Chapter Five

Fiscal Theory in Sticky Price Models

NOW, WE BUILD towards a more realistic fiscal theory of monetary policy that can be consistent with data and useful for policy analysis. To do that, I add three essential ingredients: sticky prices, long-term debt, and a surplus process with an s-shaped moving average representation.

The models so far have been frictionless, representatives of the "classical dichotomy" that changes in the price level have no effect on real quantities. Inflation is like measuring distances in feet rather than in meters. In reality, changes in the price level are often connected to changes in real quantities. Monetary economics is centrally about ways that inflations and deflations can *cause* temporary booms and recessions.

Many mechanisms have been considered to describe interactions between real quantities and the price level. I work here with the standard and simple model that prices are a bit sticky. I'm no happier about sticky prices than anyone else who works in this area, or with the specification of common sticky price models. We certainly need a deeper understanding of just why monetary shocks often seem to have real effects, yet sometimes none whatsoever as in currency reforms. But one should not innovate in two directions at once. Therefore, in this book I explore how the fiscal theory of the price level behaves if we combine it with utterly standard models of sticky prices. Equivalently, I explore how standard sticky price models behave if we give them a fiscal underpinning rather than the conventional "active" monetary policy assumption. Let us first see *how* to mix price stickiness with fiscal theory, how fiscal theory alters this most familiar model, and then add or alter ingredients.

Adding sticky prices we also see how close the statement and techniques of fiscal theory of monetary policy can be to standard new-Keynesian DSGE models. The results, however, can be quite different. By using the same specification as in standard new-Keynesian models, we see how easy it is to modify standard models to fiscal theory, and we see how the fiscal theory assumption alone changes the results.

I proceed by building models of increasing complexity, adding one ingredient at a time. Though it takes a bit more space, I find this approach helps to understand the intuition, mechanisms, and practical application of a model, which are obscured if we start with the most general case.

5.1 The Simple New-Keynesian Model

We meet the standard new-Keynesian sticky price model

$$x_t = E_t x_{t+1} - \sigma(i_t - E_t \pi_{t+1})$$
$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t.$$

The standard new-Keynesian sticky price model is

$$x_t = E_t x_{t+1} - \sigma(i_t - E_t \pi_{t+1}) \tag{5.1}$$
$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t. \tag{5.2}$$

These two equations generalize the simple model $i_t = E_t \pi_{t+1}$ of Section 2.5 to include sticky prices, which affect output x_t. Equation (5.1) is the "IS" equation, which I like to call the Intertemporal Substitution equation. Higher real interest rates induce the consumer to save more, and to consume less today than tomorrow. With no capital and no government purchases, consumption equals output. Equation (5.2) is the new-Keynesian Phillips curve. Output is high when inflation is high relative to expected *future* inflation.

To derive (5.1), start from consumer first-order conditions,

$$1 = E_t \left[\beta \left(\frac{C_{t+1}}{C_t} \right)^{-\gamma} (1 + i_t) \frac{P_t}{P_{t+1}} \right].$$

Linearize and approximate to

$$E_t \left(c_{t+1} - c_t \right) = -\sigma\delta + \sigma(i_t - E_t \pi_{t+1}).$$

where $c_t = \log C_t$, $\sigma = 1/\gamma$ and $\delta = -\log \beta$. Suppressing constants and with consumption equal to output $c_t = x_t$, we get (5.1).

The Phillips curve (5.2) comes from the first-order condition for monopolistically competitive price setters, facing costs of changing prices or a random probability of being allowed to change price. Firms set prices today knowing that prices will be stuck for a while, so they set prices with expected future prices in mind. While prices are stuck, they meet extra demand by selling more. Both equations are deviations from steady states, so x represents the output gap.

I jump to these linearized equilibrium conditions, but a big point of the new-Keynesian enterprise is that this structure has detailed microfoundations, and can thus hope to survive the Lucas (1976) critique. King (2000), Woodford (2003), and Galí (2015) are good expositions.

We can integrate the equations separately to express some of their intuition:

$$x_t = -\sigma E_t \sum_{j=0}^{\infty} (i_{t+j} - \pi_{t+j+1}) \tag{5.3}$$

$$\pi_t = \kappa E_t \sum_{j=0}^{\infty} \beta^j x_{t+j}. \tag{5.4}$$

Output is low when current and expected future real interest rates are high. Inflation is high when current and expected future output gaps are high. Equation (5.4) helps us to see that $\kappa \to \infty$ is the flexible-price limit. In that limit, output is the same for any value of inflation.

Most models add disturbances to equations (5.1) and (5.2), and study the economy's responses to shocks to these disturbances, in addition to the responses to fiscal and monetary policy shocks that I study here. IS shocks to (5.1) can be formalized as discount rate shocks, and are often viewed as a stand-in for financial shocks such as 2008. Phillips curve shocks, often called "marginal cost" shocks, are also important in the data.

I leave out such disturbances for now. Once you see *how* to adapt this model to fiscal theory, adding other disturbances is technically easy. We are often most interested in analyzing the effects of policy, so that is a good place to start. However, analyzing the effects of other shocks and how fiscal theory affects those responses is likely to be revealing. Such questions need us to move quickly to more realistic models, not just this textbook playground.

5.2 An Analytic Solution

> Inflation is a two-sided moving average of interest rates, plus a moving average of past fiscal shocks. We see the same concepts of the flexible-price model with smoothed dynamics.

We can eliminate output x_t from (5.1)-(5.2), leaving a relation between interest rates i_t and leads and lags of inflation π_t, First difference (5.2), substitute in (5.1) to obtain

$$-\frac{\beta}{\sigma\kappa} E_t \pi_{t+2} + \left(1 + \frac{1+\beta}{\sigma\kappa}\right) E_t \pi_{t+1} - \frac{1}{\sigma\kappa} \pi_t = i_t. \tag{5.5}$$

I write it this way so you can see that it reverts to $E_t \pi_{t+1} = i_t$ in the frictionless limit $\kappa \to \infty$. Thus, it generalizes the frictionless model by adding a lag polynomial on the left hand side.

Invert the lag polynomial and expand by partial fractions to obtain a solution,

$$\pi_{t+1} = \frac{\sigma\kappa}{\lambda_1 - \lambda_2} \left[i_t + \sum_{j=1}^{\infty} \lambda_1^{-j} i_{t-j} + \sum_{j=1}^{\infty} \lambda_2^{j} E_{t+1} i_{t+j} \right] + \sum_{j=0}^{\infty} \lambda_1^{-j} \delta_{t+1-j}. \tag{5.6}$$

(Algebra in Section A1.5.1 of the Online Appendix.) In words, inflation is a two-sided moving average of past and expected future interest rates. Here,

$$\lambda_{1,2} = \frac{(1+\beta+\sigma\kappa) \pm \sqrt{(1+\beta+\sigma\kappa)^2 - 4\beta}}{2}, \tag{5.7}$$

and δ_{t+1} is an expectational shock, corresponding to an undetermined initial condition in a non-stochastic difference equation, with $E_t\delta_{t+1} = 0$. I use the letter δ to indicate expectational shocks as distinct from structural ε shocks. For an impulse-response function to a shock at time 1, there is a single δ_1. Equation (5.7) describes a family of solutions indexed by the choice of δ_1. We have $\lambda_1 > 1$ and $\lambda_2 < 1$, so the moving averages as expressed converge.

From (5.7) we have

$$\frac{\sigma\kappa}{\lambda_1 - \lambda_2} = \left(1 + \frac{\lambda_1^{-1}}{1-\lambda_1^{-1}} + \frac{\lambda_2}{1-\lambda_2}\right)^{-1}. \tag{5.8}$$

This expression shows that the sum of the coefficients in (5.6) is one. A one percent permanent interest rate rise leads eventually to a one percent rise in inflation.

Recognize in (5.6) a generalization of the frictionless model

$$\pi_{t+1} = i_t + \delta_{t+1}. \tag{5.9}$$

Equation (5.6) is the same equation, with moving averages on the right-hand side as a result of sticky prices. We can anticipate that sticky prices will give us smoother and thus more realistic dynamics by putting a two-sided moving average in place of an instantaneous connection. In (5.6), past expectational shocks also affect inflation today, again describing more realistic delayed effects in place of the sudden jumps of the frictionless model.

Equation (5.6) looks like the response of inflation to a time-varying peg, but it is more general than that. It describes the relationship between equilibrium interest rates and inflation, no matter how one arrives at those quantities. For example, if one writes a monetary policy rule $i_t = \theta\pi_t + u_t$, (5.6) still holds for the equilibrium i_t and π_t.

We still have multiple equilibria and an expectational shock δ_t. Our next job is to complete the model by adding the government debt valuation equation. Conceptually, we proceed as in Section 2.5. There, we united $i_t = E_t\pi_{t+1}$ with

$$\Delta E_{t+1}\pi_{t+1} = -\Delta E_{t+1}\sum_{j=0}^{\infty}\rho^j \tilde{s}_{t+1+j} \equiv -\varepsilon_{\Sigma s,t+1}, \tag{5.10}$$

to conclude

$$\pi_{t+1} = i_t - \varepsilon_{\Sigma s,t+1}. \tag{5.11}$$

I plotted responses to monetary and fiscal shocks. We take the same steps here.

Start with short-term debt $\omega = 0$. With short-term debt, the nominal bond return equals the nominal interest rate $i_t = r^n_{t+1}$. Then, the linearized unexpected inflation identity (3.22) adds a discount rate term to (5.10), because real interest rates may vary,

$$\Delta E_{t+1}\pi_{t+1} = -\Delta E_{t+1}\sum_{j=0}^{\infty}\rho^j \tilde{s}_{t+1+j} + \Delta E_{t+1}\sum_{j=1}^{\infty}\rho^j (i_{t+j} - \pi_{t+1+j}). \tag{5.12}$$

The model thus consists of (5.5) or (5.6), and (5.12) in place of (5.10). One of the roots of (5.5) is already forward-looking, so it leaves a one dimensional family of solutions. Equation (5.12) then picks the unique path.

In addition to the smoothing effects of sticky prices, we will now see the effects of discount rates or debt service costs. Higher nominal rates can raise higher real rates, which via (5.12) adds to inflation. Other sources of higher real interest rates can raise inflation as well.

5.3 Responses to Fiscal Shocks

I add fiscal theory of the price level to the basic new-Keynesian model by adding the linearized flow equation for the real value of government debt, $\rho v_{t+1} = v_t + i_t - \pi_{t+1} - \tilde{s}_{t+1}$.

I calculate the response of the sticky price model to fiscal policy shocks. In place of a one-period price-level increase, there is now a drawn out inflation. Bondholders lose by a long period of low real interest rates – inflation above the nominal rate. Inflation fades away even though central banks do nothing.

While the present value expressions of individual equations or pairs of equations such as (5.6) or (5.12) provide a lot of intuition, they are not a practical route to solving more complex models. Instead, it is easier to write the whole model in first-order form and then solve the whole system, usually numerically, by matrix methods.

To specify and compute solutions to this model, then, I add the linearization (3.17) of the fiscal flow condition to the new-Keynesian model (5.1) - (5.2). Retaining one-period debt and hence $i_t = r^n_{t+1}$, the resulting model is

$$x_t = E_t x_{t+1} - \sigma(i_t - E_t\pi_{t+1}) \tag{5.13}$$
$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t \tag{5.14}$$
$$\rho v_{t+1} = v_t + i_t - \pi_{t+1} - \tilde{s}_{t+1} \tag{5.15}$$
$$i_{t+1} = \eta_i i_t + \varepsilon_{i,t+1} \tag{5.16}$$
$$\tilde{s}_{t+1} = \eta_s \tilde{s}_t + \varepsilon_{s,t+1} \tag{5.17}$$

We write this set of equations in matrix form, and then solve unstable eigenvalues of the system forward and stable eigenvalues backward. I defer the algebra to Online Appendix Section A1.5.2. Since $\rho \leq 1$, equation (5.15) provides the additional forward-looking root needed to determine the expectational errors δ_{t+1} and give a unique solution. The previous identities came from solving (5.15) forward on its own, so they are incorporated by this method. They remain useful guides to the intuition of model solutions. I write AR(1) processes for fiscal and monetary policy in (5.16) and (5.17) as a simple starting point. We will add much more interesting policy specifications shortly.

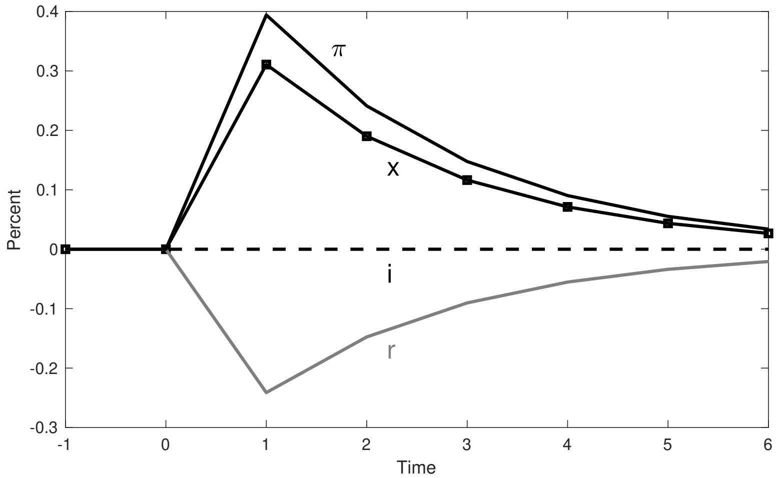

Figure 5.1: Response to a Deficit Shock $\varepsilon_{\Sigma s,1} = -1$, with No Interest Rate Movement. Sticky price fiscal theory model. Parameters $r = 0.01$, $\sigma = 0.5$, $\kappa = 0.5$.

Figure 5.1 presents the model's response to a time 1 fiscal shock, a 1% decline in the present value of surpluses, with no change in nominal interest rates. Compare this response to the response to the same shock without price stickiness in Figure 2.1, which found one period of inflation, a one time price level jump. A deficit shock still raises inflation. But with price stickiness we now have a more realistic drawn-out response, rather than a one-period price level jump.

The 1% deficit shock now only produces a 0.4% immediate rise in inflation, not 1% as before. The fiscal shock still must all come eventually out of the pockets of bondholders. With flexible prices, expected future inflation cannot devalue bondholder's wealth after they roll over bonds. If there is expected future inflation, bondholders require higher nominal interest rates ($i_t = E_t \pi_{t+1}$ works on both directions) to achieve an unchanged real return. Only an unexpected price level jump can devalue one-period bonds. With sticky prices, this intuition no longer holds. Real interest rates fall. We have expected inflation despite no change in nominal rates. A period of low real interest rates slowly eats away at the wealth of one-period bondholders. Equivalently, a period of low real interest costs brings debt back to its initial value despite lower surpluses. The real interest rate in the plot emphasizes this mechanism. In continuous time, below, the initial price-level jump disappears and this period of low real returns is the entire mechanism for reducing bondholders' wealth.

High inflation relative to future inflation means an output expansion in response to the deficit shock. This inflationary fiscal expansion thus looks a bit like "fiscal stimulus." Again, however, the sum of current and future surpluses matters here, not the current deficit. In conventional static Keynesian thinking, the flow deficit matters, not the stock of debt relative to all future repayment. A current deficit is inflationary here if and only if people think it will not be paid back by future surpluses. Deficits today followed by credible promises of future surpluses have no inflationary effect. Contrariwise, news of future unfunded deficits is inflationary even if there is no deficit today. And the "news" may not be obvious to observers. If there is a shock to expectations of future surpluses or deficits, this graph or its opposite offers an interesting picture of a boom and inflation, or recession and disinflation, that seems to come from nowhere, from animal spirits, sunspots, a run, or "sudden stop" without visible fundamental shocks to the economy or to policy, or in response to events that seem too small to cause inflation on their own. *All* that matters in this simulation is the change in the sum of surpluses, $\sum_{j=1}^{\infty} \rho^j \tilde{s}_j$, and the response is the same for any serial correlation η_s or indeed any time pattern of surpluses $\{\tilde{s}_t\}$ that have the same value of this sum.

5.4 Responses to Monetary Shocks

I calculate the response to monetary policy shocks. The responses resemble those of the frictionless model, but with dynamics drawn out by sticky prices. Output declines. First period inflation is *higher* than in the frictionless model. Higher real interest rates are a higher discount rate, or add unfunded interest costs on the debt.

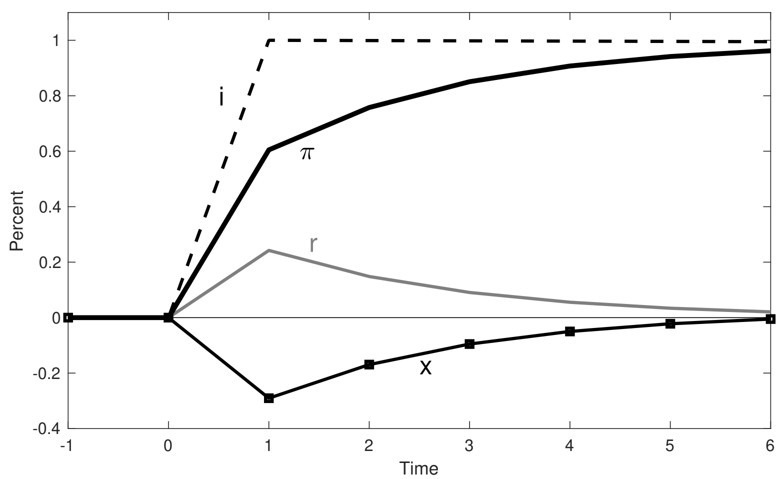

Figure 5.2: Response to an Unexpected Permanent Interest Rate Shock. No fiscal shock, simple sticky price model. Parameters $r = 0.01$, $\sigma = 0.5$, $\kappa = 0.5$.

Figure 5.2 presents responses to an unexpected permanent interest rate rise, with no change in surpluses. Compare this figure to the responses of the flexible-price model in Figure 2.1.

There are two big differences and one disappointment. First, the inflation response is drawn out as we might have expected.

Second, there is an immediate positive inflation response, $\pi_1 > 0$, on the same date as the interest rate shock, while previously inflation did not move until period 2. How can inflation move instantly without a shock to surpluses? Inflation moves because of the discount rate effect, seen in equation (5.12). Expected interest rates rise, expected inflation does not rise by the same amount, so the real interest rate rises. A higher real interest rate raises the discount rate for unchanged future surpluses. The *present value* of unchanged surpluses falls, pushing up inflation π_1. Equivalently, higher real interest rates imply higher debt service costs.

This is an important lesson. Interest costs on the debt add another *inflationary* force of higher nominal interest rates. Without additional fiscal surpluses to pay higher interest costs, greater debt service must come by inflating away the value of outstanding bonds. We will see more cases in which monetary policy has fiscal effects.

The disappointment is that sticky prices alone do not lead to a negative response of inflation to interest rates. You might have thought that higher nominal interest rates would mean higher real rates, which depress aggregate demand, and via the Phillips curve lead to less inflation. That story, reflected in the traditional Keynesian IS-LM model, does not apply in this model.

Higher interest rates do lead to lower output. With this forward-looking Phillips curve, output is low when inflation is low relative to future inflation. Equivalently, output is low when current and future real interest rates are high, as in (5.3). So, this model generates the conventional wisdom that higher interest rates with sticky prices lower output.

5.5 Long-Term Debt

I introduce long-term debt into the discrete time sticky price model. The model modifies the debt accumulation equation, and adds an expectations hypothesis model of bond prices:

$$\begin{aligned}\rho v_{t+1} &= v_t + r^n_{t+1} - \pi_{t+1} - \tilde{s}_{t+1}.\\ E_t r^n_{t+1} &= i_t\\ r^n_{t+1} &= \omega q_{t+1} - q_t.\end{aligned}$$

This modification gives a temporary inflation decline after a rise in interest rates.

Next, I add long-term debt. In Section 3.1 with flexible prices, we found that with long-term debt, an interest rate rise leads to a one-period inflation decline, shown in Figure 3.1. We have just seen how sticky prices give rise to smooth dynamics. Putting the two ingredients together, we produce smooth dynamics and negative output and inflation responses to higher interest rates.

The model consists of the usual IS and Phillips curves, (5.1)-(5.2), the linearized government debt evolution equation now with long-term debt (3.17), two bond-pricing equations to determine the government bond portfolio rate of return r^n_{t+1}, and the usual disturbance processes:

$$x_t = E_t x_{t+1} - \sigma(i_t - E_t \pi_{t+1}) \tag{5.18}$$
$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t \tag{5.19}$$
$$\rho v_{t+1} = v_t + r^n_{t+1} - \pi_{t+1} - \tilde{s}_{t+1}. \tag{5.20}$$
$$E_t r^n_{t+1} = i_t \tag{5.21}$$
$$r^n_{t+1} = \omega q_{t+1} - q_t \tag{5.22}$$
$$i_{t+1} = \eta_i i_t + \varepsilon_{i,t+1} \tag{5.23}$$
$$\tilde{s}_{t+1} = \eta_s \tilde{s}_t + \varepsilon_{s,t+1}. \tag{5.24}$$

Just adding (5.20) with $r^n_{t+1} \neq i_t$ would not be enough, as we need to determine the nominal bond return r^n_{t+1}. To this end, I assume the expectations hypothesis that the expected return on bonds of all maturities is the same in equation (5.21). I also add the linearized return definition for a bond portfolio with geometric maturity structure $B^{(t+j)}_{t-1} = \omega^j B_{t-1}$ in (5.22). This condition determines the unexpected nominal bond return by the condition that the log bond price q_t does not explode, just as the valuation equation determines unexpected inflation. Iterating (5.22) forward, and using (5.21) we have

$$r^n_{t+1} = i_t + \Delta E_{t+1} \sum_{j=1}^{\infty} \omega^j i_{t+j},$$

which determines the bond return in (5.20). Equation (5.22) is derived as equation (A1.25) in the Online Appendix. Generalization to time-varying bond risk and liquidity premiums and the actual maturity structure beckons.

Again, we solve all the flow relations together by the matrix method outlined in Online Appendix Section A1.5.2. I write (5.21) as $r^n_{t+1} = i_t + \delta_{r,t+1}$. I then substitute out r^n_{t+1}. Equations (5.20)-(5.22) become

$$\rho v_{t+1} = v_t + i_t - \pi_{t+1} - \tilde{s}_{t+1} + \delta_{r,t+1}$$
$$\omega q_{t+1} = q_t + i_t + \delta_{r,t+1}.$$

The latter gives one more explosive root to determine the additional expectational error $\delta_{r,t+1}$.

Figure 5.3 presents the response to an unexpected permanent interest rate rise in this model. Relative to the model with short-term debt in Figure 5.2, in which inflation starts rising immediately, now we have an immediate disinflation. Relative to the flexible-price long-term debt case in Figure 3.1, which produced a one-time downward price level jump, we have a drawn-out disinflation. The temporary disinflation coincides with an output decline as well, capturing standard intuition.

Higher inflation eventually reemerges. This model does not produce the view, common in the policy world, that higher interest rates permanently reduce inflation. Sims (2011) calls the pattern of lower and then higher inflation "stepping on a rake," and Sims advances it as a description of the 1970s, in which interest rate increases temporarily reduced inflation and caused recessions, but inflation then came back more strongly.

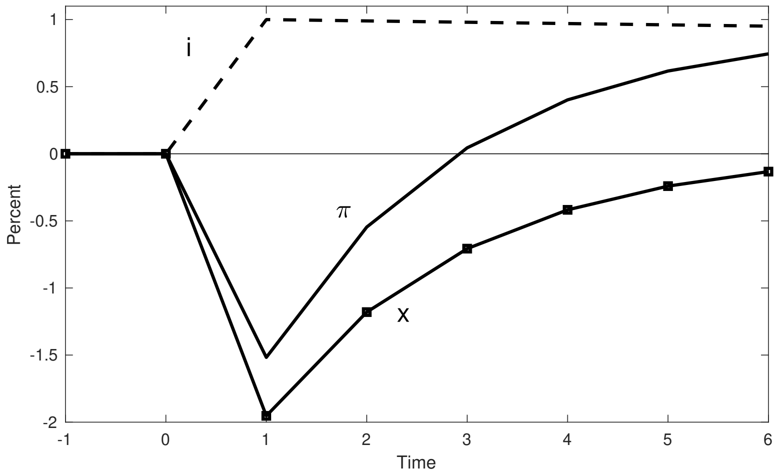

Figure 5.3: Response to an Unanticipated Permanent Interest Rate Rise. Sticky prices, no change in surpluses, and long-term debt. Parameters $\sigma = 0.5$, $\kappa = 0.5$, $\omega = 0.85$, and $r = 0.01$.

This response captures a sense of unpleasant interest rate arithmetic, parallel to the Sargent and Wallace (1981) unpleasant monetarist arithmetic. The central bank can move inflation through time, but it cannot undo a fiscal inflation. By persistently raising nominal interest rates, the central bank lowers the value of long-term nominal bonds. With no change in surpluses, that value goes to short-term bondholders, via a lower price level. The central bank can redistribute a given value among holders of bonds with different maturity.

Naturally, the longer the maturity of debt, the greater ω, the greater the disinflation. Stickier prices, however, *reduce* the initial effect, while drawing out dynamics. The flexible price version with $\kappa = \infty$ has a 5.1% deflation in period 1, rather than the 1.5% deflation plotted with sticky prices $\kappa = 0.5$. With sticky prices, we have a period of high real interest rates (interest rate above expected inflation). Again, higher real rates discount surpluses more heavily, or add to interest costs on the debt, and are a countervailing inflationary force.

That stickier prices imply less disinflation is one of many reminders that even though the *response* functions may capture common IS-LM or monetarist intuition, the *mechanism* is entirely different. That intuition would not work at all with flexible prices. Yet flexible prices produce a *greater* disinflation in this model.

The response to a fiscal shock with no interest rate change, Figure 5.1 is unaffected by long-term debt. When interest rates do not change, long term bond prices do not change.

5.6 A Monetary Policy Rule Offsets a Fiscal Shock

> We can mix monetary and fiscal shocks in this linear model. The Fed may respond to a fiscal inflation by raising interest rates. Doing so will lower inflation in the short run, but at the cost of raising inflation in the long run. We can model such a reaction with a Taylor type rule, $i_t = \theta\pi_t$. With such a rule, the central bank smooths inflation following a fiscal shock, reducing the volatility of output and inflation. The central bank cannot fully offset a fiscal inflation, but it retains full control over the long-term path of inflation.

We can mix fiscal and monetary shocks. For example, monetary policy may offset the inflationary effect of a fiscal shock by raising interest rates. By doing so, the central bank can substitute a long, slow, later inflation for current inflation in response to the fiscal shock.

A policy rule can achieve the same thing: If the central bank systematically raises interest rates in response to inflation, then it will raise interest rates in response to a fiscal shock, and automatically perform this inflation-smoothing function.

To see how this works, and also to show how one can introduce policy rules, suppose the central bank follows

$$i_t = \theta\pi_t + u_t. \tag{5.25}$$

The central bank systematically raises interest rates when inflation rises. In addition, there is a monetary policy disturbance u_t, which we often model as an AR(1)

$$u_t = \eta u_{t-1} + \varepsilon_t. \tag{5.26}$$

The model is the same as in the last section, (5.18)-(5.24), but replacing the AR(1) for interest rates (5.23) with the rule (5.25)-(5.26).

Figure 5.4 presents the response to a 1% deficit shock with this monetary policy reaction. I use the same vertical scale as in Figure 5.1, which presents the same deficit shock but with fixed interest rates, to allow an easier comparison.

The interest rate now rises to just below the inflation rate, following the rule $i_t = 0.9\pi_t$. By adding this monetary policy shock to the fiscal shock, the central bank lowers inflation a lot, from $\pi_1 = 0.4\%$ in Figure 5.1 to $\pi_1 = 0.21\%$ here. The cost is higher inflation in the long run. This is interest-rate arithmetic in action.

Output depends on inflation relative to future inflation. By making inflation much more persistent, the central bank reduces output volatility as well as the initial inflation shock. In the limit $\theta = 1$, inflation becomes a random walk, and output barely varies at all. (With $\beta < 1$ there is still a long-run inflation-output tradeoff, so a tiny output variance remains.) The central bank cannot eliminate a fiscal inflation—someone has to pay for the fiscal shock—but by drawing out that inflation, the central bank can eliminate the output consequences of the fiscal inflation. (I graph $\theta = 0.9$ rather than $\theta = 1$ so you can distinguish the interest rate and output responses, which otherwise lie on top of each other, and so that output and real rate vary at all from zero.)

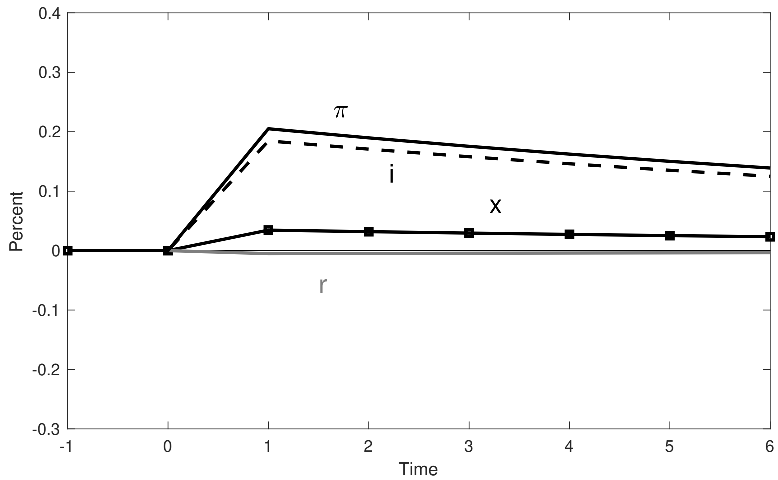

Figure 5.4: Response to a Deficit Shock $\varepsilon_{\Sigma s,1} = -1$, with a Monetary Policy Rule and Long Term Debt. Sticky price fiscal theory model. Parameters $r = 0.01$, $\sigma = 0.5$, $\kappa = 0.5$, $\omega = 0.85$, $\theta_{i,\pi} = 0.9$.

Real interest rate variation is very small in this simulation, and as the response coefficient θ approaches one, the interest rate and inflation become the same and the real rate does not change. Now the effect by which short-term bondholders lose value via low real returns disappears. Short term bond holders still lose a bit via the 20% initial (π_1) inflation. But 80% of the fiscal shock is absorbed by long-term bondholders via expected inflation between the data of the shock and the date their bonds come due. Equivalently, they lose mark-to-market value by a large fall in bond prices on the date of the shock and interest rate rise. In continuous time, below, the entire fiscal shock is borne by long-term bondholders. Again, the central bank affects the time path of inflation, output, and real interest rates, by redistributing the fiscal shock between classes of bondholders.

The example is actually negative-sum arithmetic. The total price level rise is greater in this case than in the case with no response of Figure 5.1. In the $\phi = 1$ case, inflation is permanent and the price level rise infinite. To some extent this is inevitable given that there are fewer long-term bonds available than short-term bonds. But some of the higher interest rates in this example just pointlessly raise expected inflation out in the years when there are no long-term bonds left to devalue. A more tailored response is possible, that brings long-term interest rates back down again in such a way as to minimize the total price level increase.

The central bank cannot avoid some inflation in response to the fiscal shock. But the central bank retains complete control over the long-run path of expected inflation and the long-run price level. The logic of $i_t = E_t i_{t+1}$ applies at a long horizon, when prices have adjusted and long-term bonds have matured. The central bank could follow a response like that shown, followed by swifter return of the interest rate to zero, or to negative values. The central bank could bring the price level back to its initial value. There must be some medium term inflation, but over the long term inflation will follow that interest rate where it leads.

This example is interesting to emphasize that fiscal and monetary policy rules that respond to endogenous variables are possible, and to see the beneficial properties of a Taylor-type rule. However, it is not essential that the interest rate response comes from this endogenous response. The central bank could follow the plotted interest rate path as a discretionary time-varying peg, $i_t = \eta_i i_{t-1} + \varepsilon_{i,t}$. Doing so, and choosing a suitable $\varepsilon_{i,t}$ and η_i in response to the fiscal shock produces the same result.

5.7 Continuous Time

I introduce the simple sticky price model in continuous time. The core of the model is

$$
\begin{aligned}
dx_t &= \sigma(i_t - \pi_t)dt + d\delta_{x,t} \\
d\pi_t &= (\rho\pi_t - \kappa x_t)\,dt + d\delta_{\pi,t} \\
dv_t &= (rv_t + i_t - \pi_t - \tilde{s}_t)dt + d\delta_{r,t}.
\end{aligned}
$$

It is useful to express the sticky price model in continuous time. Continuous time formulas are often simpler, as they avoid the timing conventions of discrete time, what variables are dated at t versus $t+1$, which often causes confusion. Continuous time also forces us to think more carefully about which variables can and can't jump, or follow diffusions. The price level jumps of the frictionless model are unattractive. Do we need them? The answer turns out to be no, a major point of this section. Taking the flexible-price limit makes that point clear. Macroeconomics is slowly moving from discrete time to continuous time representations, so using the representation that expresses a given result most clearly is an important skill.

The models in this section and the following build on Sims (2011) and Cochrane (2017e). Cochrane (2015a) is a short introduction to continuous time stochastic models, dz versus dt, Ito's lemma, and so forth. Cochrane (2012) shows how to do linear operator mechanics in continuous time—how to write the equivalent of $a(L)x_t = \varepsilon_t$, how to invert $a(L)$ to a moving average representation, and so forth.

The continuous time version of the model we have studied so far, with sticky prices and a geometric maturity structure, is

$$dx_t = \sigma(i_t - \pi_t)dt + d\delta_{x,t} \tag{5.27}$$
$$d\pi_t = (\rho\pi_t - \kappa x_t)\,dt + d\delta_{\pi,t} \tag{5.28}$$
$$dp_t = \pi_t dt \tag{5.29}$$
$$dv_t = (rv_t + i_t - \pi_t - \tilde{s}_t)dt + d\delta_{r,t} \tag{5.30}$$
$$dq_t = [(r+\omega)q_t + i_t]dt + d\delta_{r,t} \tag{5.31}$$
$$i_t = \theta\pi_t + u_t \tag{5.32}$$
$$du_t = -\eta_i u_t dt + d\varepsilon_{i,t} \tag{5.33}$$
$$d\tilde{s}_t = -\eta_s \tilde{s}_t dt + d\varepsilon_{s,t}. \tag{5.34}$$

These equations are linearized and all variables are deviations from steady state with $\pi = 0$, $i = r$, $\tilde{s} = rv$. Here dx_t, roughly the limit of $dx_t = x_{t+\Delta} - x_t$, is the forward differential operator used in continuous time with either diffusion or jump shocks.

Equation (5.27), $dx_t = \sigma(i_t - \pi_t)dt + d\delta_{x,t}$, is the consumer's first-order condition, linearized, and using the absence of price level jumps or diffusion terms. It is usually written $E_t dx_t = \sigma(i_t - \pi_t)dt$. I add the expectational shock $d\delta_{x,t} = dx_t - E_t dx_t$ to write an equation with the outcome dx_t on the left-hand side.

It is easiest to see this equation's analogy to its discrete time counterpart (5.1) by integrating forward to

$$x_t = -\sigma E_t \int_{\tau=0}^{\infty} (i_{t+\tau} - \pi_{t+\tau}) d\tau.$$

This is the obvious analogue of the integrated version of the discrete time IS equation, (5.3). Consumption and therefore output are low if future real interest rates are high, driving the consumer to substitute intertemporally from present to future.

Equation (5.28), $d\pi_t = (\rho\pi_t - \kappa x_t)\, dt + d\delta_{\pi,t}$, is the continuous time version of the new-Keynesian Phillips curve (5.2). The analogy to the discrete time version is again easiest to see in integral form,

$$\pi_t = \kappa E_t \int_{\tau=0}^{\infty} e^{-\rho\tau} x_{t+\tau} d\tau,$$

which parallels the discrete time version (5.4). Inflation is high if current and future output gaps are high. As $\kappa \to \infty$, output variation becomes smaller for given inflation variation, so $\kappa \to \infty$ is the frictionless limit. I use ρ with $\beta = e^{-\rho}$ to denote the discount rate in (5.28). This is a standard notation in continuous time, so I use it despite recycling symbols.

To understand the continuous time representation, you can also write the discrete time versions (5.1) and (5.2) as

$$E_t(x_{t+1} - x_t) = \sigma(i_t - E_t\pi_{t+1}) \tag{5.35}$$

$$E_t(\pi_{t+1} - \pi_t) = (e^{\rho} - 1)\pi_t - e^{\rho}\kappa x_t \tag{5.36}$$

with $\beta = e^{-\rho}$. For a time interval Δ, then,

$$E_t(x_{t+\Delta} - x_t) = \sigma(i_t - E_t\pi_{t+\Delta})\Delta \tag{5.37}$$

$$E_t(\pi_{t+\Delta} - \pi_t) = (e^{\rho\Delta} - 1)\pi_t - e^{\rho\Delta}\kappa x_t \Delta \tag{5.38}$$

As $\Delta \to 0$, the continuous time representation (5.27) and (5.28) emerges. In (5.37), $E_t\pi_{t+\Delta} - \pi_t$ also of order Δ, so it doesn't matter if we use π_t in place of$E_t\pi_{t+\Delta}$. We don't have to worry about whether inflation in the IS curve is today's inflation or expected future inflation. This is an example of the kind of timing simplification we get in continuous time.

Equation (5.29) $dp_t = \pi_t dt$, specifies that though inflation may move unexpectedly with a jump or diffusion component, the price level is continuous, differentiable, and hence completely predictable, $dp_t = E_t dp_t$. If a fraction λdt of producers changes price in each instant dt, the aggregate price level cannot jump or move unexpectedly. The discrete time models include a price level jump that devalues nominal debt. That mechanism is ruled out here, and our first task is to see what takes its place. While one can imagine models of price stickiness that allow for price level jumps and diffusions, and hence for inflation to devalue instantaneous debt, it is interesting to focus on the case in which this cannot happen. We see that the fiscal theory does not need such effects. The resulting model is elegant and intuitive.

Equations (5.30) and (5.31) are boiled down from

$$dv_t = (rv_t - \pi_t - \tilde{s}_t)dt + dR_t^n \tag{5.39}$$
$$dR_t^n = i_t + d\delta_{r,t} \tag{5.40}$$
$$dq_t = (r+\omega)q_t dt + dR_t^n \tag{5.41}$$

I use (5.40) to eliminate the symbol dR_t^n in (5.30)-(5.31). Equation (5.39) is the linearized evolution of the real market value of government debt, from (3.42), breaking the real return into a nominal return and inflation, $dR_t = dR_t^n - \pi_t dt$. Real debt increases at the steady state real return rdt, plus the real return $dR_t^n - \pi dt$, and less the scaled surplus.

The next two equations determine the nominal return dR_t^n. Equation (5.40) imposes the expectations hypothesis that the expected return is the same for all maturities. Equation (5.41) from equation (3.48) gives the evolution of the price q_t of the geometric coupon bond. This equation gives us another forward-looking root to determine the additional expectational shock $d\delta_{r,t}$.

Here, I use a geometric maturity structure as described in Section 3.6.3. A bond at time t pays coupon $e^{-\omega\tau}d\tau$ at time $t+\tau$. A nominal perpetuity that pays a constant \$1 coupon forever, with $\omega = 0$, is an important limit. The bond yield is y_t, and the bond price is $Q_t = 1/(y_t+\omega)$. The price q_t is defined as $q_t = Q_t/Q - 1$, with $Q = 1/(r+\omega)$ the steady state bond price.

In the case of instantaneous debt, $\omega \to \infty$, $dR_t^n = i_t dt$, so we drop (5.31) and (5.30) reduces to

$$dv_t = (rv_t + i_t - \pi_t - \tilde{s}_t)dt \tag{5.42}$$

Debt grows with the real interest rate, and declines with primary surpluses. The shock on the right hand side disappears.

Equation (5.32), $i_t = \theta\pi_t + u_t$, is the standard Taylor-type rule with an AR(1) disturbance as specified by (5.33), $du_t = -\eta_i u_t dt + d\varepsilon_{i,t}$. Equation (5.34), $d\tilde{s}_t = -\eta_s \tilde{s}_t dt + d\varepsilon_{s,t}$, describes an AR(1) for surpluses. We will shortly generalize both to greater reaction to endogenous variables and a surplus with an s-shaped moving average.

The $d\varepsilon_t$ shocks are structural shocks, i.e. exogenous to this model. One should add structural shocks to the IS and Phillips curves as well, and study responses to such shocks. As in discrete time, solving the model involves solving the $d\delta_t$ expectational errors in terms of the $d\varepsilon_t$ shocks. Both $d\delta_t$ and $d\varepsilon_t$ may be diffusions or compensated jump processes. (A "compensated" jump process has mean-zero innovations.) Since the model is linearized, we can compute impulse-response functions as responses to "MIT shocks," one-time unexpected shocks $d\varepsilon_0$ at time 0, and perfect foresight thereafter.

5.8 Numerical and Analytical Solutions

I set up the model for numerical solution by matrix methods, and an analytical solution by solving the IS and Phillips curves, the government debt valuation equation, and the bond pricing equation separately.

To solve the model (5.27)-(5.34) numerically, substitute out the interest rate rule, which puts it in standard form,

$$\begin{aligned}
dx_t &= \sigma[u_t - (1-\theta)\pi_t]dt + d\delta_{x,t} \\
d\pi_t &= (\rho\pi_t - \kappa x_t)\, dt + d\delta_{\pi,t} \\
dv_t &= [rv_t - (1-\theta)\pi_t + u_t - \tilde{s}_t]dt + d\delta_{r,t} \\
dq_t &= [(r+\omega)q_t + \theta\pi_t + u_t]dt + d\delta_{r,t} \\
du_t &= -\eta_i u_t dt + d\varepsilon_{i,t} \\
d\tilde{s}_t &= -\eta_s \tilde{s}_t dt + d\varepsilon_{s,t}.
\end{aligned}$$

Write this system as

$$dz_t = Az_t dt + Bd\varepsilon_t + Cd\delta_t;$$

solve the unstable eigevalues of A forward and the stable ones backward. Online Appendix Section A1.6 gives the algebra.

We can also solve the three pieces of the model analytically, as in Section 5.2 for the discrete time model. We solve forward the IS-Phillips, valuation, and bond price equations in turn. This analytical approach gives a lot of intuition about how the model works, though it doesn't scale well to more complex models and forcing processes.

I focus on impulse-responses. Before time 0, all variables are at the steady state, so deviations from steady state are zero. At time 0, people learn new paths for interest rates and surpluses, starting with $\tilde{s}_0$ and i_0. There is perfect foresight for $t > 0$ after one unexpected initial movement ($d\delta_0$, $d\varepsilon_0$) at time 0. As in discrete time, eliminate output from the IS and Phillips curves (5.27)-(5.28) to give a single second-order differential equation relating inflation and interest rates. In the perfect foresight region $t > 0$, the solution of that equation is

$$\pi_t = \left(\frac{1}{\lambda_2} + \frac{1}{\lambda_1}\right)^{-1} \left[\int_{\tau=0}^{t} e^{-\lambda_2 \tau} i_{t-\tau} d\tau + \int_{\tau=0}^{\infty} e^{-\lambda_1 \tau} i_{t+\tau} d\tau\right] + C_0 e^{-\lambda_2 t} \qquad (5.43)$$

where

$$\lambda_1 \equiv \frac{\rho + \sqrt{\rho^2 + 4\kappa\sigma}}{2}; \lambda_2 \equiv \frac{-\rho + \sqrt{\rho^2 + 4\kappa\sigma}}{2}$$

and C_0 is an arbitrary constant. (Algebra in Online Appendix Section A1.8.1.) As in discrete time, inflation is a two-sided moving average of interest rates, plus an exponentially decaying transient. There is a family of stable solutions, multiple equilibria, indexed by C_0, or equivalently by the initial value of inflation π_0.

Using $\rho = 0$, like $\beta = 1$ in discrete time, is particularly attractive. It centers the Phillips curve on expected inflation, and gives no permanent output response to inflation. The two roots then collapse to the same value, $\lambda = \sqrt{\kappa\sigma}$. I leave $\rho < 0$ despite this convenience as that is the standard parameterization, and I want to make clear that the results don't depend on $\rho = 0$.

The solution to the debt evolution equation (5.30) is

$$v_0 = dR_0^n = d\delta_{r,0} = \int_{\tau=0}^{\infty} e^{-r\tau} \left[\tilde{s}_\tau - (i_\tau - \pi_\tau)\right] d\tau. \tag{5.44}$$

This is our usual linearized present value formula. The real value of debt is the present value of surpluses, discounted at the real interest rate.

Before time 0, $v_t = 0$, the steady state value. Like the other symbols, v_0 denotes the value after the time-0 shock is realized. In the case of instantaneous debt (5.42), v_t is predetermined so $v_0 = 0$ as well. Neither the price level nor the value of instantaneous debt jumps or diffuses. Then, any change in discounted surpluses must match the change in discounted interest costs and vice versa. With long-term debt, the value of long-term bonds q_t and thus the value of debt can jump or diffuse. Surpluses or discount rates must pay off only the subsequent value of the debt. After period 0, in this simulation, the expectations hypothesis says that the expected nominal return on long-term bonds equals the interest rate, so there is no difference on the right hand side with short or long term debt.

With long term debt, we find the shock to the value of debt via bond prices by integrating forward the bond pricing equation (5.31) to

$$q_0 = d\delta_{r,0} = dR_0^n = -\int_{\tau=0}^{\infty} e^{-(r+\omega)\tau} i_t d\tau. \tag{5.45}$$

Now, the inflation path must solve (5.43), along with (5.44) and, in the case of long-term debt, (5.45). Of the many inflation paths $\{\pi_t\}$ or multiple equilibria consistent with (5.43), only one satisfies (5.44)-(5.45). Conceptually, we solve the model by plugging (5.43) into (5.44)-(5.45) and solving for the initial π_0 or C_0. In practice, that approach is arduous except for special cases, so we solve the whole system forward by matrix methods rather than solve individual equations forward and then attempt to solve them.

5.9 Fiscal Shock

I calculate responses to a fiscal shock with no change in interest rate. There is no price level jump or shock-date devaluation of short-term debt. The entire adjustment comes from a period of low real returns, with inflation greater than the nominal interest rate. As we reduce price stickiness, the price level path smoothly approaches the price-level jump of a frictionless model.

Figure 5.5 presents the continuous time model response to a 1% fiscal shock, with no change in interest rate. This is the continuous-time version of Figure 5.1. (The shock is $d\epsilon_{s,0} = -(\eta_s + r)$ so that the change in $\int_{\tau=0}^{\infty} e^{-r\tau}\tilde{s}_\tau d\tau = -1$. The value of η_s makes no difference to the plot, as the timing of surplus or deficit makes no difference.)

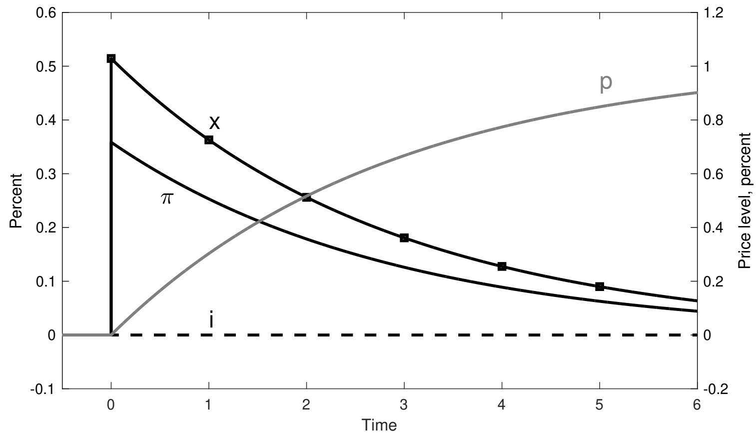

Figure 5.5: Response of the Continuous-Time Model to a 1% Deficit Shock. Parameters $\sigma = 0.5$, $\kappa = 0.25$, $r = 0.01$, $\rho = 0.01$.

This example is so simple that we can solve it analytically. With interest rates perpetually zero, the IS-Phillips curve solution (5.43) becomes simply

$$\pi_t = C_0 e^{-\lambda_2 t} = \pi_0 e^{-\lambda_2 t}. \tag{5.46}$$

With no change in interest rate, there is no change in bond price in (5.45), so the fiscal condition (5.44) is simply

$$0 = \int_{\tau=0}^{\infty} e^{-r\tau} \left[\tilde{s}_\tau - (i_\tau - \pi_\tau) \right] d\tau.$$

Substituting inflation, remembering that we have a 1% shock to the discounted sum of surpluses, and with $i_\tau = 0$,

$$1 = \int_{\tau=0}^{\infty} e^{-r\tau} \pi_\tau d\tau = \frac{\pi_0}{r + \lambda_2}.$$

Thus

$$\pi_0 = r + \lambda_2. \tag{5.47}$$

You see concretely how the fiscal condition picks the initial condition, and thus one of the family of solutions to the IS and Phillips curves.

Figure 5.5 has the same pattern as in its discrete-time counterpart: Inflation and output follow an AR(1), a drawn-out inflation in response to the fiscal shock. In this case when $\sigma = \kappa$, the inflation and output lines lie on top of each other. I use slightly different parameters than in Figure 5.1, so you can see the separate lines. Though our focus is on inflation, in reality output seems to move more than inflation so I use $\sigma > \kappa$ to mirror that fact. Since the interest rate does not move, the inflation and output responses are the same for long and short-term debt. See (5.45). As in discrete time, only the discounted sum of surpluses matter, not the persistence η_s or other features of the surplus process.

The biggest difference relative to discrete time is that *the entire loss of value to bondholders comes from the long period of interest rates lower than inflation, and none from a time-0 price level jump.* We see that fact in the price level line of the graph, which is continuous at time 0. In the valuation equation (5.44), there is no counterpart to the $\Delta E_{t+1}\pi_{t+1}$ term on the left hand side of the discrete-time identities. Instead, (5.44),

$$0 = \int_{\tau=0}^{\infty} e^{-r\tau} \left[\tilde{s}_\tau - (i_\tau - \pi_\tau)\right] d\tau.$$

tells us that the *entire* loss in surpluses $\tilde{s}_\tau$ comes from a corresponding period of real returns $i_\tau - \pi_\tau$.

This realization, much previewed, provokes a profound rethinking of the basic parable of the fiscal theory of the price level. We started in discrete time, with no pricing frictions, a constant real rate, and short-term debt. A fiscal shock was *entirely* met by a one-period unexpected price level jump which devalued outstanding short-term debt before those bondholders could roll it over. Pretty, but not at all realistic. Adding sticky prices, as in Figure 5.1, we found about half the response to the fiscal shocks now coming from the long period of nominal returns below inflation, but still half coming from the first period price level jump. Now the price level jump has disappeared completely. All of the reduction on bondholder value comes from the period of low returns – even though bondholders hold *instantaneous* debt whose interest rate can evolve constantly.

In Figure 5.5 we still see that inflation jumps. Sticky *prices* do not imply sticky *inflation*. One might imagine frictions in which inflation can only move slowly. That also seems intuitively appealing. Can the inflation rate really change overnight? But current models of sticky *prices* allow inflation to move instantly. The tiny fraction of firms that change prices each day just change prices a bit more than before. Conversely, in continuous time a jump in *inflation* does not imply any immediate change in the price level, which is why it has no immediate fiscal effect. And that observation may make an inflation jump more plausible.

The price level jump intuition is still with us. In the long run, the price level rises. If one visited this economy only every six years or so, one might apply the price-level jump vision to understand the consequence of a fiscal shock. But we watch more often, and the dynamic path is the antithesis of that parable.

(The astute reader will notice that the price level does not asymptote to 1.0 in this simulation. Integrating inflation (5.46)-(5.47), $\lim_{t\to\infty} p_t = 1 + r/\lambda_2$. The value of debt also rises temporarily as the period of deficits leads to more borrowing. What does change by 1.0? The answer is, sensibly when you think about it, the time-0 value of the bond portfolio held at time 0, i.e. without additions due to surpluses and deficits. The value of the time-0 bond portfolio $\hat{v}_t$ evolves as $d\hat{v} = (r\hat{v}_t + i_t\pi_t)dt$. A few minutes of algebra shows that $\lim_{t\to\infty} e^{-rt}\hat{v}_t = -1$ in this simulation.)

5.10 The Frictionless Limit and Limit Point

In the frictionless continuous time model, there is a time-0 jump in the price level and value of debt in response to a fiscal shock, and otherwise the interest rate equals inflation. Yet for any amount of price stickiness, there is no jump and a period of low real rates adjusts. As price stickiness decreases, however, the period of high inflation and low real interest rates becomes shorter but more extreme, and the price level smoothly approaches a time-0 jump.

Now, we face a puzzle. Even in continuous time, with flexible prices the price level can jump, and by doing so devalues outstanding debt. With flexible prices and instantaneous debt, we have

$$i_t = E_t dp_t$$

$$v_t = E_t \int_{\tau=0}^{\infty} e^{-r\tau} \tilde{s}_{t+\tau} d\tau.$$

In particular for an impulse response function with a single shock at $t = 0$,

$$i_0 = E_0 dp_0$$

$$v_0 = \int_{\tau=0}^{\infty} e^{-r\tau} \tilde{s}_{\tau} d\tau.$$

When the present value of surpluses jumps or diffuses, the value of debt must also jump or diffuse. With instantaneous debt, $V_t = B_t/P_t$, and the value of debt B_0 is predetermined. Then, the price level must jump or diffuse. Without pricing frictions, it can do so.

But for any arbitrarily small amount of price stickiness, the price level is also predetermined. We have instead

$$0 = E_0 \int_{\tau=0}^{\infty} e^{-r\tau} [\tilde{s}_{\tau} - (i_{\tau} - \pi_{\tau})] d\tau.$$

All of the adjustment comes instead from the second term on the right hand side. Rather than pick a price level at time 0, we pick an inflation *path* so that the change in interest costs / discount rate exactly matches any change in discounted surpluses.

Models in which the limit is different from the limit point are suspect! Figure 5.6 addresses this conundrum. I plot the price level path in response to the 1% fiscal shock with constant interest rates, for a variety of price stickiness parameters. You see that as we reduce price stickiness, raising κ, there is a progressively sharper and shorter period of inflation, and a sharper and shorter period of low real returns. The price level path smoothly approaches the jump at time 0.

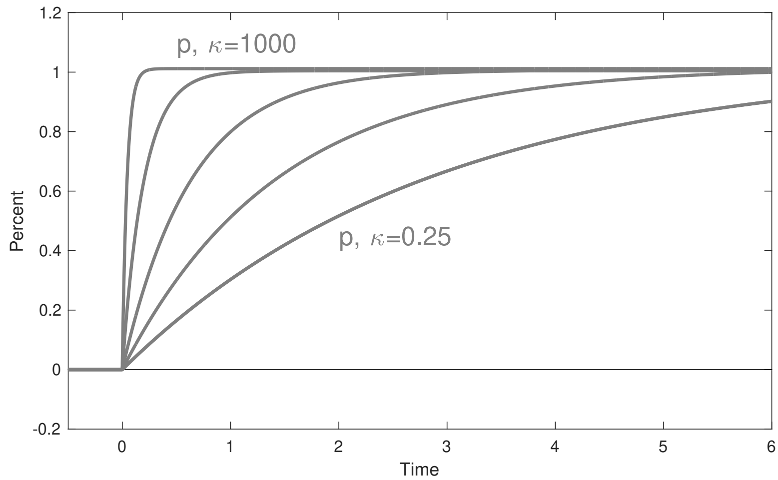

Figure 5.6: Response of the Continuous-Time Model to a 1% Fiscal Shock, with Constant Interest Rates, Varying Price Stickiness. Parameters $\sigma = 0.5$, $r = 0.01$, $\rho = 0.01$, and $\kappa = 0.25,\ 1,\ 5,\ 50,\ 1000$.

5.11 Monetary Shock

In response to an interest rate rise with no change in fiscal policy, inflation declines temporarily as in discrete time. With short-term debt, inflation immediately rises to match the nominal interest rate, a super-Fisherian response despite sticky prices. The discrete-time picture of inflation slowly rising with short-term debt resulted from one-period debt.

Figure 5.7 presents the response of the continuous time model to a permanent change in the interest rate, with no change in surpluses. This is the continuous time counterpart to Figure 5.2 with short-term debt and Figure 5.3 with long-term debt. I use $\omega = 0.1$ as $e^{-0.1} \approx 0.9$.

This response is also simple enough to be computed analytically. With a constant value of the interest rate, the inflation paths (5.43) are simply

$$\pi_t = (\pi_0 - i_0)\, e^{-\lambda_2 t} + i_0. \tag{5.48}$$

With a constant but nonzero interest rate i_0, the bond pricing equation (5.45) becomes

$$q_0 = -\frac{i_0}{r+\omega}.$$

The government debt valuation equation (5.44) is then

$$-\frac{i_0}{r+\omega} = \int_{\tau=0}^{\infty} e^{-r\tau}(\pi_\tau - i_0)d\tau.$$

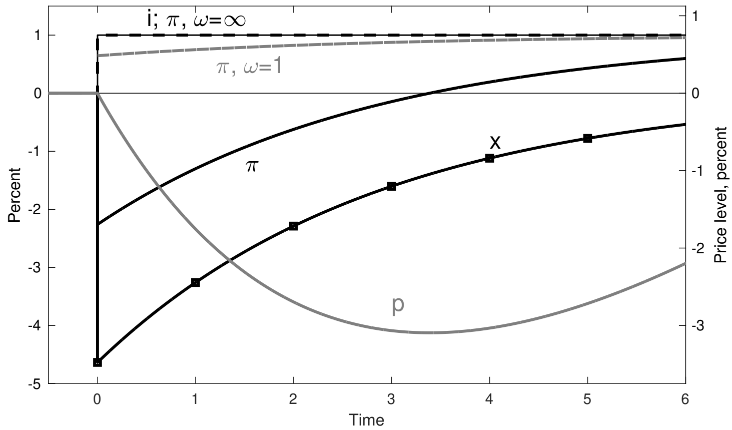

Figure 5.7: Response of the Continuous-Time Model to a Permanent Interest Rate Shock, with no Change in Surpluses. Parameters $\sigma = 0.5$, $\kappa = 0.25$, $r = 0.01$, $\rho = 0.01$, and $\omega = 0.1$.

Substituting inflation from (5.48) we find the initial value π_0

$$\pi_0 = \left(1 - \frac{r + \lambda_2}{r + \omega}\right) i_0. \tag{5.49}$$

As in discrete time, with long-term debt high interest rates result in a period of lower inflation and lower output before inflation eventually joins the interest rate in the long run. The analytical solution makes clear just how the mechanism is unlike the standard story. Equation 5.48 shows us that inflation steadily converges to the nominal interest rate. The only thing that perturbs this dynamic Fisherian response is an initial wealth shock sending initial inflation π_0 downwards.

Figure 5.7 shows the inflation response for three values of the debt maturity structure ω. The main line uses $\omega = 0.1$, i.e. the coupons of government debt for time $t + j$ decline at the rate $e^{-0.1j}$. The dashed line marked "$\pi,\ \omega = 1$" uses that value, approximating debt that is roughly one year maturity on average. We see that inflation rises initially, and then approaches the interest rate. This is essentially the same pattern as we saw for the discrete-time model with one-period debt, Figure 5.2.

The line marked "$i;\ \pi,\ \omega = \infty$" gives inflation in the case of instantaneous debt. Here, inflation equals the nominal interest rate exactly. *With short-term debt in continuous time, despite sticky prices, inflation rises instantly to exactly match a permanent interest rate increase.* We can see this result analytically in (5.49). Intuitively, with short term debt the bond price does not change. Then the government debt valuation equation (5.44) becomes

$$0 = \int_{\tau=0}^{\infty} e^{-r\tau}(i_0 - \pi_\tau)d\tau. \tag{5.50}$$

With no change in surpluses and no bond price shock, the discounted sum of interest costs on the debt must be zero. With AR(1) dynamics, interest costs cannot change sign, i.e. start negative and then turn positive before settling down to zero. So, the only inflation path consistent with (5.50) has inflation instantly and permanently equal to the interest rate.

It turns out, the discrete-time response of Figure 5.2, in which inflation rose much like the $\omega = 1$ line here, was a misleading parable of short-term debt. The discrete-time result really showed us how inflation behaves with one-year debt, and includes some of the long-term debt mechanism that reduces inflation down temporarily in response to a persistent interest rate rise.

The basic parable of fiscal theory should be instead, that *a permanent rise in the interest rate, with short-term debt and no fiscal change, gives an immediate super-Fisherian rise in inflation to exactly mirror the interest rate, no matter how sticky prices are.* Sticky prices *really* don't on their own do any good to produce a lower inflation in response to higher interest rates. Sticky *prices* do not imply sticky *inflation.* As with the fiscal shock, looking at the model in continuous time is vital for this clarification.

5.12 Fiscal Shock with Monetary Response

I calculate responses to a fiscal shock when the interest rate follows a Taylor-type rule, $i_t = \theta\pi_t$ with $\theta = 1$. Inflation and interest rates immediately and permanently rise. The rule smooths inflation and output. There is no change in real interest rate, so the fiscal shock is absorbed entirely by a fall in long-term bond prices, and repayment of long-term bonds in devalued money, rather than by a period of low real returns. The higher interest rate need not come from a rule.

We can combine shocks. Figure 5.8 presents the response of inflation to the same 1% deficit shock as Figure 5.5, adding a monetary policy rule $i = \theta\pi$ in place of a fixed interest rate. I keep the same vertical scale as in Figure 5.5 to more easily compare the two graphs. This is the continuous time counterpart to Figure 5.4, with the wrinkle that I raise the monetary policy response from $\theta = 0.9$ to $\theta = 1$. That makes the graph harder to read, since the inflation and interest rate lines lie on top of each other, but the result is more interesting.

Again, adding the interest rate response lowers inflation initially though raising inflation later on. With $\theta = 1$, the response is a step: The interest rate and inflation rise instantly and permanently.

In continuous time, $i_t = \pi_t$ means that there are no interest costs, so we can quickly solve this case analytically. With no interest costs and a 1% decline in the discounted sum of surpluses, the government debt valuation equation (5.44) reads

$$v_0 = dR_0^n = d\delta_{r,0} = -1.$$

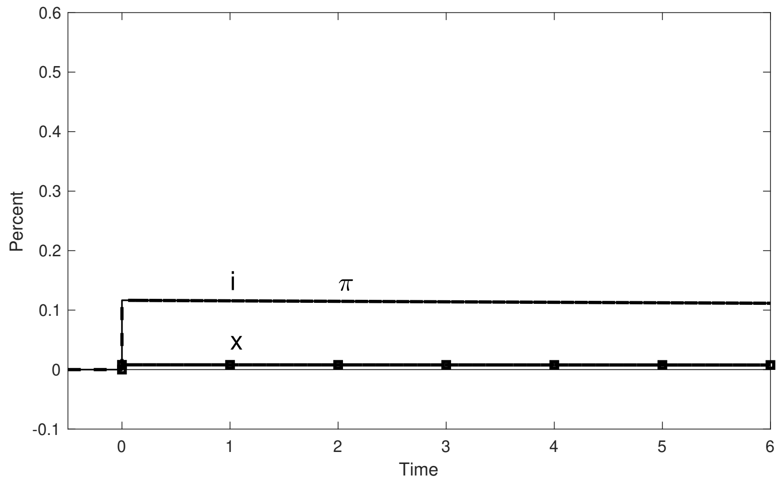

Figure 5.8: Response of the Continuous-Time Model to a 1% Deficit Shock, with a Monetary Policy Rule. Parameters $\sigma = 0.5$, $\kappa = 0.25$, $\theta = 1$, $r = 0.01$, $\rho = 0.01$.

The bond price equation (5.45) then reads

$$q_0 = d\delta_{r,0} = -1 = -\int_{\tau=0}^{\infty} e^{-(r+\omega)\tau} i_0 d\tau = -\frac{i_0}{r+\omega}.$$

Thus, inflation and the interest rate i_0 now jump up by $r+\omega$ and stay there permanently.

As in discrete time, the Taylor rule reverses how the economy absorbs the fiscal shock. With a constant nominal interest rate in Figure 5.5, a period of low real returns—inflation above the interest rate, low real interest costs on the debt—eats away at bondholder's wealth. With the $\theta = 1$ Taylor rule response here, real returns and interest costs don't move. Instead, a permanent, persistent rise in the nominal interest rate and inflation devalues long-term debt. In flow terms, debt is repaid with less valuable dollars as it comes due. In time 0 present value terms, debt suffers a shock to long-term bond prices. Though we don't have price level jumps, we do have bond price jumps, and long-term bondholders lose value in just such a jump. In the discrete time Figure 5.4, the first period of inflation also corresponded to a price-level jump that devalued short-term debt. Here that mechanism is absent, as the price level does not jump. The entire shock is absorbed by lower bond prices of long-term debt, including one-year debt. In retrospect, that's what the discrete time graph was trying to tell us.

The magnitude of the inflation and interest rate rise depends on the maturity structure of the debt. The less ω, the longer the maturity structure of the debt, the less inflation the central bank needs to allow. With a perpetuity, $\omega = 0$, a 1% decline in the present value of surpluses only requires a $r \approx 0.01\%$ permanent rise in inflation. WIth $\omega = 0.1$, in the simulation, the 1% fiscal shock gives rise to a small $r + \omega \approx 0.11\%$ permanent rise in inflation. With $\omega = 1$, roughly one-year debt, however, the 1% fiscal shock gives rise to 1.01% permanent inflation, and as $\omega \to \infty$, approaching a government financed by interest-paying money alone, an infinite amount of inflation is required. This is another instance of the general conclusion that a long maturity structure is a useful buffer. Here it gives monetary policy more power to offset fiscal shocks.

As in discrete time, the interest rate response to the fiscal shock nearly eliminates output volatility. Since with $\rho > 0$ this model retains a small permanent inflation-output tradeoff, output rises by a tiny amount. With $\rho = 0$ to represent long-run neutrality, output is completely unaffected.

It is not essential that the interest rate is generated endogenously via $i_t = \theta\pi_t$. The central bank can raise the interest rate to $i_t = r + \omega$ as a discretionary and time-varying peg. That policy produces the same results.

Again, the policy rule $i = \theta\pi$ gives more long-run inflation than is required. Bringing the interest rate back down again after the debt has all rolled over lowers long-run inflation with no time 0 fiscal consequences, and can restore zero inflation or even the original price level.

5.13 Summary: Fiscal Parables

What is fiscal theory? The response to a fiscal shock with no change in interest rates, Figure 5.5, and the response to a rise in interest rates with no change in fiscal policy, Figure 5.7 provide basic parables of how fiscal theory, married to rational expectations and new-Keynesian price stickiness, views the world. They substantially change the basic stories of the frictionless models I started with.

In the frictionless model, a fiscal shock leads to an instant price level jump, which devalues outstanding debt. With sticky prices, in Figure 5.5, the fiscal shock leads to a protracted period of inflation higher than the nominal rate, and bondholders slowly lose value. Prices never jump. The price level jump still describes the event over many years, but the dynamics to getting there are nearly the antithesis.

In the frictionless model with short-term debt, $i_t = E_t\pi_{t+1}$ says that an interest rate rise with no change in fiscal policy is immediately followed by higher inflation. Adding sticky prices and long-term debt in Figure 5.7 we still see that pattern in the very long run. But sticky prices draw out the dynamics, and long-term debt gives a mechanism by which inflation declines first, consistent with common belief.

The response to a fiscal shock with an interest rate rise in Figure 5.8 shows how we can add responses to additional insights. Monetary policy chooses when and how a fiscal policy shock will be absorbed. Figure 5.8 offers a possibility diametrically opposed to the fiscal shock with no interest rate change in Figure 5.5. By raising interest rates, the central bank smooths the fiscal inflation forward, eliminating its effect on real interest rates, and pushing the fiscal shock entirely on to a downward jump in the value of long-term bonds.

I hope that, as I have introduced them, these responses look natural and intuitive, and perhaps even empirically plausible. However, they are radical compared to standard monetary doctrines.

In response to the fiscal shock with no change in interest rates, Figure 5.5, inflation breaks out even though central banks have done nothing. (Or nothing wrong, the bankers might say!) Moreover, there is nothing that central banks can do to stop this inflation. The unfunded deficits must come out of bondholders' pockets one way or another. Central banks can only modify the inflation path, as in Figure 5.8, subject to unpleasant arithmetic. Inflation is not "always and everywhere" a monetary phenomenon.

By contrast, in conventional models, the central bank has full control of inflation. In new-Keynesian models in particular, if the bank sees underlying shocks to the economy (shocks to the IS and Phillips curve) it can set inflation exactly to zero. My fiscal shock simply does not exist. Surpluses always adjust to whatever inflation the central bank chooses.

(It is possible in a new-Keynesian model for the central bank to engineer the pattern of Figure 5.5. I dub these "open mouth operations" in Section 18.6 and construct examples. But these are still equilibrium selection choices by the central bank, not shocks coming from intractable fiscal problems. New-Keynesian models also include government spending shocks or tax rate shocks, but these are different questions. Inflation remains always and everywhere under the central bank's control.)

Most of all, in Figure 5.5 inflation goes away on its own, even if central banks continue to do nothing. To us, this may be obvious: It's a one-time fiscal shock. When the right amount of debt has been inflated away, when the price level rises sufficiently, the shock is over. But conventional doctrine holds that central banks must aggressively raise interest rates in response to undesired inflation, or inflation will spiral out of control or display multiple equilibrium sunspots. Not here.

Figure 5.7 does achieve a temporarily lower inflation in response to higher interest rates. But despite that superficial similarity, this graph too is fairly radical compared to standard views of monetary policy. Persistently high nominal, and it turns out real, interest rates do not slowly drive inflation down. They merely give a temporary period of lower inflation in return for greater inflation down the road. Without a change in fiscal policy, to pay interest costs on the debt and a windfall to bondholders, the central bank's power is much more limited than standard model suggests.

One may start to understand events and policies with these simple parables in mind, in place of MV=PY, back of the envelope IS/LM AS/AD, or the similar three-equation new-Keynesian models that one starts with otherwise. For example, consider the 2021-2022 inflation episode, covered in detail below. The \$5 trillion deficits, largely funding checks sent to people and businesses, with no discussion of repayment, look a lot like the fiscal shock of Figure 5.5. Inflation certainly did not come from a visible central bank policy change — interest rates were stuck at zero throughout inflation's emergence. That inflation did not spiral away despite a slow response by central banks again is consistent with Figure 5.5, but a puzzle to standard views.

In 2022, central banks belatedly started raising interest rates. Figure 5.7 suggests this intervention will lower inflation in the short run, but at the cost of entrenching a steady inflation, at least until a new (hopefully positive!) fiscal or other shock comes along. Figure 5.8 describes what might have happened had central banks acted sooner, but you can squint and see a description of a delayed reaction. (A better rule for empirical work is $i_t = \eta_i i_{t-1} + \theta \pi_t$ which describes central banks' slow reactions.) As I write (early 2023) that seems to be what's happening and the consensus forecast.

These responses are not realistic, and not ready for serious quantitative analysis. The models and resulting dynamics are too simple. Realism requires more detailed policy rules that react to endogenous variables. Already Figure 5.8 shows how an endogenous monetary policy response dramatically alters the response to fiscal policy. Interest rate rises cause recessions that lead to deficits, for example, that we should model. Interest rates and fiscal policies both react to the same underlying events. It is unlikely that any historical event or policy corresponds to one of these conceptual experiments. Most shocks come from other parts of the model, not fiscal and monetary policy changes. The financial crisis of 2008, for example, is such a shock.

But these are interesting conceptual questions, answers to "what would happen if" policy makers acted this way in these very simple setups, and thereby good basic parables to keep in mind.

5.14 Transitory Responses and an Attractive Mistake

One of the main lessons of this investigation is just how *hard* it is to generate a negative response of inflation to interest rates. One would have thought that sticky prices would do the trick. That turns out not to be true. Other parameters $\sigma > 0$, $\kappa > 0$ also do not help.

I presented responses to permanent interest rate increases. Perhaps sticky prices with more standard transitory interest rate increases will produce a negative inflation effect? Experience with the standard new-Keynesian model and AR(1) shocks has led to a folk theorem that permanent interest rate shocks raise inflation, but transitory shocks lower inflation. That view is an artifact of the restriction to AR(1) shocks (calculations in Section 19.3). Still, curiosity and that folk theorem motivates a look.

Figure 5.9 plots the response to a transitory interest rate shock, with no fiscal change and with short-term debt, in the continuous-time sticky price model. The response is resolutely Fisherian. Higher interest rates lead to higher inflation. Sticky prices alone do not produce a negative sign, no matter what the persistence of the interest rate shock. The standard new-Keynesian specification produces a negative inflation response to this shock, but it does so by supposing a contemporaneous fiscal tightening, absent here.

The main effect of looking at transitory shocks is that we do not see the long run, how higher interest rates eventually attract inflation.

Figure 5.10 presents an attractive mistake. This figure plots the response of this model (continuous time, short-term debt, no fiscal shock) to the plotted s-shaped interest rate path. (The path is the sum of three AR(1) with different signs and persistence. I made the interest rate path hump-shaped just to make the plot prettier. I also choose different parameters than before, including greater price stickiness via lower κ and more discounting in the Phillips curve via higher ρ to illustrate the possibility more clearly.)

This is a seductive plot. It seems we have found the answer! The interest rate

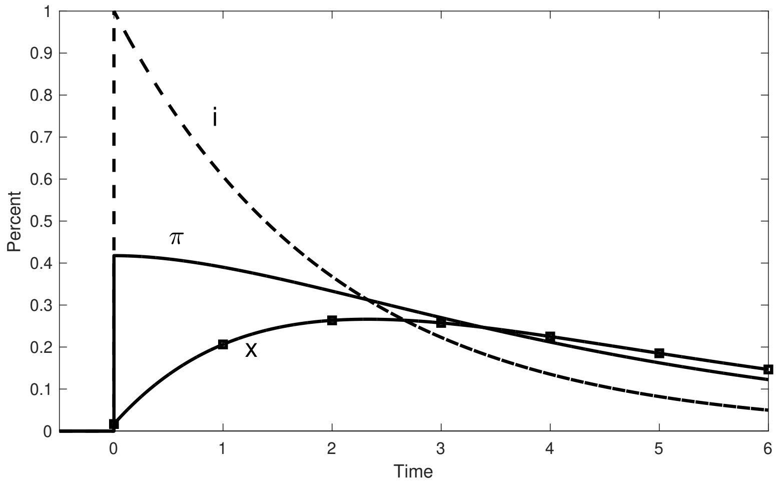

Figure 5.9: Response to a Transitory Interest Rate Shock. Continuous time model, no fiscal policy change, short-term debt. Parameters $\eta_i = 0.5$, $\kappa = 0.25$, $\sigma = 0.5$, $r = 0.01$, $\rho = 0.01$, $\omega = \infty$.

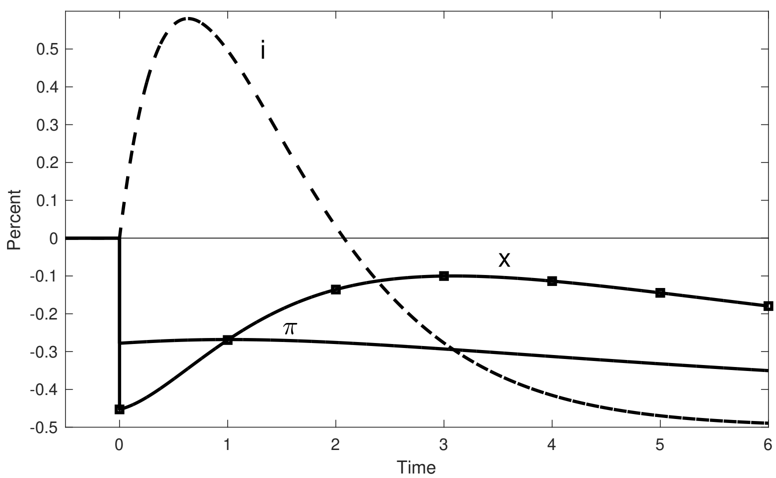

Figure 5.10: Response to a Mixed Interest Rate Shock. Continuous time model, no fiscal policy change, short-term debt. Parameters $\kappa = 0.1$, $\sigma = 0.25$, $\rho = 0.1$, $r = 0.01$, $\omega = \infty$. The interest rate follows $i_t = 30e^{-1.2t} - 29.5e^{-1.3t} - 0.05$.

rise, peaking in the first year, seems to produce an immediate and uniform decline in inflation, along with a recession. The price level (not shown) falls nearly linearly, as we often see in VARs (Ramey (2016)). Once disinflation is achieved, the central bank turns around and normalizes interest rates, following inflation downward. This looks a lot like the early 1980s, doesn't it?

Nothing of the sort is going on. This model is Fisherian in the short and long run. This is the same model that produces an upward step function of inflation, exactly equal to the interest rate, as shown in the "π, $\omega = \infty$ line of Figure 5.7. Inflation is a two-sided moving average of interest rates with positive coefficients. Inflation declines here in advance of the protracted interest rate *decline* starting in year 2. With no change in fiscal surplus, the positive interest costs from year 0 to year 2 exactly balance the discounted negative interest costs in year 3 and beyond. Lower future interest rates drag inflation down, *despite*, not *because of* the rise in interest rate from year 0 to year 2, and despite, not because of the high real interest rates of that period. If the central bank wants a disinflation in this model, it will achieve that sooner by simply lowering interest rates. The Fisherian effect will kick in faster, and it will not be fighting the fiscal consequences of higher interest costs on the debt!

Beware facile interpretations of impulse-response functions! It would be easy to read this one as saying high interest rates bring down inflation and cause a recession, and then the central bank can normalize. But that intuition is exactly wrong of the model that produces this graph.

5.15 Expected policy

I characterize the response of inflation and output to *expected* movements in fiscal and monetary policy. Fiscal shocks cause inflation when announced, not when surpluses happen. An expected interest rate rise causes inflation before the interest rates actually rise. The negative inflation effect due to long-term debt happens when higher rates are announced, not when the higher rates are realized.

VARs and early rational expectations models created the habit of calculating responses to unexpected movements. However, most policy is somewhat expected. Calculating responses to expected movements reveals a lot about how models work.

So far, I have worked out the response of inflation and output to unexpected fiscal and monetary policies. The policy variable, surplus or interest rate, moved unexpectedly, and came with news about its future path captured in an AR(1). Here, I discuss how inflation and output respond to *expected* policy movements; i.e. movements that were announced long in advance, or equivalently to announcements of policies that will be taken in the future.

The answer is easy for the responses to fiscal shocks in Figures 5.1 and 5.5. This is what happens in response to news of a shock to the discounted sum of future surpluses. The response starts on the day of the announcement or news, so it does not depend at all on when the actual surpluses happen. News of a 1% *future* decline has slightly smaller effect than the same decline happening immediately, because it is discounted. Again, this is a big difference between this fiscal theory analysis and standard static Keynesian analyses, in which any stimulus happens in the year of the deficit itself. Conversely, a long-anticipated deficit has no effect at all on inflation when it actually happens. Only bad fiscal *news* sets off inflation. We can observe year after year of deficits, as we do, but if there is no news in that, there is no inflation.

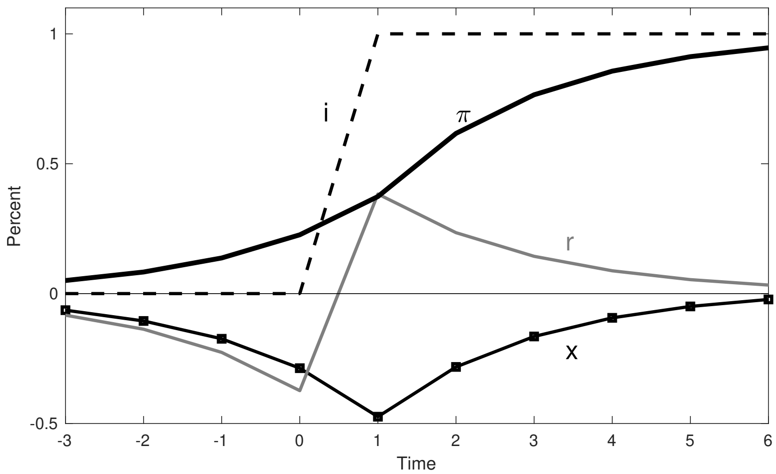

Figure 5.11: Response to a Fully Expected Rise in Interest Rates. Discrete-time fiscal theory model with price stickiness. Parameters $r = 0.01$, $\sigma = 0.5$, $\kappa = 0.5$.

Figure 5.11 presents the response to a fully expected rise in interest rates in the discrete time model. Inflation now moves *ahead* of the expected interest rate rise, reflecting the two-sided moving average in (5.6). (We can compute this response from (5.6), with all the δ_t terms equal to zero.) The expected interest rate rise also lowers output. Now output goes down in advance of the interest rate rise that causes it. We see a form of "forward guidance." Announcements of future interest rate changes, if believed, affect inflation and output today. However, fully expected higher interest rates raise, not lower, inflation.

In the frictionless model of Figure 2.1, expected and unexpected interest rates have exactly the same effect on inflation: inflation rises one period after the interest rate rise, when that rise happens, and no matter when it is annoucned. Sticky prices open up this possibility of pre-announced interest rates to affect inflation and output.

The negative response of inflation to an interest rate rise, with long-term debt and no fiscal change, Figure 5.3 and Figure 5.7 happens when the interest rate rise is announced, not when interest rates actually rise. This is a more familiar form of "forward guidance" in which the announcement of higher future interest rates causes lower inflation and output today. (Forward guidance during the zero bound era attempted the opposite sign, of course: announcing lower future interest rates to create inflation and greater output today.)

An announcement long ahead of the actual interest rate rise has a smaller effect, because the announcement only affects prices of bonds that come due after interest rates actually rise, and a delayed interest rate rise has a lesser effect on bond prices. ($q_t = -E_t \sum j = 0^{\infty} \omega^j i_{t+j}$, so leaving out the first few i has a lesser effect on the price q.)

A perfectly expected interest rate rise, or one announced before any bonds outstanding on the day of the actual rise, has no negative effect at all. Figure 5.11 is the same for long-term debt.

The lower inflation of this monetary policy shock is a fiscal effect, and works like the fiscal shock, when announced, not when interest rates actually rise. In this sense it too is utterly unlike standard static Keynesian stories, which rely on actual higher real interest rates to lower aggregate demand.

The response to expected policy variables is not often calculated, but it should be.

Most historical policy changes were at least somewhat anticipated. "Forward guidance" promises of future policy are more and more popular. Combining pure unexpected and pure expected responses lets us interpret events more realistically.

The habit of plotting responses to unexpected disturbances derives from comparing the results to vector autoregressions (VARs). VARs want to answer the causal question, what if the central bank deviates from the policy rule or raises interest rates? We cannot answer this question by simply regressing inflation on interest rates. If, say, higher interest rates are followed by higher inflation, it might be the case that the Fed raises interest rates when it sees higher inflation ahead, not that higher interest rates cause higher inflation. To estimate the answer to the causal question, VARs try to find a movement in interest rates that is not taken in response to changing expectations of future inflation or output growth. It helps in this quest to find interest rate disturbances that are unanticipated. I write "helps," as being unexpected is neither necessary nor sufficient for the causal question. An interest rate movement known ahead of time may still be orthogonal to inflation or output forecasts, and an unexpected movement can respond to contemporaneous revisions to inflation forecasts. The latter possibility is the subject of a huge orthogonalization search. Variables left out of the VAR still undermine causal interpretation.

The habit of looking at responses to unexpected shocks also derives from experience with information-based rational expectations models such as Lucas (1972), in which only unexpected monetary policy shocks have any real output effect. To characterize the economics of such a model, it makes sense to calculate the response to unexpected movements.

But those habits and implicit questions are not always relevant. Many of our central banks' policy interventions are announced months or years ahead, with no contemporaneous change in interest rates. Such "forward guidance" has become an explicit part of the Federal Reserve's "toolkit." The response of the economy to the announcement of future interest rate changes is a more relevant exercise to this policy question than "What if we surprise people with an out-of-the-blue 1% interest rise, followed by AR(1) decay?"

Unexpected and identified monetary policy shocks are also overemphasized in seeing how a model matches data. Truly exogenous and unexpected monetary policy shocks are small and rare, if they exist at all. Our central banks explain every action as a response to events, not as deliberate random experiments. Monetary policy shocks account for small fractions of the variation of interest rates, inflation, and output in most VAR estimates and model-based variance accounting.

In this way, macroeconomics anticipated by several decades the conundrum of modern applied microeconomics: One may look slivers of right hand side variation that can answer causal questions, but accounts for tiny amounts of left-hand side variation.

Matching VAR responses is also not the crucial way to test a model. Not only does the VAR by design ignore most variation in the data, one may also feel that the VARs completely miss the important effect of an intervention such as 1980. VARs isolate transitory idiosyncratic movements in the federal funds rate, not long-lasting movements that we saw in 1980. Most of all, by design, they find idiosyncratic deviations from a rule, not changes in rule or "regime" that may durably change expectations. If the art of reducing inflation is to convince people that something has changed so they should lower inflation expectations, then the response to a monetary policy "shock" orthogonal to a stable "rule" completely misses the successful policy.

We often make response plots to understand the workings of a model, and its ability to match experience more generally. That, and not matching VAR evidence, is my purpose here. I hope you have found that both expected and unexpected calculations revealing in both dimensions.

Sticky price models give output responses to expected monetary policy disturbances, unlike the early rational expectations models, so responses to expected policy are interesting characterizations of the models. And even in the information-based rational expectations models, expected monetary policy moves *inflation*.

For understanding the logic of a model, conventional impulse-response functions mix several ingredients. The policy variable also responds to the shock. An interest rate surprise raises our forecast of subsequent interest rates. Is the response of inflation and output a structural lagged response to the original shock? Or is it a structural contemporaneous response to the higher future interest rates? Does the *model* have interesting dynamics, or are the dynamics all coming from dynamics of the forcing variables?

The flexible-price model offers an example. We have $i_t = E_t\pi_{t+1}$ and thus $E_t i_{t+j} = E_t\pi_{t+j+1}$. A drawn-out response of inflation π_{t+j} to an interest rate shock $\varepsilon_{i,t+1}$ is *entirely* the result of a drawn-out interest rate response to the same shock, and a one-period response of inflation to those future interest rates. The model has no dynamics, no matter how pretty the dynamics of the plot. Figure 2.2 is a good example.

In Cochrane (1998b) I reinterpreted monetary VAR estimates through the lens that anticipated money might matter, unwinding thereby the response function to the structural effect and the effect of expected future policy. Since policy shocks are persistent, that exercise led to a much less persistent estimate of the dynamic structure in the economy than if we regard the response function as a delayed economic response to the initial shock.

The effects of expected policy changes are also rarely calculated, because the solution method leads naturally to VAR(1) representations. It's not hard to shoehorn an expected movement into an VAR(1), but people tend not to do it.

For all these reasons, it's interesting to know how the economy reacts to anticipated policy movements, or more generally it's interesting to separate announcement effects from effects of the later expected policy variable movement.

Chapter Six

Neutrality and the Response of Inflation to Interest Rates

6.1 A Complete Model of Inflation Under Interest Rate Targets

> The fiscal theory of monetary policy is stable, determinate, and neutral in the long run. It thus offers a complete theory of inflation under interest rate targets, analogous to that offered by monetary theory. Inflation is stable and determinate under an interest rate peg, just as it is under a k% money growth rule.

The frictionless fiscal theory of monetary policy model, composed of $i_t = r + E_t\pi_{t+1}$ and $\Delta E_t \pi_{t+1} = -\varepsilon_{\Sigma s,t+1}$ has three important and related properties: neutrality, stability, and determinacy. Neutrality means that real and nominal phenomena decouple: A one percentage point higher nominal interest rate means a one percentage point higher inflation rate, leaving output and real interest rates unchanged. Stability means that if the central bank sets a constant interest rate, and there are no new shocks, inflation eventually settles down, rather than spiral away positively or negatively. Determinacy means the model is complete; it does not have multiple equilibria.

The frictionless model is super-neutral: Higher interest rates cause higher inflation immediately. With sticky prices or long-term debt, we can have a short-run non-neutrality. But the model is still neutral in the long run, and stays stable and determinate. The model can still be neutral in the short run as well: Apermanent interest rise with short-term debt leads to an instant and permanent inflation rise despite sticky prices.

A basis in neutrality is an important part of a complete monetary theory. Real quantities and relative prices are eventually independent of their units of measurement. The price level has varied tremendously over long spans of time and in hyperinflations, with little correlation to real quantities. Prices include two more zeros in Japan than the U.S. with little apparent real consequence. When a country changes to a new currency, either in a currency reform or currency changes such as adopting the euro, prices by huge amounts instantly, with no real effects. A monetary theory ideally starts by describing a neutral benchmark with simple economics, and then describes short run non-neutralities with well described frictions.

The traditional $MV = PY$ monetary theory has these elegant properties. Steady states with higher money growth have higher inflation and no real effects. The central bank can set a k% money growth rate, and inflation eventually settles down to follow that growth rate. With flexible prices and perfect information, this all happens instantly. With sticky prices or information lags as in Lucas (1972), higher money growth leads first to higher real output and income, and only eventually to inflation. In Lucas' model, a fully expected money growth, such as occurs in currency reforms, is instantly super-neutral.

But central banks set nominal interest rates, and do not even pretend to control money supplies. We have needed an economic theory of inflation under *interest rate targets*, comparable to the venerable monetarist theory. We finally have such a theory.

That was not always the case. A theory of inflation under interest rate targets mirroring the elegance of $MV = PY$ has been elusive. To some extent, that lack of theoretical foundation may explain how popular monetarist theorizing remains, despite decades that central banks have targeted interest rates with no money supply control. I defer a detailed discussion of monetarist, Keynesian, and new-Keynesian alternatives to Part IV, after we fully explore fiscal theory. But a quick summary now will let us see that central accomplishment.

Friedman (1968), using adaptive expectations, taught that an interest rate peg is unstable, and Keynesian IS-LM, AS-AD modeling comes to the same conclusion. This view of the world remains the cornerstone of central bank and policy thinking. To capture this view in a very simple model, unite a static IS curve with a Phillips curve, each with adaptive expectations, that is, π_{t-1} in place of $E_t\pi_{t+1}$,

$$x_t = -\sigma(i_t - \pi_{t-1}) \tag{6.1}$$

$$\pi_t = \pi_{t-1} + \kappa x_t. \tag{6.2}$$

(Chapter 19 studies this view in detail.) Eliminating output x_t, we have the dynamic relation between inflation and interest rates.

$$\pi_t = (1 + \sigma\kappa)\pi_{t-1} - \sigma\kappa i_t. \tag{6.3}$$

Steady states have $i = \pi$, so the model displays long-run neutrality. But since $(1 + \sigma\kappa) > 1$, inflation is *unstable* around these steady states. An interest rate peg generically leads to spiraling inflation or deflation.

The Taylor rule can cure this instability. If the central bank sets $i_t = \theta\pi_t$ with $\theta > 1$, then dynamics become

$$\pi_t = \frac{1 + \sigma\kappa}{1 + \sigma\kappa\phi}\pi_{t-1}.$$

However, this model has no sensible frictionless limit or limit point. As $\kappa \to \infty$, the model's instability simply speeds up. And it fundamentally depends on adaptive expectations. We cannot even begin to talk about the price level without a fundamental break from standard economics. At a minimum, it needs a hyperinflation and currency-reform patch.

Sargent and Wallace (1975), using rational expectations, taught that an interest rate peg is indeterminate. We have seen that prediction most easily in the frictionless model, $i_t = E_t\pi_{t+1}$, but $\Delta E_t\pi_{t+1}$ can be anything. A static IS curve and a Phillips curve, but using rational expectations, captures that doctrine and the subsequent new-Keynesian view:

$$x_t = -\sigma(i_t - E_t\pi_{t+1}) \tag{6.4}$$

$$\pi_t = E_t \pi_{t+1} + \kappa x_t. \tag{6.5}$$

Now eliminating x_t, the dynamic relation between inflation and interest rates is

$$E_t \pi_{t+1} = \frac{1}{1+\sigma\kappa}\pi_t + \frac{\sigma\kappa}{1+\sigma\kappa} i_t. \tag{6.6}$$

Inflation is now stable—the first coefficient on the right hand side is less than one. Under a peg, inflation is expected to settle down. But inflation is indeterminate; there are multiple equilibria.

In new-Keynesian economics, the Taylor principle turns this stable economy into an unstable one—the opposite of its function with adaptive expectations—but with a rule against explosions the Taylor principle cures indeterminacy. With $i_t = \phi\pi_t$, and $\phi > 1$, (6.6) becomes

$$E_t \pi_{t+1} = \frac{1+\phi\sigma\kappa}{1+\sigma\kappa}\pi_t. \tag{6.7}$$

Only one path, $\pi_t = 0$ here, does not explode.

We're almost there: We have neutrality, determinacy, and stability of the outcome if not of the underlying dynamics. But central banks don't intentionally destabilize economies, or threaten hyperinflations to control multiple equilibria. Chapter 18 has an extended discussion. So, as I argue we should abandon the beautiful monetarist edifice because central banks don't control money supply, I argue we should abandon this patch, because that is not what central banks do.

As the last chapter shows extensively, fiscal theory of monetary policy solves the indeterminacy of equation (6.6) by determining unexpected inflation from the government debt valuation equation. Now we have long-run neutrality, stability and determinacy. We add short-run non-neutrality by adding well described frictions.

So, the fiscal theory of monetary policy really does fill a longstanding hole in monetary theory. Now you can do with an interest rate target pretty much everything you could do with $MV = PY$, including a peg, analogous to a k% growth rule, and you really couldn't do that before.

6.2 The Fisherian Question

> Stability under an interest rate peg implies that raising the nominal interest rate eventually raises inflation. Inflation may decline in the short run however.

Neutrality, stability, and determinacy leave an unavoidable and uncomfortable logical implication: If the central bank raises interest rates and leaves them there, inflation eventually *rises*. We have seen that property in all the response functions so far. It is a deeper property than just a particular model, flowing naturally from neutrality, stability, and determinacy. But that proposition goes against almost all current doctrine and intuition, that inflation *declines* when central banks raise interest rates.

Can we really believe long-run stability and neutrality, and the consequent prediction that higher interest rates eventually raise inflation? The equations of the model scream it, yet, it is awfully hard to change such deep seated contrary doctrines. And this model can be wrong, like any other. But remember that long-run neutrality and stability with a money supply target was once also regarded as deeply counterintuitive, as a theoretically beautiful construct of little practical importance. Milton Friedman's (1968) presidential address proclaiming long-run neutrality was counterintuitive and controversial to his audience.

Even with long-run neutrality and stability, the economy may have short-run non-neutralities. Higher interest rates might still temporarily lower inflation, as exemplified by the model with long term debt and sticky prices in Figures 5.3 and 5.7. That would help a lot to understand pervasive intuition for a negative sign. One can well imagine that practical economists and central bankers mostly observe the negative short run reaction, so naturally did not incorporate long-run stability and the eventual opposite sign into their views. Thus, it would be awfully convenient if we had a good robust model in which higher interest rates lower inflation in the short run, so we would not need to contradict such widespread intuition. The long-term debt model is a good example, but, as we will see, it is not a completely satisfactory answer. A better model of a short-term negative effect of interest rates on inflation is one of the top priorities for this line of work.

The other possibility is, the negative sign isn't there. Empirical evidence is weak, and largely consists of work that does not address the theoretical question we want to answer, as it does not hold fiscal policy constant. Thus a comparable empirical effort also strikes me as a top priority.

Generating a short-run negative sign is not easy. The force for a positive coefficient, resulting from neutrality and stability of the simplest economic model, is hard to overcome. We saw in the last chapter how sticky prices alone are not enough to generate a genuine negative sign. We need long term debt, or some new ingredient.

Even standard models struggle to produce a negative sign of inflation to an interest rate rise. The standard new-Keynesian model is also Fisherian in the long run—permanently higher interest rates lead to permanently higher inflation (Section 19.4.3, Figure 19.8). The sign on the interest rate in (6.6) is positive: higher interest rates lead immediately to higher expected inflation. When this model achieves a negative short-run response, it does so by constructing an equilibrium-selection shock coincident with the rise in interest rates, jumping to a new equilibrium to produce unexpected disinflation. The equilibrium-selection shock induces a fiscal shock, so it's identical to a joint fiscal tightening with an interest rate rise to fiscal theory. And the fiscal or equilibrium-selection shock does all the disinflationary work. Inflation declines despite the interest rate rise, not because of the rise, and inflation would decline more without it. (Chapter 18.)

The standard old-Keynesian adaptive-expectations model, epitomized by (6.1)-(6.2), achieves a negative sign. The sign on the interest rate in (6.3) is negative. But it does so by reversing the basic stability and determinacy properties of the model. The central bank raises the nominal rate, and that sends inflation and output spiraling off in the opposite direction. Eventually, the central bank then moves interest rates to follow and then exceed inflation, to stop the spiral, like a seal balancing a ball on its nose. This is the standard view of the 1980s for example.

I write "reversing" as rational expectations and flexible prices are the natural supply and demand benchmark. To get a negative sign, the adaptive-expectations approach makes a stable model unstable, makes an indeterminate model determinate, makes forward-looking people look backward. To produce a short-run negative sign it produces a long-run negative sign. It does not produce a short run negative sign while retaining a long-run positive sign consistent with long run stability. It still accounts for observed neutrality—that higher interest rates accompany higher inflation. The central bank first lowers interest rates to get inflation going, then quickly raises rates to follow inflation and keep it from exploding, like a seal balancing a ball on its nose. The positive long-run correlation goes with a negative causal relation,

This is major surgery. A little irrationality won't do; a small loading of expectations on lagged rather than future inflation. One needs enough irrationality to move the eigenvalues from stable to unstable. While a little irrationality as a way to capture short-run dynamics might be sensible, expectations that are always adaptive, people that never learn, no matter how large the inflation, is a difficult foundation for an economic theory. Conversely, as people catch on and become more rational, a moment comes that the system eigenvalue crosses one and interest rates start to directly raise inflation.

The issue should really be called "model-consistent" expectations. The issue is not so much the philosophically contentious question whether people are truly "rational" and what that means. The issue is that with adaptive expectations, the expectations *of* the model are different from the expectations *in* the model. If people in the model ever wise up, monetary policy changes sign and loses all its conventional power.

Two technical notes: First, the simply sticky price model I have used features a small permanent inflation-output tradeoff, so is not exactly neutral. From the Phillips curve (5.2),

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t,$$

permanent movements in x and π follow

$$x = \frac{1-\beta}{\kappa}\pi.$$

For typical $\beta \approx 0.99$ this non neutrality is small but not zero. One way to eliminate the long-run tradeoff is to set $\beta = 1$, so that expected future inflation shifts the Phillips curve one for one. Indexation, which adds lags of inflation to the Phillips curve, can also eliminate the ong-run output-inflation tradeoff, and improves the empirical fit (Cogley and Sbordone (2008)). But for the main purposes here, exactly one vs. close to one is not a central issue.

Second, not even $MV = PY$ is really so simple, except in the special case that velocity is truly fixed. When velocity depends on the interest rate, as it does, $MV(i) = PY$ also leads to multiple unstable equilibria. Raising money growth leads to spiraling deflation, not inflation that generically moves towards money growth. For example, if velocity rises and real money demand declines with the interest rate and expected inflation rate,

$$m_t + \alpha(p_{t+1} - p_t) = p_t + y,$$

then inflation $\pi_{t+1} = p_{t+1} - p_t$ follows

$$\pi_{t+1} = \left(1 + \frac{1}{\alpha}\right)\pi_t - \frac{1}{\alpha}(m_t - m_{t-1}).$$

Steady states with higher money growth have equally higher inflation, but the steady states are unstable. From a steady state, raising money growth leads to spiraling deflation. Adding fiscal theory prunes these multiple equilibria. So monetarism is both more theoretically analogous to interest-rate targets, and less simple and pure than you thought. It's conventional not to pay much attention to these issues, and I write above really about the general monetarist doctrine, which speaks of "stable" velocity to ignore these issues. But that gentleman's agreement doesn't mean there is a really clean theory in this case either. Chapter and Online Appendix Chapter 21.4 explore in detail.

6.3 Neutrality Intuition

> How do higher interest rates *raise* inflation? What about the intuition that higher interest rates lower aggregate demand, which lowers inflation? Higher interest rates push down current demand and up future demand, so they push down the current price level and raise the future price level. In one interpretation, then, the contrast between Fisherian predictions and this intuition is simply a confusion between current and future inflation. The intuition can also refer to adaptive expectations, but our job is to understand how a Fisherian prediction emerges from rational expectations. Doesn't a higher real interest rate raise demand for bonds, pushing inflation down and the exchange rate up? This common intuition confuses the mechanism of equilibrium formation for relations between equilibrium variables.

How do higher interest rates *raise* inflation, even in models with sticky prices? The equations are transparent, but the implications are hard to believe, so it's worth delving in to the intuition. This also helps us to understand why it's so hard to get models to display a negative sign.

What of the obvious intuition: Higher nominal rates mean higher real rates, that discourage consumption, depress aggregate demand and thus lower inflation?

Consider the consumer first-order condition $x_t = E_t x_{t+1} - \sigma(i_t - E_t \pi_{t+1})$. Raise the nominal interest rate i_t. Before prices change, a higher nominal interest rate is a higher real rate, and induces people to demand less today x_t and more tomorrow x_{t+1}. That change in demand pushes down the price level today p_t and pushes up the expected price level tomorrow p_{t+1}. If prices are flexible, and the economy has a constant real rate, for example constant endowments, that force continues until $E_t \pi_{t+1} = i_t$. Yes, a higher nominal interest rate naturally and intuitively pushes the economy to more inflation.

So, in this framing, both intuitions are correct. Higher interest rates naturally push down the current price level and current inflation, $\pi_t = p_t - p_{t-1}$. Higher interest rates push up expected future inflation $\pi_{t+1} = p_{t+1} - p_t$. So, one resolution of the puzzle is simply that we're talking to cross purposes.

But which is it, lower p_t or higher p_{t+1}? The consumer first-order condition, the intertemporal substitution effect, cannot tell us by itself. It only tells us that p_t falls relative to p_{t+1} and $E_t p_{t+1}$ rises relative to p_t. Answering that question needs a wealth effect, not just a substitution effect. If we pair the higher interest rate with no change in surpluses, and thus no wealth effect, then the initial price level p_t does not change and the entire effect of higher interest rates is a rise in p_{t+1}. A rise in surpluses, actively or passively achieved, leads to a lower price level p_t and less current inflation π_t.

Another interpretation of the lower rate intuition is that it refers to adaptive expectations. In the system (6.1)-(6.2), higher nominal interest rates mean a higher real rate automatically, because adaptive inflation expectations are stuck where they are. So output declines. This system captures standard static Keynesian thinking, as the IS curve has no intertemporal element. Then, lower output pushes down current inflation in the Phillips curve (6.2). Again adaptive expectations are stuck, so there is no way that lower output could push up expected inflation instead.

But we're trying to understand how rational expectations models push inflation up, despite sticky prices; and eventually to find a way to reverse that result. In the system (6.4)-(6.5), a higher interest rate can push up future inflation, leading to no change in the real rate. More importantly, though, if future inflation does not react completely and output declines, that pushes down current inflation, yes, but *relative to expected future inflation.* This is the central feature of the rational expectations Phillips curve, and it operates with much of the intuition of the intertemporal IS curve just above. Low output inexorably results in inflation that is rising over time.

Again, then, perhaps the reconciliation of intuition comes just from terminology: higher real interest rates push current inflation, but push up expected future inflation. And again the overall level comes from the wealth effect, not just how inflation moves over time.

A misinterpretation of the discount rate effect provides a different seductive and common story for a negative response of inflation to interest rates. The central bank raises nominal rates; prices are somewhat sticky so real rates rise. That makes government debt a more attractive asset. People want to buy it, lowering inflation. Similarly, higher real interest rates attract investors to dollar-denominated assets including government debt, raising the exchange rate.

What's wrong with this sensible-sounding story? Apply it to bonds: If bond yields rise, bonds are more attractive, so the story says that bond price should rise. That's obviously wrong: Higher bond yields mechanically mean lower prices.

Think of stocks. Higher expected returns are a higher discount rate, which other things constant should lower stock prices. But wouldn't higher expected returns attract investors, driving prices up? Here you can see the subtle trap. If expected dividends rise, and before prices change, stocks are indeed more attractive, and their expected return rises, drawing people in. But people buy, and drive the price up until the expected return is unchanged. We only see a higher expected return in equilibrium if the price stays low, and people don't want to rush in because of risk. The story describes how prices adjust to equilibrium, not the effect on equilibrium prices of a higher equilibrium and observed rise in expected return. Intuition is slippery, and can confuse how equilibrium is formed with relations we see among equilibrium variables.

The same logic applies to government debt. If expected surpluses rise, and the price level does not move, government debt has a larger expected return. People buy, driving down the price level until the expected return has returned to normal. This is the mechanism for equilibrium formation, which we cannot not observe, in response to a fiscal shock. But in this case, we observe constant expected returns. If we observe a higher expected return, with no change in surplus, that means government debt is less valuable.

Thus, higher real interest rates with no change in surpluses are inflationary, not deflationary. Higher real rates, which may come from higher nominal rates and sticky prices, lower the value of debt.

In sum, verbal intuition is tricky. It is easy to confuse an effect on current inflation with an effect on future inflation, and it is easy to confuse the forces that push prices to equilibrium with relations we expect to see among equilibrium variables.

6.4 Higher or Lower Inflation?

> Do higher interest rates raise or lower inflation? I summarize the lessons of the simple sticky price model for this question with a list of considerations: Is the interest rate rise permanent, or temporary? Is the policy likely to be reversed, if inflation goes temporarily the other way? Is there a lot of long-term domestic-currency debt outstanding? Is the interest rate rise a surprise or widely anticipated? Are prices sticky? Is fiscal policy likely to react either to the same events or to the monetary policy intervention? How will fiscal policy react to larger interest costs? Each of these considerations is important to the sign of the effect of interest rates on inflation.

Does raising interest rates raise or lower inflation, and conversely? These models do not offer a mechanistic answer. Sometimes they predict a positive sign and sometimes a negative sign. That is useful. The point of an economic model is to spell out the preconditions for an effect. Historical experience does not always tell us the full set of necessary ingredients for an outcome, or how that outcome would have changed with different ingredients. A good theory help us to avoid exporting experience from one circumstance to another where the preconditions have changed.

Listing the preconditions may help to believe what the equations are telling us. Contrary episodes or a policy situation may not satisfy the model's preconditions. Evaluating whether each sign and magnitude emerges when theory says it should is a useful way to test the theory and to design improvements. In practice, a good theory will tell us *how* to raise inflation by raising interest rates, or *how* to lower inflation by raising interest rates, guiding us to the right preconditions to obtain a desired result.

Fiscal policy is the most important precondition for any these calculations. We have examined changes in interest rates that come with *no change in fiscal sur-*

pluses. When we analyze events or policies, or consult our intuition built up from historical experience and alternative models, that separation is surely false. Monetary policy changes often come with fiscal policy changes. Countries experiencing inflation are likely to tighten fiscal policy at the same time as they tighten monetary policy. Countries experiencing deflation are likely to lower interest rates and to run deficits. Monetary policy also induces fiscal changes: If an interest rate rise causes a recession, automatic stabilizers, stimulus, and bailouts are likely to follow. These are likely to be an extra *inflationary* force of higher interest rates. I add such feedback in the next chapter. Conversely, some countries lower interest rates precisely to reduce interest costs on the debt, and thus to allow for greater deficits. That mechanism is likely to lead to a negative sign, greater inflation from lower interest rates.

If fiscal authorities react to higher real or nominal interest costs by reducing primary deficits, that adds a deflationary effect of an interest rate rise. If fiscal authorities react to a reduction in real interest costs by abandoning fiscal reforms, a reduction in rates that monetary authorities hope to create disinflation will fail to do so. If fiscal authorities see an interest-rate rise and say, "Whew, the central bank is going to solve inflation for us, we can relax," or if the monetary authorities tighten in response to what they see as too much fiscal stimulus, then we may see fiscal inflation, not temporary monetary disinflation, when the central bank raises rates. If the fiscal authorities cooperate with a joint monetary–fiscal contraction, then the inflation decline can be larger.

For an interest rate rise to *lower* inflation via the long-term debt channel, the interest rate rise must be *persistent* and *unexpected*. It must lower long-term bond prices and raise long-term interest rates, by raising far-future expected inflation. Only a credible and persistent interest rate rise will do that. It's easy for us to write down a persistent process, but harder for the central bank to communicate that expectation and commit to its communications. It will fail if people think this is a trial or experimental effort, or if they worry that the bank will quickly back down if events don't conform to the banks' forecasts. If the rate rise is expected, bond prices will have already declined and the deflationary effect will have passed. This is a testable prediction: Only interest-rate rises that raise long term rates should have disinflationary effects, and should have such effects in proportion to the size of the rise in long term rates. Thus, a sudden shock, that is believed to be long-lasting, a belief reflected in bond prices, is most likely to be disinflationary. The 1980–82 shock, for example, is widely thought to have had more commitment behind it than earlier attempts to lower inflation by raising interest rates. It had a visible and greater impact on long-term rates.

The disinflation gets going when the interest rate rise is *announced*, or really when it becomes believed and expected. The actual initial interest rate rise may be small or delayed. Small rate changes, or simply the appointment of a well known hawk can trigger the bond price decline and the beginning of disinflation.

There must be *long-term debt outstanding*. Many countries in fiscal stress have moved to short-term financing, so there just isn't that much long-term debt left. They should see higher interest rates (with no fiscal change) raise inflation.

The interest rate rise only affects *domestic-currency* debt. A government that has largely borrowed in foreign-currency debt cannot change the value of that debt by interest rate rises. Thus, a country that borrows more abroad is likelier to see inflation rise rather than decline when it raises interest rates.

This list of preconditions makes another point. It would be wonderful to have a good model of short-term non-neutrality that makes sense of the widespread belief in a robust negative sign, but the long-term debt mechanism is far from perfect for this purpose. The negative effect only holds for unexpected interest rate rises, when they are announced. But maybe expected interest rate rises can also lower inflation, and maybe they do so when interest rates actually rise, not just when they are announced. Maybe higher interest rates lower inflation even when governments borrow short-term, or with foreign debt. Maybe the size of a negative effect is larger than the model predicts, and is not tied to the maturity structure as the model predicts. US government debt is relatively short-term, and has been shorter in the past. Maybe transitory interest rate rises that do not lower long term bond prices also lower inflation. Maybe the negative effect is stronger, not weaker, with stickier prices.

These features are not necessarily counterfactual. They are just unknown. This model is new. Nobody has looked to see if the negative effect of interest rates, orthogonalized to fiscal policy, on inflation is quantitatively linked to announcement, maturity, persistence, and price stickiness as the model predicts. Looking would be a valuable empirical project. But the standard intuition, contrary to all these predictions, may also have a grain of truth. I suspect we need a better model of a short-run negative response.

The long-term debt response function also lowers inflation immediately, where the common intuition we would like to see if the model can produce lowers inflation gradually. That fact reveals a deep gulf between the economic mechanism of the long-term debt response and usual intuition. The negative long-term debt response operates much like a fiscal shock. It is a wealth or a level effect. It has nothing to do with the conventional adaptive expectations intuition, in which higher real interest rates lower aggregate demand, and thereby lower *future* inflation. It does not produce something like the adaptive-expectations dynamics in the short run, which then turn around and become stable when some suitable friction or information problem is resolved. It will be a long time before a long time before anyone writes opeds, Fed chairs explain, and we teach to undergraduates that the central mechanism by which the Fed can temporarily lower inflation is by pledging through persistently higher interest rates to inflate away long-term debt, which thereby makes short-term debt more valuable as a claim to unchanged fiscal surpluses. I suspect we need a better model of a short-run negative response. Some speculations follow.

If the government wants to *raise* inflation by raising interest rates, the rise should be *preannounced* far ahead of time to avoid the contrary long-term debt mechanism which relies on unexpected inflation. If the move is announced before a lot of debt is sold, the inflation decline induced by the long-term debt mechanism is reduced or eliminated. It helps if there is not much long-term debt outstanding so the initial negative effect can be smaller. The United States' slow, widely preannounced, and credible interest rate rises of the 2015–2019 period, which featured more inflation than in Europe which did not follow that policy, together with the maturity-shortening effect of the Federal Reserve's QE program, is a suggestive example, or at least a contrast with the shock treatment of 1980–1982.

There are likely other short-run non-neutralities pushing inflation the other way, probably involving financial channels. Whatever they are, a policy that raises inflation by raising interest rates must avoid them. Preannouncement and slow and steady interest rate rises seem plausible antidotes to these as well.

To raise inflation, the interest rate rise should also be *persistent* in our model. Inflation rises in advance of expected interest rate rises, and falls in advance of the expected end of a rate rise. In the previous graphs, permanent and expected rate rises have greater positive effects. Standard new-Keynesian models also give greater positive effect to permanent rate rises. Here too, permanence is harder in the real world than writing down an AR(1) coefficient. The rate rise must be credible and committed. Where disinflationary rate rises need only the commitment to ride out a recession, inflationary rate rises (or disinflationary rate reductions, especially) must have a strong commitment to ride out a short-term contrary movement, and all sorts of pressures to abandon the policy, not the least from economists in the adaptive-expectations tradition.

The *discount rate* or *interest cost* effect adds an inflationary force of interest rate rises. Higher real rates raise the discount rate for surpluses, or directly the interest costs on the debt. Loyo (1999) finds higher interest rates raised inflation in Brazil by this mechanism. Under fiscal stress, the central bank tried to defend the currency and to lower inflation by repeated interest rate rises. Each one seemed to quickly and perversely lower the exchange rate and result in more inflation. If one wishes inflation, then convincing fiscal authorities *not* to raise taxes or cut spending to pay additional interest costs on the debt is helpful. Contrariwise, of one wishes to disinflate with interest rate rises, the more fiscal support the better. Convince fiscal authorities to at least pay higher interest costs on the debt and the real windfall to bondholders.

The discount rate or interest cost effect is more important for highly indebted countries. At 100% debt-to-GDP ratio, each one percentage point rise in real interest rates adds 1% of GDP to interest costs. If the debt to GDP ratio is 10% of GDP, the same rate rise only adds 0.1% of GDP to interest costs. So highly indebted countries, with much short-term debt and sticky prices, are more likely to see higher interest rates translate into higher, not lower inflation, and vice versa. The *stickier* prices are, then, the more likely it is that higher interest rates *raise* inflation.

6.5 Higher Inflation in Practice

> The long quiet zero bound is evidence for stability, and against adaptive expectations' instability or new Keynesian indeterminacy. Stability implies that higher interest rates lead to higher inflation. Interest rate pegs that failed typically are accompanied by fiscal problems. The Fisherian prediction was seriously considered as a way to raise inflation at the zero bound.

Is inflation stable and determinate under an interest rate peg? It's generally hard to tell, since central banks don't leave interest rates alone for a long time, in an environment with stable fiscal policy and no other large shocks. Recently, however, we witnessed a crucial experiment. Following the 2008 financial crisis, the interest rate in the US was stuck at zero for nearly a decade, that in Europe and the UK for more than a decade, and in Japan for 25 years. Despite the widespread predictions of deflation spiral or multiple equilibrium volatility, nothing happened. The failure of inflation to spiral away when central banks moved inflation slowly in 2021-2022 is a second suggestive episode.

What about many interest rate pegs that failed in a poof of inflation? The proposition says that inflation is stable so long as there are no fiscal shocks. Many of those episodes featured large and long lasting fiscal problems. Indeed, many governments peg interest rates at low levels as a method of financial repression to lower interest costs on large debts. They typically also allocate credit, so the interest rate is not the market-clearing rate of the model. Artificially low interest rates are a stock tool of wartime finance. The US interest rate peg from World War II to the early 1950s was explicitly undertaken to lower interest costs on the debt. An overvalued foreign exchange rate peg, which likewise should raise the value of the currency, often comes as a means of diverting foreign exchange earnings to a fiscally strapped government.

What about Turkey, in the early 2020s loudly trying to lower inflation by lowering interest rates, and failing to do so? An unsteady long-term fiscal policy, including a lot of foreign currency borrowing, are natural questions to ask. Who wants to buy long-term Turkish government debt?

Contrariwise, not all interest rate pegs led to inflation. Many lasted a surprisingly long time, given models that they should immediately lead to instability or indeterminacy.

All this this is anecdote, not proof, but that there exist episodes consistent with stability, and troubling to pervasive instability and indeterminacy is news. I cover these and other episodes in Chapters 22 and 23.

One may swallow hard, look at the zero bound era, and accept that stability and determinacy are not completely implausible. But the proposition that higher interest rates eventually lead to higher inflation seems much harder. Yet it follows inescapably from determinacy and stability. Can the central bank really raise interest rates and thereby raise inflation, perhaps after a long wait, and vice versa? As we didn't really know much about long-run stability until central banks left interest rates alone for a decade in the 2010s, recognize that we really have no experience to build on. No central bank has ever tried it, with the required background of stable fiscal policy.

It's not an academic question. The possibility of raising interest rates to raise inflation was in the air through the zero-bound era of the 2010s. The United States, Japan, and Europe, despite long periods of near-zero or even negative interest rates, and forward guidance of more to come, still had inflation below their targets. Most policy discussion remained anchored on how to raise inflation by applying ever more monetary "stimulus" or "accommodation," immense quantitative easing bond purchases, banning cash to allow sharply negative interest rates, or undertaking additional "fiscal stimulus." None of these measures made any visible dent in inflation. To some, that just meant governments needed to do more of what was not yet working.

A few academics and commentators started to question the uniform and mechanical negative sign, and suggested that perhaps a steady and widely preannounced interest rate rise might *raise* inflation, at least eventually. Prominent examples include Kocherlakota (2010), Williamson (2013),Bullard (2015), Schmitt-Grohé and Uribe (2014), Cochrane (2014c), Smith (2014) and Garín, Lester, and Sims (2018) and Uribe (2022). These authors pointed out that a long-run Fisherian response is a robust prediction of standard new-Keynesian models.

In the early 2020s, a range of opinion in Brazil and Turkey, each dealing with persistent inflation, started to think that perhaps lowering interest rates is the secret to lowering inflation. Turkey embarked on that policy, and it produced only large inflation. But do these economies have the preconditions for that strategy to work? Arguably, neither has the stable fiscal situation required for a Fisherian response, nor are the interest rate reductions steady, preannounced, and clearly destined to survive intermediate run turmoil.

6.5.1 Summary and Implications

I struggle with the long run Fisherian prediction, and the limits of the negative short-run effect in the long-term debt model. Perhaps central banks just exploit a negative short-run effect and we never see, and they never experience the long run. The long-term debt model is inadequate for that purpose however. I speculate how we might construct a better model of a short-run negative effect. Empirical work is not definitive. The short-run negative effect may be more ephemeral than current doctrine suggests.

You can see by the length of this discussion—indeed, of this book—that I struggle with the clear predictions of the theory developed so far. They are simple and logically transparent, but they quite different from conventional doctrine and its stylized narrative of history and policy choices: Higher interest rates eventually produce higher inflation. There is a limited mechanism for a short-run negative sign, but it only works for surprises, with long-term debt outstanding, and it produces an instant disinflation that melts away, rather than a slowly increasing disinflation. Moreover, it operates via a wealth effect, not via higher real interest rates that depress aggregate demand.

The Fisherian prediction that an interest rate peg is stable, and that higher interest rates eventually lead to higher inflation is hard to avoid. It results at heart from neutrality, that real quantities eventually do not depend on units of measurement, stability, and determinacy. Any model with those properties will be Fisherian. In our models, it comes centrally from rational expectations. Even that is too strong a requirement – it comes from model-consistent expectations; that the expectations in the model are not systematically different from the expectations of the model; and it comes from forward-looking behavior. That's a lot weaker than "rationality." And it only requires this behavior of the very long run, that people catch on eventually, which is even more robust.

The Fisherian prediction has been part of rational expectations models for at least 40 years, including new-Keynesian models beginning in the late 1980s. It is not centrally a fiscal theory issue. But it is so counter to conventional doctrine that it was first ignored for decades, and when recognized in the early 2010s it stood out like a sore thumb, and a new discovery.

This is a great story of the interplay between theory and experience. Sometimes the equations and logic of a simple model speak up and make startling new predictions. Money is neutral in the long run, inflation cannot permanently lower unemployment, clocks run slower on trains that approach the speed of light. Great advances came from taking those predictions seriously. For each success, though, a thousand equations have screamed nonsense. So an agonizing soul searching is appropriate, as well as a dedicated search among model ingredients alternatives that can at least rationalize the pervasive belief in standard doctrines.

The most sensible reconciliation of this theory and conventional doctrine, as above, is that central banks regularly exploit a short-run negative reaction, so we never see, and they never experience, the positive long-run relation. The bank raises interest rates to push inflation down fast, then lowers rates to the Fisherian long run level. The path of interest rates and inflation from such episodes looks almost identical to the path generated by the conventional adaptive expectations model. Whether inflation would converge to interest rates or diverge in the long run then makes essentially no difference to central bank policy or what we observe.

The relevance of conventional doctrine is enhanced if the long run takes a really long time. Then it can be treated as an important theoretical benchmark of less practical importance, like the related doctrine of purchasing power parity for exchange rates.

That reconciliation still suggests, however, that a widely preannounced interest rate rise and patience would be useful for governments that wish more inflation at the zero bound, or when otherwise the negative short-run relationship cannot be exploited.

One trouble with this explanation, however, is that the long-term debt mechanism requires a persistent rate rise to lower inflation. The bank cannot raise rates and then *predictably* lower them once inflation has subsided. At best, then, one must tell a story that higher rates push down inflation, and then fiscal policy or economic growth comes to the rescue with greater surpluses to keep inflation down. That is, below, essentially the story I offer of the 1980s.

This observation, and the many limitations of the long-term debt model alluded to above, put a better model of a short-term negative effect of interest rates on inflation on the front burner for investigation. The large literature on channels of monetary policy suggests many other mechanisms that might work, and continue to work when we raise the bar by disallowing contemporaneous fiscal shocks.

In these little models, the Phillips curve is the central source of dynamics. Indeed, the adaptive vs. rational expectations question comes down really to the sign of output in the Phillips curve. With rational expectations, high output comes with inflation that is large relative to expected future inflation, but that means that inflation *decreases* over time. With adaptive expectations, high output comes with inflation that is large relative to past inflation, but that means that inflation *increases* over time. Empirical work seems to favor the latter. Many new-Keynesian models add lags in the Phillips curve for a better fit. Perhaps more elaborate and better fitting Phillips curves will help. The search and matching view has revolutionized the economics of unemployment, but not yet really changed the nature of Phillips curves used in macroeconomics.

More elaborate models with capital with adjustment costs, credit constraints, balance sheet channels, portfolio adjustment costs, financial frictions, individual heterogeneity, strategic complementarity and more offer a smorgasbord of potential mechanisms. For example, higher real interest rates mean that households with adjustable rate mortgages have to pay higher rates, and if the corresponding receipt of more interest revenue by lenders does not offset, inflation may decline. Liquidity effects in financial markets and for government bonds are intriguing, putting a vestige of monetarism back in the model, though the supply of liquidity must be constrained for that to have any effect. Integrating such models with fiscal theory is more low-hanging fruit.

However, though it is likely to be possible to find *sufficient* conditions to deliver a negative sign, with enough model complications, our goal should be to find minimum *necessary* conditions, that apply most broadly and robustly.A negative response of inflation to interest rates should be a robust and deeply rooted phenomenon, one that will not vanish if, for example, the US changes the downpayment rules on mortgages. In his Nobel Lecture, **?**, Bob cites David Hume for understanding the neutrality and non-neutrality of money in 1752. Velde (2009) documents a beautiful non-neutrality episode in 1724 France, with a monetary and financial system utterly unlike our own.

And the challenge is more difficult than for monetary theory. From $MV = PY$, it is enough for price stickiness to slow down the flow from M to P. If more money instantly leads to more nominal income PY, then all we need to do is to give some of that to real income Y rather than inflation P in the short run. The interest-rate based doctrine needs to change sign. If higher nominal rates lead to less than proportional inflation in the short run, and thus higher output, as in Figure 5.2, then we have a similar non-neutrality. But higher interest rates do not lower inflation. If we wish higher interest rates to produce *declining* inflation, not just a downward jump in inflation that melts away, then we need the real rate to decline by more than the nominal rate rises! Start with $i_t = r_t + E_t\pi_{t+1}$. If you want higher i_t to lower $E_t\pi_{t+1}$, then you need r_t to go down by more than i_t rises. That's a lot of non-neutrality.

I long for the clarity and simplicity of Bob Lucas's (1972) model of money non-neutrality, adding one explicit friction to an otherwise neutral and stable model.

Perhaps rather than add ingredients, we should jettison one. In these models, the Phillips curve is the central source of inflation dynamics. Yet the Phillips curve has not achieved great theoretical and empirical clarity, despite decades of dedicated work by top macroeconomists. It may make sense that firms sell more when output prices are high, or that worker work harder when wages are high. But these are relative prices, where the Phillips curve states that output and employment increase when all prices and wages rise together. So, any Phillips curve needs some confusion or correlation of relative prices with the overall price level.

Perhaps we can start to study the dynamic relationship between inflation and nominal interest rates apart from the Phillips curve. Our goal, after all, is to understand the dynamic relationship between interest rates and inflation. The Phillips curve came from thinking about output and employment *effects* of inflation. We are reversing the logic, using the IS equation to describe how interest rates lower output, and then the Phillips curve to describe how output affects inflation. The Phillips curve wasn't designed to be the central mechanism for nominal dynamics. Perhaps inflation dynamics should come before the Phillips curve.

For example, in 2021-2022 most commentary in and around central banks centered on "supply chain" shocks and relative price movements, which particular goods or sectors were going up or down, as both underlying cause and key variables for the dynamics of inflation. A good example is **?**. In this view, large good or sector specific supply shocks or demand shocks move relative prices; interacted with prices that are more sticky downward than upward, they and not the immense fiscal expansion, account for inflation. More importantly, these relative prices – average vs. marginal rents, house prices vs. rents, etc.—are key state variables that analysts look at for forecasting future inflation. The old Phillips curve, with a single output gap or unemployment capturing the entire effect of the real economy on inflation, is pushed to the background. This extensive commentary and forecasting is only beginning to enter academic modeling. Related, there is new interest in describing inflation dynamics in production networks, for example **?** and **?**. **?** argue that some inflation is optimal when there are reallocation shocks and downward nominal stickiness. Perhaps reallocations, networks, supply and demand shocks interacted with sticky prices and wages, will completely take over from the IS and Phillips curve as our basic model of inflation dynamics.

There is, of course, another possibility: A strong, reliable, negative short run effect of higher nominal interest rates without a change in fiscal policy might not be true. Empirical work is surprisingly unclear, given the strong and widespread belief that higher interest rates lower inflation. In VARs, interest rate shocks give rise to transitory interest rate movements, so we don't see a measurement of long-term neutrality. And even the short-run negative sign is hard to see. Starting with Sims (1980), VAR estimates routinely find a "price puzzle" that tighter monetary policy raises inflation. The opposite prior being so strong, the result was chalked up to reverse causality, the Fed tightening in response to information about future inflation. It took a lot of delicate shock-identification carpentry to see the hoped-for result, prominently in Christiano, Eichenbaum, and Evans (1999), and even then a monetary tightening only slowly sets off a small downward drift in the price level, with most variance of inflation ascribed to inflation shocks.(Most of the literature also identifies tightening with a change in monetary aggregates.) In a survey and replication of this vast literature,Ramey (2016) finds that the "price puzzle" that higher interest rates result in higher inflation remains the central finding of VARs. Rusnak, Havranek, and Horvath (2013) conduct a meta-analysis of monetary VARs, using a variety of statistical techniques to correct for the selection and publication biases that want to produce a negative effect of interest rates on inflation, with the same result. Uribe (2022), looking with a different prior in mind, presents VAR evidence that permanent interest rate rises increase inflation in the United States. Cochrane (1994a) discusses a key problem why this seemingly simple response is hard to measure: Central banks always react to events, and never randomly change interest rates. One must look for interest rate changes that do not respond to the specific variable one wishes to examine, a so-far unused suggestion.

And even so, one may feel that the VARs completely miss the important effect of an intervention such as 1980. Most VARs isolate transitory idiosyncratic movements in the federal funds rate, not long-lasting movements that we saw in 1980, or that both the Fisherian prediction and the long-term debt model require. Most of all, by design, they find idiosyncratic deviations from a rule, not changes in rule or "regime" that may durably change expectations. If the art of reducing inflation is to convince people that something has changed so they should lower inflation expectations, then the response to a monetary policy "shock" orthogonal to a stable "rule" completely misses the successful policy.

Most of all, no current VAR attempts to find monetary policy shocks orthogonal to fiscal policy. Empirically evaluating the long-term debt model, checking if the inflation response varies with debt maturity, shock persistence, and anticipation as it should, remains low-hanging fruit, though admittedly fruit that is challenging to harvest. For this reason, I emphasize episodes such as the zero bound era rather than formal econometric evidence in this book.

In sum, nothing is easy in economics. The answer to "What happens if the central bank raises rates and leaves them there forever?" is not easily answered by historical experience of transitory and reversible rate changes, and monetary and fiscal policy that continuously reacts to events. Models robustly point to a long-run positive relationship, counter to policy world intuition. This section summarizes my struggles to see if the prediction is believable, by recognizing the long list of its preconditions, that may make it true but not visible or frequently exploitable. The predictions are not as simple as just raise or lower rates and inflation will painlessly follow, and the facts are not nearly as contradictory as conventional wisdom would have one believe. But the point of economics is not to judge "beliefs" but to look at the world through the eyes of models.

Chapter Seven

Policy Rules and an S-Shaped Surplus

So far, I have taken fiscal surpluses $\tilde{s}_t$ as an exogenous process, modeled if at all by an AR(1). Yet Chapter 4 explained at length just how awful an AR(1) is as a model of surpluses; I have been opining that one can and should allow endogenous fiscal policy, responding to recessions with deficits for example; and we want to study fiscal policy rules. So, in this chapter I consider how to specify fiscal policy in the same sticky price fiscal theory framework. I also add more flexible monetary policy rules for setting interest rates, and build as far to a complete model as this book will go.

7.1 A Surplus Process

I introduce a simple parametric surplus process that allows an s-shaped response, allows some unexpected inflation, and retains active fiscal policy. The surplus responds to a latent variable:

$$\tilde{s}_{t+1} = \alpha v_t^* + u_{s,t+1}$$
$$\rho v_{t+1}^* = v_t^* + \beta_s \varepsilon_{s,t+1} - \tilde{s}_{t+1}$$
$$u_{s,t+1} = \eta_s u_{s,t} + \varepsilon_{s,t+1}.$$

We may interpret the latent variable v_t^* as the value of debt if the surplus responds to changes in the value of debt that come from past deficits but does not respond to changes in the value of debt brought about by arbitrary unexpected inflation or deflation. In equilibrium, debt v_t is equal to v_t^*.

Our next step is to write a reasonable, flexible, realistic, and tractable surplus process. Building to larger models, we want a process written in first-order, VAR(1) form, describing variables at time $t+1$ in terms of variables at time t and shocks at time $t+1$, as most recently the standard new-Keynesian IS and Phillips equations (5.18)-(5.22). We want a process that allows an s-shaped moving average, one in which today's deficits ($s < 0$) are followed by future surpluses ($s > 0$) that can at least partially pay off the debt.

The natural way to induce an s-shaped moving average in a VAR(1) structure is to add a latent state variable, which I denote v_t^*. I write

$$\tilde{s}_{t+1} = \alpha v_t^* + u_{s,t+1} \tag{7.1}$$
$$\rho v_{t+1}^* = v_t^* + \beta_s \varepsilon_{s,t+1} - \tilde{s}_{t+1} \tag{7.2}$$