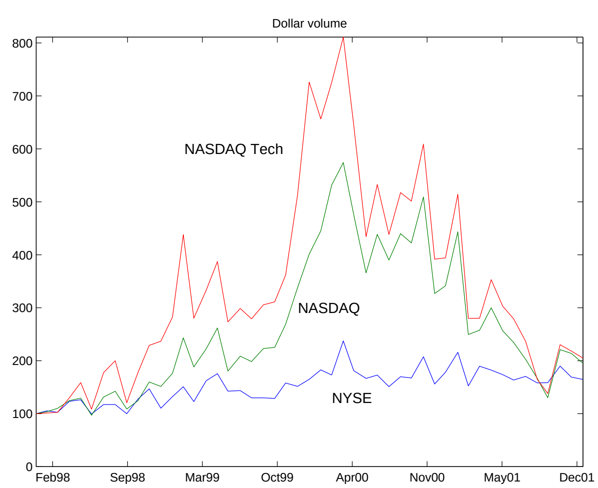
a) On the day that Palm went public, what happened to 3Com's price?

b) Did short sales constraints mean that nobody in fact was able to short Palm stock?

c) Was Palm more or less liquid than 3Com?

d) If you want to buy Palm because you think it will go up next week, why not buy 3Com instead? After all, 3Com owns 95% of Palm so it will go up too. (Be specific about facts.)

e) What implication did Cochrane draw from this graph?



Dollar volume on NYSE, NASDAQ and NASDAQ with SIC code 737. Series are normalized to 100 on Jan 1 1998.

ANSWER

a) Fell, from 95 to 81.

b) No, at the peak palm was 147% shorted

c) fun question. Bid ask was larger, but turnover vastly more. We discussed and decided there was more demand to trade, despite higher costs, so less liquid. Mentioning the fact of high turnover and high bid ask spread is the key answer.

e) The volume here is visually nearly identical to a price graph. "overpricing" comes with massive trading volume.

11. (10) a) A broker-dealer lost money and is running short of cash. Why does it not just issue more equity?

b) Derivatives are exempt from bankruptcy – they get paid first. Why does this make sense? Since the firm typically is running a matched book, with no overall derivative exposure, why does it case a problem in bankruptcy?

c) Why does it hurt the bank if you pull securities of your brokerage account? After all, they're just executing trades for a fee; the securities you own are yours. Missing a few weeks of fee income isn't going to make them bankrupt.

d) Why, according to Gorton and Metrick, did a run at Lehman spark a crisis, but a run at MF Global did not?

ANSWER

a) Debt overhang. Having lost money, the debt is trading below par. New equity first bails out that debt before making profits.

b) It makes sense to keep them from running. However, they get the right to replace their contracts, so the firm pays the bid ask spread on the whole book.

c) They are rehypothecated, and used by the firm as collateral for its own trading.

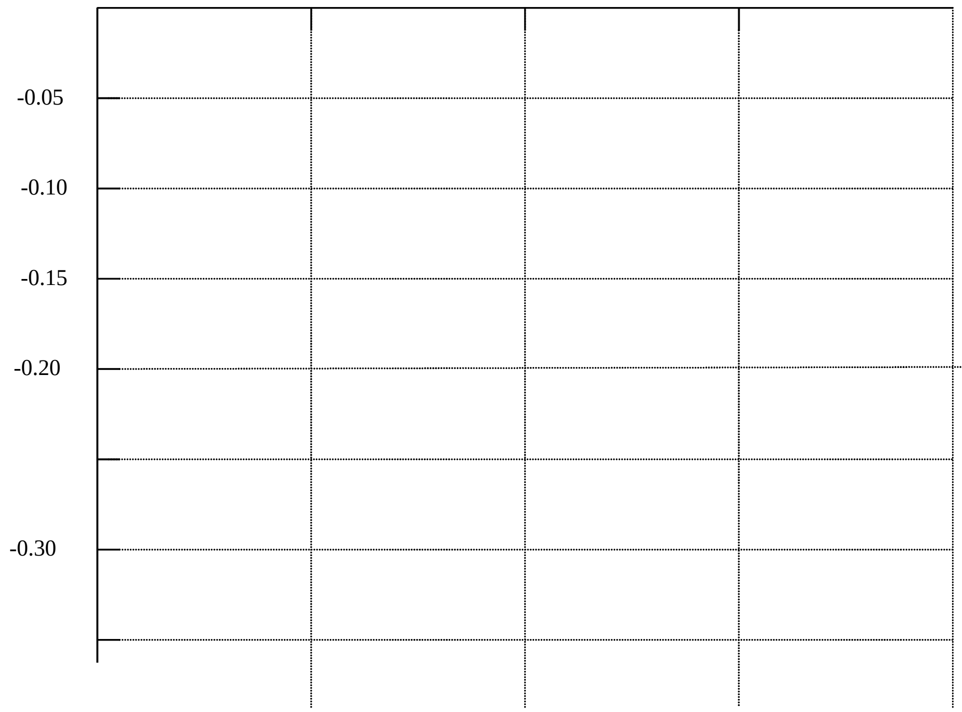
d) Gorton and Metric's big point is that the problem is "systemic runs" when the "system" is insolvent. This happens when losses at one institution spark an e coli outbreak, people become worried about other institutions or assets. MF global was transparently a bet on Greece, and since nobody learned anything about Greece or other investment banks from its failure, it didn't cause any systemic problems. n outbreak you avoid the whole salad bar.

12. (15) The price of one, two and three year bonds is $p_0^{(1)} = -0.05,\ p_0^{(2)} = -0.15, p_0^{(3)} = -0.30$

a) Find today's yields and forward rates

b) Plot the expected evolution of these bonds' prices over time, according to the expectations hypothesis.

c) Plot the expected evolution of these bonds' prices for the first year, according to the Fama Bliss regressions, specializing all the coefficients to 1 and 0 as appropriate.



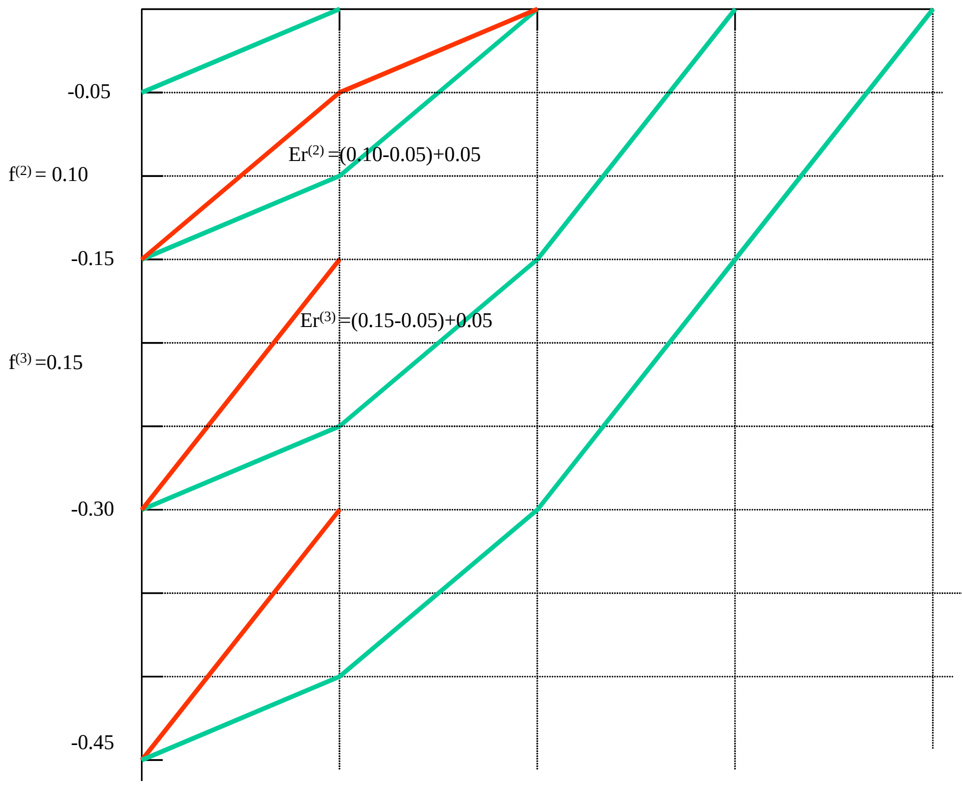
Answer: This includes a 4 year bond that I deleted from the question.

$$
\begin{aligned}
y^{(1)} &= 0.05 \\
y^{(2)} &= 0.075 \\
y^{(3)} &= 0.10
\end{aligned}
$$

$$
\begin{aligned}
f^{(2)} &= 0.10 \\
f^{(3)} &= 0.15
\end{aligned}
$$

I drew expectations just by having each line rise at the same rate as the one year rate that year. (green)

for FB, $rx_{t+1}^{(n)} = 0 + 1(f_t^{(n)} - y_t^{(1)})$ as plotted



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12. (5) Cochrane and Piazzesi run regressions

$$rx_{t+1}^{(n)} = a_n + \beta_{n,1} y_t^{(1)} + \beta_{n,2} f_t^{(2)} + \beta_{n,3} f_t^{(3)} + \beta_{n,4} f_t^{(4)} + \beta_{n,5} f_t^{(5)} + \varepsilon_{t+1}^{n}$$

They find betas in a tent shape across the right hand variables. What pattern do they find in these betas *across maturity* n? Write an equation that captures this pattern.

ANSWER: They found that the betas have the same shape for each maturity, just scaled up. So, they obey

$$rx_{t+1}^{(n)} = a_n + b_n \left(\gamma_1 y_t^{(1)} + \gamma_2 f_t^{(2)} + \gamma_3 f_t^{(3)} + \gamma_4 f_t^{(4)} + \gamma_5 f_t^{(5)} \right) + \varepsilon_{t+1}^{n}$$

13. (15) Suppose the one-year rate is an MA(1),

$$y_t^{(1)} = \varepsilon_t + \varepsilon_{t-1}.$$

$E_t(\varepsilon_{t+1}) = 0; E(\varepsilon_t) = 0$ as usual. Form a term-structure model, by supposing that the expectations hypothesis holds. (You're looking for yields and forward rates of all maturities as a function of two "factors" ε_t and ε_{t-1}.)

a) Find forward rates (at time t, for maturity 2,3,4,...n)

b) Find yields (at time t, for maturity 2,3,4,...n).

c) plot the yield and forward curves on a day in which $\varepsilon_t = 1; \varepsilon_{t-1} = 1$.

(Hint: You may think you got it wrong because the answer is too simple. Don't worry, it really is simple. This problem does NOT involve a lot of algebra.)

ANSWER

$$y_t^{(1)} = \varepsilon_t + \varepsilon_{t-1}$$
$$f_t^{(2)} = E_t y_{t+1}^{(1)} = \varepsilon_t$$
$$f_t^{(3)} = E_t y_{t+2}^{(1)} = 0$$
$$f_t^{(n)} = E_t y_{t+n-1}^{(1)} = 0$$

$$y_t^{(1)} = \varepsilon_t + \varepsilon_{t-1} = 2$$
$$y_t^{(2)} = \tfrac{1}{2}\left(y_t^{(1)} + f_t^{(2)}\right) = \tfrac{1}{2}\left(\varepsilon_t + \varepsilon_{t-1} + \varepsilon_t\right) = \varepsilon_t + \tfrac{1}{2}\varepsilon_{t-1} = 1.5$$
$$y_t^{(3)} = \tfrac{1}{3}\left(y_t^{(1)} + f_t^{(2)} + f_t^{(3)}\right) = \tfrac{1}{3}\left(\varepsilon_t + \varepsilon_{t-1} + \varepsilon_t\right) = \tfrac{2}{3}\varepsilon_t + \tfrac{1}{3}\varepsilon_{t-1} = 1$$
$$y_t^{(n)} = \tfrac{1}{n}\left(y_t^{(1)} + f_t^{(2)} + f_t^{(3)} + .. + f_t^{(n)}\right) = \tfrac{1}{n}\left(\varepsilon_t + \varepsilon_{t-1} + \varepsilon_t\right) = \tfrac{2}{n}\varepsilon_t + \tfrac{1}{n}\varepsilon_{t-1} = \tfrac{3}{n}$$

(Optional: Factors The factors are ε_t and ε_{t-1}. You can find them just by

$$\varepsilon_t = f_t^{(2)}$$
$$\varepsilon_{t-1} = y_t^{(1)} - f_t^{(2)}.$$

You already have the loadings.

$$\begin{bmatrix} y_t^{(1)} \\ y_t^{(2)} \\ y_t^{(3)} \\ y_t^{(n)} \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ \frac{1}{2} \\ \frac{2}{n} \end{bmatrix} \varepsilon_t + \begin{bmatrix} 1 \\ \frac{1}{2} \\ \frac{1}{3} \\ \frac{1}{n} \end{bmatrix} \varepsilon_{t-1}$$

14 (5) You form an optimal portfolio of the 25 Fama French size and b/m sorted returns. You use the mean-variance formula

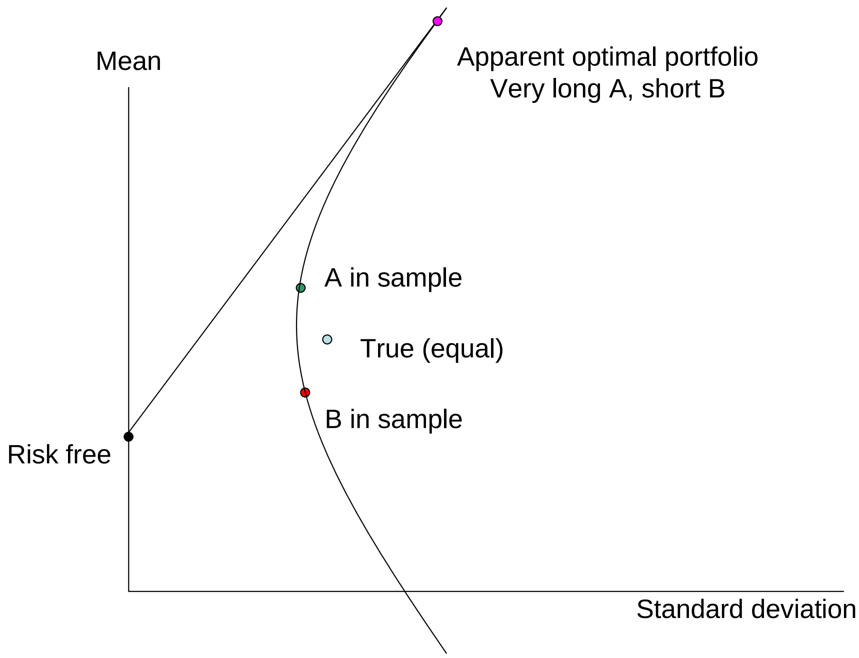
$$\text{“optimal”: } w = \frac{1}{\gamma}\Sigma^{-1}E(R^e)$$

Here are the results, in percent. Did something go wrong, and if so what? Explain, using an equation or a graph.

	low	2	3	4	high
small	-149	51	69	96	52
2	-19	-57	190	-13	-60
3	29	-34	-31	-93	41
4	116	-39	-42	35	-2
large	87	-19	8	-22	2

rmrf	hml	smb
-94	77	-69

ANSWER: This happens typically. What went “wrong” was taking mean returns at face value.



15. (5) You have risk aversion $\gamma = 1$, and returns are independent over time. Your best guess is that the mean annual premium μ is 4% with volatility σ =20%,

a) What should your allocation to stocks be?

b) In fact you don't really know what the mean return is. Reflecting on it, your uncertainty abut the mean return $\sigma(\mu)$ is 10 percentage points, and both the actual return and your uncertainty about it are normally distributed. (Numbers are easy to calculate, not realistic.) How does this consideration change your optimal allocation to stocks?

ANSWER

$$\begin{aligned}\frac{0.04}{0.2^2} &= \frac{0.04}{0.04} = 1 \\ \frac{0.04}{0.2^2 + 0.1^2} &= \frac{0.04}{0.04 + 0.01} = \frac{0.04}{0.05} = 0.8\end{aligned}$$

Verbal answers to the effect that "parameter uncertainty is extra variance and scale back the allocation to stocks" are worth some partial credit.

Business 35150 **John H. Cochrane**

2011 Final Exam

Name (Print clearly):

Section:

Mailfolder location:

Directions

DO NOT START UNTIL WE TELL YOU TO DO SO. *Read these directions* in the meantime.

Please *do not tear your exam apart.* Answer the questions in the space provided. There are some extra pages at the end if you run out of space (but if you do, it means you're writing too much.)

You can rip off the formula sheet and blank pages at the back for throw-away scrap paper if you wish.

Show your work. An answer that comes out of the blue or is the right answer but coming from the wrong equation will be graded as wrong. Also, by showing your work you may get partial credit.

Keep your answers *short.* We are only looking for the right answer; we will grade off for a memory dump of unrelated stuff as it reveals you don't know what's relevant to the question. Put your answers in a *box* or *underline* to make sure we find them. Make sure you answer each direct question. The questions are not clever or subtle. In each case, we just want to know the one obvious point.

For fact questions, *quote the author and paper*, or *state that the fact comes from a problem set* if such a source is relevant.

This is an *closed-book, closed-note* exam. You may use a calculator, but you do not need one; all the answers come out to simple numbers. You may not use a laptop computer, PDA, ipad, cell phone, etc.

Each question has a suggested time, which is also the number of points it will count in grading. Small times (5 min) require shorter answers. The suggested times add up to 2:45; you have the full 3 hours to complete the exam.

AM section: Do not discuss the contents of this exam with anyone until 9 PM. There are two sections of this class, and any information passed to the other section is not only a serious honor code violation, it lowers your grade directly.

Booth honor code required statement: I pledge my honor that I have not violated the honor code during this examination.

Signature:

1. (10) a) You show a buddy the following table from class. (Numbers slightly simplified)

$$R_{t+1} = a + b \times D/P_t + \varepsilon_{t+1}$$

b	t	R^2
4.0	2.47	0.09

Your buddy says "Come on now, that's not important. Ok, you got a t statistic above two, but the R^2 is only 9%. That's not important in any economic or practical sense. It's tiny." What facts might you cite to defend the importance of this regression? (Hint: What's the definition of R^2? Is there a better number? Use the numbers in the table where possible.)

b) Many commenters say that events such as the internet boom, with huge price multiples (P/E, P/D, M/B), are "bubbles" in stock prices; prices were high just because people expected stock prices to keep going up; they irrationally expected good returns forever, not because high prices rationally signaled higher profits. What facts from this course support this interpretation? What facts argue against it?

2 (15)

We studied a simple vector autoregression (VAR) which came out roughly as follows

$$\begin{aligned} r_{t+1} &= 0.1 \times dp_t + \varepsilon^r_{t+1} \\ \Delta d_{t+1} &= 0 \times dp_t + \varepsilon^d_{t+1} \\ dp_{t+1} &= 0.94 \times dp_t + \varepsilon^{dp}_{t+1} \end{aligned}$$

We identified combinations of the shocks $\varepsilon^r_{t+1}, \varepsilon^d_{t+1}, \varepsilon^{dp}_{t+1}$ that corresponded to "expected return" shocks and "expected cashflow" shocks. Sketch the response of r_t, d_t, dp_t, $t = 0, 1, 2, ...$ to each kind of shock (a) expected return, and b) expected cashflow), occurring at $t = 1$. (Hint: the identity $\varepsilon^r_{t+1} = -\rho\varepsilon^{dp}_{t+1} + \varepsilon^d_{t+1}$, resulting from $r_{t+1} = -\rho dp_{t+1} + dp_t + \Delta d_{t+1}$ will prove useful. Be sure to distinguish period 0, before the shock period 1, when the shock hits, and period 2,3,4,... which respond to the shock.)

3. (15) a) True/False and why. The "value" puzzle is not there in US data before 1963, because in that period "value" stocks did not outperform "growth" stocks.

b) Is the basic point of the Fama-French model that stocks with higher book/market ratios and smaller size (market capitalization) have higher average returns?

c) Which gets better returns going forward, stocks that had great past sales growth, or stocks that had poor past sales growth? Does this pattern correspond sensibly to betas?

d) If we form a momentum portfolio, from stocks that did well last year, are the returns on that portfolio correlated with the returns on value stocks over the next year? I.e. if value stocks go up, do momentum stocks tend to go up, down, or remain the same?

4. (10) a) Companies should issue stock and invest when the cost of capital is low, meaning expected returns are low. Thus, portfolios of companies that are repurchasing stock should have higher returns going forward than portfolios of companies that are issuing stock. If you form a portfolio of companies that are issuing vs. repurchasing stock, do you see a spread of average returns in the right direction?

b) Wait a minute – those issuing companies have high stock prices and the repurchases low stock prices. Surely the big issuers are growth stocks and the repurchasers are value stocks. Is any pattern you found in a subsumed by value and growth effects?

5. (15) You're valuing two private-equity investments, which will pay off x_{t+1} when they go public next year. (Ok, lame attempt to make this look "real-world.") Here is how they behave in a scenario analysis. c_{t+1} is consumption next year. Consumption this year is $c_t = \$1,000$. (The numbers are unrealistic to make the problem easy to solve. You should not use approximations in this problem.)

a) Use the consumption-based model to find the value of each investment today. Use $\beta = 1, \gamma = 1$.

b) Compare the values i) to each other and ii) to the expected value of the payoffs. Explain why each one is higher, lower, or the same as the other, and higher, lower, or the same as the expected value of the payoff.

Probability:	1/3	1/3	1/3	Expected value	Value
c_{t+1} :	\$1,500	\$1,000	\$500	\$1000	—
x^A_{t+1}	\$90	\$60	\$30	\$60	
x^B_{t+1}	\$30	\$60	\$90	\$60	

6. (5) Your assignment is to evaluate the CAPM using the FF 25 portfolios in a subsample. One group member uses a pure time-series regression. She reports that the CAPM is lousy; the market premium is positive, but the alphas are huge; some alphas are even bigger than the average excess returns. The other group member uses a cross-sectional regression. He reports that no, the CAPM is doing fine. The alphas are small and a plot of actual $E(R^{ei})$ vs. predicted $\hat{\gamma} + \beta_i \hat{\lambda}$ returns looks great. How can both of these results happen? Illustrate your answer on a graph of average returns vs. betas.

7. (10) a)You investigate a hedge fund with high CAPM alpha, but find that you can explain its entire performance with loadings on hml, smb and umd (momentum) factors, which you measure using tradeable ETFs. The fund objects: "You're not allowed to use momentum as a factor, it's not even remotely a state variable of concern to investors." Is this a valid objection?

b) Fund managers claim that fees and turnover do not reduce returns to investors, because higher fees and turnover generate higher alpha. A cynical Chicago economist predicts that higher fees and turnover have no relationship to returns to investors, because the fund will keep all the alpha. What are the facts (roughly)? How much does a 1% change in fees change returns to investors? How much does turnover – selling one stock and buying another – change returns to investors? What regression did Carhart run to measure these quantities?

8. (5) Berk argues that there can be alpha even if mutual fund returns to investors do not persist over time, and that flows following past returns are not irrational. Which of the following considerations is key to this argument, and explain why. (Focus on the right one, briefly comment on the wrong ones).

a) Managers can only achieve alpha up to a certain scale

b) Managers raise their fees (as a percent of assets under management) if they do well

c) Momentum in underlying stocks explains the appearance of persistent returns

9. (15)

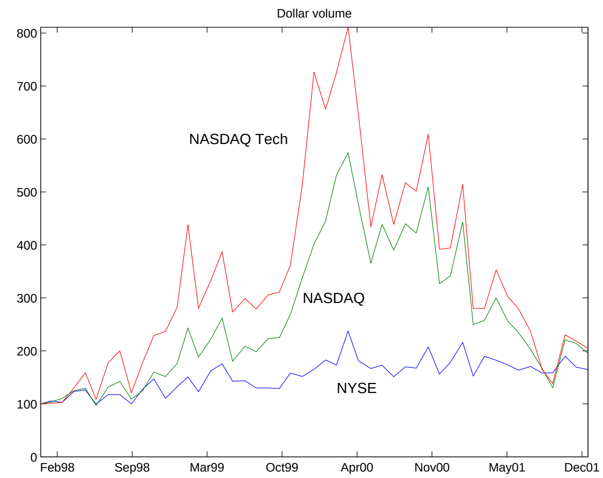
a) On the day that Palm went public, what happened to 3Com's price?

b) Did short sales constraints mean that nobody in fact was able to short Palm stock?

c) Was Palm more or less liquid than 3Com?

d) If you want to buy Palm because you think it will go up next week, why not buy 3Com instead? After all, 3Com owns 95% of Palm so it will go up too. (Be specific about facts.)

e) What implication did Cochrane draw from this graph?



Dollar volume on NYSE, NASDAQ and NASDAQ with SIC code 737. Series are normalized to 100 on Jan 1 1998.

10. (10)

a) Suppose your brokerage is part of an investment bank that might default on its debt. Why might you pull out your securities? After all, they're yours, not the bank's.

b) Why does it hurt the bank if you pull out of your brokerage account? After all, they're just executing trades for a fee; the securities you own are yours. Missing a few weeks of fee income isn't going to make them bankrupt.

c) Gorton and Metrick call short-term debt "information insensitive." What does this mean, and what can change that fact (hint: Gorton and Metrick made an analogy to a food-poisoning outbreak.)

11. (15) The current log forward curve is (in years)

$y_t^{(1)}$	$f_t^{(2)}$	$f_t^{(3)}$	$f_t^{(4)}$	$f_t^{(5)}$
1%	2%	3%	4%	5%

a) According to the expectations hypothesis, what is the expected interest rate four years from now $E_t\left(y_{t+4}^{(1)}\right)$?

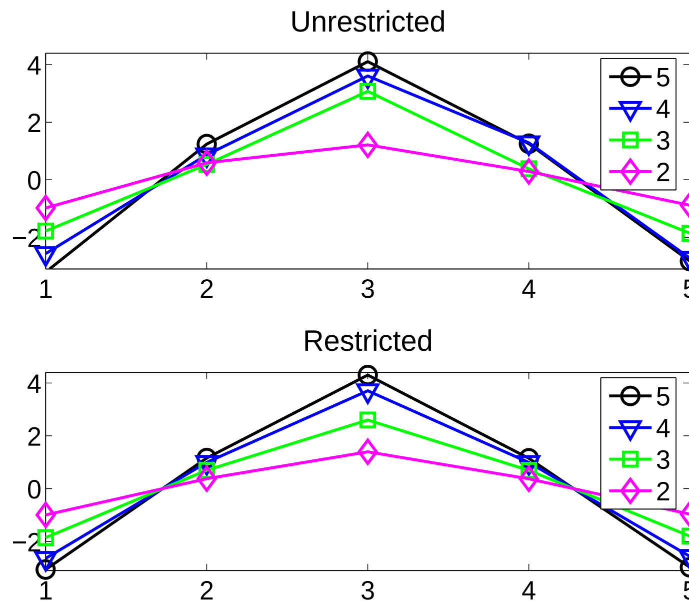
b) Do Fama and Bliss' regressions support or contradict this conclusion?

c) According to the expectations hypothesis, what is the expected value of next year's three-year forward rate $E_t(f_{t+1}^{(3)})$?

d) According to Fama and Bliss' one-year regressions, and approximating the numbers by 1 and 0, what is the expected return on three year bonds $E_t r_{t+1}^{(3)}$?

e) According to Fama and Bliss' one-year return regressions, and approximating the numbers by 1 and 0, what is the expected value of next year's three-year forward rate $E_t f_{t+1}^{(3)}$? Compare to c.

12. (10) What does this picture represent? Be explicit, with equations. What is the point of the picture?



13. (15) Suppose the one-year rate is an MA(1),

$$y_t^{(1)} = \varepsilon_t + \varepsilon_{t-1}.$$

$E_t(\varepsilon_{t+1}) = 0; E(\varepsilon_t) = 0$ as usual. Form a term-structure model, by supposing that the expectations hypothesis holds.

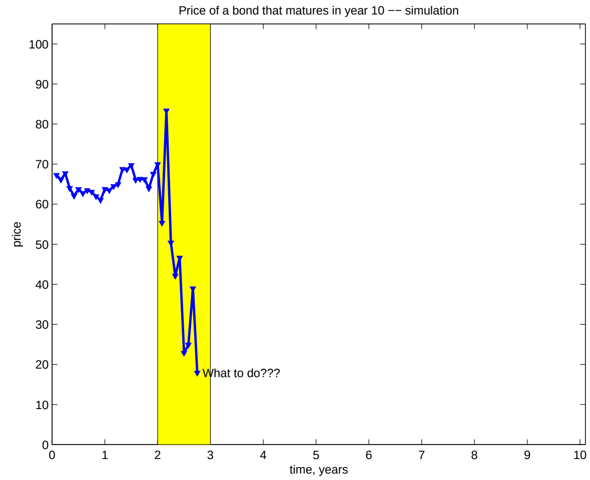
a) Find forward rates (at time t, for maturity 2,3,4,...n)

b) Find yields (at time t, for maturity 2,3,4,...n).

c) Express the factors in terms of yields or forward rates. Then express the yield-curve model as a function of factors, i.e. loadings on the factors. (There are many ways to do this. Choose an easy one!)

(Hint: You may think you got it wrong because the answer is too simple. Don't worry, it really is simple. This problem does NOT involve a lot of algebra.)

14. (5) You are a very risk averse investor with a 10 year horizon, and you have decided that a 10 year zero coupon TIP is the right security. A new financial crisis comes along. Long-term rates rise, so your bond plummets in value. Interest-rate volatility spikes. Your long-term bond investment has underperformed the 7-year bond benchmark by 5%.



Your investment adviser wants you to bail out to less risky short term bonds, or more profitable investments. He reminds you of $w = \frac{1}{\gamma} E_t(R^e)/\sigma_t^2(R_{t+1}^e)$. The bond-return forecasting model says $E_t(R^e)$ is dreadful for next year, and with $\sigma_t(R_{t+1}^e)$ higher surely you should rebalance away from this bond. Is he right? *If not, why not and what should you do?* (This is a short answer question, not a lot of algebra. Explain with reference to portfolio formulas, and intuition.)

15. (5) You have risk aversion $\gamma = 1$, and returns are independent over time. Your best guess is that the mean annual premium μ is 4% with volatility σ =20%,

a) What should your allocation to stocks be?

b) In fact you don't really know what the mean return is. Reflecting on it, your uncertainty about the mean return $\sigma(\mu)$ is 10 percentage points, and both the actual return and your uncertainty about it are normally distributed. (Numbers are easy to calculate, not realistic.) How does this consideration change your optimal allocation to stocks?

16. (5) You create a "small business" factor, a portfolio of assets that has 90% correlation with shocks to the profits of small and privately-held businesses. Alas, after running regressions of your portfolio return on rmrf, smb, hml, you find the alpha of your factor return is zero.

$$R^{\text{efactor}} = 0 + \beta \times rmrf_t + h \times hml_t + s \times smb_t + \varepsilon^{\text{factor}};\ R^2 = 0.4$$

This means we do not need your new factor to price assets, and the expected returns of your factor can be achieved by the combination of FF 3 factors. Might a fund based on your new factor be interesting to investors nonetheless? If so, how? (Hint: The R^2 matters here)

2011 Final Exam Answers

1. (10)

a) You show a buddy the following table from class. (Numbers slightly simplified)

$R_{t+1} = a + b \times D/P_t + \varepsilon_{t+1}$

b	t	R^2
4.0	2.47	0.09

Your buddy says "Come on now, that's not important. Ok, you got a t statistic above two, but the R^2 is only 9%. That's not important in any economic or practical sense. It's tiny." What facts might you cite to defend the importance of this regression? (Hint: What's the definition of R^2? Is there a better number? Use the numbers in the table where possible.)

b. Many commenters say that events such as the internet boom, with huge price multiples (P/E, P/D, M/B), are "bubbles" in stock prices; prices were high just because people expected stock prices to keep going up so they irrationally expected good returns forever, not because high prices rationally signaled higher profits. What facts from this course support this interpretation? What facts argue against it?

ANSWER:

a) There are several observations we made to show the economic importance of forecastability. The one I was steering you towards was a, but b and c can also be mentioned.

i) The variation in expected returns, while small compared to the variation in total returns, is large compared to expected returns. From the table we can compute $R^2 = \frac{var(E_t(R_{t+1}))}{var(R_{t+1})} = 0.09$ so $\sigma(b \times dp) = \sqrt{0.09}\sigma(R) =: 0.3 \times 0.20 = 6\%$

ii) Both coefficient and R^2 are larger at longer horizons

iii) The volatility test says this predictability is just enough to account for the large variance of dividend yields.

b)

i) It is true that price dividend ratios do not, in fact, forecast lower future dividends.

ii) a) Price dividend ratios do, however forecast lower returns. So the data are also consistent with lower discount rates. This is the main point – as stated the bubble said expected returns are constant.

Also possible to mention:

b) Price dividend ratios do not forecast forever higher price dividend ratios

c) we studied volatility tests

$$var\left(p_t - d_t\right) \approx cov\left(p_t - d_t, \sum_{j=1}^{k} \rho^{j-1}\Delta d_{t+j}\right) - cov\left(p_t - d_t, \sum_{j=1}^{k} \rho^{j-1} r_{t+j}\right) + cov\left(p_t - d_t, \rho^k(p_{t+k} - d_{t+k})\right)$$

The return forecast coefficient on the right is just about the same as the variance on the left, so return predictability is enough to account for volatility.

The remaining argument is over whether the expected return variation is "rational." An answer that mentions Cochrane Stocks as money and large amounts of trading and difficulty shorting houses gets an extra gold star.

2 (15)

We studied a simple vector autoregression (VAR) which came out roughly as follows

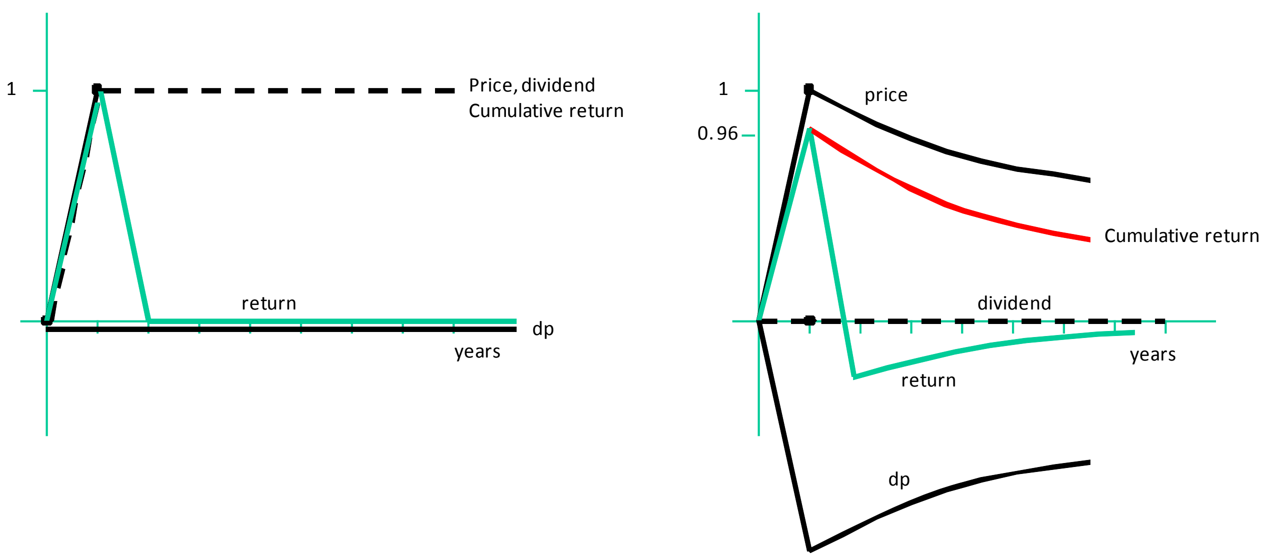
$$\begin{aligned} r_{t+1} &= 0.1 \times dp_t + \varepsilon^r_{t+1} \\ \Delta d_{t+1} &= 0 \times dp_t + \varepsilon^d_{t+1} \\ dp_{t+1} &= 0.94 \times dp_t + \varepsilon^{dp}_{t+1} \end{aligned}$$

We identified combinations of the shocks $\varepsilon^r_{t+1}, \varepsilon^d_{t+1}, \varepsilon^{dp}_{t+1}$ that corresponded to "expected return" shocks and "expected cashflow" shocks. Sketch the response of r_t,d_t, dp_t, $t = 0, 1, 2, ...$ to each kind of shock (a) expected return, and b) expected cashflow), occurring at $t = 1$. (Hint: the identity $\varepsilon^r_{t+1} = -\rho\varepsilon^{dp}_{t+1} + \varepsilon^d_{t+1}$, resulting from $r_{t+1} = -\rho dp_{t+1} + dp_t + \Delta d_{t+1}$ will prove useful. Be sure to distinguish period 0, before the shock period 1, when the shock hits, and period 2,3,4,... which respond to the shock.)

ANSWER

The "expected cashflow" shock moves Δd by 1 without changing dp. Hence it moves r by 1 as well, by the ε identity. In subsequent periods, nothing changes. Thus, you get the plot on the left.

The "expected return" shock moves dp by 1, without changing Δd. Hence, r moves by -0.96. dp then follows AR(1) reversion at rate 0.94^j. This means r also follows 0.1×0.94^j. The right hand sketch shows a negative shock to dp, flipped upside down is fine too.



3. (15) a) True/False and why. The "value" puzzle is not there in US data before 1963, because in that period "value" stocks did not outperform "growth" stocks.

b) Is the basic point of the Fama-French that stocks with higher book/market ratios and smaller size (market capitalization) have higher average returns?

c) Which gets better returns going forward, stocks that had great past growth in sales, or stocks that had poor past growth in sales? Is this a new factor or is the pattern of returns consistent with some pattern of factor exposures?

d) If we form a momentum portfolio, from stocks that did well last year, are the returns on that portfolio correlated with the returns on value stocks over the next year? I.e. f value stocks go up, do momentum stocks tend to go up, down, or remain the same?

ANSWER

a) False. Yes, the value effect is not present in the earlier period, but expected returns are roughly the same. It's the betas that changed and *every puzzle is a joint puzzle of expected returns and betas.*

b)NO. It is a fact, but the Fama french *model* explains this fact with size and b/m factors.

c)Fama and french multifactor. Good growth = bad returns. This is explained by h loadings.

d) High momentum stocks have low h values – "act like" growth stocks with high prices. So momentum is negatively correlated with value.

4. (10) a) Companies should issue stock and invest when the cost of capital is low, meaning expected returns are low. Thus, portfolios of companies that are repurchasing stock should have higher returns going forward than portfolios of companies that are issuing stock. If you form a portfolio of companies that are issuing vs. repurchasing stock, do you see a spread of average returns in the right direction?

ANSWER FF dissecting anomalies. Yes, net issues do correspond to low returns and vice versa.

b) Wait a minute – those issuing companies have high stock prices and the repurchasers low stock prices. Surely the big issuers are growth stocks and the repurchasers are value stocks, so any pattern you found in a is subsumed by value and growth effects?

ANSWER FF used characteristic adjusted portfolios, so this is above any BM effect.

5. (15) You're valuing two private-equity investments, which will pay off x_{t+1} when they go public next year. (Ok, lame attempt to make this look "real-world.") Here is how they behave in a scenario analysis. c_{t+1} is consumption next year. Consumption this year is $c_t = \$1,000$. (The numbers are unrealistic to make the problem easy to solve. You should not use approximations in this problem.)

a) Use the consumption-based model to find the value of each investment today. Use $\beta = 1, \gamma = 1$.

b) Compare the values i) to each other and ii) to the expected value of the payoffs. Explain why each one is higher, lower, or the same as the other, and higher, lower, or the same as the expected value of the payoff.

Probability:	1/3	1/3	1/3	Expected value	Value
c_{t+1} :	\$1,500	\$1,000	\$500	\$1000	—
x^A_{t+1}	\$90	\$60	\$30	\$60	
x^B_{t+1}	\$30	\$60	\$90	\$60	

ANSWER

Probability:	1/3	1/3	1/3	Expected value	Value	$E(x)/R^f$
c_{t+1} :	\$1,500	\$1,000	\$500	\$1,000	-	
x^A_{t+1}	\$90	\$60	\$30	\$60	\$60	\$73.67
x^B_{t+1}	\$30	\$60	\$90	\$60	\$86.67	\$73.67

a)

$$
\begin{aligned}
& \frac{1}{3}\times\left(\frac{1500}{1000}\right)^{-1}\times 90+\frac{1}{3}\times\left(\frac{1000}{1000}\right)^{-1}\times 60+\frac{1}{3}\times\left(\frac{500}{1000}\right)^{-1}\times 30 \\
& \frac{1}{3}\times\left(\frac{1000}{1500}\right)\times 90+\frac{1}{3}\times\left(\frac{1000}{1000}\right)\times 60+\frac{1}{3}\times\left(\frac{1000}{500}\right)\times 30 \\
& \frac{2}{3}\times 30+1\times 20+(2)\times 10 \\
20+20+20 &= 60
\end{aligned}
$$

$$
\begin{aligned}
& \frac{1}{3}\times\left(\frac{1500}{1000}\right)^{-1}\times 30+\frac{1}{3}\times\left(\frac{1000}{1000}\right)^{-1}\times 60+\frac{1}{3}\times\left(\frac{500}{1000}\right)^{-1}\times 90 \\
& \frac{1}{3}\times\left(\frac{1000}{1500}\right)\times 30+\frac{1}{3}\times\left(\frac{1000}{1000}\right)\times 60+\frac{1}{3}\times\left(\frac{1000}{500}\right)\times 90 \\
& \frac{2}{3}\times 10+1\times 20+(2)\times 30 \\
6.2/3+20+60 &= 86\ 2/3
\end{aligned}
$$

b) They covary differently with consumption growth. project A pays off when consumption is already good, and does badly in bad times. B pays off when consumption is bad, so provides “consumption insurance.”

c) That again explains why B is more than its expected value, but what about A? It’s paying its expected value even though it’s risky. The key here is that the risk free rate is negative. $E(m)$ is not the same thing as expected consumption growth.

$E(m)$

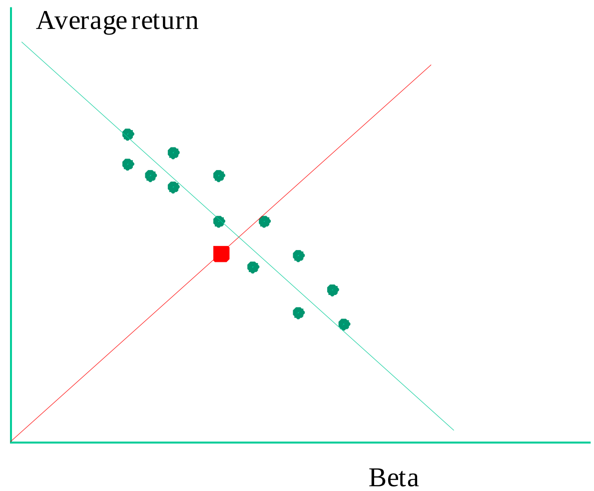
$$
\begin{aligned}
E(m) &= \frac{1}{3}\times\left(\frac{1500}{1000}\right)^{-1}+\frac{1}{3}\times\left(\frac{1000}{1000}\right)^{-1}+\frac{1}{3}\times\left(\frac{500}{1000}\right)^{-1} \\
E(m) &= \frac{1}{3}\times\left(\frac{1000}{1500}\right)+\frac{1}{3}\times\left(\frac{1000}{1000}\right)+\frac{1}{3}\times\left(\frac{1000}{500}\right) \\
E(m) &= \frac{1}{3}\times\left(\frac{2}{3}\right)+\frac{1}{3}+\frac{1}{3}\times(2)=\frac{1}{3}\times\left(\frac{2}{3}+1+2\right)=\frac{11}{9} \\
R^f &= \frac{9}{11}
\end{aligned}
$$

Thus, $E(x)/R^f = 11/9\ E(x) = 11/9 * 60 = 11/3 * 20 = 220/3 = 73.67$

6. (5) Your assignment is to evaluate the CAPM using the FF 25 portfolios in a subsample. One group member uses a pure time-series regression. She reports that the CAPM is lousy; the market premium is positive, but the alphas are huge; some alphas are even bigger than the average excess returns. The other group member uses a cross-sectional regression. He reports that no, the CAPM is doing fine. The alphas are small and a plot of actual $E(R^{ei})$ vs. predicted $\hat{\gamma}+\beta_i\hat{\lambda}$ returns looks great. How can both of these results happen? Illustrate your answer on a graph of average returns vs. betas.

ANSWER

The problem is the free intercept. Here’s what happened. Either include the interest rate as a test asset, use GLS, or don’t allow a free intercept.



7. (10) a) You investigate a hedge fund with high CAPM alpha, but find that you can explain its entire performance with loadings on hml, smb and umd (momentum) factors, which you measure using tradeable ETFs. The fund objects: "You're not allowed to use momentum as a factor, it's not even remotely a state variable of concern to investors." Is this a valid objection?

b) Fund managers claim that fees and turnover do not reduce returns to investors, because higher fees and turnover generate higher alpha. A cynical Chicago economist predicts that higher fees and turnover have no relationship to returns to investors, because the fund will keep all the alpha. What are the facts (roughly)? How much does a 1% change in fees change returns to investors? How much does turnover – selling one stock and buying another – change returns to investors? What regression did Carhart run to measure these quantities?

ANSWER: performance evaluation factors are only about "can I program a computer to do that without paying your fees," they don't have to be "state variables of concern to investors."

ANSWER: If fees and turnover produced more alpha. We should expect in fact that after fee returns are not affected by fees.

ANSWER: roughly 1%. roughly 1% roundtrip transactions costs. A cross sectional regression

$$\alpha_i = a + b(fees_i) + c(turnover_i) + \varepsilon_i i = 1, 2...N$$

8. (5) Berk argues that there can be alpha even if mutual fund returns to investors do not persist over time, and that flows following past returns are not irrational. Which of the following considerations is key to this argument, and explain why. (Focus on the right one, briefly comment on the wrong ones).

a) Managers can only achieve alpha up to a certain scale

b) Managers raise their fees (as a percent of assets under management) if they do well

c) Momentum in underlying stocks explains the appearance of persistent returns

ANSWER:

a. As funds rush in, returns to investors decline. It's important that b does not happen, otherwise we wouldn't need new funds to give more money to the managers. c is irrelevant, that was Carhart's point not Berk and Green's.

9. (15)

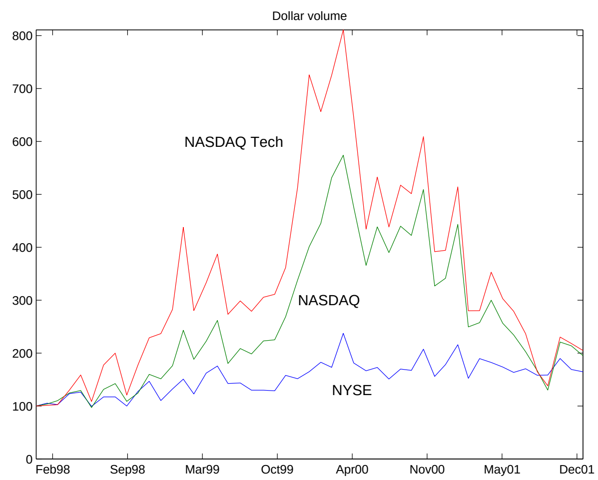
a) On the day that Palm went public, what happened to 3Com's price?

b) Did short sales constraints mean that nobody in fact was able to short Palm stock?

c) Was Palm more or less liquid than 3Com?

d) If you want to buy Palm because you think it will go up next week, why not buy 3Com instead? After all, 3Com owns 95% of Palm so it will go up too. (Be specific about facts.)

e) What implication did Cochrane draw from this graph?



Dollar volume on NYSE, NASDAQ and NASDAQ with SIC code 737. Series are normalized to 100 on Jan 1 1998.

ANSWER

a) Fell, from 95 to 81.

b) No, at the peak palm was 147% shorted

c) fun question. Bid ask was larger, but turnover vastly more. We discussed and decided there was more demand to trade, despite higher costs, so less liquid. Mentioning the fact of high turnover and high bid ask spread is the key answer.

e) The volume here is visually nearly identical to a price graph. "overpricing" comes with massive trading volume.

10. (10)

a) Suppose your brokerage is part of an investment bank that might default on its debt. Why might you pull out your securities? After all, they're yours, not the bank's.

b) Why does it hurt the bank if you pull out of your brokerage account? After all, they're just executing trades for a fee; the securities you own are yours. Missing a few weeks of fee income isn't going to make them bankrupt.

c) Gorton and Metrick call short-term debt "information insensitive." What does this mean, and what can change that fact (hint: Gorton and Metrick made an analogy to a food-poisoning outbreak.)

ANSWER

a) The bank has used your securities for its own loans, "rehypothecated" so it's going to be hard to get them back in bankruptcy.

b) See above. If you pull out the bank loses financing for its own trading and must dump assets

c) It means that most of the time you don't do a lot of credit checking to lend overnight with full collateral. However, once one big bank is in trouble, it makes you wonder about all the others. Like e coli, once you hear there's an outbreak you avoid the whole salad bar.

11. (15) The current log forward curve is (in years)

$y_t^{(1)}$	$f_t^{(2)}$	$f_t^{(3)}$	$f_t^{(4)}$	$f_t^{(5)}$
1%	2%	3%	4%	5%

a) According to the expectations hypothesis, what is the expected interest rate four years from now $E_t\left(y_{t+4}^{(1)}\right)$?

b)Do Fama and Bliss' regressions support or contradict this conclusion?

c) According to the expectations hypothesis, what is the expected value of next year's three-year forward rate $E_t(f_{t+1}^{(3)})$?

d) According to Fama and Bliss' one-year regressions, and approximating the numbers by 1 and 0, what is the expected return on three year bonds $E_t r_{t+1}^{(3)}$?

e) According to Fama and Bliss' one-year return regressions, and approximating the numbers by 1 and 0, what is the expected value of next year's three-year forward rate $E_t f_{t+1}^{(3)}$? Compare to c.

ANSWER

a) $E_t y_{t+4}^{(1)} = f_t^{(5)} = 5\%$.

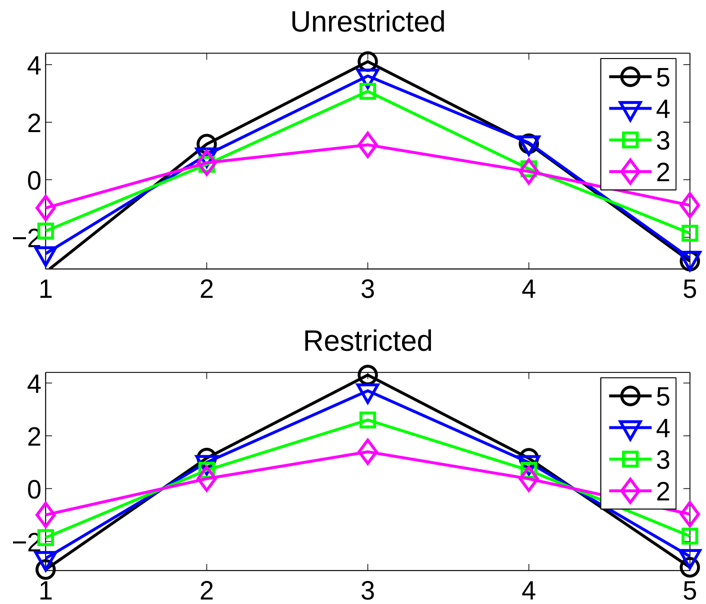
b) Yes, the regression was roughly $y_{t+4}^{(1)} - y_t^{(1)} = a + 1.0 \times (f_t^{(5)} - y_t^{(1)}) + \varepsilon_{t+1}$

c) $E_t f_{t+1}^{(3)} = f_t^{(4)} = 4\%$

d) $E_t rx_{t+1}^{(3)} = f_t^{(3)} - y_t^{(1)} = 3\% - 1\% = 2\%$; $E_t r_{t+1}^{(3)} = E_t rx_{t+1}^{(3)} + y_t^{(1)} = 3\%$

e)$E_t(f_{t+1}^{(3)}) = f_t^{(3)} = 3\%$. We did this on a problem set. Fama-Bilss is "prices don't change." To reach this conclusion you have to unwind return regressions to find the forward rate implications, $E_t(f_{t+1}^{(3)}) = E_t(p_{t+1}^{(2)} - p_{t+1}^{(3)}) = E_t(p_{t+1}^{(2)} - p_t^{(3)} - \left(p_{t+1}^{(3)} - p_t^{(4)}\right)) + \left(p_t^{(3)} - p_t^{(4)}\right) = E_t(r_{t+1}^{(3)} - r_{t+1}^{(4)}) - f_t^{(4)} = (\text{Fama-Bliss}) = (f_t^{(3)} - f_t^{(4)}) - f_t^{(4)}$

12. (10) What does this picture represent? Be explicit, with equations.



ANSWER

These are Cochrane-Piazzesi regressions. The top one is unrestricted

$$rx_{t+1}^{(n)} = a^{(n)} + \beta_1^{(n)} y_t^{(1)} + \beta_2^{(n)} f_t^{(2)} + .. + \beta_5^{(n)} f_t^{(5)} + \varepsilon_{t+1}^{(n)}$$

this is a plot of $\beta^{(n)}$, each line a different $\not n$. The bottom one is restricted,

$$rx_{t+1}^{(n)} = b_n \left[\gamma_0 + \gamma_1 y_t^{(1)} + \gamma_2 f_t^{(2)} + .. + \gamma_5 f_t^{(5)} \right] + \varepsilon_{t+1}^{(n)}$$

The plot is $b_n \gamma$. The point of the plot is that the two representations are very nearly identical.

13. (15) Suppose the one-year rate is an MA(1),

$$y_t^{(1)} = \varepsilon_t + \varepsilon_{t-1}.$$

Form a term-structure model, by supposing that the expectations hypothesis holds.

a) Find forward rates (at time t, for maturity 2,3,4,...n)

b) Find yields (at time t, for maturity 2,3,4,...n).

c) Express the factors in terms of yields or forward rates. Then express the yield-curve model as a function of factors, i.e. loadings on the factors. (There are many ways to do this. Choose an easy one!)

(Hint: You may think you got it wrong because the answer is too simple. Don't worry, it really is simple. This problem does NOT involve a lot of algebra.)

ANSWER

$y_t^{(1)} = \varepsilon_t + \varepsilon_{t-1}$

$f_t^{(2)} = E_t y_{t+1}^{(1)} = \varepsilon_t$

$f_t^{(3)} = E_t y_{t+2}^{(1)} = 0$

$f_t^{(n)} = E_t y_{t+n-1}^{(1)} = 0$

$$y_t^{(1)} = \varepsilon_t + \varepsilon_{t-1}$$
$$y_t^{(2)} = \frac{1}{2}\left(y_t^{(1)} + f_t^{(2)}\right) = \frac{1}{2}\left(\varepsilon_t + \varepsilon_{t-1} + \varepsilon_t\right) = \varepsilon_t + \frac{1}{2}\varepsilon_{t-1}$$
$$y_t^{(3)} = \frac{1}{3}\left(y_t^{(1)} + f_t^{(2)} + f_t^{(3)}\right) = \frac{1}{3}\left(\varepsilon_t + \varepsilon_{t-1} + \varepsilon_t\right) = \frac{2}{3}\varepsilon_t + \frac{1}{3}\varepsilon_{t-1}$$
$$y_t^{(n)} = \frac{1}{n}\left(y_t^{(1)} + f_t^{(2)} + f_t^{(3)} + .. + f_t^{(n)}\right) = \frac{1}{n}\left(\varepsilon_t + \varepsilon_{t-1} + \varepsilon_t\right) = \frac{2}{n}\varepsilon_t + \frac{1}{n}\varepsilon_{t-1}$$

Factors The factors are ε_t and ε_{t-1}. You can find them just by

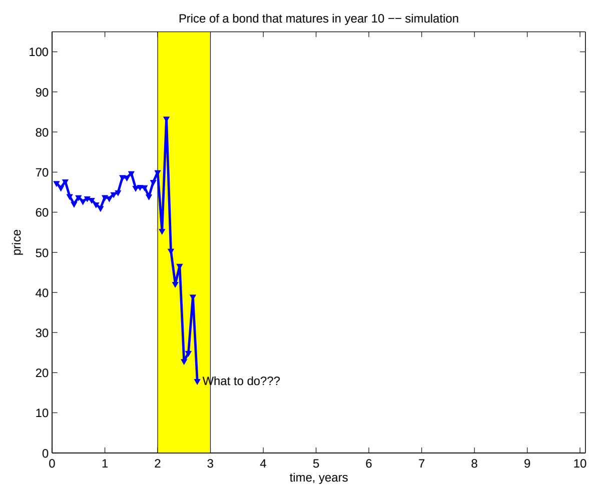
$\varepsilon_t = f_t^{(2)}$

$\varepsilon_{t-1} = y_t^{(1)} - f_t^{(2)}$.

You already have the loadings.

$$\begin{bmatrix} y_t^{(1)} \\ y_t^{(2)} \\ y_t^{(3)} \\ y_t^{(n)} \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ \frac{2}{3} \\ \frac{2}{n} \end{bmatrix} \varepsilon_t + \begin{bmatrix} 1 \\ \frac{1}{2} \\ \frac{1}{3} \\ \frac{1}{n} \end{bmatrix} \varepsilon_{t-1}$$

14. (5) You are a very risk averse investor with a 10 year horizon, and you have decided that a 10 year zero coupon TIP is the right security. A new financial crisis comes along. Long-term rates rise, so your bond plummets in value. Interest-rate volatility spikes. Your long-term bond investment has underperformed the 7 year bond benchmark by 5%.



Your investment adviser wants you to bail out to less risky short term bonds, or more profitable investments. He reminds you of $w = \frac{1}{\gamma} E_t(R^e)/\sigma_t^2(R_{t+1}^e)$. The bond-return forecasting model says $E_t(R^e)$ is dreadful for next year, and with $\sigma_t(R_{t+1}^e)$ higher surely you should rebalance away from this bond. Is he right? *If not, why not and what should you do?* (This is a short answer question, not a lot of algebra. Explain with reference to portfolio formulas, and intuition.)

ANSWER

No. He left out the "hedge demand" terms. One period mean-variance is wrong for long-term bond investors.

15. (5) You have risk aversion $\gamma = 1$, and returns are independent over time. Your best guess is that the mean annual premium μ is 4% with volatility σ =20%,

a) What should your allocation to stocks be?

b) In fact you don't really know what the mean return is. Reflecting on it, your uncertainty about the mean return $\sigma(\mu)$ is 10 percentage points, and both the actual return and your uncertainty about it are normally distributed. (Numbers are easy to calculate, not realistic.) How does this consideration change your optimal allocation to stocks?

ANSWER

$$\begin{aligned} \frac{0.04}{0.2^2} &= \frac{0.04}{0.04} = 1 \\ \frac{0.04}{0.2^2 + 0.1^2} &= \frac{0.04}{0.04 + 0.01} = \frac{0.04}{0.05} = 0.8 \end{aligned}$$

Verbal answers to the effect that "parameter uncertainty is extra variance and scale back the allocation to stocks" are worth some partial credit.

16. (5) You create a "small business" factor, a portfolio of assets that has 90% correlation with shocks to the profits of small and privately-held businesses. Alas, after running regressions of your portfolio return on rmrf, smb, hml, you find the alpha of your factor return is zero.

$$R^{\text{efactor}} = 0 + \beta \times rmrf_t + h \times hml_t + s \times smb_t + \varepsilon^{\text{factor}};\ R^2 = 0.4$$

This means we do not need your new factor to price assets, and the expected returns of your factor can be achieved by the combination of FF 3 factors. Might a fund based on your new factor be interesting to investors nonetheless? If so, how? (Hint: The R^2 matters here)

ANSWER

Small business owners would love to short this portfolio!

MORE QUESTIONS

9) (10) Below, find an excerpt from Fama and French's Table 3, which studies gross (before fees) mutual fund returns.

i) What does this table tell you about the median fund's alpha?

ii) Are there more "good funds" with positive alpha than we would expect due to chance if all funds had zero alpha?

iii) Are there more "bad funds" with negative alphas?

iv) How does this calculation address the retort "sure, the average fund doesn't have much alpha, but the good funds have alpha."

Give some numbers in your answers: What does 1.68 mean? What does 2.04 mean? What do -1.71 and -2.19 mean? Showing you understand what "Sim" and "act" mean is important (and not just .

Percentile	Simulated	Actual
5	-1.71	-2.19
50	-0.01	-0.06
90	1.30	1.59
91	1.38	1.68
95	1.68	2.04

Table 3 - Percentiles of t(α) estimates for actual and simulated fund returns...[3-factor adjusted] gross fund returns....

ANSWER

i) The median fund has -0.06 alpha (before costs).

ii) There are slightly more good funds than you'd expect. 1.68 means that if all funds really have exactly zero alpha, then we expect to see that 5% of the funds in a sample will have an alpha t statistic greater than 1.68 just due to chance. In fact, 5% of funds had a t stat greater than 2.04, and 9% had a t stat greater than 1.68. Thus there are 4% too many funds with alpha greater than 1.68 than there should be.

iii) Similarly, -1.71 means that 5% of funds should have results this bad due to chance. The actual cutoff is -2.19.

iv) Thus, the *before fee* alpha distribution is very slightly wider than a view that all true alphas are zero and observed alphas are luck can account for. However, it's "very slightly". Yes there are some "good funds," but they're awfully hard to find.

5) (15) Suppose log consumption is as in the table, and suppose people know what's going to happen ahead of time. (I'm trying to illustrate a "recovery," "boom," "recession" and "recovery," superimposed on growth.) Use the consumption-based model to find as many of the interest rates as possible. Use discount rate $\delta = 2\%$, and risk aversion $\gamma = 2$, approximate as necessary to get round answers.

t:	1	2	3	4	5	6
$\log(c_t)$:	0.01	0.03	0.04	0.03	0.03	0.05
r_t^f :						

ANSWER

$$r_t^f = \delta + \gamma E_t \Delta c_{t+1} = 0.02 + 2 \times \Delta c_{t+1}$$

The trick is you have to compute Δc_{t+1}, not use the level of c_t.

t:	1	2	3	4	5	6
$\log(c_t)$:	0.01	0.03	0.04	0.03	0.03	0.05
Δc_{t+1}	0.02	0.01	-0.01	0.00	0.02	
r_t^f :	0.05	0.04	0.00	0.02	0.04	
r_t^f :	5%	4%	0%	2%	4%	

9) (10) You're investigating a hedge fund that invests in illiquid securities. It offers a mean return of 10%, low market beta, and advertises a low annualized standard deviation of 10% and hence a very good annualized Sharpe ratio of 1.0. However, you discover that its returns follow an MA(1), with serial correlation $cov(r_t, r_{t+1})/var(r_t) = 0.5$; $cov(r_t, r_{t+2}) = 0$, $cov(r_t, r_{t+k}) = 0, k > 1$

a) Suggest two ways to get a more realistic beta.

b) Does the serial correlation suggest that the mean return is biased, and how ?

c) Does the serial correlation suggest that the variance of returns is biased, and how?

d) If your answer to b and/or c is yes, find a better estimate of the fund's i) mean, ii) variance and hence iii) Sharpe ratio? (Hint for b-d: Think about the mean and variance of long horizon returns $(r_{t+1} + r_{t+2} + ... + r_{t+k})$, .)

ANSWER

a) Add lagged market returns. Do betas at longer horizons

b) No

c) Yes

d) Mean long horizon returns still scale with k as they should.

$$\begin{aligned} E(r_{t+1} + r_{t+2} + ... + r_{t+k}) &= kE(r) \\ E(r_{t+1} + r_{t+2} + ... + r_{t+k})/k &= E(r) \end{aligned}$$

the variance of long horizon returns does not scale with k

$$\begin{aligned} \sigma^2(r_{t+1} + r_{t+2} + .. + r_{t+k}) &= k\sigma^2(r) + 2(k-1)cov(r_t, r_{t+1}) \\ \frac{\sigma^2(r_{t+1} + r_{t+2} + ... + r_{t+k})}{k} &= \sigma^2(r)\left[1 + 2\frac{(k-1)}{k}\rho\right] \\ \frac{\sigma^2(r_{t+1} + r_{t+2} + ... + r_{t+k})}{k} &\rightarrow \frac{\sigma^2(r)}{k}\left[1 + 2\rho\right] \end{aligned}$$

thus, the variance of long horizon returns is really ($\rho = 0.5$) twice as large as suggested by the variance of short horizon returns. Thus, i) the true mean is still 10% ii) the true variance is twice as large, 2×0.10^2. iii) the true annualized Sharpe ratio is $1/\sqrt{2} \approx 1/1.4 = 0.7$

How could charging more money or churning the portfolio *not* reduce returns to investors?

5. (15) Suppose log consumption is as in the table, and suppose people know what's going to happen ahead of time. (I'm trying to illustrate a "recovery," "boom," "recession" and "recovery," superimposed on growth.) Use the consumption-based model to find as many of the interest rates as possible. Use discount rate $\delta = 2\%$, ($\beta = e^{-0.02} \approx 0.98$) and risk aversion $\gamma = 2$, approximate as necessary to get round answers, and give your answer in percent. You can add rows for intermediate results if you wish.

t:	1	2	3	4	5	6
$\log(c_t)$:	0.01	0.03	0.04	0.03	0.03	0.05
r_t^f :						

d) If you sort stocks into "winners" that went up from year -5 to one year ago, and losers that went down from year -5 to one year ago, which ones do better for the next year? Is this consistent with some pattern of factor exposures?

ANSWER: FF again. This is the long term reversal effects. It also corresponds to h loadings – winners act like growth stocks.

5. (15) Should you keep the smb factor? You run regressions

$$R_t^{ei} = \alpha_i + b_i rmrf_t + h_i hml_t + s_i smb_t + \varepsilon_{it} \;\; t = 1, 2, ..T \text{ for each } i \tag{1}$$

$$R_t^{ei} = \alpha_i + b_i rmrf_t + h_i hml_t + \varepsilon_{it} \;\; t = 1, 2, ..T \text{ for each } i \tag{2}$$

$$smb_t = \alpha_s + b_s rmrf_t + h_s hml_t + \varepsilon_{st} \;\; t = 1, 2, ..T \tag{3}$$

(note α_i, b_i, h_i are not necessarily the same in (4) and (5).) You find that the α in (5) are about the same as in (4), and you find that $\alpha_s = 0$. On the other hand, $E(smb)$ is quite high and statistically significant (well, suppose that is the case), the t statistics on s_i are very strong, the R^2 in (4) is much higher than in (5) and a joint test that all $s_i = 0$ decisively rejects. So, should you keep the smb factor or not?

ANSWER

What model you use depends on what the purpose is! Given these facts, you can drop smb for the purpose of understanding mean returns, i.e.

$$E(R^{ei}) = \alpha_i + b_i E(rmrf_t) + h_i E(hml_t) + s_i E(smb_t)$$

Even though $E(smb) \neq 0$, the results of (5) and (6) mean that the b and h will change, so the alphas do not change. For the purpose of understanding return *variance* however, the s t stats and R^2 loudly warn you not to drop *smb*. Also, including *smb* will improve standard errors, including standard errors of α_i, so it might be a good idea in any case.

8. (10) Mitchell and Pulvino ran regressions of merger arbitrage returns on the market. This is an excerpt from their Tables II and IV. These are monthly returns, so 0.0053 means 0.53% per month and 0.0101 means 1.01% per month.

a) Given these results, how should you benchmark a risk-arbitrage manager?

b) How much return does merger arbitrage earn after benchmarking?

c) Is it better to follow the VWRA strategy or the RAIM strategy?

	VWRA	RAIM	CRSP	Riskfree
Average returns	16.05%	10.64%	12.25	6.22
Std. Dev	9.29%	7.74%	15.08%	0.73%
Sharpe	1.06	0.57	0.40	0.00

Dependent variable	$\alpha_{MktHigh}$	β_{MktLow}	$\beta_{MktHigh}$	Adj.R^2
RAIM returns	0.0053	0.4920	0.0167	0.124
	(0.0011)***	(0.0673)***	(0.0292)	
VWRA returns	0.0101	0.4757	-0.0678	0.065
	(0.0013)***	(0.0840)***	(0.0364)	

VVWRA = Value Weighted Risk Arbitrage portfolio; RAIM = Risk Arbitrage Index Manager Standard errors in parentheses. *** denotes significant at the 1% level.

ANSWER a) the point is that the payoffs behave like an index put, so you should benchmark to a strategy that writes puts. This also means to forgive them if they do badly in bad times! b) These are not returns, so the intercept is not alpha. The answer is “we don’t know from the provided information.” You need to do either a contingent-claim valuation or put actual option returns on the right hand side. c) The VWRA returns ignore transactions costs, so you’d follow them if you could but you can’t. Answer: that’s a silly question

11) (15) Here is a table of Fama-Bliss bond forecast regressions The coefficients in the first row add up to one. The coefficients in the other rows do not add up to one.

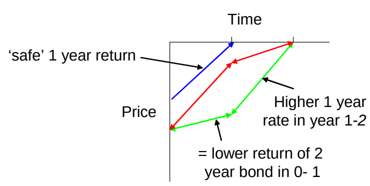
a) Why not? What does add up? Explain either in words or using a graph.

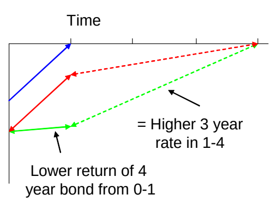
b) Be specific in the case of the second row and first column. If this coefficient is 1.12, exactly what coefficient is -0.12 ? (Hint: break up $f_t^{(3)} - y_t^{(1)} = rx_{t+1}^{(3)}$ + something else. Then if you run a regression of both sides on $f_t^{(3)} - y_t^{(1)}$, you have $1 = b+$ another coefficient.)

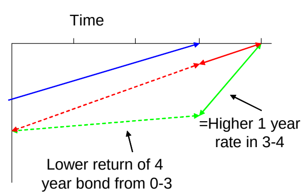
	$rx_{t+1}^{(n)} = a + b\left(f_t^{(n)} - y_t^{(1)}\right) + \varepsilon_{t+1}$			$y_{t+n-1}^{(1)} - y_t^{(1)} = a + b\left(f_t^{(n)} - y_t^{(1)}\right) + \varepsilon_{t+1}$			
n	b	$\sigma(b)$	R^2	a	b	$\sigma(b)$	R^2
2	0.83	0.27	0.12		0.17	0.27	0.01
3	1.12	0.36	0.13		0.47	0.31	0.04
4	1.34	0.45	0.14		0.75	0.23	0.12
5	1.02	0.52	0.06		0.87	0.16	0.16
	forecasting *one* year returns on n-year bonds			forecasting *one* year rates n years from now			

ANSWER

a) The main point is to identify that the one-year change in longer-term yields is complementary on the left hand side, and the multi-year return on long term bonds held to one year before maturity is complementary on the right hand side. The graph is good.







b) To really get this right you have to identify the long/short positions. Left side:

$$
\begin{aligned}
f_t^{(3)} - y_t^{(1)} &= p_t^{(2)} - p_t^{(3)} - y_t^{(1)} \\
&= -\left(p_{t+1}^{(2)} - p_t^{(2)}\right) + \left(p_{t+1}^{(2)} - p_t^{(3)}\right) - y_t^{(1)} \\
&= 2\left(y_{t+1}^{(2)} - y_t^{(2)}\right) + rx_{t+1}^{(2)}
\end{aligned}
$$

If you run a regression of left and right side on $f^{(3)} - y^{(1)}$ you find that 1 (left) = (right) coefficient of $2\left(y_{t+1}^{(2)} - y_t^{(2)}\right)$ on $f^{(3)} - y^{(1)}$ plus the coefficient of $rx_{t+1}^{(2)}$ on $f^{(3)} - y^{(1)}$.

I did not ask for the right side, but here it is.

$$\begin{aligned} f_t^{(3)} - y_t^{(1)} &= p_t^{(2)} - p_t^{(3)} - y_t^{(1)} \\ &= -p_{t+2}^{(1)} + p_t^{(2)} + p_{t+2}^{(1)} - p_t^{(3)} - y_t^{(1)} \\ &= p_{t+2}^{(1)} - p_t^{(3)} + p_t^{(2)} + \left(y_{t+2}^{(1)} - y_t^{(1)}\right) \\ &= \left(r_{t\to t+2}^{(3\to 1)} - r_{t\to t+2}^{(2\to 0)}\right) + \left(y_{t+2}^{(1)} - y_t^{(1)}\right) \end{aligned}$$

Corresponding to the two-year change in $y^{(1)}$ then is the excess return for buying a 3 year bond and holding for two years, over the return for buying a 2 year bond and holding for 2 years.

12) (15) You form a model of the term structure of interest rates by supposing the one-year rate is an AR(1),

$$y_{t+1}^{(1)} - \delta = \rho(y_t^{(1)} - \delta) + \varepsilon_{t+!}$$

and that the expectations hypothesis holds. If your model is right, what should the eigenvalue decomposition of forward rates look like? Specifically, if you took data as predicted from your model,

$$f_t \equiv \begin{bmatrix} y_t^{(1)} \\ f_t^{(2)} \\ f_t^{(3)} \end{bmatrix}$$

and performed $Q\Lambda Q' = eig(cov(f_t, f_t'))$, what would the Q and Λ look like? Note: you do not have to give the exact values of Q and Λ. It is enough to answer that columns of Q have a specific pattern, show where any zeros are, and say what if any parts of Q or Λ are arbitrary and don't matter.

ANSWER: Forward rates should follow a one-factor model.

$$f_t^{(2)} = E_t(y_{t+1}^{(1)}) = \delta + \rho(y_t^{(1)} - \delta)$$

$$f_t^{(3)} = E_t(y_{t+2}^{(1)}) = \delta + \rho^2(y_t^{(1)} - \delta)$$

$$f_t^{(N)} = E_t(y_{t+N-1}^{(1)}) = \delta + \rho^{N-1}(y_t^{(1)} - \delta)$$

thus Λ will have only one non-zero element

$$\Lambda = \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

and Q will look like

$$Q = k \begin{bmatrix} 1 & | & | \\ \rho & q_2 & q_3 \\ \rho^2 & | & | \end{bmatrix}$$

where k is an arbitrary constant – *the pattern is* $1, \rho, \rho^2$. Since they multiply zeros, *the second and third columns of* Q *are irrelevant.*

Optional. The exact values are harder, so I didn't ask for them. The scale of Q is determined by $Q'Q = 1$ or $k^2(1 + \rho^2 + \rho^4) = 1$ so

$$Q = \frac{1}{\sqrt{1 + \rho^2 + \rho^4}} \begin{bmatrix} 1 & | & | \\ \rho & q_2 & q_3 \\ \rho^2 & | & | \end{bmatrix}$$

We can find λ_1 by the requirement that

$$\sigma^2_{y^{(1)}} = \frac{1}{1-\rho^2}\sigma^2_\varepsilon = q^2_{11}\lambda_1 = \frac{1}{1+\rho^2+\rho^4}\lambda_1$$
$$\lambda_1 = \left(1+\rho^2+\rho^4\right)\sigma^2_{y^{(1)}}$$

As the number of included maturities increases, these actually get simpler. In the limit of an infinite number of forward rates, we would have

$$Q = \frac{1}{\sqrt{1+\rho^2+\rho^4+...}}\begin{bmatrix} 1 & | & | & ... \\ \rho & q_2 & q_3 & \\ \rho^2 & | & | & \\ ... & & & ... \end{bmatrix}$$
$$= \sqrt{1-\rho^2}\begin{bmatrix} 1 & | & | & ... \\ \rho & q_2 & q_3 & \\ \rho^2 & | & | & \\ ... & & & ... \end{bmatrix}$$
$$\lambda_1 = \left(1+\rho^2+\rho^4+...\right)\sigma^2_{y^{(1)}} = \frac{1}{(1-\rho^2)}\sigma^2_{y^{(1)}}$$

14) (5) Great news is published in the 2012 *Journal of Finance*. It turns out there was a bug in the programs, and the CAPM works beautifully. All the other "state variables" are unimportant for explaining mean returns. For example, the value premium $E(hml)$ is fully explained by its beta on the market $E(hml) = \beta_{hml,rmrf}E(rmrf)$. Furthermore, it turns out (one more bug!) that mean and variance of returns are constant after all. $E_t(R^e_{t+1})$ is the same all the time. Does this mean we should all go back to simple mean-variance portfolio theory and ignore all those troubling hedging and market timing terms? (You should point to some equations in your answer,but no big algebra is required.)

ANSWER:

$$R^i = R^f + \frac{\gamma^m}{\gamma^i}R^{em} + \frac{1}{\gamma^i}\left(\eta^{i\prime} - \eta^{m\prime}\right)R^{ez};\; R^{ez} \equiv \beta_{y,R'}R^e$$
$$E(R^e) = cov(R^e, R^m)\gamma^m - cov(R^e, y')\eta^m$$

The capm working means $\eta^m = 0$. But you may still have $\eta^i \neq 0$. If you are in financial services, your portfolio should underweight financial services, even though there is no alpha.

10. (10) Below, find an excerpt from Fama and French's Table 3.

i) What does this table tell you about the median fund's alpha?

ii) Are there more "good funds" with positive alpha than we expect due to chance if all true alphas were zero?

iii) Are there more "bad funds" with negative alphas?

iv) How does this calculation address the retort "sure, the average fund doesn't have much alpha, but the good funds have alpha."

Give some numbers in your answers: What does 1.68 mean? What does 2.04 mean? What do -1.71 and -2.19 mean? Showing you understand what "Sim" and "act" mean is important.

Percentile	Simulated	Actual
5	-1.71	-2.19
50	-0.01	-0.06
90	1.30	1.59
91	1.38	1.68
95	1.68	2.04

Table 3 - Percentiles of t(α) estimates for actual and simulated fund returns...[3-factor adjusted] gross fund returns....

5. (15) Should you keep the smb factor? You run regressions

$$R_t^{ei} = \alpha_i + b_i rmrf_t + h_i hml_t + s_i smb_t + \varepsilon_{it} \ t = 1, 2, ..T \text{ for each } i \quad (4)$$

$$R_t^{ei} = \alpha_i + b_i rmrf_t + h_i hml_t + \varepsilon_{it} \ \ t = 1, 2, ..T \text{ for each } i \quad (5)$$

$$smb_t = \alpha_s + b_s rmrf_t + h_s hml_t + \varepsilon_{st} \ t = 1, 2, ..T \quad (6)$$

(note α_i, b_i, h_i are not necessarily the same in (4) and (5).) You find that the α in (5) are about the same as in (4), and you find that $\alpha_s = 0$. On the other hand, $E(smb)$ is quite high and statistically significant (well, suppose that is the case), the t statistics on s_i are very strong, the R^2 in (4) is much higher than in (5) and a joint test that all $s_i = 0$ decisively rejects. So, should you keep the smb factor or not?

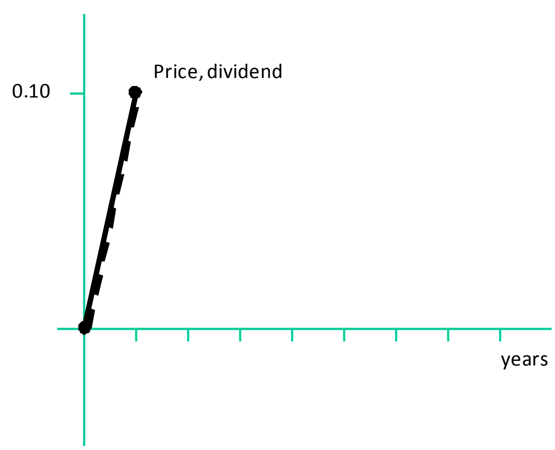
d) If you sort stocks into "winners" that went up from year -5 to one year ago, and losers that went down from year -5 to one year ago, which ones do better for the next year? Is this consistent with some pattern of betas?

2 (15) a) Suppose that the stock market rises 10%, and dividends also rise 10%. Sketch how this event changes your forecast of future i) dividends ii) returns iii) prices. (All variables in logs)

b) Now suppose that the stock market rises 10% but dividends don't change. Make the same sketch.

(Note: art is enough, but if you want to work it out with equations, the formula sheet has the ones you need.)

A



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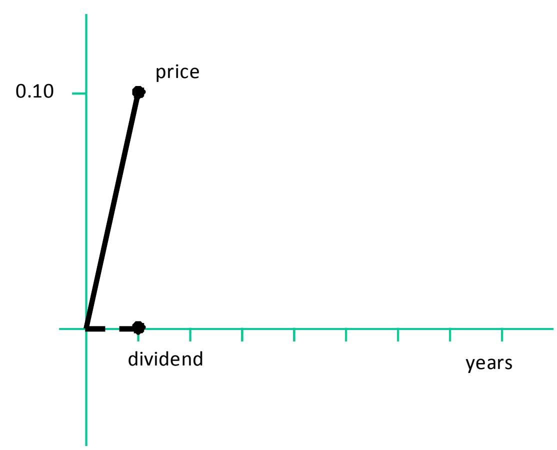
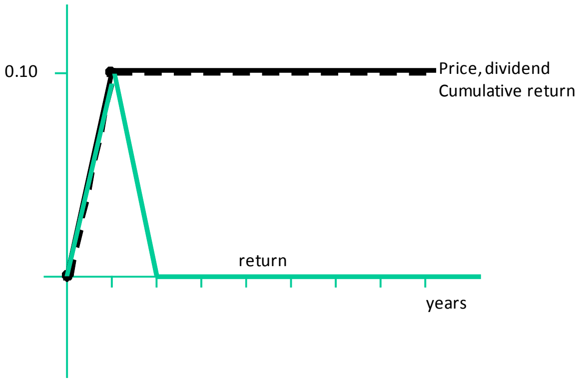
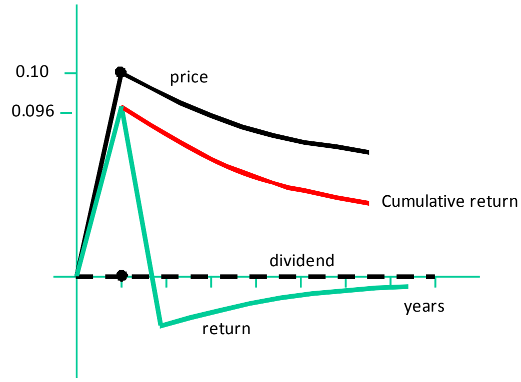


ANSWER





I hope you remember the results, but you can work it out from formulas. If $\Delta p_{t+1} = 0.1$,then

$$\begin{aligned}\Delta p_{t+1} &= -(d_{t+1} - p_{t+1}) + (d_t - p_t) + \Delta d_{t+1} \\ &= -dp_{t+1} + dp_t\end{aligned}$$

this means that dp_{t+1} goes down by -0.1. Returns are

$$r_{t+1} = -\rho dp_{t+1} + dp_t + \Delta d_{t+1}$$

The return shock is thus $r_{t+1} = \rho = 0.96$. If prices go up 10%, why isn't the return 10%, you ask? Because of dividends. The price component of returns goes up 10%, but the dividend component of returns stays the same, so returns don't rise as much as the prices.Now we have the response of dp_t

$$dp_t = (0, -0.1, -0.1\phi, -0.1\phi^2 ...)$$

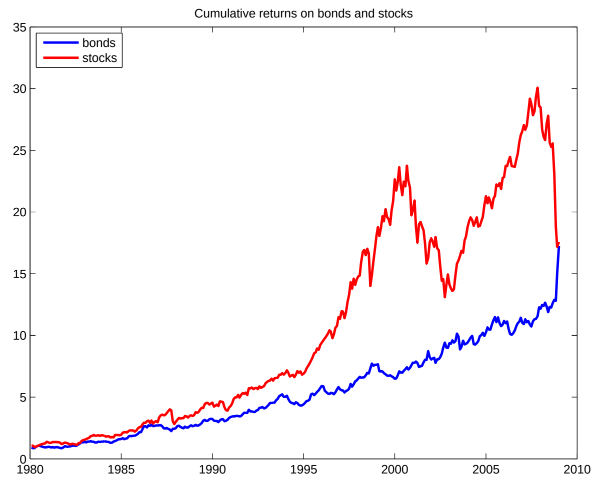
and the responses of the other variables follow

$$\begin{aligned}r_{t+1} &= -\rho(dp_{t+1}) + dp_t = 0.1 \times \left\{\rho, -(1-\rho), -(1-\rho)\phi, -(1-\rho)\phi^2 ...\right\} \\ \sum r_t &= 0.1 \times \left\{\rho, \rho - (1-\rho), \rho - (1-\rho)(1+\phi), \rho - (1-\rho)\left(1+\phi+\phi^2\right)...\right\}\end{aligned}$$

$$\begin{aligned}\Delta p_{t+1} &= -(d_{t+1} - p_{t+1}) + (d_t - p_t) + \Delta d_{t+1} \\ &= -dp_{t+1} + dp_t \\ &= \{0.1, -0.1(1-\phi), -0.1\phi(1-\phi)\} \\ & \quad p_t = \left\{0.1, 0.1\phi, 0.1\phi^2\right\}\end{aligned}$$

2009 Final Exam

(5) Here is a plot of cumulative returns on stocks and long term bonds since 1980



and the corresponding numbers. (Yes, long term bond yields fell dramatically last fall, giving that last boost to bond returns.)

	stocks	bonds	difference
mean	11.2	10.6	0.56
σ	15.7	12.4	18.9
$\sigma/\sqrt{T}$	2.91	2.30	3.51

A commentator made this graph, and said that we are in a new era; looking forward we should not expect the strong equity premium experienced up to 1980.

a) To make the counter-argument that stocks might look good relative to long term bonds looking forward, despite their recent poor performance, what other numbers would you look at? (Hint: this is not about t statistics, or about whether the longer data sample is useful.)

b) Doesn't "momentum" mean that stocks should keep going down for a while?

2. (25) Suppose that returns are predictable and dividend growth is not predictable,

$$\begin{aligned} E_t r_{t+1} &= x_t \\ x_t &= \phi x_{t-1} + \delta_t \\ \Delta d_{t+1} &= v_{t+1}, \end{aligned}$$

δ_t and v_t are all uncorrelated with each other and over time. When asked for numerical values, use $\phi = 0.94$, $\rho = 0.96$, $\sigma(\delta) = 0.01$,(1%) $\sigma(v) = 0.10$.(10%). Only rough answers are required, for example, you may approximate $0.94 \times 0.96 \approx 0.9$ and $1/0.94 \approx 1.06$.

a) Using the approximate present value formula, find the log price-dividend ratio at time t. You're looking for a very short equation with $p_t - d_t$ on the left and variables people know at t such as x_t on the right.

b) What does this model predict for the coefficient of log returns on the log dividend yield? I.e. what is b_r in

$$r_{t+1} = a_r + b_r (d_t - p_t) + \varepsilon^r_{t+1}.$$

Give both formula and number.

c) Find the unexpected return at time t. You're looking for a very short equation with unexpected return $r_t - E_{t-1} r_t$ on the left and shocks v_t, δ_t on the right.

d) In this model, what fraction of dividend yield variance comes from expected return shocks (δ), and what fraction from expected dividend growth shocks (v)? (Give the number.)

e) In this model, what fraction of the variance of unexpected returns comes from expected return shocks δ vs. what fraction comes from dividend growth shocks v? Again, find the number. If this answer is different than in part d, how do you reconcile the two answers? (Hint: what if prices move but dividends also move so there is no change in d-p?)

3. (10) (3-4 are facts questions – cite results in papers.) a) Which gets better returns going forward, stocks that had great past growth in sales, or stocks that had poor past growth in sales? Is this pattern consistent with some pattern of factor exposures (b, h, s, etc.)?

b) If you sort stocks into "winners" that went up from year -5 to one year ago, and losers that went down from year -5 to one year ago, which ones do better for the next year? Is this consistent with some pattern of factor exposures?

c) If we form a momentum portfolio, from stocks that did well last year, are the returns on that portfolio correlated with the returns on value stocks over the next year? If value stocks go up, do momentum stocks tend to go up, down, or remain the same?

4. (15) a) Here's an idea: Companies should issue stock and invest when the cost of capital is low, meaning expected returns are low. Thus, portfolios of companies that are repurchasing stock should have a lot higher returns going forward than portfolios of companies that are issuing stock. Does this idea work?

b) Wait a minute – those issuing companies have high stock prices and the repurchasers low stock prices. Surely the big issuers are growth stocks and the repurchasers are value stocks, so are we just finding that value stocks have high average returns?

c) Suppose that the answer to b is no – stocks with large issues have lower average returns, even controlling for their book/market and size. Would this fact mean that we need to form a new "issues factor," iss_t, a portfolio of all high issues firms minus low issues firms, and then run factor models that include this factor

$$R_t^{ei} = \alpha_i + b_i rmrf_t + h_i hml_t + s_i smb_t + a_i iss_t + \varepsilon_{it}?$$

Explain exactly how an extra iss_t factor might not be necessary, and what regressions you would run to check if the factor is necessary.

5. (20) The "Fed model" says that stock dividend yields and bond yields move together. Let's explore this idea. Suppose the world only lasts two periods, t and $t+1$. Log consumption growth $\Delta c_{t+1} = c_{t+1} - c_t$ has mean $E(\Delta c_{t+1}) = \mu$, variance $\sigma^2(\Delta c_{t+1}) = \sigma^2$ and is normally distributed. The investor has power utility $u(C) = \frac{C_t^{1-\gamma}}{1-\gamma}$ so the discount factor is $m_{t+1} = \beta \left(\frac{C_{t+1}}{C_t}\right)^{-\gamma} = e^{-\delta} e^{-\gamma \Delta c_{t+1}}$

a) Find the log dividend yield $d_t - p_t$ of an asset whose dividend is the same as aggregate consumption. I.e, you pay a price P_t and you get C_{t+1} at time $t+1$, with no further payments. Hint: the answer has $\delta, \gamma, \mu, \sigma^2$ in it.

b) Find the log yield $y_t^{(1)}$ of a one-period bond, which pays one for sure at time $t+1$. (Hint:$y_t^{(1)} = -p_t^{(1)}$, so find $p_t^{(1)}$ first. Don't confuse levels and logs.)

c) Are the bond yield and dividend yield the same? (Hint "it depends," but on what?)

6. (5) Your assignment is to evaluate the CAPM using the FF 25 portfolios on postwar data. One group member uses a pure time-series regression. She reports that the CAPM is lousy; the market premium is positive, but the

alphas are huge; some alphas are even bigger than the average excess returns. The other group member uses a cross-sectional approach with a free intercept. He reports that no, the CAPM is doing fine. The alphas are reasonable, though in this sample it seems the market premium came out negative. Can both of these results happen, or did one of them make a mistake? If a mistake, who made the mistake? (Illustrate your answer with an appropriate graph. Label the axes.)

7. (10) a) Fund managers claim that fees and turnover do not reduce returns to investors. How could charging more money *not* reduce returns to investors?

b) What are the facts (roughly)? How much does a 1% change in fees change returns to investors? How much does turnover – selling one stock and buying another – change returns to investors? What kind of regression would we run to answer these questions? (Write it out)

c) Carhart shows that winning funds go on to earn better returns in the next year, but that a "momentum factor" explains the pattern. Does this mean that momentum funds do well? (Cite facts from Carhart.)

8. (10)

a) Everyone else looks for skill by seeing if funds sorted on some characteristic do better than others in the future. Fama and French claim to be able to detect the presence of skill *without* knowing which funds are skilled. How can they do that ? (Simple answer, please. Just say what kind of statistic can answer this question, you don't have to recap the paper.)

b) Berk claims that it is perfectly rational for flows to chase past performance, even though returns to investors in the future are no better for funds that have done well in the past. How can this be true? (Hint: What would happen if funds did not flow to the winners?)

9. (5) A merger arbitrageur buys targets of cash deals, but cleverly offsets the target's beta by shorting index futures. He says the risk is now truly idiosyncratic. Does this work?

10. (5) A hedge fund claims to be "market neutral," since it follows dynamic long-short strategies in US equities. It's proud of its "consistent" performance in excess of the risk free rate, which is its benchmark for performance fees. The fund provides you with regressions in monthly data showing (after fees)

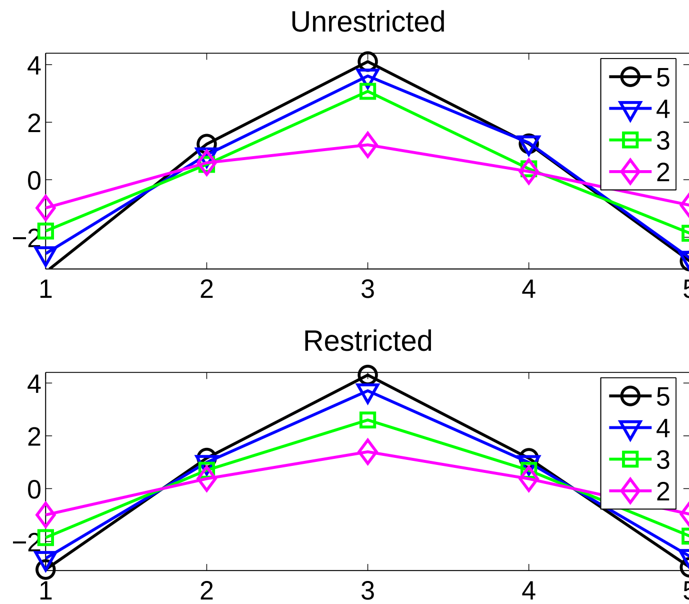
$$R_t^{ei} = \alpha_i + 0.02\ R_t^{em} + \varepsilon_{t+1} \quad (t = 1.12)$$

What other regressions (or an equivalent assessment using holdings data) would you like to see before agreeing that this fund deserves the risk free rate as a benchmark?

11. (5) a) Lamont and Thaler think Palm investors are behaving irrationally. What should these investors have done with their money rather than buy Palm?

b (5) Name two pieces of evidence that Cochrane cites for the "convenience yield" theory as opposed to the "morons" theory or the theory that "short sales constraints means that pessimists can't express their views" in explaining the high price of Palm over 3com.

12. (10) What does this picture represent? Be explicit, with equations.

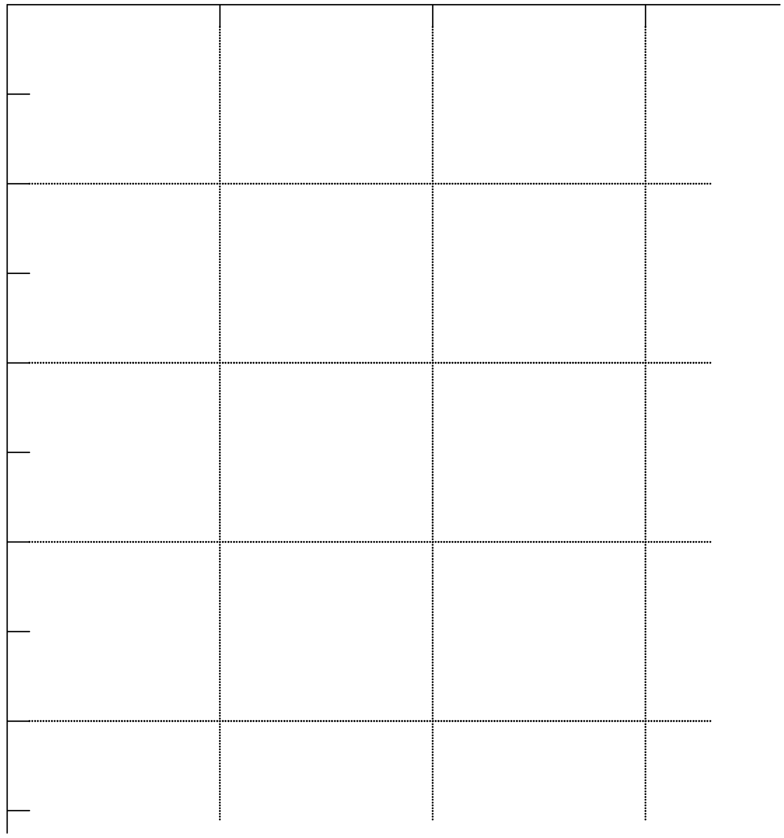


13. (15) The current one year log yield is 5%, and the current 2 and 3 year log forward rates are 10% and 15% respectively.

a) Find current log prices and yields.

b) Plot the expected log bond prices through time according to the expectations hypothesis. Label the numbers on the graph.

c) Plot expected log bond prices for the first year according to Fama and Bliss' regressions, simplifying coefficients to 1 and 0.



14. (5) If the Euro interest rate is 10% and the dollar interest rate is 5%, roughly by how much do you expect the Euro to depreciate relative to the dollar over the next year? Explain both the expectations hypothesis answer and the facts as learned in class (which may be the same thing).

15. (15) To investigate the carry trade, Verdelhan, Lustig and Roussanov (no, you didn't read this) formed 6 portfolios of "high interest spread" countries to "low interest spread" countries, and tracked the excess returns to a dollar investor from investing in their currencies over time. The mean annual returns on these 6 portfolios (percent per year) are

$$\begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ -2.92 & 0.02 & 1.40 & 3.66 & 3.54 & 5.90 \end{bmatrix}$$

they performed an eigenvalue decomposition of the covariance matrix of these returns,

$$Q\Lambda Q' = cov(R^e, R^{e\prime})$$

Here are their results:

$$Q = \begin{bmatrix} 0.43 & 0.41 & -0.18 & 0.31 & 0.72 & 0.03 \\ 0.39 & 0.26 & -0.14 & -0.02 & -0.44 & 0.75 \\ 0.39 & 0.26 & -0.46 & -0.38 & -0.31 & -0.57 \\ 0.38 & 0.05 & 0.72 & -0.56 & 0.16 & -0.01 \\ 0.42 & -0.11 & 0.38 & 0.66 & -0.37 & -0.31 \\ 0.43 & -0.82 & -0.28 & -0.10 & 0.18 & 0.11 \end{bmatrix}$$

$$100 \times \frac{\lambda_i}{\sum \lambda_i} = \begin{bmatrix} 70 & 12 & 6.2 & 4.5 & 3.8 & 3.2 \end{bmatrix}$$

(the latter matrix is scaled up to sum to 100 for better viewing).

a) What does the Q matrix mean, precisely? I.e., what are the two interpretations of the columns or rows of Q?

b) Suppose you want to invest in the carry trade. What do these results tell you about the risks you face? For example, are the portfolio returns pretty uncorrelated? If not, can you describe the nature of some of the risk from these results?

c)How many factors would use to describe currency returns?

16. (15) You are considering investing in two assets, and of course the market index. You have a mean-variance objective with risk aversion $\gamma = 2$. Your assessment of the market portfolio is a mean $E(R^{em}) = 8\%$, volatility $\sigma(R^{em}) = 20\%$. You run CAPM regressions for the two assets

$$R_t^{ei} = \alpha_i + \beta_{im} R_t^{em} + \varepsilon_t^i$$

with result $\beta_1 = 1, \beta_2 = 1$; $\sigma(\varepsilon) = 10\%$ for both assets, and the residuals ε are uncorrelated. Your fundamental analysis gives $\alpha_1 = -2\%$, $\alpha_2 = 4\%$. Find the optimal allocation to the market index and to the two assets. All numbers are on an annualized basis. (Hint: Be careful about units, i.e. should you express 10% as 10, 1.10, 0.10?)

a) (optional) Express the answer any way you want, i.e. in terms of beta-neutral portfolios.

b) Find the weights in the actual assets, i.e. $R^{ep} = R^f + xR^{em} + yR^{e1} + zR^{e2}$

Answer Sketch

1. a) Look at dividend yields and bond yields. D/P is higher and bond yields much lower so expected returns are much better for stocks relative to bonds. More generally, we learned to look at price-based forecasts rather than past returns to learn about expected returns.

b) Momentum refers to a cross-sectional pattern, not the market. Stocks that did well go up, stocks that did badly go down. But there is almost no tendency for the market as a whole to keep going down after a fall. ($R_{t+1} = a + bR_t + \varepsilon_{t+1}$ has tiny b, R^2).

2. a)

$$p_t - d_t = E_t\left(\sum_{j=1}^{\infty} \rho^{j-1}\Delta d_{t+j} - \sum_{j=1}^{\infty} \rho^{j-1} r_{t+j}\right) = -\sum_{j=1}^{\infty} \rho^{j-1}\phi^{j-1} x_t = -\frac{1}{1-\rho\phi} x_t$$

b)

$$\begin{aligned} E_t r_{t+1} &= x_t = (1-\rho\phi)(d_t - p_t) \\ b_r &= 1 - \rho\phi \ \approx 0.1 \end{aligned}$$

c)

$$\begin{aligned} r_t - E_{t-1} r_t &\approx (E_t - E_{t-1}) \left[\Delta d_t + \sum_{j=1}^{\infty} \rho^{j-1}\Delta d_{t+j} - \sum_{j=1}^{\infty} \rho^{j-1} r_{t+j}\right] \\ &\approx (E_t - E_{t-1}) \left[\Delta d_t - \frac{1}{1-\rho\phi} x_t\right] \\ &= v_t - \frac{1}{1-\rho\phi}\delta_t \end{aligned}$$

d) 100% from expected returns.There are lots of ways to see this. Most easily, from 2a, you see that $p_t - d_t$ is proportional to x_t, so all variation in $p_t - d_t$ must come from variation in x_t, which IS expected returns. More subtly, you can make contact with the variance decompositions we studied in class. $b_d = 0$, so $cov(p_t - d_t, \Delta d_{t+j}) = 0$.

e)

$$\begin{aligned}
\sigma^2\left(r_t - E_{t-1}r_t\right) &= \sigma_v^2 + \left(\frac{1}{1-\rho\phi}\right)^2 \sigma_\delta^2 \\
&= \sigma_v^2 + \left(\frac{1}{1-0.96\times 0.94}\right)^2 \sigma_\delta^2 \\
&= \sigma_v^2 + \left(\frac{1}{0.1}\right)^2 \sigma_\delta^2 \\
&= \sigma_v^2 + 100\times\sigma_\delta^2 \\
&= 0.10^2 + 100\times(0.01)^2 \\
&= 0.01 + 0.01
\end{aligned}$$

So it's half and half. Why the difference? Return variance also comes from *current* dividend growth in it. If prices and dividends move there is no movement in pd, but there is a big return. pd only moves on news of expected *future* dividend growth.

3. a) Fama and French Multifactor anomalies. Low sales growth companies have high returns, which corresponds to value betas.

b) FF winners do badly, again corresponding to value betas.

c) Winners act like growth stocks, so have negative correlation with hml. This is what made momentum such a disaster for Fama and French.

4. a) Yes, Fama and French Dissecting anomalies. This is the "net issue" effect.

b) FF "characteristic adjusted" the returns meaning their portfolios are net of similar stocks by value characteristics. Thus, this helps in addition to value to forecast returns.

c) It's quite possible that the new sort is explained by value betas, just as say sales growth was. So, first run

$$R_t^{ei} = \alpha_i + b_i rmrf_t + h_i hml_t + \varepsilon_{it}$$

and see if h is high where issue-sorted average returns are high. Then, it's likely that issue returns are correlated with hml, so run the standard "can we drop a factor" test,

$$iss_t = \alpha_{ac} + b_{ac} rmrf_t + h_{ac} hml_t + \varepsilon_{act}$$

and see if that α is zero. More generally, you want to see if the α with the new factor included are lower than the old FF3F alphas. (Since they "characteristic adjusted" it's likely that this test will not work, but it's still possible. The main point is that average returns *can* still exist without being anomalies, if the high average returns correspond to high values of known betas.)

5. a)

$$\begin{aligned}
\frac{P_t}{C_t} &= E\left[\beta\left(\frac{c_{t+1}}{c_t}\right)^{-\gamma}\frac{c_{t+1}}{c_t}\right] \\
&= E\left[e^{-\delta}e^{(1-\gamma)\Delta c_{t+1}}\right] \\
&= E\left[e^{-\delta}e^{(1-\gamma)E(\Delta c_{t+1})+\frac{1}{2}(1-\gamma)\sigma^2_{\Delta c_{t+1}}}\right] \\
p_t - c_t &= -\delta + (1-\gamma)E(\Delta c_{t+1}) + \frac{1}{2}(1-\gamma)^2\sigma^2_{\Delta c_{t+1}} \\
d_t - p_t &= \delta + (\gamma-1)E(\Delta c_{t+1}) - \frac{1}{2}(\gamma-1)^2\sigma^2_{\Delta c_{t+1}} \\
d_t - p_t &= \delta + (\gamma-1)\mu - \frac{1}{2}(\gamma-1)^2\sigma^2
\end{aligned}$$

b)

$$
\begin{aligned}
P_t^{(1)} &= E\left[\beta\left(\frac{c_{t+1}}{c_t}\right)^{-\gamma}\right] \\
p_t^{(1)} &= -\delta-\gamma E(\Delta c_{t+1})+\frac{1}{2}\gamma^2\sigma^2_{\Delta c_{t+1}} \\
y_t^{(1)} &= \delta+\gamma E(\Delta c_{t+1})-\frac{1}{2}\gamma^2\sigma^2_{\Delta c_{t+1}} \\
y_t^{(1)} &= \delta+\gamma\mu-\frac{1}{2}\gamma^2\sigma^2
\end{aligned}
$$

c) No. They're related, especialy for very high γ. But if $\gamma = 1$, you have a constant dividend yield and a varying bond yield.

6) This happened on a problem set. The cross section has a huge intercept, a negative market premium and thus small alphas. The graph should have $E(R^{ei})$ vs. β_i.The time series imposes a positive market premium – look for a line going through the origin and rmrf exactly – and thus has huge alphas. In a cross sectional regression, momentum would have looked good, because you could fit a negative HML premium!

7) a) the claim is that fees pay for superior ability, and that turnover is losing dogs and buying good stocks so helps.

b) Carhart table. A 1% increase in costs corresponds at least to 1% loss to investors and maybe more. The tecnhique is Cross-sectional (FMB) regression. In a simple form,

$$E(R^{ei}_{t+1}) = a + b\times E\left(\text{costs}^i_t\right)+\varepsilon_i$$

c) Carhart reports that sorts on pr1yr betas don't give anything. It looks like trading on momentum has too much costs for the funds

8)

a) They look at the distribution of alpha t statistics. Roughly, if there is no skill, we should only see 5% of alpha t stats more than 2. In fact we see a very small excess number with t stats greater than 2, and a very large excess number with t stats less than -2. This indicates that the distribution of "true skill" is centered below zero with about a 1% standard deviation

b) According to Berk there is skill, revealed in past returns. Then funds flow in until the subsequent returns are back to normal, after fees and crowding of the trade. If funds had not flown in, we would have seen higher returns.

9. This doesn't control for implicit put option since the "down beta" will be bigger. Cite put option fact, maybe with graph, per Mitchell Pulvino is good for half credit.

10. Up/ down betas especially, betas with 3 months of lags, betas on additional factors like hml, smb, umd, term, def. Cite Asness and lecture.

11. a) Buy 3 com and wait.

b) i) High prices occur with high volume; ii) 3com price fell on opening iii) price deviation went down as short volume expanded.

12. These are Cochrane-Piazzesi regressions. The top one is unrestricted

$$rx^{(n)}_{t+1} = a^{(n)}+\beta^{(n)}_1 y^{(1)}_t+\beta^{(n)}_2 f^{(2)}_t+..+\beta^{(n)}_5 f^{(5)}_t+\varepsilon^{(n)}_{t+1}$$

this is a plot of $\beta^{(n)}$, each line a different $\not{n}$. The bottom one is restricted,

$$rx_{t+1}^{(n)} = b_n \left[\gamma_0 + \gamma_1 y_t^{(1)} + \gamma_2 f_t^{(2)} + .. + \gamma_5 f_t^{(5)}\right] + \varepsilon_{t+1}^{(n)}$$

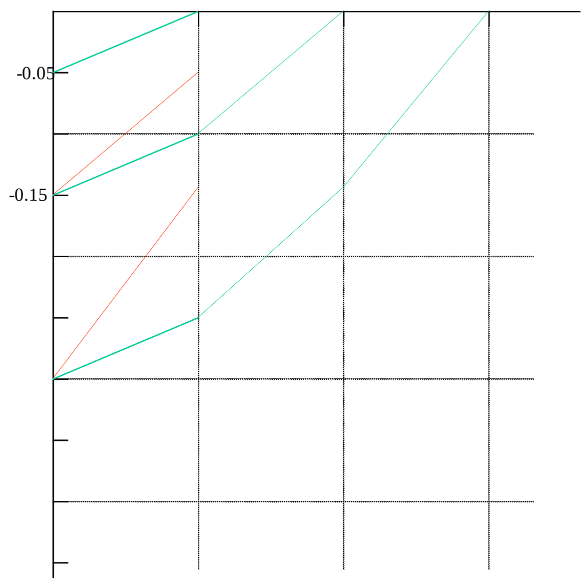
The plot is $b_n \gamma$.

13. a)

$$\begin{aligned} y^{(1)} &= 0.05;\ f^{(2)} = 0.10;\ f^{(3)} = 0.15 \\ p^{(1)} &= -0.05;\ p^{(2)} = -0.15; p^{(3)} = -0.30 \\ y^{(1)} &= 0.05; y^{(2)} = 0.075;\ y^{(3)} = 0.10 \end{aligned}$$

b)

c) $f^{(2)} - y^{(1)} = 5\%$ so the 2 year bond returns $rx^{(2)} = 5\%$ or $r^{(2)} = 10\%. f^{(3)} - y^{(1)} = 10\%$ so $r^{(3)} = 15\%$



14. Expectations says -5%, the facts say somewhere between 0 and +5%.

15. a) The columns of Q are loadings – how much each portfolio moves if a factor moves – and also weights – how to construct factors from the portfolio returns. The first is a “dollar depreciation factor.” The second reflects the chance that all the high interest rate countries go one way and all the low rate countries go another way, a “country risk” factor.

b) There is a big risk that they all go down together. Then there is a risk that the high expected returns go down and the low expected returns go up, or vice versa. This makes variance larger.

c) Level or "dollar appreciation”, slope, and curve. I’d stop there, as the variances get small and the patterns disappear, but so long as you say "variance" and "pattern" you can stop wherever you want.

16.

$$w^m = \frac{1}{\gamma}\frac{E(R^{em})}{\sigma^2(R^{em})} = \frac{1}{\gamma}\frac{E(R^{em})}{0.20^2} = \frac{1}{2}\frac{0.08}{0.04} = 1$$

$$
\begin{aligned}
w_\alpha &= \frac{1}{\gamma}\Sigma^{-1}\alpha = \frac{1}{2}\begin{bmatrix} 0.1^2 & 0 \\ 0 & 0.1^2 \end{bmatrix}^{-1}\begin{bmatrix} \alpha_1 \\ \alpha_2 \end{bmatrix} \\
&= \frac{1}{2}(100)I\begin{bmatrix} -0.02 \\ 0.04 \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \end{bmatrix}
\end{aligned}
$$

Thus, you should hold

$$
\begin{aligned}
R^{ep} &= R^{em} - 1(R^{e1} - R^{em}) + 2(R^{e2} - R^{em}) \\
&= 0R^{em} - 1R^{e1} + 2R^{e2}
\end{aligned}
$$

Fall 2007 Final Exam

1.(10) a) Historically, in round numbers, what are the historical mean, standard deviation and Sharpe ratio of US stocks at a i) annual horizon 2) monthly horizon ? (You may assume returns are independent over time to connect horizons.)

b) In annual regressions, we found

$$\begin{aligned} r_{t+1} &= a_r + 0.1 \times (d_t - p_t) + \varepsilon^r_{t+1}. \\ (d_{t+1} - p_{t+1}) &= a_{dp} + 0.9 \times (d_t - p_t) + \varepsilon^{dp}_{t+1}. \end{aligned}$$

In round numbers, what do you expect for the regression coefficient of *two-year* total log returns on the log dividend yield?

c) If stock market prices really go up this year – good returns – will this price movement continue upwards ("momentum"), largely disappear in the long run ("mean-reversion"), or is it expected to stay where it is ("random walk")? What other piece of information would really help to answer this question?

2) (15) Suppose that returns are predictable and dividend growth is not predictable,

$$\begin{aligned} E_t r_{t+1} &= x_t \\ x_t &= \phi x_{t-1} + \delta_t \\ \Delta d_{t+1} &= v_{t+1}, \end{aligned}$$

ε_t, δ_t and v_t are all uncorrelated with each other and over time. Use $\phi = 0.94$, $\rho = 0.96$, $\sigma(\delta) = 0.01$,(1%) $\sigma(v) = 0.10$.(10%).

a) Using the approximate price-dividend formula find the log price-dividend ratio at time t. (As always with these formulas, ignore constants and means.) You're looking for an equation with $p_t - d_t$ on the left and variables you can observe at t such as x_t on the right.

b) Using the approximate return formula, find the unexpected return at time t. You're looking for a formula with unexpected return on the left and shocks on the right.

c) In this model, what fraction of dividend yield variance comes from expected returns, and what fraction from expected dividend growth?

d) In this model, what is the standard deviation of unexpected returns? What fraction of unexpected return variance comes from expected returns vs. what fraction comes from dividend growth? If this answer is different than for dp in part d, how do you reconcile the two answers?

(Feel free to approximate the numerical part of your answer, for example $0.96 \times 0.94 \approx 0.90$; $1/0.96 \approx 1.04$, etc. One digit accuracy is enough, i.e. 15% not 15.340489%)

3) (5) a) Which gets better returns going forward, stocks that had great past growth in sales, or stocks that had poor past growth in sales? Is this pattern consistent with some pattern of betas?

b) If you sort stocks into "winners" that went up from year -5 to one year ago, and losers that went down from year -5 to one year ago, which ones do better for the next year? Is this consistent with some pattern of betas?

4) (5) a) Does Fama and French's "dissecting anomalies" find any characteristics that can tell you which stocks have high vs. low returns, beyond the value, size, and momentum that we know about, or are all the other characteristics subsumed by value, size, and momentum characteristics (outside tiny stocks)?

b) How is their "characteristic adjustment" different from a "beta adjustment?"

5) (5) You form a new "factor", amz, formed from first letters of stock tickers. It's the returns on the (a-l) portfolio minus returns on the (m-z) portfolio. You run a regression of the 26 letter-of-the-alphabet portfolios on the market and your new factor amz,

$$R^{ei}_{t+1} = \alpha_i + \beta_{im} rmrf_{t+1} + \gamma_i amz_{t+1} + \varepsilon^i_{t+1}$$

You get very strong γ_i estimates with t statistics of 5, with big positive γ_i values for the a,b,c stocks, and big negative γ_i values for x,y,and z stocks. Is there some way to rescue the CAPM, or do we need to add amz as a factor?

6) (10) Suppose the representative investor has quadratic utility $u(c) = -\frac{1}{2}(c^* - c_t)^2$ rather than power utility. a) How does the riskfree interest rate relate to consumption in this case? b) Assuming c_t and c_{t+1} are less than c^*, does the interest rate rise or decline if $E_t c_{t+1}$ becomes larger? Explain the intuition. c) What is the percent interest rate if $c_t = c^*$, but $c_{t+1} < c^*$ (and hence $E_t c_{t+1} < c^*$)? Explain the intuition.

7) (10) Your assignment is to evaluate the CAPM using the FF 25 portfolios on postwar data. One group member uses a pure time-series regression. She reports that the CAPM is lousy; the market premium is positive, but the alphas are huge; some alphas are even bigger than the average excess returns. The other group member uses a cross-sectional approach with a free intercept. He reports that no, the CAPM is doing fine. The alphas are reasonable, though in this sample it seems the market premium came out negative. Can both of these results happen, or did one of them make a mistake? If a mistake, who made the mistake? (Illustrate your answer with an appropriate graph. Label the axes.)

8) (5) Carhart sorted mutual funds by past one year returns. If you sort by past 5 year returns instead, a) Do the past good funds do better in the future? b) Is this effect stronger or weaker than Carhart's sort on past one year returns – does a 5 year return produce a better indicator of skill? c) Do you see a change in the pattern of 4 factor alphas going forward?

9) (5) How do mutual fund managers benefit from connections formed during school? Cite facts.

10) (5) A merger arbitrageur says, "We know about beta. We hedge using index futures. On cash deals, we offset the acquirer beta. On stock deals, we offset the difference between acquirer and target beta." Does this make their risk truly idiosyncratic, so that cash (interest rate) is the right benchmark?

11) (5) What's the difference between "backfill bias" and "survivor bias?" How bad are these for hedge funds?

12) (5) A hedge fund claims to be "market neutral," since it follows dynamic long-short strategies in US equities. It's proud of its "consistent" performance in excess of the risk free rate, which is its benchmark for performance fees. The fund provides you with regressions in monthly data showing (after fees)

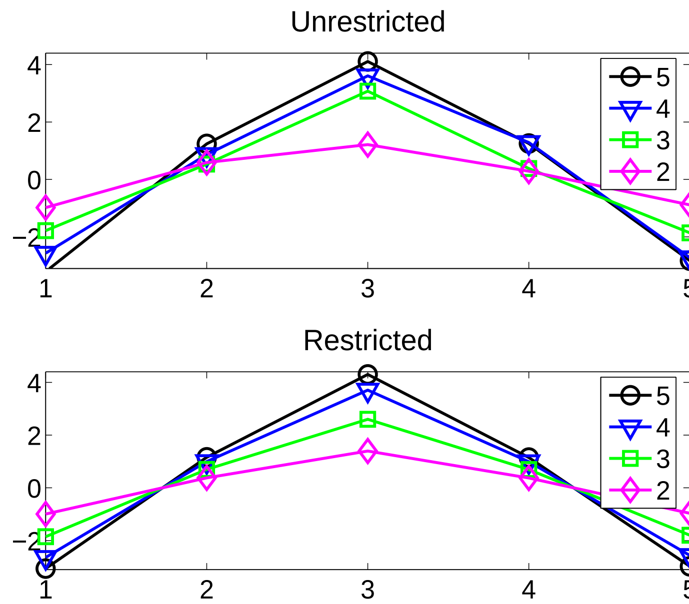
$$R_t^{ei} = \alpha_i + 0.02\ R_t^{em} + \varepsilon_{t+1} \quad (t = 1.12)$$

What other regressions (or an equivalent assessment using holdings data) would you like to see before agreeing that this fund deserves the risk free rate as a benchmark?

13) (5) Lamont and Thaler think Palm investors are behaving irrationally. What should these investors have done with their money rather than buy Palm?

14) (5) What is the single most important piece of evidence that Cochrane cites for the "convenience yield" theory as opposed to the "morons" theory or the theory that "short sales constraints means that pessimists can't express their views" in explaining the high price of Palm over 3com?

15. (10) What does this picture represent? Be explicit, with equations.

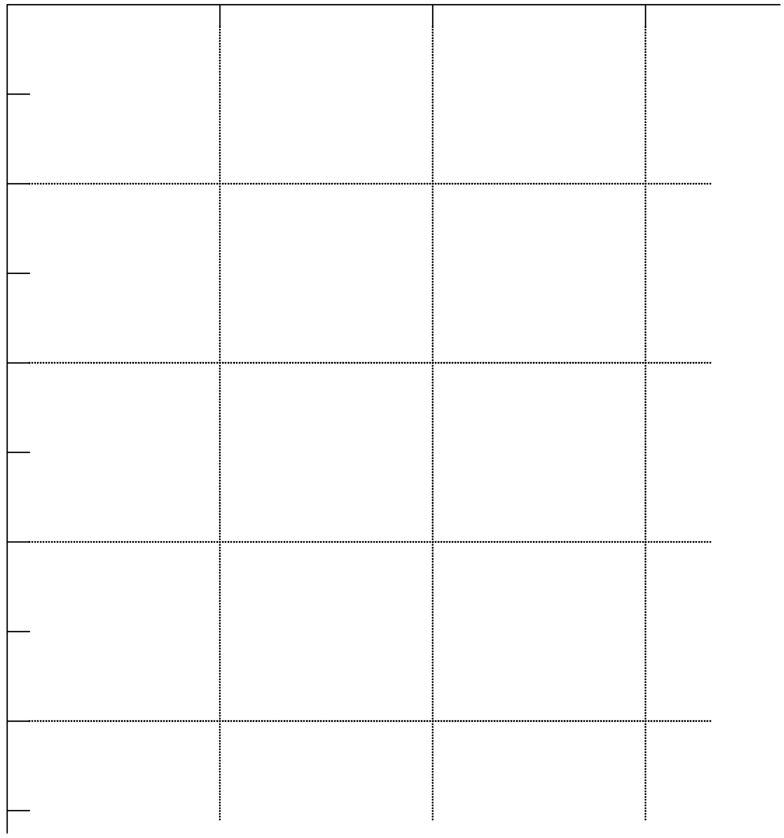


16. (15) The current one year log yield is 5%, and the current 2 and 3 year forward rates are 10% and 15% respectively.

a) Find current log prices and yields

b) Plot the expected log bond prices through time according to the expectations hypothesis.

c) Plot the first year of expected log bond prices through time according to Fama and Bliss' regressions, simplifying coefficients to 1 and 0.



17. (5) If the Euro interest rate is 10% and the dollar interest rate is 5%, roughly by how much do you expect the Euro to depreciate relative to the dollar over the next year? Explain

18. (5) Brandt and Kavaiecz show that this morning's signed volume helps to predict this afternoon's yields. What is their interpretation of this phenomenon, and the other possible interpretations? Give one (there are three) central pieces of evidence for their interpretation.

19. (15) You are considering investing in two assets, and of course the market index. You have a mean-variance objective with risk aversion $\gamma = 2$. Your assessment of the market portfolio is a mean $E(R^{em}) = 8\%$, volatility $\sigma(R^{em}) = 20\%$. You run CAPM regressions for the two assets

$$R_t^{ei} = \alpha_i + \beta_{im} R_t^{em} + \varepsilon_t^i$$

with result $\beta_1 = 1, \beta_2 = 1$; $\sigma^2(\varepsilon) = 10\%$ for both assets, and the residuals are correlated with $\rho_{\varepsilon_1,\varepsilon_2} = -0.5$. Your fundamental analysis gives $\alpha_1 = -1\%$, $\alpha_2 = 4\%$. Find the optimal allocation to the market index and to the two assets. All numbers are on an annualized basis. (Hint: Be careful about units, i.e. should you express 10% as 10, 1.10, 0.10?) Explain any non-intuitive findings intuitively.

207 Answer Sketch

1. a) Dividend yields and bond yields. D/P is higher and bond yileds much lower so expected returns are much better for stocks.

b) Momentum refers to a cross-sectional pattern, not the market. Stocks that did well go up, stocks that did badly go down, but not for the market as a whole.

a) Annual somewhere near 8%/16% for SR = 0.5. Monthly, did you scale correctly, $8\%/12 = 2/3\%$, $16/\sqrt{12} = 4.6\%$ $.5/\sqrt{12} = 0.14$

b) $0.1 + 0.9 \times 0.1 = 0.19$

c) There is very slight mean reversion of prices taken alone. You want to see dividends – if there is no movement in dividends then mean reversion is much stronger effect. Reference $r_{t+1} = a + b(d/p)_t + \varepsilon_{t+1}$, and better yet the impulse-response functions from problem set 2.

2) a)

$$p_t - d_t = E_t\left(\sum_{j=1}^{\infty}\rho^{j-1}\Delta d_{t+j} - \sum_{j=1}^{\infty}\rho^{j-1}r_{t+j}\right) = -\sum_{j=1}^{\infty}\rho^{j-1}\phi^{j-1}x_t = -\frac{1}{1-\rho\phi}x_t$$

b)

$$\begin{aligned}
r_t - E_{t-1}r_t &\approx (E_t - E_{t-1})\left[\Delta d_t + \sum_{j=1}^{\infty}\rho^{j-1}\Delta d_{t+j} - \sum_{j=1}^{\infty}\rho^{j-1}r_{t+j}\right] \\
&\approx (E_t - E_{t-1})\left[\Delta d_t - \frac{1}{1-\rho\phi}x_t\right] \\
&= v_t - \frac{1}{1-\rho\phi}\delta_t
\end{aligned}$$

c) 100% from expected returns.

d)

$$\begin{aligned}
\sigma^2\left(r_t - E_{t-1}r_t\right) &= \sigma_v^2 + \left(\frac{1}{1-\rho\phi}\right)^2\sigma_\delta^2 \\
&= \sigma_v^2 + \left(\frac{1}{1-0.96\times 0.94}\right)^2\sigma_\delta^2 \\
&= \sigma_v^2 + \left(\frac{1}{0.1}\right)^2\sigma_\delta^2 \\
&= \sigma_v^2 + 100\times\sigma_\delta^2 \\
&= 0.10^2 + 100\times(0.01)^2 \\
&= 0.01 + 0.01 \\
&= 0.02 \\
\sigma\left(r_t - E_{t-1}r_t\right) &= \sqrt{0.02} = 0.14
\end{aligned}$$

So it's i) 14%, ii) about half and half ($.10^2$ and $.14^2$) iii) this one has current dividend growth in it. If prices and dividends move there is no movement in pd, but there is a big return.

3. a) FF low sales have high returns, which corresponds to value betas. b) FF winners do badly, again corresponding to value betas.

4. a) Yes, stock issues is the strongest. b) They don't run betas on hml and smb, they instead subtract off the returns of portfolios with similar values of book/market and size.

5. γ is Totally irrelevant, like the "industry portfolio" example. If amz is uncorrelated with the market and has mean zero, this is completely consistent with the CAPM. The point is to see if you fall into the trap of thinking betas and the TS regression matters, rather than focus on alphas. It's nice if they go on to say how to test – run amz on rmrf – but not necessary.

6 a)

$$R_t^f = 1/E_t(m_{t+1}) = \frac{1}{E_t\left(\beta \frac{c^*-c_{t+1}}{c^*-c_t}\right)} = \frac{1}{\beta}\frac{c^*-c_t}{E_t\left(c^*-c_{t+1}\right)}$$

b) as c_{t+1} rises the denominator declines so the interest rate rises. As in the power case, we have to bribe people not to consume now if they know they will be rich in the future.

c) $R^f = 0$ meaning $r^f = -100\%$. At c^* you are completely stuffed today – additional consumption has no value. But you're going to be a bit hungry tomorrow. You would give up a million bucks today to have a breadcrumb tomorrow. You'll only accept this situation if the interest rate is -100% – if saving makes no difference at all.

7) This happened on a problem set. The cross section has a huge intercept, a negative market prmium and thus small alphas. The graph should have $E(R^{ei})$ vs. β_i.The time series imposes a postive market premium – look for a line going through the origin and rmrf exactly – and thus has huge alphas.

8. Problem set. 5 year returns give about the same ER spread, and no change in 4 factor alphas.

9. Frazzini et al. Point to numbers in Table IV, given as part of cheat sheet.

10. This doesn't control for implicit put option since the "down beta" will be bigger. Cite put option fact, maybe with graph, per Mitchell Pulvino is good for half credit.

11. Malkiel, numbers from cheat sheet.

12. Up/ down betas especially, betas with 3 months of lags, betas on additional factors like hml, smb, umd, term, def. Cite Asness and lecture.

13. Buy 3 com and wait. b) High prices occur with high volume.

14. CP regressions. Top is unrestricted

$$rx_{t+1}^{(n)} = a^{(n)} + \beta_1^{(n)} y_t^{(1)} + \beta_2^{(n)} f_t^{(2)} + .. + \beta_5^{(n)} f_t^{(5)} + \varepsilon_{t+1}^{(n)}$$

this is a plot of $\beta^{(n)}$, each line a different $\cancel{n}$. bottom is restricted,

$$rx_{t+1}^{(n)} = b_n \left[\gamma_0 + \gamma_1 y_t^{(1)} + \gamma_2 f_t^{(2)} + .. + \gamma_5 f_t^{(5)}\right] + \varepsilon_{t+1}^{(n)}$$

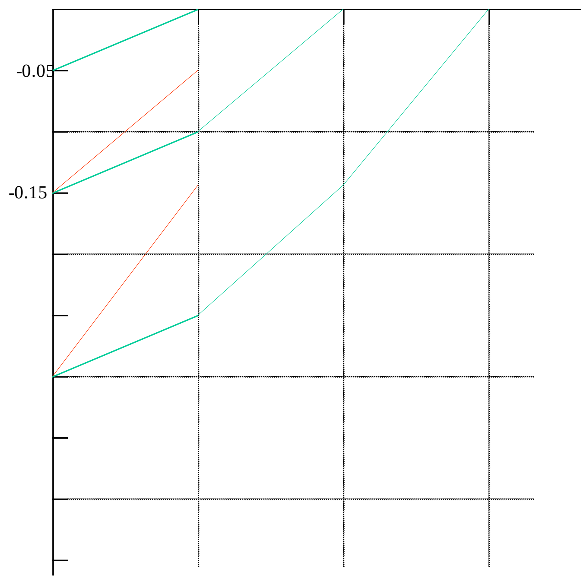
plot is $b_n \gamma$.

16. a)

$$\begin{aligned} y^{(1)} &= 0.05;\ f^{(2)} = 0.10;\ f^{(3)} = 0.15 \\ p^{(1)} &= -0.05;\ p^{(2)} = -0.15; p^{(3)} = -0.30 \\ y^{(1)} &= 0.05; y^{(2)} = 0.075;\ y^{(3)} = 0.10 \end{aligned}$$

b)

c) $f^{(2)} - y^{(1)} = 5\%$ so the 2 year bond returns $rx^{(2)} = 5\%$ or $r^{(2)} = 10\%.f^{(3)} - y^{(1)} = 10\%$ so $r^{(3)} = 15\%$



17. Expectations says -5%, facts say nothing to +5%.

18. "Price discovery" vs "price pressure". Cheat sheet table 5 year volume forecasts the 1 year change in yield, evidence for the former.

19.

$$w^m = \frac{1}{\gamma}\frac{E(R^{em})}{\sigma^2(R^{em})} = \frac{1}{\gamma}\frac{E(R^{em})}{0.20^2} = \frac{1}{2}\frac{0.08}{0.04} = 1$$

$$\begin{aligned}
w_\alpha &= \frac{1}{\gamma}\Sigma^{-1}\alpha = \frac{1}{2}\frac{1}{0.01}\begin{matrix} 1 & \rho \\ \rho & 1 \end{matrix}^{-1}\begin{bmatrix} \alpha_1 \\ \alpha_2 \end{bmatrix} \\
&= \frac{1}{2}\frac{1}{0.01}\frac{1}{1-\rho^2}\begin{matrix} 1 & -\rho \\ -\rho & 1 \end{matrix}\begin{bmatrix} \alpha_1 \\ \alpha_2 \end{bmatrix} \\
&= \frac{1}{2}\frac{1}{0.01}\frac{1}{0.75}\begin{matrix} 1 & 0.5 \\ 0.5 & 1 \end{matrix}\begin{bmatrix} -0.01 \\ 0.04 \end{bmatrix} \\
&= \frac{1}{2}\frac{1}{0.01}\frac{1}{0.75}\begin{matrix} 0.01 \\ 0.035 \end{matrix} \\
&= \frac{1}{2}\frac{1}{0.75}\begin{bmatrix} 1 \\ 3.5 \end{bmatrix} = \begin{bmatrix} 0.67 \\ 2.33 \end{bmatrix}
\end{aligned}$$

This expresses how much you should invest in $\alpha^i = R^{ei} - \beta_i R^{em}$. This is fine. You can also express the answer in terms of the fundamental assets.

Review Problems

0.1 Stock market prediction – models

1. Suppose expected returns as an AR(1) and assume dividend growth is not predictable,

$$\begin{aligned} r_{t+1} &= x_t + \varepsilon_{t+1} \\ x_{t+1} &= \phi x_t + \delta_{t+1} \\ \Delta d_{t+1} &= v_{t+1}. \end{aligned}$$

$x_t =$ expected returns, since $E_t(r_{t+1}) = x_t$.

   (a) Use the approximate price-dividend formula

$$p_t - d_t = E_t \left(\sum_{j=1}^{\infty} \rho^{j-1} \Delta d_{t+j} - \sum_{j=1}^{\infty} \rho^{j-1} r_{t+j} \right)$$

   What is the relation between dividend-price ratios and expected returns x_t? (A: $d_t - p_t = \frac{1}{1-\rho\phi} x_t$.)

   (b) In this model, what will be the regression coefficient of actual returns on the dividend-price ratio? (A: $1-\rho\phi$ but you have to say why.)

   (c) Is the coefficient of returns on the dividend price ratio larger or smaller as expected returns become more persistent? Give the intuition for your answer. (A: as ϕ rises, the coefficient declines. An expected return that lasts a long time has a greater impact on d-p. A bigger impact of 1% Er on d-p means a smaller coefficient of actual return on d-p.)

2. (25) Suppose that returns are predictable and dividend growth is not predictable,

$$\begin{aligned} E_t r_{t+1} &= x_t \\ x_t &= \phi x_{t-1} + \delta_t \\ \Delta d_{t+1} &= v_{t+1}, \end{aligned}$$

δ_t and v_t are all uncorrelated with each other and over time. When asked for numerical values, use $\phi = 0.94$, $\rho = 0.96$, $\sigma(\delta) = 0.01$,(1%) $\sigma(v) = 0.10$.(10%). Only rough answes are required, for example, you may approximate $0.94 \times 0.96 \approx 0.9$ and $1/0.94 \approx 1.06$.

   (a) Using the approximate present value formula, find the log price-dividend ratio at time t. You're looking for a very short equation with $p_t - d_t$ on the left and variables people know at t such as x_t on the right.

   (b) What does this model predict for the coefficient of log returns on the log dividend yield? I.e. what is b_r in

$$r_{t+1} = a_r + b_r (d_t - p_t) + \varepsilon^r_{t+1}.$$

   Give both formula and number.

   (c) Find the unexpected return at time t. You're looking for a very short equation with unexpected return $r_t - E_{t-1} r_t$ on the left and shocks v_t, δ_t on the right.

   (d) In this model, what fraction of dividend yield variance comes from expected return shocks (δ), and what fraction from expected dividend growth shocks (v)? (Give the number.)

(e) In this model, what fraction of the variance of unexpected returns comes from expected return shocks δ vs. what fraction comes from dividend growth shocks v? Again, find the number. If this answer is different than in part d, how do you reconcile the two answers? (Hint: what if prices move but dividends also move so there is no change in d-p?)

ANSWER

(a)

$$p_t - d_t = E_t\left(\sum_{j=1}^{\infty}\rho^{j-1}\Delta d_{t+j} - \sum_{j=1}^{\infty}\rho^{j-1}r_{t+j}\right) = -\sum_{j=1}^{\infty}\rho^{j-1}\phi^{j-1}x_t = -\frac{1}{1-\rho\phi}x_t$$

(b)

$$\begin{aligned} E_t r_{t+1} &= x_t = (1-\rho\phi)(d_t - p_t) \\ b_r &= 1-\rho\phi \ \approx 0.1 \end{aligned}$$

(c)

$$\begin{aligned} r_t - E_{t-1}r_t &\approx (E_t - E_{t-1})\left[\Delta d_t + \sum_{j=1}^{\infty}\rho^{j-1}\Delta d_{t+j} - \sum_{j=1}^{\infty}\rho^{j-1}r_{t+j}\right] \\ &\approx (E_t - E_{t-1})\left[\Delta d_t - \frac{1}{1-\rho\phi}x_t\right] \\ &= v_t - \frac{1}{1-\rho\phi}\delta_t \end{aligned}$$

(d) 100% from expected returns.There are lots of ways to see this. Most easily, from 2a, you see that $p_t - d_t$ is proportional to x_t, so all variation in $p_t - d_t$ must come from variation in x_t, which IS expected returns. More subtly, you can make contact with the variance decompositions we studied in class. $b_d = 0$, so $cov(p_t - d_t, \Delta d_{t+j}) = 0$.

(e)

$$\begin{aligned} \sigma^2\left(r_t - E_{t-1}r_t\right) &= \sigma_v^2 + \left(\frac{1}{1-\rho\phi}\right)^2\sigma_\delta^2 \\ &= \sigma_v^2 + \left(\frac{1}{1-0.96\times 0.94}\right)^2\sigma_\delta^2 \\ &= \sigma_v^2 + \left(\frac{1}{0.1}\right)^2\sigma_\delta^2 \\ &= \sigma_v^2 + 100\times\sigma_\delta^2 \\ &= 0.10^2 + 100\times(0.01)^2 \\ &= 0.01 + 0.01 \end{aligned}$$

So it's half and half. Why the difference? Return variance also comes from *current* dividend growth in it. If prices and dividends move there is no movement in pd, but there is a big return. pd only moves on news of expected *future* dividend growth.

0.2 Stock market prediction – concept questions

1. How is it possible that almost all of the variance of *price-dividend ratios* is due to changing expected returns, with no contribution from expected dividend growth, but as much as 40% of the variance of *returns* IS due to dividend growth? (A: Look at the formula. Return includes contemporaneous dividends.)

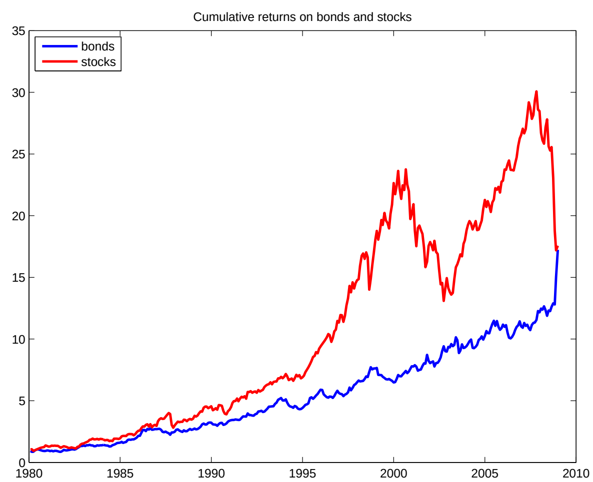
2. We know that variables such as D/P ratios can forecast future returns.

   (a) Does that mean that markets are inefficient? (A No, of course. Why not? "Model of market equilibrium," "time-varying risk premium.")

   (b) As an investor, can you exploit this predictability? (Does market timing make sense?) (A: Yes, if you have a mean-variance objective, and are not exposed to the time-varying risk driving the price. How? Give a formula, see last class.)

3. "The high returns of the last 10 years are an indication that average stock returns and the equity premium are increasing over time. As the equity premium increases, you are more and more likely to see higher returns, just as global warming makes warm summers more and more likely." Phrase a reaction in terms of concepts or examples we studied in this class. (A: Yes, but a rise in expected returns means a decline in ex post returns for a while.)

4. Well, that's the way I used to ask it. Here's a new version of the same question. (5) Here is a plot of cumulative returns on stocks and long term bonds since 1980



and the corresponding numbers. (Yes, long term bond yields fell dramatically last fall, giving that last boost to bond returns.)

	stocks	bonds	difference
mean	11.2	10.6	0.56
σ	15.7	12.4	18.9
$\sigma/\sqrt{T}$	2.91	2.30	3.51

A commentator made this graph, and said that we are in a new era; looking forward we should not expect the strong equity premium experienced up to 1980.

   (a) To make the counter-argument that stocks might look good relative to long term bonds looking forward, despite their recent poor performance, what other numbers would you look at? (Hint: this is not about t statistics, or about whether the longer data sample is useful.)

   (b) Doesn't "momentum" mean that stocks should keep going down for a while?

ANSWER

   (a) Look at dividend yields and bond yields. D/P is higher and bond yields much lower so expected returns are much better for stocks relative to bonds. More generally, we learned to look at price-based forecasts rather than past returns to learn about expected returns.

(b) Momentum refers to a cross-sectional pattern, not the market. Stocks that did well go up, stocks that did badly go down. But there is almost no tendency for the market as a whole to keep going down after a fall. ($R_{t+1} = a + bR_t + \varepsilon_{t+1}$ has tiny b, R^2).

5. Several prominent financial economists have opined that markets must not be efficient, using the following logic. They start with the present value relation

$$P_t = E_t \sum_{j=1}^{\infty} \frac{1}{R^j} D_{t+j}$$

and express it as

$$\frac{P_t}{D_t} = E_t \sum_{j=1}^{\infty} \frac{1}{R^j} \frac{D_{t+j}}{D_t}$$

They run regressions $\frac{D_{t+j}}{D_t}$on $\frac{P_t}{D_t}$.

$$\frac{D_{t+j}}{D_t} = a + b\left(\frac{P_t}{D_t}\right) + \varepsilon_{t+j}$$

They find no association whatsoever between high current prices or low P/D and high subsequent dividends or dividend growth. They conclude that prices are unrelated to fundamental values of discounted cashflows; prices have a lot of "excess volatility" that must be due to shifts in "investor sentiment." Is this a correct conclusion from $b = 0$, or is there another? (A: variance decompositions, p/d forecasts returns)

6. Suppose you forecast annual returns (jan-jan, feb-feb, etc.) with monthly observations of a variable such as interest rates, dp, etc. The return observations overlap.

$$r_{t\to t+12} = a + bx_t + \varepsilon_{t+12}$$

Is the b estimate biased? Is R^2 optimistic? Is the t statstic ok? Describe one simple way to fix any of b, R^2 and t that need fixing. (A: No. No. No. Use nonoverlapping data for t's.)

7. I run regressions of log dividend yield, log returns, and log dividend growth on the lagged log dividend yield, with the following results

$$\begin{aligned} dp_{t+1} &= a_{dp} + 0.95 \times dp_t + \varepsilon^{dp}_{t+1} \\ r_{t+1} &= a_r + 0.10 \times dp_t + \varepsilon^{r}_{t+1} \\ \Delta d_{t+1} &= a_d + 0 \times dp_t + \varepsilon^{d}_{t+1} \end{aligned}$$

(a) Plot the response of dp,r, and d are to the shocks ε^{dp} and ε^{d}.

(b) How can we label the shocks, based on your answer to a? (A: see problem set)

8. Roughly speaking, what is the coefficient of a regression of percent returns on the percent dividend yield? (Roughly speaking, i.e. 0.01, 0.05, 0.10, 1, 3, 10, etc. and get the sign right.) (A: Percent, the coefficient is about 4. In logs, it's about 0.1)

9. (This is a problem set question, a bit too complex for an exam.) The Gordon growth formula $P/D = 1/(r-g)$ assumes you know both r and g. Suppose your best guess is $r = 6\%$, $g = 4\%$, so $r - g = 0.02$ and $P/D = 50$. But you're not sure, really; there is half a chance that you're a percentage point off on r or g, so there is a 1/2 chance that $r - g = 0.01$ and a 1/2 chance that $r - g = 0.03$.

(a) Calculate $P/D = E\left[1/(r-g)\right]$ in this case.

(b) Does uncertainty make prices higher or lower than if you knew r and g for sure? Why? (Hint: what does the graph of $1/(r-g)$ look like?)

(c) If dividends are \$1 today, what does the probability distribution of dividends 10, 20 and 50 years out look like if i) $g = 4\%$ for sure ii) $g = 3\%$ or 5% with 1/2 probability each? (This should help you to understand why the price rises when there is more uncertainty.)

(d) Is there a way of stating uncertainty in a way that more uncertainty in dividends, keeping r constant, does not affect the price? (Hint: Think about uncertainty in levels of cash flows vs. uncertainty in growth rates of cash flows.)

ANSWER:

(a)

$$\frac{1}{2}\frac{1}{0.01} + \frac{1}{2}\frac{1}{0.03} = 66.7$$

(b) The price is higher. *Uncertainty in r and g make prices higher, not lower.* The mathematical reason is that $1/(r-g)$ is a nonlinear function, and $E(f(x)) \neq f(E(x))$. If f is concave (bowed up) as in this case, $E(f(x)) > f(E(x))$. In graph form, In finance terms, growth is like an option, and options are more

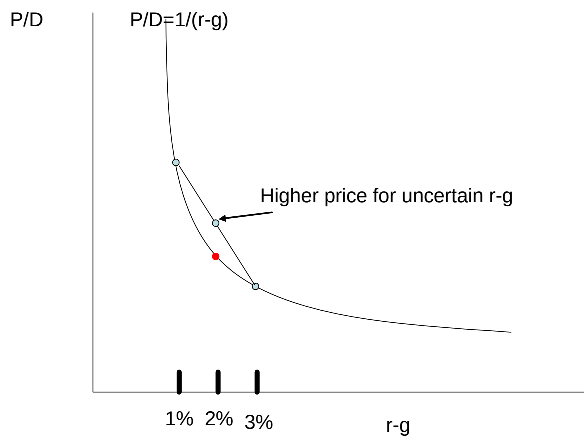


Figure 1:

valuable with more volatility.

(c) Here are the dividends. Table entries are $(1+g)^N$ You can see that a skewness develops. At 50 years, a 1% better growth rate raises dividends from 7.11 to 11.47, but a 1% worse growth rate lowers dividends by less, to 4.38. Thus the expected dividend at 7.93 is bigger then the dividend at the average growth, 7.11.

	1+g			avg of
years	1.03	1.04	1.05	1.03,1.05
10	1.34	1.48	1.63	1.49
20	1.81	2.19	2.65	2.23
50	4.38	7.11	11.47	7.93

In this sense, asset pricing is always linear in *cash flows* but not linear in *compounded growth rates.* If you state your uncertainty as uncertainty in the *level of future cashflows* rather than in *growth rates*, then adding variation does not change the price (so long as the discount rate is not affected by more variance).

Note: this problem is inspired by Lubos Pastor and Pietro Veronesi's recent paper "Was There a Nasdaq Bubble in the Late 1990s?" They argue that a large and perfectly "rational" component of the high prices in the late 1990s was uncertainty about future growth rates. A small chance of large growth is worth a lot. When uncertainty fell, with no change in the mean forecast, they say, the "bubble" burst.

10. We found a coefficient of about 0.1 in a regression of log stock returns on the log dividend yield,

$$r_{t+1} = a + 0.1 \times (d_t - p_t) + \varepsilon_{t+1}.$$

If you ran a regression of one-year returns, *one year ahead of time*, on the dividend yield,

$$r_{t+2} = a + b \times (d_t - p_t) + \varepsilon_{t+2},$$

roughly what coefficient b should you expect to find? (Answer: $b \times \phi$)

11. If you see good market stock returns in a year with no dividend growth, does this tell you that future stock returns are going to be higher ("momentum"), lower ("mean-reversion," "reversal"), or does this kind of movement have no real signal about future returns? (Answer: this was the lesson of the impulse-response functions. Price moves with no change in dividends are transitory.)

0.3 Discount factors, consumption model, theory

1. (5) Historically, in round numbers, what are the historical mean, standard deviation and Sharpe ratio of US stocks at a monthly horizon?

2. Show that a discount factor linear in the market return

$$m_{t+1} = a - bR^m_{t+1}$$

implies the CAPM

$$E(R^{ei}) = \beta_i E(R^{em})$$

(A: Start with $0 = E(mR^e)$, and use the definition of covariance. $0 = E(m)E(R^e) + cov(m, R^e), E(R^e) = -cov(m, R^e)/E(m)$....)

3. Do you expect interest rates to be higher in good times or bad times? Back up your view with an equation and an explanation. (A: we developed an equation relating interest rates to consumption growth)

4. (This is another ex problem set question, a bit too long for an exam, but bits and pieces are certainly fair game.) An investor lives for two periods, time 0 and time 1. He has a utility function over consumption c_0 in period zero and random consumption c_1 in period 1 given by

$$-\frac{1}{2}E\left[(c^* - c_0)^2 + 0.95 \times (c^* - c_1)^2\right]$$

c^* is a parameter (number), c_0 and c_1 are consumption in the first and second periods of life. We learn from a detailed statistical analysis that his consumption follows a random walk,

$$c_1 = c_0 + \epsilon_1;$$

the random shock ϵ_1 is normally distributed with mean 0 and variance σ^2. (A useful preliminary: As of time zero, i.e., knowing c_0, what is the mean and variance of c_1?) We observe consumption at period 0, c_0. It is less than c^*; $c_0 < c^*$. Your answers to the following questions can contain c_0. Find the price at time 0 (i.e. knowing c_0) of the following securities.

   (a) A one period zero coupon bond. (You may assume zero inflation if this worries you.) (A:$p = E(m \times 1) = E\left[0.95\frac{c_1 - c^*}{c_0 - c^*}\right] = 0.95\frac{E(c_1) - c^*}{c_0 - c^*} = 0.95$.)

(b) A "Stock," which pays a random dividend equal to c_1 and nothing thereafter. (Note: for any random variable x, $E(x^2) = E(x)^2 + \sigma^2(x)$.)

A:

$$p = E(mc_1) = 0.95E\left[\frac{c_1 - c^*}{c_0 - c^*}c_1\right] = 0.95E\left[\frac{c_1^2 - c^*c_1}{c_0 - c^*}\right] = 0.95\frac{E(c_1)^2 + \sigma^2(c_1) - c^*E(c_1)}{c_0 - c^*}$$
$$= 0.95\frac{c_0^2 + \sigma^2 - c^*c_0}{c_0 - c^*} = 0.95\frac{\sigma^2 + c_0\left(c_0 - c^*\right)}{c_0 - c^*} = 0.95\left[\frac{\sigma^2}{c_0 - c^*} + c_0\right]$$

Finally,

$$P = 0.95\left[c_0 - \frac{\sigma^2}{c^* - c_0}\right]$$

(c)

i. Is the stock price greater or less than the price of a bond with c_0 face value?

ii. How and why does the price depend σ^2?

iii. How and why does the price depend on c^*? In particular, explain what happens as c_0 gets closer and closer to c^*?

(A: The two terms in the stock price are the expected payoff $c_0 = E(c_1)$, discounted at the risk free rate and then a risk correction. The risk correction is larger when there is more variance, σ^2 or when c is closer to c^* making the investor more risk averse. At $c = c^*$, the investor is infinitely risk averse – any variation in consumption makes him worse off. Therefore the price goes to -∞. Obviously, this is not a realistic model, but it is fun to trace through the implications of the model as stated.)

5. When we do portfolio problems, we found for a single risky asset

$$w = \frac{1}{\gamma}\frac{E(R^e_{t+1})}{\sigma^2(R^e_{t+1})}.$$

Thus, the risk aversion necessary to have the investor put all weight in stocks $w = 1$ is

$$\gamma = \frac{E(R^e_{t+1})}{\sigma^2(R^e_{t+1})} = \frac{0.08}{0.16^2} = 3.125.$$

What's the deal with the "equity premium puzzle?" We seem fine with a sensible 3.125 risk aversion coefficient. (A: Consumption in this model is $C = kW$ so the model tells you to change consumption by 16% per year, which nobody does. People treat stock market wealth declines as if stocks will bounce back. Maybe they will. This portfolio rule ignores that fact.)

6. The equity premium puzzle says that we seem to need a huge risk aversion coefficient to explain the market Sharpe ratio.

(a) Show where this conclusion comes from. (A: from $0 = E(mR^e)$ to $|E(R^e)|/\sigma(R^e) \leq \sigma(m)/E(m) \approx \gamma\sigma(\Delta c)$. Then $0.5 = \gamma \times 0.02$)

(b) Why don't we just accept high risk aversion and get on with it? (A: riskfree rates. $r^f = -\delta + \gamma E(\Delta c)$)

7. We did a lot of deriving CAPM, ICAPM, and so forth. Do you always have to believe in these models in order to use the time-series regression technique? (Hint: Carhart, "performance evaluation")

8. In deriving a factor model with two factors f_{it} and f_{2t}, you get to the point

$$E(R^{ei}_{t+1}) = \beta_{i,1}\lambda_1 + \beta_{i,2}\lambda_2.$$

How (and with what additional assumptions) do you get to

$$E(R^{ei}_{t+1}) = \beta_{i,1}E(f_1) + \beta_{i,2}E\left(f_2\right)?$$

(A: are the factors also returns?)

9. You want to check this story I keep telling that "recession" should be the extra factor in addition to the market return. Using gdp growth ΔGDP_{t+1} as a recession measure, how would you check whether the market return and GDP growth explain the FF 25 portfolios. (A: you need a cross sectional regression because GDP is not a factor. You run time series regressions to find β of assets on GDP growth, the a cross sectional regression of average returns i on βi.)

10. T/F/U and why. "The CAPM and ICAPM and multifactor models are superior to the consumption based model, because they get rid of all that silly utility function and marginal this and that stuff." (A: they are special cases, not alternative models.)

11. Suppose you run a single return on a tradeable factor, (the factor is also an excess return)

$$R^{ei}_{t+1} = \alpha_i + \beta_i f_{t+1} + \varepsilon^i_{t+1}$$

What is the best Sharpe ratio you can get by investing in the asset and hedging the factor? When will a group of such assets display an approximate factor-pricing model? (SR^2 $= SR(f)^2 + \alpha'\Sigma^{-1}\alpha$. APT, small error means small alpha)

12. If you run a time series regression of, say, Microsoft, on the market and the tech portfolio,

$$R^{ems}_{t+1} = \alpha_{ms} + \beta_{ms} R^{em}_{t+1} + \delta_{ms} R^{eHT}_{t+1} + \varepsilon^{ms}_{t+1}$$

you undoubtedly will get a large δ_{ms} and a better R^2. Does this mean you should include a "hitech" factor in asset pricing models? (Hint: "factor" vs. "priced factor")

13. If you find that the FF3F model is true, is there a way nonetheless to capture the cross-section of average returns with a single factor model, and if so how? (A Yes. Mean-variance efficient theorem)

14. After adding a momentum factor, you suspect that you can drop the size factor, at least for the purpose of explaining the cross section of average returns. Is it enough to look whether $E(smb) = 0$? Do you need to test $s_i = 0$? How do you test whether you can drop smb? (A: alpha of smb on the others = 0)

15. You're considering a risky investment project. The project will produce a single risky payment c in one year. Here's what you know about c:

$$\begin{aligned} E(c) &= \$1,000 \\ \sigma(c) &= \$100 \\ corr(c, R^m) &= 0.5 \end{aligned}$$

Use the CAPM and standard numbers $E(R^m) = 9\%, R^f = 1\%, \sigma(R^m) = 16\%$ to value this project. (Hint: The standard deviation of the *cash flow* c is not the same thing as the standard deviation of the *return.* The CAPM specifies how *returns* behave.)

(A: As per the hint, you start with

$$E(R) = R^f + \beta \left[E(R^m) - R^f\right]$$

The trouble is finding β. Let P denote the price, then $R = c/P$; $E(R) = E(c)/P$ and $\sigma(R) = \sigma(c)/P$. Then

$$\beta = \frac{corr \times \sigma(R) \times \sigma(R^m)}{\sigma^2(R^m)} = \frac{corr}{\sigma(R^m)}\sigma(R) = \frac{0.5}{0.16}\frac{\sigma(c)}{P}$$

Now, plug in to the CAPM,

$$\frac{E(c)}{P} = R^f + \frac{0.5}{0.16}\frac{\sigma(c)}{P}\left[E(R^m) - R^f\right].$$

Solve for P

$$\left\{E(c) - \frac{0.5}{0.16}\sigma(c)\left[E(R^m) - R^f\right]\right\}\frac{1}{P} = R^f$$

$$\begin{aligned} P &= \frac{E(c) - \frac{0.5}{0.16}\sigma(c)\left[E(R^m) - R^f\right]}{R^f} \\ &= \frac{1000 - 0.5 \times 100 \times 0.08/0.16}{1.01} \\ &= 965.346 \end{aligned}$$

A common mistake is to use $\sigma(R) = \sigma(c)/E(c)$ which gives a similar, but incorrect answer.)

Time series, cross-sectional regression; fund evaluation

1. Consider the following four year history of a fund's total return and the S&P500 return.

Year	Fund return	S&P500 return
1	10%	7%
2	30%	25%
3	-10%	-11%
4	5%	0%

   (a) Is the fund average return statistically greater than zero? (A: No. you need a calculator to find $\sigma(R)/\sqrt{4}$)

   (b) Is the fund average return statistically greater than the S&P500 average return? (A: Yes. Find mean and variance of the *difference* between the fund and s&p. Give the appropriate t)

   (c) The fund salesperson greets your scepticism with "Forget means and variances. Look, we beat the S&P index in *every single year.* This fund is so good it's an arbitrage opportunity." Is this statement true? (A: No. "Arbitrage" means it will beat the S%&P in every conceivalble state. S&P500 + writing a 12% out of the money put would also have beat the S&P500 in each year of this sample!)

2. The GRS test checks i) whether the factors are significant in a time series regression ii) whether the regression is stable over time iii) whether you need to add a nonlinear term to the regression iv) whether an additional factor should be brought in v) whether the alphas are jointly zero vi) whether the cross-sectional intercepts are zero. (A:v)

3. The GRS test only applies to time-series regressions. How would you do the same thing for a cross-sectional regression? (Describe the procedure, and describe what formulas you'd look up. You do not have to give a big equation here.) (A: $\alpha' cov(\alpha)^{-1}\alpha$)

4. How can a model such as the FF model produce very small alphas, on the order of 0.1% per month, and yet still reject the GRS test? (A: $\alpha'\Sigma^{-1}\alpha$)

5. T/F/U and why. I run time-series regressions of the 25 FF portfolios on the market and the new JC factor, obtaining R^2 of over 95% in each case. Therefore, the JC factor model does a great job of explaining the cross-section of average returns. (A: distinguish between explaining variance and explaining mean. R2 in the TS regression does not tell me anything about the facts of $E(R)$, alpha and beta.)

6. T/F/U and why. I run cross-sectional regressions of the average returns of the 25 FF portfolios on their betas with respect to the new JC factor, and I get an R^2 of 95%. Therefore, the JC factor model does a great job of "explaining returns" across assets. (A: this time it is explaining mean returns but not necessarily the variance of returns. It could be a poor factor model, with poor R^2 in the time-series regressions.)

7. How are the *cross-sectional implications of a time series regression* different from a *cross-sectional regression?* Draw a graph, using the CAPM to show how these two concepts can be different. Hint: Put $E(R^{ei})$ on the y axis. (Just the time-series vs. cross section graph. The first draws the cross-sectional line through the market return and the risk free rate.)

8. Suppose a new factor increases the R^2 of time series regressions, but has no effect on the alphas.

   (a) How will including this new factor affect your explanation of the cross-section? (A: not at all)

   (b) Should you include the new factor. (Hint: for which purposes? It can make standard errors smaller, and improve portfolio evaluation, but you don't need to include it for risk adjustment, i.e. explaining alphas.)

9. (30 min) You're evaluating a portfolio manager. Write down the regressions you would run and state what numbers coming out of those regressions (coefficients, t stats, R2, etc.) you would look at to answer the following questions.

   (a) Does he seem to be following a momentum strategy? (A: performance evaluation with a momentum factor)

   (b) Does he earn average returns in excess of those you can get with investments in a market index and a value fund? (A: performance evaluation with rmrf and hml.)

   (c) You believe he has alpha, but is that alpha generated by taking large bets or by following his benchmark closely and regularly finding small mispricings? (A: Look at the residual $\sigma(\varepsilon)$ and R^2)

   (d) He seems to have a steady alpha, but is he likely to suffer unusually large losses in the next downturn? (A: Up and down betas, or betas on put writing.)

   (e) Suppose you only care about mean and variance, not any of this multifactor stuff. Should *you* invest in this fund? (Make heroic assumptions about data availability.) (A: One way to do it is run his return on *your* return and look for alpha.)

   for f and g "what regression would you run" no longer applies]

   (f) You run a 3 factor performance attribution regression, using tradeable benchmarks, and you find a 99% R^2. Does this mean you should not invest in the fund, since you can replicate its returns so well with three factors? (A: Not if there is huge, believable alpha.)

   (g) The fund claims, and you believe, a 2% alpha relative to its benchmark. In what fraction of the years should you expect the fund to nonetheless underperform the benchmark? (A: We need to know $\sigma(\varepsilon)$. If $\sigma(\varepsilon) = 1\%$, then we expect underperformance in about 15% of the years.)

10. How would you go about answering the following questions? If you want to run a regression or other statistical procedure *write down the equation* you would run. Be very careful about putting the E, i, and ts in the right place! Say what you would look at. (You don't have to spit up formulas.)

    Example: Can you tell that the Euro is likely to depreciate relative to the dollar by looking to see whether Euro interest rates are higher than US interest rates?

    Answer: I would run a regression

$$\Delta s_{t+1} = a + b(r_t^{EU} - r_t^{US}) + \varepsilon_{t+1} \; t = 1, 2, ...T$$

    Δs_{t+1} =exchange rate (euros/dollar) at time $t+1$ minus the exchange rate at time t; r_t =one-year interest rates at time t for the period t to $t+1$. I would look at the coefficient b. If it is 1, then Euros do go down one for one when the interest rate is higher. If it is zero, then a higher Euro interest rate does not correspond to depreciation. The t-statistic on b would tell me if the answer is statistically reliable. The R^2 would tell me how strong the forecast is. If it is low, then there is a lot of exchange rate risk that I can't anticipate by looking at interest differentials.

    (a) You're evaluating a fund. What is the fund's style? Is this fund basically just following a momentum strategy? Does it earn average returns in excess of those you can get with passive investments? Is it very active or pretty much following a mechanical strategy? Suppose you only care about mean and variance, not any of this multifactor stuff. Should *you* invest in this fund? (Make heroic assumptions about data availability here.) (A:This is a shorter version of the last question. $R^{ei}_{t+1} = \alpha_i + \beta' F_{t+1} + \varepsilon^i_{t+1}$.)

(b) We've long talked about labor income as a factor – people don't want stocks that go down when they lose their jobs, in addition to avoiding stocks that go down a lot when the market goes down. Given (say) excess returns for 25 size and B/M sorted portfolios, data on labor income growth, (and any other data you specify) how would you check this model? (A: factor model with labor income. First time-series, then cross-sectional regression.)

(c) An investor owns a business and comes to you for portfolio advice. He doesn't think he's much different or smarter than the average investor, and so he likes to hold an index portfolio. However, he owns a business and wonders how this circumstance should affect his portfolio. Fortunately he has a good time series of returns for his business. What should he do? (Hint: Hedge portfolio. Run returns of his business on a good set of assets (industry portfolios). Then short the "mimicking portfolio" for his business income, the fitted value of this regression.)

(d) Some people think that stock returns are worse when there is a lot of inflation. Can you tell based on inflation when it's a good time to go more into stocks relative to bonds? (A: $R^e_{t+1} = a + b(\pi_t) + \varepsilon_{t+1}$)

(e) Some people say that "neglected" stocks are underpriced, and deliver higher returns on average than other stocks. Using the number of analysts that cover a stock as a measure of "attention" or "neglect," How would you check this? (There are two steps: First, do the neglected stocks really generate higher returns? Second, if so, is this return really a compensation for risk? You can stop at the CAPM for the latter question.) (A: Form portfolios sorted by # of analysists: "most followed", to "least followed." Then find average returns of those portfolios. Next, run time series regressions of portfolio returns on factors.)

(f) Fama and French say that hml, smb are proxies for "distress." Perhaps the excess returns on a junk bond index can capture "distress." How would you check whether this new factor can account for the average returns of the FF 25 portfolios? (A: time series regression. I'd try $R^{ei}_t = a_i + b_i rmrf_t + j_i junk_t + \varepsilon^i_t$ and look at the pattern of j, as well as the GRS test.)

(g) How would you check whether the new factor *drives out* the FF factors – does it work so well that we could drop hml and smb and just use the market and the new factor? (A: The acid test of course is $hml_t = a_h + b_h rmrf_t + j_h junk_t + \varepsilon_t$ and see if that α is zero, our test for "can you drop a factor.")

0.4 Readings

1. Write no more than three sentences describing the biggest point in each of the following papers we read

   (a) Fama-French multifactor anomalies

   (b) Carhart on funds

   (c) Mitchell and Pulvino on merger arb

   (d) Agarwal and Naik on hedge fund index returns

   (e) Asness on hedge fund betas

   (f) Lamont/Thaler on 3com/Palm

   (g) Cochrane on convenience yield

2. Short answers. Mention authors if one of our papers bears on a question.

   (a) How do value *funds* compare to growth *funds* in performance relative to appropriate style benchmarks? (A: Davis, growth funds actually generate a little 3 factor alpha)

   (b) Is it true that mutual funds that did well last year are no more likely to do well next year? (A: No, Carhart, the winners have higher returns next year. But not as much as you might think: The top 1/30 of funds did, say, 100% last year, so doing on average 5% next year is not much continuation)

(c) Which portfolio-sorting variables can be explained reasonably well with a factor model including the market return, a value-growth index and a small-big index? Which portfolio-sorting variables cannot be explained well with these factors? (A: Fama French. Size, Book/market, E/P, C/P, sales growth, and past 5 year return work well. Momentum fails – produces betas of the wrong sign.)

(d) If you sort stocks by their returns from year -5 to year -1, do past winners do better or worse than average? Does risk exposure to one of FF's factors explain this fact? (Worse, reversion, yes)

(e) If you sort stocks by their returns from year -5 to year 0, do past winners do better or worse than average? Does risk exposure to one of FF's factors explain this fact? (About the same, as reversion and momentum offset)

(f) If you sort stocks by their returns over the last year, do past winners do better or worse than average? Does risk exposure to one of FF's factors explain this fact? (Winners, momentum, and the 3 factor model fails)

(g) In years that momentum stocks (past winners) do poorly, do value (low M/B) stocks do well, do poorly, or is there no correlation? (FF, momentum stocks do have strong hml betas, just of the wrong sign. Winner portfolios have negative hml betas.)

(h) Carhart finds that a "momentum factor" helps to explain the returns of portfolios of funds formed on the basis of past returns. Does Carhart's model result in zero alphas? (A: no, slightly negative, and very negative for the worst ones.)

(i) All the investment gurus tell you to look for companies with solid sales growth. Do companies with good sales growth give better than average returns? Are their returns explained by some risk story? (A: No, the other way around. FF show that hml betas account for the pattern)

(j) What signals have we studied in this class that seem to give a) alpha relative to the CAPM b) alpha relative to the FF3F model?

(k) Does Carhart say that the apparent persistence in fund performance is because the good funds are following momentum strategies? What evidence does he give on this question? (A: duration; the alphas decay quickly. B: He reports that sorts on momentum betas do not produce mean returns or alphas.)

(l) Fund managers claim that fees and turnover do not reduce returns to investors. How could charging more money *not* reduce returns to investors? (If the fees go to pay talent that produces higher pre-fee alpha)

(m) A hedge fund claims to be "market neutral" and it charges 2% plus 20% of all profits above the risk free rate. It follows long-short strategies holding illiquid securities. You're sceptical, so a hedge fund provides you with a regressions in monthly data showing

$$R_t^{ei} = \alpha_i + \beta_i R_t^{em} + \varepsilon_{t+1}$$

They show $\beta = 0.02$ with $\sigma(\beta) = 0.05$. What other regressions would you like to see before agreeing that this fund is in fact market neutral and deserves the risk free rate as a benchmark? (Ignore style drift and additional factors such as smb hml. Base your answer on facts from our readings.) (A: Up/Down or option betas, betas on other common styles such as hml, smb, etc.)

(n) A hedge fund claims to be market-neutral because all of its positive equity positions are matched by negative (short) equity positions. Is this claim valid? (A: beta isn't zero if the longs have differen betas than the shorts)

(o) What is the single most important piece of evidence that Cochrane cites for the "convenience yield" theory as opposed to the "morons" theory or the theory that "short sales constraints means that pessimists can't express their views" in explaining the high price of Palm over 3com? (Well, I think it's turnover. If you cite another fact that's ok too)

(p) TFU and why. Fama and Bliss show that the expectations hypothesis is completely wrong. (A: yes a one year, but not at 5 years.)

(q) Brandt and Kavaiecz show that this morning's signed volume helps to predict this afternoon's yields; "buying pressure" seems to drive prices up. They have a different interpretation. What is it? What are the three central pieces of evidence for their interpretation?

3. More *short* answer questions. Where appropriate, mention authors and be specific about evidence. (We're interested in evidence, not opinions!)

   (a) (5) All the investment gurus tell you to look for companies with solid sales growth. Do companies with good sales growth give better than average returns, or vice versa? (A: FF evidence, worse)

   (b) Can a strategy of buying stocks in companies with good sales growth be replicated fairly well with standard factor portfolios? (A: FF. Yes, the 3 factor regression has high R2)

   (c) Do stocks that went up over the last 5 years tend to go up more (momentum), go down (reversal) or stay about the same? (A: FF. Actually about the same. Momentum and reversal offset)

   (d) One might imagine that companies issue more shares when stock prices are high, meaning expected returns are low. Do large share issues correspond to lower subsequent returns? If so, one might think that the "high prices" are also captured in the company's book/market ratio. Is this true? (FF dissecting anomalies. Share issues are one of the strongest additional signals, even controlling for size and BM)

   (e) The conventional (academic) wisdom is that mutual funds that earned good returns last year are no more or less likely to earn good returns this year.

      i. Is this conventional wisdom true? (A: Carhart)

      ii. Do funds that did well last year differ systematically in their factor exposures from funds that did poorly last year?

   (f) Lamont and Thaler think Palm investors are behaving irrationally. What should these investors have done with their money rather than buy Palm? (A: 3com and wait)

   (g)

Term structure

1. (a) If the Euro interest rate is 10% and the dollar interest rate is 5%, roughly by how much do you expect the Euro to depreciate relative to the dollar over the next year? Explain. (A: Expectations says the Euro should depreciate 5%, but the evidence says as much as 5% appreciation.)

2. The current log yield on 1, 2 and 3 year bonds is 20%, 15%, 10% – an inverted yield curve

   (a) Find current log prices and forward rates.

   (b) Find the expected one year return on 2 and 3 year bonds, and the expected one and two year yields one year from now,

      i. According to the expectations hypothesis

      ii. According to Fama and Bliss. Simplify their regression coefficients to 1 or 0, as appropriate.(Hint: you can figure out the expected two year yield one year from now from the expected return on the three year bond.)

   (c) Plot the expected bond prices through time in each case. (Your plot has time on the x axis and bond price on the y axis. You do not have to find the FB path for the 3 year bond past time 1).

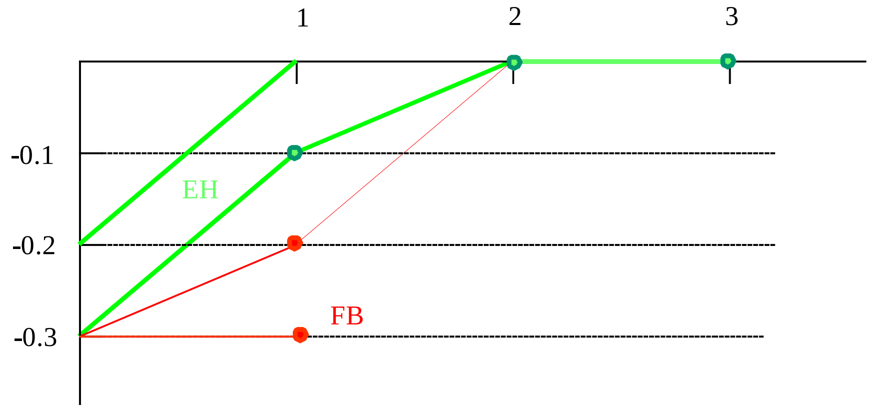
   A:

   (a) $p_t^{(1)} = -0.20$, $p_t^{(2)} = -0.30, p_t^{(3)} = -0.30, f^{(1\to 2)} = 0.10, f^{(2\to 3)} = 0.0$

(b)

i. $E_t y_{t+1}^{(1)} = 10\%, E_t r_{t+1}^{(2)} = 20\%, E_t r_{t+1}^{(3)} = 20\%, E p_{t+1}^{(2)} = -0.1, E y_{t+1}^{(2)} = 0.05$

ii. $E_t y_{t+1}^{(1)} = 20\%$, $E_t r_{t+1}^{(2)} = y_t^{(1)} + 1 \times (f^{(1\to 2)} - y^{(1)}) = 10\%. E_t r_{t+1}^{(3)} = 0\%$ $E_t p_{t+1}^{(2)} = -0.30, E_t y_{t+1}^{(2)} = 15\%$
$E_t(E_{t+1} r_{t+2}^{(2)} - y_{t+1}^{(1)}) = 1 \times E_t(f_{t+1}^{(1\to 2)} - y_{t+1}^{(1)}) = (10\% - 20\%) = -10\%$



Note: 2 and 3 year EH paths are the same

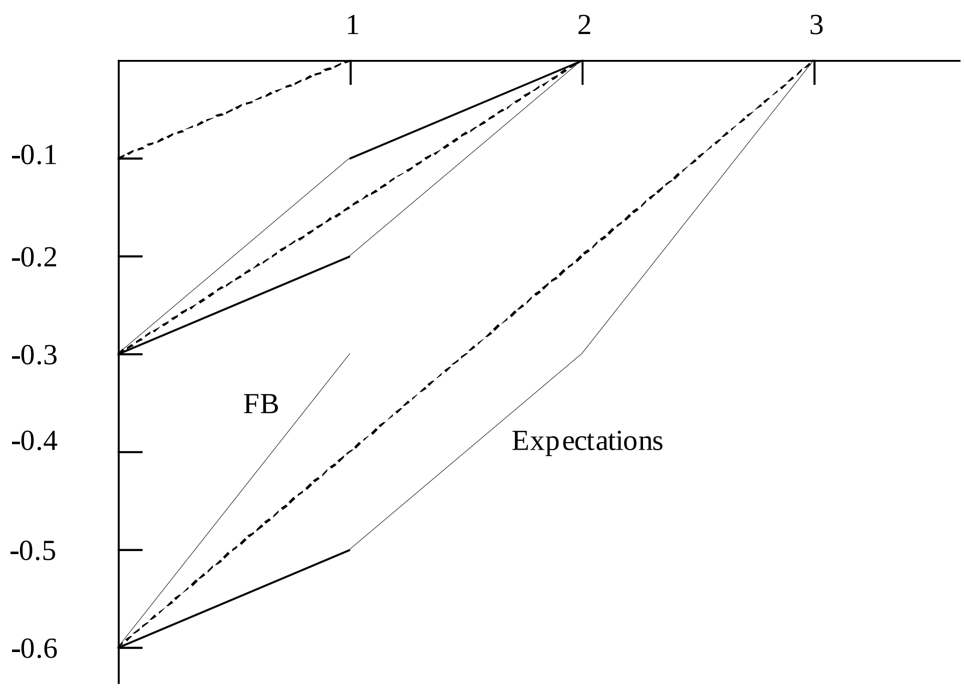
3. The current log yield on 1 2 and 3 year bonds is 10%, 15%, 20%.

(a) Find current log prices and forward rates.

(b) Find the expected log one year return on 2 and 3 year bonds i) According to the expectations hypothesis and ii) According to Fama and Bliss. Simplify their regression coefficients to 1 or 0, as appropriate.

(c) Plot the expected log bond prices through time in each case. (Your plot has time on the x axis and bond price on the y axis. You do not have to find the FB path for the 3 year bond past time 1).

(d) Find the expected one and two year log yields one year from now in each case.



4. We can construct a "term structure model" using the expectations hypothesis. Suppose the one year rate follows an AR(1),

$$y_{t+1}^{(1)} = \rho y_t^{(1)} + \varepsilon_{t+1}.$$

(Normally we would also have a constant, but I'm keeping it simple for an exam question. With no constant, the one year yield will sometimes be positive and sometimes negative. Don't let that worry you – adding a constant like 5% would make it look more realistic.) Now,

(a) Find 2, 3, 4 year yields from the expectations hypothesis that long yields are the average of expected future short yields .$y_t^{(n)} = E_t\left(y_t^{(1)} + y_{t+1}^{(1)} + .. + y_{t+n-1}^{(1)}\right)$ (A:$y_t^{(2)} = \frac{(1+\rho)}{2} y_t^{(1)}, y_t^{(3)} = \frac{(1+\rho+\rho^2)}{3} y_t^{(1)}$ but you have to get there)

(b) What kind of model is this – "one factor" "two factor" etc.? (one. Why?)

(c) How well do you think this model would fit the term structure? (A: see comments on similar model in class. Does this one give the right pattern of $E(y^{(n)})$?)

5. (Multiple choice) The Euro is now at 1.45 Euro/$. The European one month government bond rate is 8%, while the US one month T-bill rate is 5%. The currently available empirical evidence suggests

(a) You should put your money in US bonds, since a large fall in the Euro is expected.

(b) The Euro must be expected to depreciate just enough to offset the higher interest rate. The expected return (in dollars) is the same.

(c) The interest rate differential does not predict any change in exchange rate, so the expected dollar return on Euro bonds is 3% higher than on US bonds.

(d) The expected dollar return on Euro bonds is *more* than 3% higher than on US bonds.

(e) You should buy Euro*dollar* bonds, to avoid the exchange rate risk.

In your answer, also evaluate the size of the risks corresponding to the dollar vs. DM strategy. (A:d, but note lots of risk (low R^2). Refer to evidence)

6. Suppose you see the following pattern of log or continuously compounded yields on zero coupon bonds.

(a) Fill in the table. Find i) The log price of zero coupon bonds ii) Log forward rates (You only need to find the one-year forward rates, e.g. from year 0 to year 1, from year 1 to year 2 etc.)

Maturity:	1 month	1	2	3	4
Yield:	5%	5%	6%	7%	6%
Prices:					
Forward rates(N-1 to N):	xx				

(b) What does the expectations hypothesis say is the expected one year holding period return on a 4 year zero coupon bond, given these yields?

(c) What does Fama and Bliss' empirical evidence say is that expected holding period return? (Use round numbers for the regression estimates that summarize FB's main point.)

(d) What does the expectations hypothesis say is the expected one year rate three years from now (the rate from year 3 to year 4)?

(e) What does Fama and Bliss' evidence say about this rate? Again, use a round number that summarizes FB's main point

A.

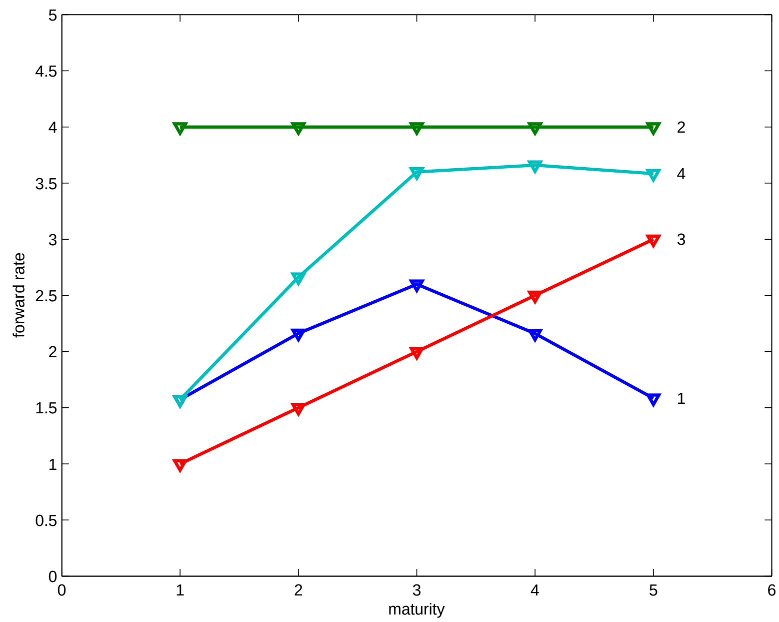
Maturity:	1 month	1	2	3	4
Yield:	5%	5%	6%	7%	6%
Prices:	-0.05/12 = -0.0042	-0.05	-0.12	-0.21	-0.24
Forward rates(N-1 to N):	xx	5%	7%	9%	3%

b) 5% c) Using 1.0 for the coefficient, 3% d) 3% e) 3%; expectations seems to work in this long run; i.e. using 1.0 for the coefficient.

7. Suppose we eigenvalue decompose the covariance matrix of yields y_t into $cov(y_t) = Q\Lambda Q'$. What are the two interpretations of the colums of Q? What is the significance of $z_t = Q'y_t$?

8. What are the differences between factor decompositions of i) yields ii) changes in yields iii) bond returns iv) bond *expected* returns, as measured by regressions of returns on all forward rates?

(a)



Rank forward curves 1-4 by which provides the strongest signal of one year excess returns on 5 year bonds i) according to Fama and Bliss' regressions ii) according to Cochrane and Piazzesi's regressions. (There may be ties.) (A: FB look for slope, CP look for tent shapes.)

Portfolios

1. Let's take at face value Fama and French's evidence that the hml factor is necessary to explain average returns. Does this mean that *any* investor with quadratic utility and a one period horizon, currently holding a market index, should add some hml to his or her portfolio? (A: No. It depends on what additional risks the investor has as well as his preferences. If he owns a "value" business, he should short value.)

2. Assume that you know for sure the FF model is right, so that all stocks average returns depend on hml and smb betas. Is there a circumstance in which you should nonetheless use the CAPM to evaluate good and bad prospects? (Hint: portfolio. If you're a MV investor, you can use the CAPM relative to your portfolio)

3. (20) You are considering an investment in an actively-managed fund. For asset allocation, you have decided to focus on a two-factor model using rf, $rmrf$ and hml as your risk factors. (Pretend that you can freely invest in all three of these)

(a) Before you think about the new investment, you form an optimal mean-variance portfolio of the two factors. Use $\gamma = 2$, and suppose you measure $E(rmrf) = 6\%$, $E(hml) = 4\%$, $\sigma(rmrf) = \sigma(hml) = 10\%$, $\sigma(rmrf,\ hml) = 0$ (not realistic numbers, but easy to compute.) *What portfolio weights do you put in each asset? How much money* do you invest in each one?

(b) Now you consider the new fund. The fund has a 12% mean return over treasuries $E(R) = R^f + 12\%$. You run a performance evaluation model using $rmrf$ and hml,

$$R_{t+1} - R_t^f = \alpha + b\ rmrf_{t+1} + c\ hml_{t+1} + \varepsilon_{t+1}$$

You discover $b = 1$, $c = 1$ and $\sigma(\varepsilon) = 5\%$. *What α do you find?* Now, how should you adapt your portfolio?

(c) *Describe your portfolio weights* in $rf, rmrf, hml$, and the "portable alpha" version of the fund.

(d) *Describe your portfolio weights* in "long-only assets", R^f, rm, h, l, and the fund R. ($hml \equiv h - l$. These weights should sum to one)

(e) These seem like suspiciously large investments. The fund's expected return and alpha claim is based on a 5 year track record. You don't believe that many funds earn more than 1% alpha, so your prior can be summarized as $\sigma(\alpha) = 1\%$. Use these facts to *produce an alpha* that is scaled back to a more sensible value. (You don't need to recalculate the portfolio – you've shown you know how to do that. Just state what alpha you would use.)

(f) (Answers:

$$w = \frac{1}{\gamma} cov(R^e)^{-1} E(R^e) = \frac{1}{\gamma} \begin{bmatrix} \frac{E(rmrf)}{\sigma^2(rmrf)} \\ \frac{E(hml)}{\sigma^2(hml)} \end{bmatrix} = \frac{1}{2} \begin{bmatrix} \frac{0.06}{0.01} \\ \frac{0.04}{0.01} \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$$

Thus, the portfolio is

$$R^f + 3rmrf + 2hml$$

rmrf and hml are zero cost, so you put \$100 in Rf, and sign up for 3 units of rmrf and 2 units of hml. If you want to express these in long-only terms, you have -\$200 in Rf, \$300 in Rm, \$200 in h and -200 in l, but I did not ask for that.

With these numbers, we have $\alpha = 12 - 6 - 4 = 2\%$.

$$w_\alpha = \frac{1}{\gamma} \Sigma^{-1} \alpha = \frac{1}{2} \frac{.02}{.05 \times .05} = \frac{1}{0.25} = 4$$

This is the weight in $\alpha + \varepsilon$, equivalently the weight in the hedged position $R_{t+1} - R_t^f - 1 \times rmrf_{t+1} - 1 \times hml_{t+1}$. Thus, no change to a by the inclusion of b

$$\alpha = \frac{\left(\Sigma_\alpha^{-1} \hat{\alpha} + \Sigma_p^{-1} 0\right)}{\left(\Sigma_\alpha^{-1} + \Sigma_p^{-1}\right)} = \frac{\Sigma_\alpha^{-1}}{\Sigma_\alpha^{-1} + \Sigma_p^{-1}} \hat{\alpha}$$

$\Sigma_a = \sigma^2(\varepsilon)/T = 0.05^2/5$:

$$\alpha = \frac{\frac{5}{0.05^2} \hat{\alpha}}{\frac{5}{0.05^2} + \frac{1}{0.01^2}} = \frac{\frac{1}{0.05}}{\frac{1}{0.05} + \frac{1}{0.01}} \hat{\alpha} = \frac{\frac{1}{5}}{\frac{1}{5} + \frac{1}{1}} \hat{\alpha} = \frac{1}{1+5} \hat{\alpha} = \frac{\hat{\alpha}}{6} = 2/6\%$$

More questions, but not from current readings or material

1. (a) What is the evidence that mutual funds do some last-minute buying of stocks to improve their end-of-year NAV? Which kinds of funds are most likely to do this?

   (b) How do value mutual *funds* compare to growth *funds* in performance relative to appropriate style benchmarks? (A: Davis)

(c) Brandt and Kavaiecz show that this morning's signed volume helps to predict this afternoon's yields.

i. What is their interpretation of this phenomenon, and the other possible interpretations?

ii. What are three central pieces of evidence for their interpretation?

2. How could you modify our standard utility function $u(c) = -\frac{c^{1-\gamma}}{1-\gamma}; u'(c) = c^{-\gamma}$ to model an investor who absolutely does not want to lose more than 50% of wealth under any circumstances, and will give up any upside necessary to protect against that loss? How would making this modification alter typical portfolio problems we worked out for power utility investors?

ANSWER $u(c) = (c-h)^{1-\gamma}/(1-\gamma)$ (In previous years we studied this problem.)

3. A stock will either go to \$130 or to \$90, with probability 1/2 of each event. (Call the two states u and d). You cannot invest in this stock and you do not know its current price, but you can invest in two call options. One option has strike \$70 and is currently trading for \$30 and one has strike \$110 currently trading for \$5.

(a) Find a discount factor m_{t+1} that prices the two call options.

(b) You are a one-period investor with log utility $u(W_{t+1}) = \ln(W_{t+1})$ and initial wealth $W_t = \$100$. Find the optimal allocation to the options. (Hint: first find optimal wealth in the two states tomorrow. Then figure out how to obtain this optimal wealth by investing in the two options. Hint 2: this is the same setup used in binomial option pricing.)

Answer: a) $m^u = \frac{1}{2}$; $m^d = \frac{3}{2}$. b) $W_{t+1,u} = \$200$; $W_{t+1,d} = \$66.67$;The portfolio is 10/3 = 3.33 of the option with strike \$70 and 0 of the option with strike \$110.

ANSWER:

(a)

$$
\begin{aligned}
30 &= E(mC_{70}) = \frac{1}{2} \times m_u \times (130-70) + \frac{1}{2} \times m_d \times (90-70) \\
5 &= E(mC_{110}) = \frac{1}{2} \times m_u \times (130-110) + \frac{1}{2} \times m_d \times 0
\end{aligned}
$$

$$
\begin{aligned}
5 &= \frac{1}{2} \times m_u \times 20 \\
\frac{1}{2} &= m_u
\end{aligned}
$$

(b)

$$
\begin{aligned}
u'(W_{t+1}) &= W_{t+1}^{-1} = \lambda m_{t+1} \\
W_{t+1,u}^{-1} &= \lambda \times \frac{1}{2};\; W_{t+1,u}^{-1} = \lambda \times \frac{3}{2} \\
W_{t+1,u} &= \lambda^{-1} \times 2;\; W_{t+1,d} = \lambda^{-1} \times \frac{2}{3}
\end{aligned}
$$

We need to choose λ so that the value of final wealth equals initial wealth,

$$
\begin{aligned}
\$100 &= E(m_{t+1}W_{t+1}) \\
&= \frac{1}{2} \times \frac{1}{2} \times \lambda^{-1} \times 2 + \frac{1}{2} \times \frac{3}{2} \times \lambda^{-1} \times \frac{2}{3} \\
&= \lambda^{-1}
\end{aligned}
$$

$$
W_{t+1,u} = \$100 \times 2 = \$200;\; W_{t+1,d} = \$100 \times \frac{2}{3}
$$

(c) What portfolio gets this?

$$\begin{aligned} W^u &= 200 = hC_{70} + kC_{110} = h \times (130 - 70) + k \times (130 - 110) = 60h + 20k \\ W^d &= 100\frac{2}{3} = hC_{70} + kC_{110} = h \times (90 - 70) = 20h \end{aligned}$$

$$\begin{aligned} 100\frac{2}{3} &= h \times 20 \\ \frac{10}{3} &= h \end{aligned}$$

$$\begin{aligned} 200 &= h \times 60 + k \times 20 \\ 200 &= \frac{10}{3} \times 60 + k \times 20 \\ 200 &= 10 \times 20 + k \times 20 \\ k &= 0 \end{aligned}$$

Check: 10/3 of C_{70} delivers

$$\begin{aligned} W^u &= \frac{10}{3} \times (130 - 70) = \frac{600}{3} = 200 \\ W^d &= \frac{10}{3} \times (90 - 70) = \frac{200}{3} \end{aligned}$$

4. Instead of the standard binomial model, imagine a 'trinomial' model: You know the stock price today S. The stock price can go up to S^u, down to S^d, or stay the same S, with equal probability. There is also a bond, and the risk-free interest rate is zero.

   (a) Using the fundamental equation $p = E(mx)$, show that there are many stochastic discount factors m in this model with the given information. (This is an economy with incomplete markets and the discount factor m can only be uniquely pinned down by asset market data in a complete market.) .

   (b) Suppose that an at-the-money call option is traded, and its' price is $C = 0.5 \times S$. Now the markets are complete – there are as many assets as there are states of nature. Find the unique stochastic discount factor m.

   (c) Suppose an investor with log utility of consumption tomorrow, $u(c) = log(c)$, optimally invests \$1000 today. What is this investor's consumption in each of three states of the world tomorrow?

ANSWER:

$$\begin{aligned} S &= \frac{1}{3}m_u S_u + \frac{1}{3}m_d S_d + \frac{1}{3}m_s S \\ 1 &= \frac{1}{3}m_u + \frac{1}{3}m_d + \frac{1}{3}m_s \\ \frac{1}{2}S &= \frac{1}{3}m_u\left(S_u - S\right) \end{aligned}$$

Solve,

$$\begin{aligned} m_u &= \frac{3}{2}\frac{S}{S_u - S} \\ m_d &= \frac{3}{2}\frac{S}{S - S_d} \\ m_s &= \frac{3}{2}\left(2 - \left(\frac{S_u}{S_u - S} + \frac{S_d}{S - S_d}\right)\right) \end{aligned}$$

$$\begin{aligned} u'(W) &= \frac{1}{W} = \lambda m \\ W_i &= \lambda^{-1} m_i^{-1} \\ W_0 &= \sum \pi_i m_i \lambda^{-1} m_i^{-1} = \lambda^{-1} \\ W_i &= W_0 m_i^{-1}. \end{aligned}$$

(On an exam, I'd give you numbers for S_u, S_d, S and you'd give each W)