The Fiscal Theory of the Price Level

John H. Cochrane

Hoover Institution, Stanford University

PRINCETON UNIVERSITY PRESS
PRINCETON AND OXFORD

Published by Princeton University Press,
41 William Street, Princeton, New Jersey 08540

In the United Kingdom: Princeton University Press,
6 Oxford Street, Woodstock, Oxfordshire OX20 1TR

press.princeton.edu

Library of Congress Control Number:
0000000000
ISBN: 0000000000000
ISBN (pbk): 0000000000000

British Library Cataloging-in-Publication Data is available

Editorial: ...
Production Editorial: ...
Text Design: ...
Jacket/Cover Design: ...
Jacket/Cover Credit: ...
Production: ...
Publicity: ...

This book has been composed ....

Printed on acid-free paper.

Typesetting and Printing information ...

To Eric W. Cochrane and Lydia G. Cochrane.
Their memory and example are with me always.

A prince, who should enact that a certain proportion of his taxes be paid in a paper money of a certain kind, might thereby give a certain value to this paper money.

Adam Smith, *Wealth of Nations*, Vol. I, Book II, Chapter II.

Contents

Preface

THIS BOOK IS a midpoint, I hope, of a long intellectual journey. It started in the fall of 1980, drinking a beer and eating nachos on a sunlit afternoon in Berkeley, with my good friends and graduate school study group partners, Jim Stock, Eric Fisher, Deborah Haas-Wilson, and Steve Jones. We had been studying monetary economics, and thinking about what happens as electronic transactions reduce the demand for money. When money demand and money supply converge on fast-moving electronic claims to a single dollar bill, framed at the Federal Reserve, will supply and demand for that last dollar really determine the price level? If the Fed puts another dollar bill up on the wall, does the price level double? Jim and I, fallen physicists, joked about a relativistic limit. Signals are limited by the speed of light, so maybe that puts a floor to money demand.

The conversation was playful. Clearly, long before the economy is down to the last dollar bill, each of us holding it for a microsecond at a nanodollar interest cost, the price level becomes unhinged from money supply. Such a "cashless limit" is a good example of a result in economics that one should not take seriously. But is there a theory of the price level that continues to work as we move to electronic transactions and a money-less economy, or equivalently as money pays interest? Why is inflation apparently so stable as our economy moves in that direction? Or must economic and financial progress be hobbled to maintain money demand and thereby control inflation? Having no ready answers, the conversation moved on, but the seed was planted.

Berkeley was, it turns out, a great place to be asking such questions. Our teachers, and especially George Akerlof, Roger Craine, and Jim Pierce, mounted a sustained and detailed critique of monetarism. They had their own purposes, but the critique stuck, and my search continued for an alternative theory of the price level. Berkeley also gave us an excellent grounding in microeconomics and general equilibrium, for which I thank in particular Rich Gilbert, Steve Goldman, and Gerard Debreu, together with unmatched training in empirical economics and econometrics, for which I thank especially Tom Rothenberg.

I spent a year as a research assistant at the Council of Economic Advisers, working under Bill Poole on many policy-oriented monetary economics issues. This was a formative experience, and one of those projects turned into my Ph.D. dissertation. I owe Bill many thanks for his gracious guidance.

I was then supremely lucky to land a job at the University of Chicago. Chicago was a natural fit for my intellectual inclinations. I like the way standard economics works. You start with supply, demand, and frictionless markets. You add frictions and complications carefully, as needed. It also often turns out that if you work a little harder, a simple supply and demand story explains many puzzles, and you don't need the frictions and complications. For my tastes, economists too often give a clever name to a puzzle, proclaim that no standard economic model can explain it, and invent a new theory. Ninety-nine revolutions are pronounced for each one that succeeds.

This statement may sound contradictory. In this book I argue that the fiscal theory is a genuinely new theory that unseats its predecessors at the foundation of monetary economics. Yet fiscal theory is, at least as I present it, much in the Chicago tradition. It allows a less-is-more approach, in which with a little bit of hard supply and demand work takes you further than you might have thought.

These were, in hindsight, glorious years for macroeconomics at Chicago. Efficient markets, Ricardian equivalence, and rational expectations were just in the past. Dynamic programming and time-series tools were cutting through long-standing technical limitations. Kydland and Prescott (1982) had just started real business cycle theory, showing that you can make remarkable progress understanding business cycles in a neoclassical framework, if you just try hard enough and don't proclaim it impossible before you start. For me, it was a time of great intellectual growth, learning intertemporal macroeconomics and asset pricing, privileged to hang out with Lars Hansen, Gene Fama, Bob Lucas, and many others, and to try out my ideas with a few generations of amazing students.

But monetarism still hung thick in the air at Chicago, and monetary doubts nagged me. I wrote some papers in monetary economics, skeptical of the standard stories and the VAR literature that dominated empirical work. Still, I didn't find an answer to the big price level question.

A watershed moment came late in my time at the Chicago economics department. I frequently mentioned my skepticism of standard monetary stories. The conversations usually didn't get far. Then one day, Mike Woodford responded that I really should read his papers on fiscal foundations of monetary regimes, which became Woodford (1995) and Woodford (2001). I did. There it was at last: a model able to determine the price level in a completely cashless economy. I knew in that instant this was going to be a central idea that I would work on for the foreseeable future. I was vaguely aware of Eric Leeper's (1991) original paper, but I didn't understand it or appreciate it until I went back to it much later. Papers are hard to read, and I was not well-read in the new-Keynesian tradition to which Leeper rightly addressed his paper. Social networks are important to point us in the right direction.

It is taking a lot longer than I thought it would! I signed up to write a *Macroeconomics Annual* paper (Cochrane (1998a)), confident that I could churn out the fiscal theory analogue of the Friedman and Schwartz (1963) *Monetary History* in a few months. Few forecasts have been more wrong. That paper solved a few puzzles, and paved the way for many more, but I'm still at the larger question more than two decades later.

I thought then, and still do, that the success of fiscal theory will depend on its ability to organize history, to explain events, and to coherently analyze policy; on its usefulness; not by theoretical disputation or formal time-series tests, just as Friedman's monetarism and Keynes' Keynesianism had done. Nonetheless, my first years with the fiscal theory were dragged into theoretical controversies. One has to get a theory out of the woods where people think it's logically wrong or easily dismissed by armchair observations before one can get to the business of matching experience.

"Money as Stock" (Cochrane (2005b)) addressed many controversies. (I wrote it in the same year as "Stocks as Money" (Cochrane (2003)), an attempt at CV humor as well as to point toward a theory that integrates fundamental value with value deriving from transactions frictions, which applies to stocks as well as to government debt.) I owe a debt of gratitude to critics who wrote scathing attacks on the fiscal theory, for otherwise I would not have had a chance to rebut the similar but more polite dismissals that came up at every seminar.

I then spent quite some time understanding and then documenting the troubles of the reigning new-Keynesian paradigm, including "Determinacy and Identification with Taylor Rules " (Cochrane (2011a)), "The New-Keynesian Liquidity Trap" (Cochrane (2017c)), and "Michelson-Morley, Occam, and Fisher" (Cochrane (2018)). The first paper emphasized flaws in the theory, while the second two pointed to its failures to confront the long zero interest rate episode. To change paradigms, people need the carrot of a new theory that plausibly accounts for the data, but also a stick, to see the flaws of the existing paradigm and how the new theory mends those flaws.

Matching the fiscal theory with experience turns out to be more subtle than noticing correlations between money and nominal income. The present value of surpluses is hard to measure independently. In the wake of the decades-long discussion following Friedman and Schwartz (1963), we approach causality and equilibrium-formation discussions in a sophisticated way. Easy predictions based on natural simplifying assumptions quickly go wrong in the data. For example, deficits in recessions correlate with less, not more, inflation. I spent a lot of time working through these puzzles. "A Frictionless View of U.S. Inflation " (Cochrane (1998a)) already suggests that a surplus process with an s-shaped moving average representation and discount rate variation in the present value formula are crucial to understanding that pattern. "Long-Term Debt and Optimal Policy" (Cochrane (2001)) took on the surplus process more formally, but with a cumbersome argument using spectral densities. Only in "Fiscal Roots" (Cochrane (2021a)) did I really express how discount rate variation rather than expected surplus variation drives inflation in postwar U.S. recessions. Only while dealing with some contemporary "puzzles" have I realized just how bad a mistake it is to write a positively autocorrelated process for government surpluses. Though "A Frictionless View" pronounced observational equivalence, only now have I come to my current understanding of its implications, and that it is a feature, not a bug.

It turned out to be useful that I spent most of my other research time on asset pricing. Indeed, I sometimes refer to fiscal theory as "asset pricing imperialism." I recognized the central equation of the fiscal theory as a valuation equation, like price = present value of dividends, not an "intertemporal budget constraint," a point that forms the central insight of "Money as Stock" (Cochrane (2005b)) and surmounts a first round of objections to fiscal theory. Intellectual arbitrage is a classic source of progress in economic research. I also learned in finance that asset price-dividend ratios move largely on discount rate news rather than expected cash flow news (see "Discount Rates" Cochrane (2011c) for a review). More generally, all the natural "tests of the fiscal theory" you might want to try have counterparts in the long difficult history of "tests of the present value relation" in asset pricing.

Dividend forecasts, discounted at a constant rate, look nothing like stock prices. So don't expect surplus forecasts, discounted at a constant rate, to look like the value of debt, and their differences to quickly match inflation. The resolution in both cases is that discount rates vary. This analogy let me cut through a lot of knots and avoid repeating two decades of false starts. Again, it took me an embarrassingly long time to recognize such simple analogies sitting right in front of me. I wrote about time-varying discount rates in asset prices in Cochrane (1991b) and Cochrane (1992). I was working on volatility tests in 1984. Why did it take nearly 30 years to apply the same lesson to the government debt valuation equation?

"Interest on Reserves" (Cochrane (2014b)) was another important stepping stone. The Fed had just started trying to run monetary policy with abundant reserves, and controlling market interest rates by changing the interest rate the Fed pays on reserves. But the Fed also controls the size of reserves. Can the Fed control the interest rate on reserves, and simultaneously the quantity? Will doing so transmit to other interest rates? It took some puzzling, but in a fiscal theory framework, I came to an affirmative conclusion. This paper introduced the expected-unexpected inflation framework, and much of the merging of fiscal theory with new-Keynesian models that occupy the first part of this book. It only happened as John Taylor and Mike Bordo invited me to present a paper at a Hoover conference to mark the 100th birthday of the Federal Reserve. The opportunity, and obligation, to write a paper that connects with practical policy considerations, and to present it to a high-powered group of economists and Fed officials, brought me back to thinking in terms of interest rate targets. I should have been doing so all along—Eric Leeper's papers have for decades—but such is life.

Another little interaction that led to a major step for me occurred at the Becker-Friedman Institute conference on fiscal theory in 2016. I had spent most of a year struggling to produce any simple sensible economic model in which higher interest rates lower inflation, without success. Presenting this work at a previous conference, Chris Sims mentioned that I really ought to read a paper of his, "Stepping on a Rake," (Sims (2011)). Again, I was aware of Chris's paper, but had found it hard. After Chris nagged me about it a second time, I sat down to work through the paper. It took me six full weeks to read and understand it, to the point that I wrote down how to solve Chris's model, in what became Cochrane (2017e). He had the result, and it became important to the unified picture of monetary policy I present here. Interestingly, Chris's result is a natural consequence of the analysis in my own "Long-Term Debt" paper, Cochrane (2001). We really can miss things that are right in front of our noses. The simple exposition of the result in this book is a nice case of how economic ideas get simpler over time and with rumination.

Marty Eichenbaum and Jonathan Parker then kindly agreed to my proposal for a *Macroeconomics Annual* essay, "Michelson-Morley, Fisher, and Occam" (Cochrane (2018)), putting together these thoughts along with an overview of how the zero bound era provides a decisive test of theories. The result is rather sprawling, but the chance to put it together and to get the incisive feedback of the top economists at that event was important to producing the (I hope) cleaner vision you see here.

These events allowed me to complete a view that has only firmed up in my mind in the last year or so, which I call the "fiscal theory of monetary policy" expressed most recently in Cochrane (2021b) and in this book. Monetary policy implemented by interest rate targets remains crucially important. The fiscal theory neatly solves the determinacy and equilibrium selection problems of standard new-Keynesian models. You can approach the data armed with interest rate rules and familiar models. You really only change a few lines of computer code. The results may change a lot, especially by emphasizing fiscal–monetary interactions. Without the conferences, and Chris's and others' sharp insights, none of it would have happened.

My fiscal theory odyssey has also included essays, papers, talks, op-eds, and blog posts trying to understand experience and policy with the fiscal theory, and much back and forth with colleagues. This story-telling is an important prelude to formal work, and helps to focus and distill formal work. Story-telling is hard too. Is there at least a possible, and then a plausible story to interpret events via the fiscal theory, on which we can build formal model descriptions? That's what "Unpleasant Fiscal Arithmetic" (Cochrane (2011e)), "Inflation and Debt" (Cochrane (2011d)), "Michelson-Morley, Fisher, and Occam" (Cochrane (2018)), and "The Fiscal Roots of Inflation" (Cochrane (2021a)) attempt, building on "Frictionless View" (Cochrane (1998a)), among others. This book contains many more stories and speculations about historical episodes, which I hope inspire you to do more serious theoretical and empirical work.

I owe a lot to work as referee and journal editor, especially for the *Journal of Political Economy*. Editing and refereeing forced me to understand many important papers that I might otherwise have put aside or read superficially in the usual daily crush. Discussing papers at conferences had a similar salutary effect. "Determinacy and Identification" is one example that can stand for hundreds. I grasped a central point late one night while working on Benhabib, Schmitt-Grohé, and Uribe (2002). Their simple, elegant paper finally made clear to me that in new-Keynesian models, the central bank deliberately destabilizes an otherwise stable economy. I immediately thought, "That's crazy." And then, "This is an important paper. The JPE has to publish it."Several of my papers were born that moment. Research is all a conversation.

I also owe a deep debt to generations of students. I taught a Ph.D. class in monetary economics for many years. Discussions with really smart students helped me to understand the standard models and key parts of fiscal theory alternative. Working through Mike Woodford's book (Woodford (2003)), and working through papers such as Werning (2012), to the point of understanding their limitations is hard work, and only the pressure of facing great students forced the effort. There are important externalities between teaching, service, and research.

More recently, writing a blog has allowed me to try out ideas and have a discussion with a new electronic community. My understanding of the Fisherian question – does raising interest rates maybe raise inflation? – developed in that forum.

Over the years, I benefited from the efforts of many colleagues who took the time to engage in discussions, write me comments, discuss my and other papers at conferences, write referee and editor reports, and listen to and contribute to many seminars where I presented half-baked versions of these ideas. Research *is* a conversation.

I owe debts of gratitude to institutions as well as to people. Without the Berkeley economics department, I would not have become a monetary skeptic, or, probably, an economist at all. Without Chicago's economics department and Booth school of business, I would not have learned the dynamic general equilibrium tradition in macroeconomics, or asset pricing. Without the Hoover Institution, I would not have finished this project, or connected it to policy.

I am also grateful to many people who have sent comments on this manuscript and the recent work it incorporates, including Jean Barthélemy, Christopher Ball, Marco Bassetto, Michael Ben-Gad, Tom Coleman, François Gourio, Jon Hartley, Zhengyang Jiang, Greg Kaplan, Marek Kapička, Bob King, Mariano Kulish, Eduardo Leitner, Fulin Li, Gideon Magnus, Livio Cuzzi Maya, Simon Mongey, Edward Nelson, Jón Steinsson, George Tavalas, Harald Uhlig, anonymous reviewers, and the members of Kaplan's reading group at the University of Chicago, especially Chase Abram, Arisha Hashemi, Leo Aparisi de Lannoy, Santiago Franco, Zhiyu Fu, Agustín Gutiérrez, Sangmin S. Oh, Aleksei Oskolkov, Josh Morris-Levenson, Hyejin Park, and Marcos Sora. Ross Starr pointed me to the lovely Adam Smith quotation. I am especially grateful to Eric Leeper, who capped off decades of correspondence and friendship with extensive comments on this manuscript, some of which substantially changed my thinking on basic issues.

Why tell you these stories? At least I must express gratitude for those sparks, for the effort behind them, and for the institutions that support them. By mentioning a few, I regret that I will seem ungrateful for hundreds of others. Still, in my academic middle age, I think it's useful to let younger readers know how one piece of work came about. Teaching, editorial and referee service, conference attendance and discussions, seminar participation, working with students, writing reference letters, and reading and commenting on colleague's papers all are vital parts of the collective research enterprise, as is the institutional support that lets all this happen. I hope to have returned some of these favors in my own correspondence on others' work. I hope also to give some comfort to younger scholars who are frustrated with their own progress. It does take a long time to figure things out.

My journey includes esthetic considerations as well. I pursued fiscal theory in part because it's simple and beautiful, characteristics which I hope to share in this book. That's not a scientific argument. Theories should be evaluated on logic and their ability to match experience, elegance be darned. But it is also true that the most powerful and successful theories of the past have been simple and elegant, even if they initially had a harder time fitting facts. I hope that clarity and beauty attracts you and inspires you, as it does me, to the hard work of seeing how this theory might fit facts and analyze policy.

I was attracted to monetary economics for many reasons. Monetary economics is (even) more mysterious at first glance than many other parts of economics, and thus beautiful in its insights. If a war breaks out in the Middle East and the price of oil goes up, the mechanism is no great mystery. Inflation, in which all prices and wages rise together, is more mysterious. If you ask the grocer why the price of bread is higher, the grocer will blame the wholesaler. The wholesaler will blame the baker, who will blame the wheat seller, who will blame the farmer, who will blame the seed supplier and workers' demands for higher wages, and the workers will blame the grocer for the price of food. If the ultimate cause is a government printing up money to pay its bills, there is really no way to know this fact but to sit down in an office with statistics, armed with economic theory. Investigative journalism will fail. The answer is not in people's minds, but in their collective actions. It is no wonder that inflation has led to so many witch hunts for "hoarders," "speculators," "greed," "middlemen," "profiteers," and other phantasms.

This Book

I am reluctant to write this book, as there is so much to be done. Perhaps I should title it "Fiscal Theory of the Price Level: A Beginning." I think the basic theory is now settled, and theoretical controversies over. We know how to include fiscal theory in standard macroeconomic models including sticky prices and monetary and financial frictions. But just how to use it most productively, which frictions and specifications to include, and then how to understand episodes, data, institutions, and guide policy, has just started.

We have only started to fit the theory to experience. This is as much a job of historical and institutional inquiry and story-telling as it is of model specification, formal estimation, and econometric testing. Friedman and Schwartz do not offer a test of monetarism. Keynes did not offer a statistical test of the General Theory. They were pretty influential, because they were useful.

Our task is likewise to make fiscal theory useful: to understand its message, to construct plausible stories, then to construct formal models that embody the stories, to quantitatively account for data and episodes, and to analyze policy. This book offers a beginning, and some effort to light the way. It is full of suggestions, but these are suggestions of paths to follow and episodes to analyze, not reports of concluded voyages.

I argue that an integration of fiscal theory with new-Keynesian and DSGE models is a promising path forward, and I provide a recipe for such integration. But just how do such merged models work exactly? Which model ingredients will fit the data and best guide policy decisions? How will their operation differ with fiscal foundations? The project is conceptually simple, but the execution has only just begun. In particular, the mechanism by which higher interest rates may temporarily lower inflation, and the Fisherian implications of rational expectations, are deeply troublesome questions. These are central parts of the rest of the model, not really part of the fiscal theory contribution. It is unsettling that such basic ingredients are still so uncertain. But that is an invitation as well. The international version, extending the theory to exchange rate determination, has barely begun.

We have also only started to apply fiscal theory to think about how monetary institutions could be better constructed. How should the euro be set up? What kinds of policy rules should central banks follow? What kind of fiscal commitments are important for stable inflation? Can we set up a better fiscal and monetary system that produces stable prices and without requiring clairvoyant central bankers to divine the correct interest rate? I offer some ideas, but we have a long way to go.

I also pursue a different direction than much current fiscal theory literature. In an effort to identify fiscal versus monetary regimes, that literature ties monetary and fiscal policy, which we see, to equilibrium selection policies, which we do not see. It assumes that in a fiscal regime, the government cannot commit to raise future surpluses when it runs a current deficit, so all deficits are inflated away. "Fiscal dominance" is a bad state, in which intractable deficits force large and volatile inflation.

This book emphasizes a few innovations that together fundamentally alter this approach. The observational equivalence theorem, the s-shaped surplus process, writing models in a way that separates observed fiscal and monetary policy from equilibrium selection policy, along with attention to discount rates and long-term debt, open the door to understanding the whole sample with fiscal theory, to regard the fiscal theory as the only theory of the price level, and to consider fiscal–monetary institutions that can produce low and quiet inflation.

You may find this book chatty, speculative, and constantly peering forward murkily. Some sections will surely turn out to be wrong. I prefer to read short, clear, definitive books. But this is the fiscal theory book I know how to write. I hope you will find it at least interesting, and the speculative parts worth your time to work out more thoroughly, if only to disprove them or heavily modify them.

The point of this book is to spur us to *use* fiscal theory. There are many articles and books with lots of equations, but it's not clear how to apply the equations to issues of practice. As a result, many theories have had more limited impact than they should. Many other books and popular articles have lots of beautiful prose, but one is often left wondering just how it all fits together, and whether contrary ideas could be just as persuasive. This book spends hundreds of pages trying to understand deeply very simple models, and to draw their lessons for history and policy. The models are there, with equations. But the models are simplified down to their minimal essence, to understand what they are trying to tell us. I hope that this middle ground is at least rewarding to the reader. This simplicity is not the end goal, though. Having really understood simple foundations, one should build up again more complex and realistic models.

For years I put off writing this book because I always wanted to finish the next step in the research program first. But life is short, and for each step taken I can see three others that need taking. It's time to encourage others to take those steps. It is also time to put down here what I understand so far so we can all build on it.

On the other hand, every time I give a fiscal theory talk, we go back to basics, and answer questions from 25 years ago: "Aren't you assuming the government can threaten to violate its intertemporal budget constraint?" (No.) "Doesn't Japan violate the fiscal theory?" (No). That's understandable. The basic ideas are spread out in three decades' worth of papers, written by a few dozen authors. Simple ideas are often hidden in the less-than-perfect clarity of first papers on any subject, and in the extensive defenses against criticisms and what-ifs that first papers must include. Responses to such questions are buried in the back ends of papers that rightly focus on positive contributions. By putting what we know and have digested in one place, in simple frameworks, I hope to move the conversation to the things we genuinely don't know, and broaden the conversation beyond the few dozen of us who have worked intensely in this field.

The fiscal theory has been until recently a niche pursuit, an alternative to standard theory. Real progress comes when a group of critical mass works on an issue. I hope in writing this book to help get that snowball rolling, to the point that fiscal theory becomes the standard way to think about monetary economics. This book is littered with suggestions for papers to write and puzzles to solve, which I hope will offer some of that inspiration.

Where's the fire? Economic theories often emerge from historical upheavals. Keynes wrote the *General Theory* in the Great Depression. Friedman and Schwartz offered an alternative explanation of that searing episode, and Friedman saw the great inflation in advance. Yet inflation was remarkably quiet in the developed world, for the 30 years from 1990 to 2020 when the fiscal theory I describe was developed. Fiscal theory is in some ways a slow rumination over 1980, started by a sequence of Tom Sargent and Neil Wallace articles studying that era. But its development was not propelled by a continuing policy problem.

I finish this book in the late 2021 inflation surge, which may make fiscal theory more immediately relevant to policy. This inflation spurt seems clearly related to the massive fiscal expansion of the COVID-19 recession. If inflation continues, it is likely to have fiscal roots. In the shadow of large debt and deficits, taming inflation will require stronger fiscal–monetary coordination. So fiscal theory may soon have important policy application, either to forestall or to remedy inflation.

We are, however, at a less public and well-recognized crisis in monetary economics. Inflation was *too* quiet in the 2010s. Current economic theory doesn't understand that quiet. Nobody expected that if interest rates hit zero and stayed there for a decade or more, *nothing* would happen, and central banks would agonize that 1.7% inflation is below a 2% target. Clearly predicting big events that did not happen is just as much a failure as not predicting the inflation that did break out in the 1970s, or its end in the 1980s. (Chapter 20.)

More deeply, it's increasingly obvious that current theory doesn't hold together logically, or provide much guidance for how central banks should behave if inflation or deflation do break out. Central bankers rely on late 1970s IS-LM intuition, expanded with some talk about expectations as an independent force. They ignore the actual operation of new-Keynesian models that have ruled the academic roost for 30 years. They tell stories of great power and minute technocratic control that are far ahead of economists' models or solid empirical understanding.

If you think critically as you study contemporary monetary economics, you find a trove of economic theories that are broken, failed, internally inconsistent, or describe economies far removed from ours. Going to the bank once a week to get cash to make transactions? Who does that anymore? IS-LM-based policy models with "consumption," "investment," etc., as basic building blocks, not people making consistent, intertemporal, cross-equation, and budget-constrained decisions? The Fed threatening hyperinflation to make people jump to the preferred equilibrium?

So the intellectual fire is there. And, given government finances around the world, the painful lessons of a thousand years of history, and the simple logic of fiscal theory, a real fire may come sooner than is commonly expected.

As it evolved, this book took on a peculiar organization. I write for a reader who does not already know fiscal theory, has only a superficial knowledge of contemporary macroeconomics and monetary theory, in particular new-Keynesian DSGE style modeling, and is not deeply aware of historical developments and controversies. Thus, I develop fiscal theory first, standing on its own. I make some comparisons with monetarist and new-Keynesian thought, but a superficial familiarity should be enough to follow that, or the reader may just ignore that discussion. Only toward the end of the book do I develop the standard new-Keynesian model, monetary models, and theoretical controversies, discussions of active versus passive policies, on versus off equilibrium, and so forth. The controversies are really all what-ifs, responses to criticisms, what about other theories, and so on. If the fiscal theory takes off as I hope it will, alternative theories and controversies will fade in the rear-view mirror. The front of the book – what is the fiscal theory, how does it work, how does it explain facts and policy – will take precedence. But if you're hungry to know just how other theories work, how fiscal theory compares to other theories, or answers to quibbles, just keep going.

I also develop ideas early on using very simple models, and then return to them in somewhat more general settings, rather than fully treat an idea in generality before moving on. If on reading you wish a more general treatment of an issue, it's probably coming in a hundred pages or so. The benefit of this strategy is that you will see hard issues show up first in simple clear contexts. The cost is a bit of repetition as you see the same idea gain nuance in more general contexts.

Economists who think rigorously in the general equilibrium tradition may find the presentation frustratingly informal. The point of this book is to make fiscal theory accessible, to develop stories and intuition for how it can help us to understand the world. For this reason, I focus on bits and pieces of fully fleshed out models, only occasionally spelling out the full details. For example, we spend a lot of time looking at the government debt valuation equation, which states that the real value of government debt equals the present value of primary surpluses. That equation by itself is not a model. It is one equilibrium condition of a model. The surpluses and discount rate are endogenous variables. Though it is easy to slip into saying that changing expected surpluses or discount rates "cause" changes in inflation, that is sloppy thinking. We are really evaluating equilibrium inflation given equilibrium surpluses and discount rates. Likewise, price equals expected discounted dividends does not mean that expected dividends "cause" price changes. That too is one equilibrium condition of a full model, relating endogenous variables. Yet looking at this equilibrium condition in isolation has been enormously productive for asset pricing.

I write this disclaimer because many economists (including some who generously sent comments on this book) are so well-trained in general equilibrium that they find it hard and frustrating to look at bits and pieces of models that are not fully fleshed out. Start with preferences, technology, market structure, fundamental shocks, and write the whole bloody model already, they advise. Being of the Chicago/Minnesota school that believes this is the "right" way to do economics, at least eventually, I am sympathetic. But I have found that at this stage, full models hide much intuition. Moreover, while in this framework one should never think of x causing y unless x is a truly exogenous structural shock, the actual exogenous structural shocks to the economy are awfully hard to pin down. So, brace yourself. We will largely look at a few equilibrium conditions and see how they work and organize the world. By and large, though, the models in this book are so simple that if you know enough to ask these questions, you know enough to fill in the details on your own—a representative agent, constant endowment, complete markets, and so on.

Relative to most of the literature in macroeconomics and monetary economics that appears in academic journals, the models in this book are simple and stripped down. I think a good deal of macro theory has built complicated elaborations and frictions while we still are not completely sure of basic stability and determinacy questions. Shouldn't we first settle whether an interest rate peg is stable and determinate?

General readers may be intimidated that this book has a lot of equations. Fear not. One really doesn't need any more economics or math than is covered in a good undergraduate economics course to understand them all. One can get by with a good deal less. The hard equations are mostly general cases, building blocks for future research but not necessary to understand most of the book. You don't have to actually *do* much math at all, or derive any equations. We mostly just stare at equations and untangle their meaning. But equations and the models they embody are central to the enterprise. Without the equations, you can't check that the story is internally consistent. Mathematical models do not prove economics is right, but economic theories that cannot be written in models are almost certainly wrong. Popular writing in monetary economics is particularly full of beautiful prose that falls apart when looked at analytically. Time and again, in writing this book, I wrote a section of beautiful prose, convinced of one or another effect. I then went back to flesh out some equations, only to discover that most of my beautiful intuition was wrong. The remaining verbal sections may suffer a similar fate. They are written to encourage others to do some of that difficult fleshing out.

So while a reader can understand most of what I have to say simply glossing over the math, the core point of this book is a set of simple models, whose operation is not obvious at a verbal level, but that help us to understand the world. Economic theory consists of quantitative parables, and examples in which one applies those parables to illuminate a complex world.

While this book has a lot of equations, it could have a lot more. An Online Appendix is available on my website, https://www.johnhcochrane.com/. This appendix contains detailed algebra for many of the more complex expressions that I present here. It also contains a number of extensions and additional topics: A Chapter "How Not to Test the Fiscal Theory" devoted to some common mistakes, and detailed treatment of multiple equilibria in models with money. The "Fiscal Theory of the Price Level" tab of my website also includes additional material, related essays, updates, and typos or other corrections.

Monetary economics, and this book, offer a surprisingly high ratio of talk to equations. We fancy ourselves a science in which equations speak for themselves. They do not. (They often do not speak directly in physical sciences either.) You will see that circumstance throughout this book. The equations are quite simple, but there is lots of debate about what they mean and how to read them, interpret them, or apply them. Seeing the world through the lens of the model, finding what specifications might match an episode or policy question, is harder than solving equations. This comment should be encouraging if you don't view yourself as a top-notch mathematician. The math is simple. Seeing how the math describes the world is hard.

John H. Cochrane
December 2021

Notation

I TRY TO USE capital letters for nominal variables and levels, and lowercase letters for real variables, logs, and rates of return. Variables without subscripts are steady-state values, though sometimes I use them to refer to the variable in general rather than at a specific date, or to indicate that a variable is constant over time. I use the same symbol for variables and for their deviations from steady state, so you have to look in context. If it's a deviation from steady state, then there are no constants and $0 = 0$ is a solution. I use a comma to separate an identifying subscript from a time subscript, e.g. $\varepsilon_{i,t}$ is an interest rate i shock at time t. I do not use a comma when an identifying subscript uses two letters, e.g. $i_t = \theta_{i\pi}\pi_t$. I follow the usual convention of dating variables when they are known. Thus the nominal interest rate i_t and a real risk free rate r_t are returns for an investment from t to $t+1$, as are risky returns r_{t+1} or R_{t+1}. I only define widely used symbols here. When symbols are defined and only used within a section, I omit them here.

Roman letters

A. Transition matrix, e.g. $z_{t+1} = Az_t + B\varepsilon_{t+1} + C\delta_{t+1}$ or $dz_t = Az_t dt + Bd\varepsilon_t + Cd\delta_t$.

$a(L)$. Lag polynomial, e.g. $s_t = a(L)\varepsilon_t$.

a_x. Vector that selects a variable from a vector, e.g. $x_t = a_x' z_t$.

B_t. Face value of nominal debt. $B_{t-1}^{(t)}$ is one-period debt issued at $t-1$ due at time t. B_t used with no superscript can mean one period debt when there is no long-term debt in the model, or an aggregate quantity of debt.

B. Part of the matrix representation of a model, e.g. $z_{t+1} = Az_t + B\varepsilon_{t+1} + C\delta_{t+1}$.

b_t. Real (indexed) debt.

$b_{y,x}$. Regression coefficient, e.g. $y_t = a + b_{y,x}x_t + u_t$.

C_0. An arbitrary constant in the solution to a differential equation.

C. Part of the matrix representation of a model, $z_{t+1} = Az_t + B\varepsilon_{t+1} + C\delta_{t+1}$.

c_t. Real consumption, e.g. $u(c_t)$. Where necessary for clarity, I use capital letters for the level and lowercase letters for the log, $c_t = \log(C_t)$.

D. Differential operator, $D = d/dt$.

D_t. Fraction of debt coming due at time t that is repaid in a partial default.

d. Differential operator, e.g. dx_t. Also dividends, e.g. $R_{t+1} = (p_{t+1} + d_{t+1})/p_t$.

dz_t. Compensated jump or diffusion, e.g. $dx_t = \mu_t dt + \sigma_t dz_t$; $E_t dz_t = 0$.

E. Expectation. $E_t(x_{t+1})$ conditional expectation at time t.

$f'(k)$. Marginal product of capital.

g_t. Real GDP growth rate. Also used as a government spending or other Phillips-curve disturbance.

i_t. Net or log nominal interest rate.

i_t^m. Interest rate on money, e.g. interest on excess reserves.

i_t^*. Interest rate target, equilibrium interest rate, e.g. $i_t = i_t^* + \phi(\pi_t - \pi_t^*)$.

I. Identity matrix.

I_t. Investor information set, e.g. $E(x_{t+1}|I_t)$.

j. Used as index for sums, e.g. $\sum_{j=1}^{\infty} \beta^j s_{t+j}$.

L. Lag operator, e.g. $x_{t-1} = Lx_t$. Also used to express money demand, e.g. $M_t/P_t = L(y, i_t)$.

$\mathcal{L}$. Continuous time lag operator, $\mathcal{L}(D)$, corresponding to $a(L)$. For example, if $ds_t = -\eta s_t dt + d\varepsilon_t$, then $(\eta + D)s_t = D\varepsilon_t$. The moving average representation is $s_t = \int_{\tau=0}^{\infty} e^{-\eta\tau} d\varepsilon_{t-\tau} = 1/(\eta + D)D\varepsilon_t$.

M_t. Money. Usually only money issued by the government, i.e. cash and reserves. M_t is held from time t to time $t+1$. M^d, M^s money demand and supply. Mb, Mi monetary base and inside money.

m_t. Log(M_t).

n. Population growth rate, e.g. $r = \delta + \gamma\,(g - n)$.

P_t. Price level, dollars per goods.

P_t^*. Price level target.

p_t. Log price level, $p_t = \log(P_t)$, or proportional deviation from steady state. Also stock price.

Q_t. Nominal bond price. $Q_t^{(t+j)}$ price at time t of a zero coupon bond that comes due (pays \$1) at time $t+j$. Q_t is also the price of a bond with geometrically declining coupon.

q_t. Log bond price, or proportional deviation of bond price from steady state, $q_t = \log(Q_t)$, or $q_t = Q_t/Q$.

R_{t+1}. Real gross rate of return. Ten percent is 1.10, not 0.10 or 10.

R_{t+1}^n. Nominal gross rate of return.

r_{t+1}. Real net or log rate of return. Ten percent is 0.10. When riskfree, r_t.

r_{t+1}^n. Nominal net or log return.

r. A constant or steady state real rate of return.

T. Upper time limit for sums, integrals, transversality conditions.

s_t. Real primary surplus or surplus to GDP ratio.

$\tilde{s}_t$. Real primary surplus expressed in units of a fraction of the real value of debt, e.g. $\tilde{s}_t = s_t/V$.

$u(c)$. Utility.

u_t. Serially correlated disturbances. Additional subscripts distinguish variables when needed, e.g. $i_t = \theta\pi_t + u_{i,t}$, $u_{i,t+1} = \eta_i u_{i,t} + \varepsilon_{i,t+1}$. Also used to denote an arbitrary regression disturbance, e.g. $y_t = a + b_{y,x}x_t + u_t$.

V, v. Velocity, $MV = Py$. When a function of other variables $V(i, \cdot)$. $v = \log(V)$. Also steady states of the value of government debt V_t, v_t.

V_t, v_t Real value of government debt, e.g. $V_t = B_t/P_t$. May have units of debt to GDP, $V_t = B_t/(P_t y)$.

v_t. Log real value of government debt, or proportional deviation, e.g. $v_t = \log(V_t)$ or $v_t = V_t/V - 1$.

V_t^*, v_t^*. Latent variable for active fiscal policy, equal to debt in equilibrium.

W_t . Value of a continually reinvested portfolio, usually of government debt. $W_{t+1} = R_{t+1}W_t$.

$W_t^{(j)}$. Weights in the long-term debt formula (7.17).

x_t. Real GDP gap or deviation from trend used in sticky price models.

y_t. Real GDP or income. Also yield on long-term bonds, $Q_t = 1/(y_t + \omega)$.

z_t. A generic compensated jump or diffusion, e.g. $dx_t = \mu_t dt + \sigma_t dz_t$. Also a vector of state variables, e.g. $z_t = [\ x_t \quad \pi_t \quad i_t\ ]'$.

Greek letters

α. Coefficient of surplus on debt for active fiscal policy, $s_t = \alpha v_t^* + \ldots$ Also interest-elasticity of money demand, $M = PyV^{-\alpha i}$.

β. Subjective discount factor. Utility is $\sum_{t=0}^{\infty} \beta_t u(c_t)$.

β_s, β_i. Regression coefficients of unexpected inflation target on surplus and interest rate shocks. $\Delta E_{t+1}\pi_{t+1}^* = -\beta_s \varepsilon_{s,t+1}$.

γ. Coefficient of surplus on debt in passive fiscal policy, $s_t = \gamma v_t + \ldots$. Also coefficient of risk aversion, $u'(c_t) = c_t^{-\gamma}$.

Δ. Used as a difference operator. $\Delta x_t = x_t - x_{t-1}$. Applied to an expectation, it takes a difference of expectations of the same variable, e.g. $\Delta E_{t+1}(y_{t+1}) = E_{t+1}(y_{t+1}) - E_t(y_{t+1})$. In continuous time Δ_t is the corresponding expectation operator. If $dx_t = \mu dt + \sigma dz_t$ then $\Delta_t(dx_t) = \sigma dz_t$. Also used to denote a small discrete time difference, $x_{t+\Delta} - x_t$, or a small difference in a variable, $c_t + \Delta c$.

δ. Subjective discount rate, $\beta = e^{-\delta}$.

δ_{t+1}. Expectational errors, e.g. $\pi_{t+1} = E_t\pi_{t+1} + \delta_{\pi,t+1}$ or $d\pi_t = E_t d\pi_t + d\delta_{\pi,t}$. I use δ_{t+1} to distinguish expectational errors from shocks ε_{t+1}. A complete model derives expectational errors, and takes shocks as exogenous.

ε_{t+1}. A shock. When necessary a first subscript denotes the variable, e.g. $\varepsilon_{i,t+1} = \Delta E_{t+1} i_{t+1}$. $\varepsilon_{\Sigma s,t+1}$ denotes the shock to the present value of surpluses $\varepsilon_{\Sigma s,t+1} = \Delta E_{t+1}\sum_{j=0}^{\infty} \rho^j \tilde{s}_{t+j}$.

ζ_i. Partial adjustment coefficient in an interest rate policy rule. $di_t = -\zeta_i[i_t - (\theta_{i\pi}\pi_t + \theta_{ix}x_t + u_{i,t})]dt + \theta_{i\varepsilon}d\varepsilon_{i,t}$.

η. Serial correlation parameters, e.g. $u_{t+1} = \eta u_t + \varepsilon_{t+1}$ or $du_t = -\eta u_t dt + \sigma dz_t$.

Note the units are different in discrete and continuous time. The discrete version of the latter is $u_{t+1} - u_t = (1-\eta)u_t + \varepsilon_{t+1}$.

θ. Used for the parameters of policy rules, with subscripts when needed to distinguish variables, e.g. $i_t = \theta_{i\pi}\pi_t + u_{it}$. Also used as a moving average coefficient, e.g. $a(L) = 1+\theta$ and other parameters as defined and used within a few sections.

κ. Price stickiness parameter of the new-Keynesian Phillips curve, e.g. $\pi_t = \beta E_t \pi_{t+1} + \kappa x_t$.

Λ_t. Stochastic discount factor, e.g. $1 = E_t[(\Lambda_{t+1}/\Lambda_t)R_{t+1}]$; $\Lambda_t = \beta^t u'(c_t)$.

Λ. Matrix of eigenvalues.

λ_t. Log stochastic discount factor in Sims's model (5.123).

λ. Eigenvalues of a transition matrix.

μ_t. Drift term of diffusion/jump processes, e.g. $dx_t = \mu_t dt + \sigma_t dz_t$. Also average money growth rate.

Π_t. Gross inflation rate, $\Pi_t = P_t/P_{t-1}$.

π_{t+1}. Net or log inflation rate, $\pi_{t+1} = P_{t+1}/P_t - 1$ or $\pi_{t+1} = \log(P_{t+1}/P_t)$. In continuous time with differentiable prices, $\pi_t = d\log(P_t)/dt$.

π_t^e. Expected inflation. When rational, $\pi_t^e = E_t\pi_{t+1}$.

π_t^*. Inflation target.

ρ. Constant of linearization in present value formulas, e.g. $\rho v_{t+1} = v_t + r_{t+1}^n - \pi_{t+1} - g_{t+1} - \tilde{s}_{t+1}$. Units of a discount factor, $\rho = e^{-r}$. Also the discount rate in the continuous time Phillips curve, $E_t d\pi_t = (\rho\pi_t - \kappa x_t)\, dt$.

σ. Intertemporal substitution elasticity, e.g. $x_t = E_t x_{t+1} - \sigma(i_t - E_t\pi_{t+1})$. Also standard deviation and coefficient in diffusion processes, e.g. $dx_t = \mu_t dt + \sigma_t dz_t$.

τ. Tax rate, e.g. $s_t = \tau y_t$. Also an index for integrals, e.g. $\int_{\tau=0}^{\infty} e^{-r\tau} s_{t+\tau} d\tau$.

ϕ. Used for policy rules that select equilibria in new-Keynesian models, e.g. $i_t = i_t^* + \phi(\pi_t - \pi_t^*)$. I distinguish θ for rules that relate equilibrium quantities and ϕ for rules that select equilibria.

Φ. $\Phi(\Pi)$ is a nonlinear version of $i = \phi\pi$ or $\pi_{t+1} = \phi\pi_t$.

Ω_t. Full economic-agent information set.

ω. Parameter of a geometric maturity structure of debt; $B_t^{(t+j)} = B_t\omega^j$ in discrete time, $B_t^{(t+j)} = e^{-\omega t}B_t$ in continuous time. Note the units are different in discrete versus continuous time.

$\omega_{j,t}$. Maturity structure when not geometric. $\omega_{j,t} = B_t^{(t+j)}/B_t^{(t+1)}$.

ϖ. Used for the continuous time case when needed to contrast with the discrete time case $\omega = e^{-\varpi}$. When clear from context I use ω for both cases.

Symbols

$\{\cdot\}$. Denotes a sequence. $\{s_t\} = s_0, s_1, s_2, \ldots s_t \ldots$

$\cdot$. Denotes an unstated list of variables. $f(\cdot) = f(x, y, z, \ldots)$

$*$. Used to denote target or equilibrium values, i_t^*, π_t^*, etc.

Part I

The Fiscal Theory

Introduction

WHAT DETERMINES the overall level of prices? What causes inflation, deflation, or currency appreciation and devaluation? Why do we work so hard for pieces of paper? A \$20 bill costs 10 cents to produce, yet you can trade it for \$20 worth of goods or services. And now, \$20 is really just a few bits in a computer, for which we work just as hard. What determines the value of a dollar? What is a dollar, really?

As one simple story, the fiscal theory of the price level answers: Money is valued because the government accepts money for tax payments. If on April 15 you have to come up with these specific pieces of paper, or these specific bits in a computer, and no others, then you will work hard through the year to get them. You will sell things to others in return for these pieces of paper. If you have more of these pieces of paper than you need, others will give you valuable things in return. Money gains value in exchange because it is valuable on tax day. This idea seems pretty simple and obvious, but as you will see it leads to surprising conclusions.

The fiscal theory is additionally interesting by contrast with more common current theories of inflation, and how its simple insight solves the problems of those theories. Briefly, there are three main alternative theories of the price level. First, money may be valued because it is explicitly backed: The government promises 1/32 of an ounce of gold in return for each dollar. This theory no longer applies to our economies. We will also see that it is really an interesting instance of the fiscal theory, as the government must have or obtain gold to back dollars.

Second, intrinsically worthless money may be valued, if people need to hold some money to make transactions, and if the supply of that money is restricted. This is the most classic view of fiat money. ("Fiat" means money with no intrinsic value, redemption promise, or other backing.) But current facts challenge it: Transactions require people and business to hold less and less money. More importantly, our governments and central banks do not control internal or external money supplies. Governments allow all sorts of financial and payments innovation, money multipliers do not bind, and central banks follow interest rate targets, not money supply targets.

Third, starting in the late 1970s a novel theory emerged to describe that reality, and in response to the experience of the 1970s and 1980s. In this theory, inflation is controlled when the central bank follows an interest rate target, so long as the target varies more than one for one with inflation, following what became known as the Taylor principle. We will analyze the theoretical problems with this view in detail below. Empirically, the fact that inflation remained stable and quiet even though interest rates did not move in long-lasting zero bound episodes contravenes this theory.

The fiscal theory is an alternative to these three great classic theories of inflation. The first two do not apply, and the third is falling apart. Other than the fiscal theory, then, I argue that there is no simple, coherent, economic theory of inflation that is vaguely compatible with current institutions.

Macroeconomic models are built on these basic theories of the price level, plus descriptions of people's saving, consumption, production, and investment behavior, and potential frictions in product, labor, or financial markets. Such models are easily adapted to the fiscal theory instead of alternative theories of inflation, leaving the rest of the structure intact. Procedurally, changing this one ingredient is easy. But the results of economic models often change a lot if you change just one ingredient.

Let's jump in to see what the fiscal theory *is*, how it works, and then compare it to other theories.

Chapter One

A Two-Period Model

THIS CHAPTER introduces the fiscal theory, and previews many following issues, with a simple two-period model. The model has perfectly flexible prices, constant interest rates, short-term debt, and no risk premiums. I add these elements later, as they add important realism. But by starting without them we see that they are not necessary in order to determine the price level, nor do they change the basic logic of price level and inflation determination.

1.1 The Last Day

We look at a simple one-period frictionless fiscal theory of the price level

$$\frac{B_0}{P_1} = s_1.$$

In the morning of day 1, bondholders wake up owning B_0 one-period zero-coupon government bonds coming due on day 1. Each bond promises to pay \$1. The government pays bondholders by printing up new cash. People may use this cash to buy and sell things, but that is not important to the theory.

At the end of the day, the government requires people to pay taxes $P_1 s_1$ where P_1 denotes the price level (dollars needed to buy a basket of goods) and s_1 denotes real tax payments. For example, the government may levy a proportional tax τ on income, in which case $P_1 s_1 = \tau P_1 y_1$ where y_1 is real income and $P_1 y_1$ is nominal income. Taxes are paid in cash, and soak up money.

The world ends on day 2, so nobody wants to hold cash or bonds after the end of day 1. Figure 1.1 illustrates the timing of events in this little story.

In equilibrium, then, cash printed up in the morning must all be soaked up by taxes at the end of the day,

$$B_0 = P_1 s_1$$

or

$$\frac{B_0}{P_1} = s_1. \tag{1.1}$$

Debt B_0 is predetermined. The price level P_1 adjusts to satisfy (1.1). *We just determined the price level.* This is the fiscal theory.

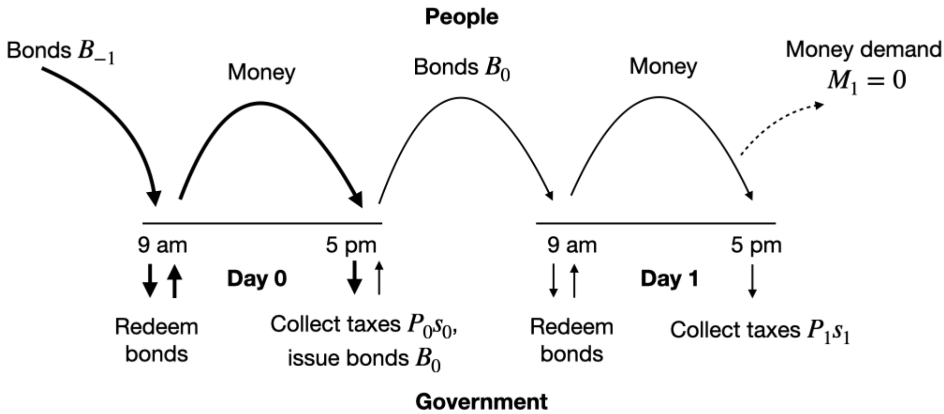

Figure 1.1: Timing of the Two-Period Model

1.2 Intuition of the One-Period Model

> The mechanism for determining the price level can be interpreted as too much money chasing too few goods, as aggregate demand, or as a wealth effect of government bonds. The fiscal theory does not feel unusual to people, even economists, who live in it. The fiscal theory differs on the measure of how much money is too much, and the source of aggregate demand. Fiscal theory builds on a completely frictionless foundation.

If the price level P_1 is too low, more money was printed up in the morning than will be soaked up by taxes in the evening. People have, on average, more money in their pockets than they need to pay taxes, so they try to buy goods and services. There is "too much money chasing too few goods and services." "Aggregate demand" for goods and services is greater than "aggregate supply." Economists trained in either the Chicago or Cambridge traditions living in this economy would not, superficially, notice anything unusual.

The difference from the standard (Cambridge) aggregate-demand view lies in the source and nature of aggregate demand. Here, aggregate demand results directly and only as the counterpart of the demand for government debt. We can think of the fiscal theory mechanism as a "wealth effect of government bonds," again tying the fiscal theory to classical ideas. Too much government debt relative to fiscal surpluses acts like net wealth which induces people to try to spend, raising aggregate demand.

The difference from the standard (Chicago) monetary view lies in just what is money, what is the source of money demand, and therefore how much money is too much. Here, inflation results from more money in the economy than is soaked up by net tax payments, not by more money than is needed to mediate transactions or to satisfy asset, liquidity, precautionary, etc. sources of demand for money. Here, only outside money, only government liabilities, drives inflation, along with government bonds that promise such money. If in this economy someone were to set up a bank, issuing notes and making loans, to a monetarist those notes would count as money that causes inflation. They are irrelevant to the price level in the fiscal theory.

We may view the fiscal theory as a backing theory of money. Dollars are valuable because they are backed by the government's fiscal surpluses. Many financial liabilities are valuable because they are a claim to some assets. Many currencies have been explicitly backed by assets such as gold. Dollars say "This note is legal tender for all debts, public and private," so you have the right to pay your taxes with dollars. That right constitutes a backing. Dollars backed by gold can be soaked up by giving people the gold in return for dollars. Our dollars can be soaked up by taxes.

My story that money is valued because the government accepts its money in payment of taxes goes back to Adam Smith himself:

> A prince, who should enact that a certain proportion of his taxes be paid in a paper money of a certain kind, might thereby give a certain value to this paper money. (*Wealth of Nations*, Vol. I, Book II, Chapter II.)

My story about money printed up in the morning and soaked up in the afternoon helps to fix intuition, but it is not essential. People could redeem debt for money five minutes before using the money to pay taxes. Or, people could just pay taxes directly with maturing government bonds. "Cash" can be reserves, electronic accounts at the Fed. A "morning" versus "afternoon" is not a necessary part of the model. It can all happen continuously, or in an instant.

How people make transactions is irrelevant to the price level in this model. People could make transactions with maturing bonds, foreign currency, or Bitcoin. People could make transactions with debit cards or credit cards linked to bank accounts, netted at the end of the day with no money changing hands, which is roughly how we do things today. People could wire claims to funds that hold government bonds, private bonds, mortgages, or stocks. The dollar can be a pure unit of account, with nobody ever holding actual dollars.

The simple model shows that the fiscal theory can determine the price level in a completely frictionless economy. In this model, money has no extra value from its use in transactions or other special features. Money does not pay a lower return than other assets, people do not carry around an inventory of money, and the government does not limit the supply of inside liquid assets. This model has perfectly flexible prices, and markets clear instantly. Backing theories naturally continue to determine the price level in a frictionless context: If money is valued because it is a claim to something else that has value in a frictionless model, then money has value absent any transactions, liquidity, pricing, or other frictions.

By contrast, the monetarist story of money supply and demand and the Keynesian story of interest rate targets and Phillips curves, which are the two standard theories of the price level, require monetary or pricing frictions to determine the price level at all. This is a beautiful aspect of fiscal theory, and makes it an attractive starting point for monetary economics today. Electronic transactions and financial innovation undermine money demand. Our central banks do not limit money supply. The internet might undermine sticky prices and wages.

One can add monetary, financial, and pricing frictions to fiscal theory, and I will do so presently, in order to create a more realistic model. But the fiscal theory allows us to *start* to analyze the price level with a simple frictionless, flexible price, backing model, and to add frictions as needed for realism, or not to add them when not needed. One can often understand basic mechanisms of more complex models with the simple supply and demand logic of the frictionless underpinning.

In this simple model *all* the government does is collect taxes, requiring that people surrender money or maturing government bonds. The government can use some of that money to make cash transfers, with $P_t s_t$ denoting primary surpluses, the difference between taxes and transfers.

That we pay taxes in dollars is not essential. The government could accept goods or foreign currency for tax payments and then sell those to soak up dollars. What matters to price level determination is that the government uses tax revenues in excess of spending to soak up any excess dollars at the end of the day, and thereby maintain their value. While not necessary, offering the right to pay taxes with money, or requiring such payment, is a useful way of communicating and pre-committing to fiscal backing.

Equation (1.1) is one equilibrium condition of a model, not a full model on its own, just as price = present value of dividends is one equilibrium condition of a full model. Both are useful if one remembers that limitation. I return below to describe a complete economic model, though most readers who know enough to ask the question can fill in the details quickly on their own (representative agent, constant endowment, flexible prices, etc.). The "government" here unites treasury and central bank balance sheets. All debt B is debt in private hands, cancelling out central bank holdings of treasury debt.

1.3 A Two-Period Model and Present Value

We add an initial period time 0, and bond sales B_0. The price level in each period is determined by

$$\frac{B_0}{P_1} = s_1$$
$$\frac{B_{-1}}{P_0} = s_0 + \beta E_0 (s_1) .$$

The price level P_0 adjusts so that the real value of nominal debt equals the *present value* of real primary surpluses. The theory need not predict a strong relation between inflation and contemporaneous debt and deficits. A higher discount rate, lower β, can lead to inflation even with no change in surpluses.

Next, let us add the previous day, time 0. This addition allows us to think about where the debt B_0 came from, and what the effects are of changing this second policy lever.

The time 0 flow equilibrium condition is

$$B_{-1} = P_0 s_0 + Q_0 B_0. \tag{1.2}$$

Money printed up in the morning of day 0 to pay maturing nominal bonds B_{-1} is soaked up by surpluses $P_0 s_0$, and now also by nominal bond sales B_0 at the end of time 0, at nominal bond price Q_0. In this flexible-price, constant interest rate world, that bond price is

$$Q_0 = \frac{1}{1+i_0} = \beta E_0\left(\frac{P_0}{P_1}\right) = \frac{1}{R}E_0\left(\frac{P_0}{P_1}\right). \tag{1.3}$$

The first equality defines the notation i_0 for the nominal interest rate. The second and third equalities are the bond pricing equation, with subjective discount factor β or real gross interest rate R. It is a nonlinear version of the statement that the nominal interest rate equals the real interest rate plus expected inflation.

Substituting (1.3) in (1.2), we have

$$\frac{B_{-1}}{P_0} = s_0 + \beta E_0\left(\frac{1}{P_1}\right) B_0,$$

and using time 1 equilibrium,

$$\frac{B_0}{P_1} = s_1, \tag{1.4}$$

we have

$$\frac{B_{-1}}{P_0} = s_0 + \beta E_0\left(s_1\right). \tag{1.5}$$

- *The price level P_0 adjusts so that the real value of nominal debt equals the present value of real primary surpluses.*

I refer to (1.5) as the "government debt valuation equation." It works like stock, in which the stock price adjusts so that the value of a given number of shares equals the present value of dividends.

The present value on the right-hand side of (1.5) has immediate, fortunate, and important consequences: The theory does not necessarily predict a strong contemporaneous relationship between inflation, debt, and deficits. Governments can run large deficits $s_0 < 0$ or have large debts B_{-1} with no inflation, if people believe that the governments will pay back new B_0 and old B_{-1} debt with subsequent surpluses, s_1. Conversely, inflation can break out today (0) if people see intractable future fiscal problems (s_1) despite healthy debt and deficits today. Higher discount rates—higher real interest rates, a lower β—can also induce inflation, with no change in surpluses.

1.4 Monetary Policy, Fiscal Policy, and Inflation

> If the government sells additional debt B_0 without changing surpluses, it lowers the bond price, raises the nominal interest rate, raises no additional revenue, and raises the expected price level and future inflation. The government may follow a nominal interest rate target, by offering to sell debt B_0 at a fixed interest rate i_0, with no change in surpluses. I call these operations "monetary policy." Monetary policy can set a nominal interest rate target and thereby determine expected inflation. Fiscal policy sets unexpected inflation. An interest rate rise with no change in fiscal policy raises inflation one period later, with no contemporaneous change in inflation in this model, a natural neutrality benchmark. This combination is the simplest example of what I shall call the "fiscal theory of monetary policy." Inflation is stable and determinate under an interest target, even an interest rate peg.

The government has two policy levers, debt B_0 and surpluses s_0, s_1. The two-period model lets us think about the effect of selling debt B_0.

The price levels at the two dates are given by

$$\frac{B_0}{P_1} = s_1 \tag{1.6}$$

$$\frac{B_{-1}}{P_0} = s_0 + \beta E_0 \left(s_1\right). \tag{1.7}$$

Suppose the government sells more debt B_0 at time 0, without changing surpluses s_0 and s_1. The price level P_0 does not change, and P_1 rises.

To understand the result, write

$$\frac{B_{-1}}{P_0} = s_0 + \frac{1}{1+i_0}\frac{B_0}{P_0} = s_0 + \beta E_0 \left(\frac{P_0}{P_1}\right)\frac{B_0}{P_0} = s_0 + \beta E_0 \left(s_1\right). \tag{1.8}$$

Money printed up to redeem debt B_{-1} is soaked up by surpluses s_0 or by new bond sales B_0. In turn, bond sales B_0 raise real revenue equal to $\beta E_0(s_1)$. With P_0 determined and no change in s_0 or s_1, greater bond sales B_0 just lower the bond price $Q_0 = 1/(1+i_0)$, raise the nominal interest rate, and raise the price level P_1, but generate no extra time-0 revenue.

Selling more debt B_0 without changing surpluses s_1 is like a share split. When a company does a 2-for-1 share split, each owner of one old share receives two new shares. People understand that this change does not imply any change in expected dividends. The price per share drops by half and the total value of the company is unchanged. Here a doubling of B_0 with no change in surpluses halves the bond price $Q_0 = 1/(1+i_0)$. In the morning of time 1, additional bonds B_0 with no more surplus s_1 are like a currency reform. They imply an instant and proportionate change in price level. Doubling debt B_0 doubles the price level P_1.

Rather than auction a fixed quantity of nominal debt B_0, the government can announce an interest rate target i_0, and allow people to buy all the bonds B_0 they want at that price. It can offer a flat supply curve rather than a vertical supply curve of nominal debt. Now the B_0 terms of (1.8) describe the number of bonds that people will choose to buy at the fixed price.

Buying and selling government bonds in return for cash, or offering a fixed nominal interest rate, without changing fiscal policy, is a reasonable abstraction of a central bank. Thus, I will call this interest rate target "monetary policy." We learn that a central bank can set the nominal interest rate even in this frictionless and cashless world. The interest rate target sets the expected rate of inflation, by

$$\frac{1}{1+i_0} = \beta E_0 \left(\frac{P_0}{P_1} \right). \tag{1.9}$$

Thus the central bank can determine *expected* inflation via its interest rate target. The "fiscal" theory of the price level does not mean that central banks are powerless!

An increase in the interest rate target i_0 has no effect on the price level P_0 and no effect on contemporaneous inflation P_0/P_{-1}. It raises expected inflation entirely by raising the expected future price level P_1 (really, by lowering $E(1/P_1)$). This "Fisherian" response contrasts with the usual presumption that raising nominal interest rates lowers inflation, at least for a while. We will study many mechanisms that produce a negative response. However, first recognize how natural a positive response is: This is a frictionless model, with flexible prices and without monetary distortions. Monetary policy ought to be "neutral" in this model. It is. Higher nominal interest rates coincide with higher inflation in the long run of almost all models, once real interest rates settle down, prices adjust, and output returns to normal. With no frictions, this model's immediate positive reaction of expected inflation to an interest rate rise embodies a natural neutrality proposition.

Monetary policy sets expected inflation, so fiscal policy sets unexpected inflation. We can see this result easily by multiplying (1.6) by P_0 and taking innovations,

$$\frac{B_0}{P_0}(E_1 - E_0)\left(\frac{P_0}{P_1} \right) = (E_1 - E_0)(s_1). \tag{1.10}$$

The combination (1.9)-(1.10) completely determines expected and unexpected inflation. It is the simplest example of what I shall call the "fiscal theory of monetary policy." The government, and its idealized central bank, can follow an interest rate target. The interest rate target determines expected inflation. Fiscal policy determines unexpected inflation. The interest rate target may, but need not, vary with inflation. Inflation is stable and determinate even with an interest rate peg.

1.5 Fiscal Policy Debt Sales

> If the government sells more debt B_0 and at the same time promises proportionally greater surpluses s_1, the policy has no effect on the price level P_1, and raises revenue at time 0. That revenue can fund a deficit, less s_0, or lower the price level P_0. A bond sale *with* more surplus is like an equity issue, not a share split.

> A government that runs a deficit, lower s_0, can fund that deficit by such borrowing with greater s_1, leaving constant P_0 and P_1, and raising the real value of debt Q_0B_0/P_0. If the government does not change s_1, then it funds the deficit s_0 by inflating away outstanding debt. If the government lowers s_1 as well as s_0, then it produces a large inflation P_0 and the value of debt declines following the deficit s_0. Since we see larger values of debt following deficits in most time-series data, the former reaction dominates, which requires an "s-shaped" surplus process. Fiscal theory can produce a completely steady price level despite wide variation in deficits and debt.

Now, let us think about debt sales B_0 that *are* accompanied by changes in surpluses and deficits s_0 and s_1. Suppose that at time 0, the government sells more debt B_0, but this time it promises additional surpluses s_1. Look again at (1.6) and (1.8),

$$\frac{B_0}{P_1} = s_1 \tag{1.11}$$

$$\frac{B_{-1}}{P_0} = s_0 + \frac{1}{1+i_0}\frac{B_0}{P_0} = s_0 + \beta E_0\left(\frac{P_0}{P_1}\right)\frac{B_0}{P_0} = s_0 + \beta E_0\left(s_1\right). \tag{1.12}$$

In (1.11), if the government raises B_0 and s_1 proportionally, there is no effect on the price level at time 1, P_1. Looking from right to left in (1.12), this action raises the real value of debt at the end of period 0, and raises the real revenue the government obtains by selling that debt. The additional revenue can fund a deficit, a decline in s_0, with no change in P_0 or P_1. Or additional revenue can be used to lower the price level P_0.

The debt sale B_0 with corresponding rise in surplus s_1 is like an equity issue, in contrast to a share split. In an equity issue, a firm also increases shares outstanding, but it promises to increase future dividends. By doing so, the firm raises revenue and does not change the stock price. The value of the company increases. The revenue can be used to fund investments – a negative s_0 – that generate the larger dividends.

Turning it around, let us think about the government's options for financing a deficit. Suppose that the government at day 0 runs a deficit $s_0 < 0$, or reduces the surplus s_0 from what was previously expected. How does the government finance this deficit? Examine three options, also summarized in Table 1.1.

Strategy	Time 0 surplus s_0	Time 1 surplus s_1	Value of debt $Q_0B_0/P_0; B_0/P_1$
Borrow	lower	rise by $-R\Delta s_0$	rise
Inflate	lower	no change	no change
AR(1)	lower	lower	lower

Table 1.1: Strategies for Financing a Deficit at Time 0

First, as in the last scenario, the government can borrow, and thereby have no effect on either price level, P_0 or P_1. In order to raise real revenue from borrowing, the government must promise larger future surpluses to repay additional debt. The government lowers s_0 by $\Delta s_0 < 0$ but raises s_1 by $-R\Delta s_0$. The price level P_0 at time 0 does not change, since the present value of surpluses $s_0 + \beta E_0 s_1$ does not change. The dollars printed to redeem debt B_{-1} that are not soaked up by the lower surplus s_0 are now soaked up by the larger real value of bond sales $Q_0 B_0 / P_0$, which is also the real value of nominal debt at the end of the period. If the government sells extra nominal debt B_0, so that $(B_0 + \Delta B_0)/P_1 = s_1 + R\Delta s_0$, then the price level P_1 does not change either, as in the previous scenario.

Second, the government can inflate away outstanding debt. If s_0 declines and there is no change in s_1, then the price level P_0 rises. The real value of debt at the end of period 0, $Q_0 B_0 / P_0 = \beta E_0 (s_1)$, a claim to unchanged surpluses at time 1, does not change. As the price level P_0 rises, the value of the dollars that redeem bonds B_{-1} falls by exactly the fall in the surplus s_0.

Third, suppose the government lowers s_1 along with the lower s_0. A typical AR(1) model of serially correlated deficits produces this result. We might imagine that the initial deficit comes with persistent bad fiscal news. Now the present value $s_0 + \beta E_0 (s_1)$ drops even more than the initial deficit s_0. The time 0 inflation is even larger, inflating away the larger present value of both deficits s_0 and s_1. The deficit in time 0 is accompanied by a *decline* in the end-of-period value of debt.

In typical advanced countries and episodes, including postwar U.S. time series, larger deficits lead to a rise in the value of debt, not equality, and not a decline. Deficits are not strongly correlated with inflation. Inflation shocks are much smaller than deficit shocks. These observations, and especially the first, tell us that fiscal policy largely consists of borrowing, credibly promising future surpluses to repay debt, and therefore on average doing so. They tell us that fiscal policy does not routinely inflate away debt. I call the result an “s-shaped” surplus process: If today’s surplus s_0 declines, the surplus must turn around and rise later on. A government that wants steady inflation, and can do so, will arrange its fiscal affairs in this way. There is some unexpected inflation, but we will have to see its fiscal roots on top of this dominant pattern.

The fiscal theory *can* describe an economy with widely varying debt and deficits, yet little or no inflation at all. The fiscal theory does *not* imply that large variation in debt and deficits must result in inflation.

Other countries and time periods are different. On occasion we see deficits associated with inflation. On occasion we see large inflation or currency devaluation associated with deficit shocks that seem too small to provoke them. The persistent deficit model can capture these episodes.

Discussion of debts, deficits, inflation, stimulus, and fiscal theory often refers to “Ricardian” and “non-Ricardian” policies. Briefly, and acknowledging different uses of the term, a “Ricardian” debt issue includes a full expectation of repayment by future surpluses. It therefore has no “wealth effect” on aggregate demand, and thereby no stimulus or inflation. Keynesian analysis of fiscal stimulus by borrowed money typically asserts that people don’t pay attention to the future taxes that retire debt, so debt has a wealth effect on consumption. The consequent negative wealth effect of those taxes when they arrive is not commonly analyzed. Moreover, if the debt sale raises revenue, bondholders must fully expect future surpluses, or they must be somehow symmetrically irrational in the other direction, buying debt despite no expectation that it will be repaid.

By analyzing explicitly nominal debt, fiscal theory allows such "non-Ricardian" debt sales, without a rise in expected future surpluses, like share splits. Yet there need be no irrationality or asymmetry in people's expectations. Fiscal theory allows "Ricardian" debt sales as well, and everything in between. How much a debt sale inflates, raises revenue, and is or is not expected to be paid off is not a constant, but varies with many circumstances. Fiscal theory is sometimes called "non-Ricardian," but I find this name confusing. Fiscal theory allows the possibility of selling debt without changes in future surpluses, but does not require it. And the value of debt is always equal to the present value of future surpluses. The price level adjusts.

1.6 Debt Reactions and a Price Level Target

I introduce a price level target P_1^* and a fiscal rule $s_1 = B_0/P_1^*$. This rule produces $P_1 = P_1^*$ in equilibrium. Additional nominal debt sales B_0 now generate surpluses to pay them off at the price level P_1^*. The decision to borrow or inflate away a deficit is implemented by borrowing or not borrowing rather than by changing a promised stream of surpluses.

I introduced these fiscal exercises by thinking about changes in the sequence of surpluses $\{s_0, s_1\}$. But that's not the way economists or policy people usually think of fiscal affairs. It is more common to think in terms of a surplus or deficit today, s_0, borrowing or inflation today, and to characterize the future by fiscal reactions to outstanding debt, the price level, and other state variables, endogenous variables, or shocks. More generally, macroeconomics and finance are often expressed in terms of state variables and actions that are functions of state variables rather than sequences of actions. This dynamic programming approach is often useful conceptually and not just (very) useful for solving models. If you're like me, you think of the cost of buying something in terms of the value of wealth, not a specific alternative future purchase.

Think then of surplus at time 1 that responds to the quantity of debt B_0, according to a rule

$$s_1 = \frac{B_0}{P_1^*}. \tag{1.13}$$

I will call the variable P_1^* a price level target. With this rule, equilibrium inflation is given uniquely from $B_0/P_1 = s_1$ by $P_1 = P_1^*$. Parameterizing the surplus decision this way, we think of the government as committing to respond to an increase in the quantity of debt B_0 by raising real surpluses to repay that debt, rather than think of the larger surplus s_1 with a direct connection to the previous deficit s_0. This fiscal rule does not respond to the future price level itself P_1, or to deviations of that level from the target, $P_1 - P_1^*$.

The price level target may represent an inflation target, a gold price target, or an exchange rate target. It may represent less formal rules and traditions, expectations, or it may simply model fiscal behavior and therefore expectations of fiscal behavior, as the Taylor rule $i_t = \theta_\pi \pi_t$ started as a description of Fed behavior. It can capture the idea that governments often respond to inflation with "austerity" and to deflation with "stimulus." If inflation breaks out, $P_1 > P_1^*$, for example, the government deliberately runs a larger real surplus to fight inflation and bring the price level down to P_1^*, even though the government can avoid formal default by running a smaller surplus $s_1 = B_0/P_1$. If deflation breaks out, $P_1 < P_1^*$, this government refuses to raise surpluses to pay an unexpected windfall to bondholders. It runs deliberate "helicopter money" unbacked fiscal stimulus instead. I use a subscript P_1^* to allow the price level target to vary over time and according to information or, later, other variables at time 1, perhaps as a transitory deviation from stated long-run price level or inflation targets. Many more interpretations of the price level target specification follow. For now, let's just follow (1.13) as a potentially interesting possibility for fiscal policy.

Consider again the government's options to finance a deficit s_0. We rephrase the previous three options. In addition to (1.13) and consequent $P_1 = P_1^*$, we have at time 0,

$$\frac{B_{-1}}{P_0} = s_0 + \beta E_0 s_1 = s_0 + \beta E_0 \left(\frac{B_0}{P_1^*}\right).$$

To finance a deficit s_0 by borrowing, without affecting the price level at either date, the government issues more nominal debt B_0, without changing the price level target P_1^*. The greater borrowing produces larger surpluses s_1. But now rather than try to communicate promises about a specific stream of surpluses, the government communicates a commitment to raising whatever surpluses are needed to repay debt at the price level target P_1^*. In an intertemporal model, the government does not need to be specific about just when the surpluses will arrive.

To finance the deficit by inflating away outstanding debt, the government simply does not sell more debt B_0. With lower surpluses s_0, the money printed up to redeem bonds B_{-1} is left outstanding. People try to spend that money, driving up the price level P_0. Again, rather than make promises about future surpluses, the government acts, by not selling more debt. To super-inflate, as in the AR(1) surplus model, the government sells even less debt B_0, which lowers surpluses s_1.

1.7 Fiscal Policy Changes Monetary Policy

> The fiscal policy rule with a price level target $s_1 = B_0/P_1^*$ dramatically changes the effects of monetary policy. A rise in debt B_0, or a rise in nominal interest rate i_0, with no change in this fiscal policy *rule*, lowers P_0 with no effect on P_1 rather than raise P_1 with no effect on P_0. Higher interest rates lower inflation, immediately.

The effects of monetary policy depend crucially on the fiscal policy rule. We will see this lesson repeatedly. The price level target rule offers a simple and important example.

Suppose that the central bank sells more debt B_0 without directly changing fiscal policy. Specifying fiscal policy as an unchanged s_0 and s_1, this action had no effect on P_0 and raised P_1. But suppose fiscal policy is specified by s_0 and the rule $s_1 = B_0/P_1^*$, and let us think of "unchanged fiscal policy" as not making changes to this rule. This question gives exactly the opposite result: Larger debt B_0 has no effect on $P_1 = P_1^*$, and it lowers the price level P_0 immediately. The larger debt B_0 generates more surplus s_1. It therefore generates more revenue from bond sales at time 0, and more revenue soaks up dollars, lowering P_0.

Fixing surpluses $\{s_0,\ s_1\}$, the demand curve for nominal debt was unit elastic, giving the same total real revenue $\beta E_0(s_1)$ for any amount of bonds B_0 that the government sells. Now, the debt demand curve is flat, giving the same real price Q_0/P_0 for any amount sold B_0. The more bonds B_0 sold, then, the more total real revenue such sales produce. If the bonds are sold as "fiscal policy," to finance a larger deficit s_0, they allow the government to run such a deficit without inflation. If the bonds are sold as "monetary policy," with no change in s_0, and no change in the *fiscal rule*, bond sales soak up more cash at time 0 and lower the price level P_0.

Likewise, fixing surpluses $\{s_0,\ s_1\}$, a higher interest rate target i_0 raised expected inflation, via $1/(1+i_0) = \beta E_0(P_0/P_1)$, having no effect on the price level P_0 and raising the expected price level P_1. Now a higher interest rate still raises expected inflation. But with $P_1 = P_1^*$ unchanged, the rise in interest rate i_0 raises expected inflation by lowering the price level P_0, and thus lowers current inflation P_0/P_{-1}. We have a model that overturns the Fisherian prediction, a model in which higher interest rates lower inflation!

Monetary policy drives down inflation because of a different *fiscal* policy rule. Monetary policy does not change the fiscal rule, but monetary policy can change a variable that fiscal policy responds to, and thereby indirectly change fiscal policy. We see a simultaneous fiscal tightening along with the interest rate rise. That fiscal tightening produces the lower time 0 inflation. But it is only an expected future fiscal tightening. We do not see a deficit in the period 0 when the interest rate rises. Looking at data, it could be hard to see what's going on.

This is an important story, by highlighting the importance of the fiscal policy rule for the effects of monetary policy. It also highlights one of the central mechanisms in many models for producing a negative inflation response to interest rates: Higher interest rates induce a future fiscal contraction.

1.8 Budget Constraints and Active Versus Passive Policies

I preview two theoretical controversies.

$B_0/P_1 = s_1$ is an equilibrium condition, not a government budget constraint. The government could leave cash M_1 outstanding overnight. People who don't want to hold cash overnight drive the equilibrium condition.

The government may choose to set surpluses s_1 so that $B_0/P_1 = s_1$ for any P_1. In this case the fiscal theory does not determine the price level. This is called a "passive" fiscal policy. Such a policy is a choice, however, not a budget constraint. It is also not a natural outcome of a proportional tax system.

This simple model helps us to preview a few common theoretical concerns.

First, isn't the fiscal theory equation $B_0/P_1 = s_1$ or $B_{-1}/P_0 = s_0 + \beta E_0(s_1)$ the government's budget constraint? Shouldn't we solve it for the surplus that the government must raise to pay off its debts, given the price level P_1? Economic agents must obey budget constraints, for any price. Budget constraints limit quantities given prices, they don't determine prices given quantities. You and I can't fix the real amount we want to repay for a mortgage, and demand that the price level adjust so we can afford a mansion. Are we specifying, weirdly or perhaps incorrectly, that the government is some special agent that can threaten to violate its budget constraint at off-equilibrium prices?

No. Equation (1.1),

$$\frac{B_0}{P_1} = s_1 \tag{1.14}$$

is not a budget constraint. The condition that holds at any price level is

$$B_0 = P_1 s_1 + M_1 \tag{1.15}$$

where M_1 is money left over at the end of the day after paying taxes, plus any of the debt B_0 that people may have chosen not to redeem. (I assume no default here. We'll add that later.) For any given B_0 and P_1, government choices of $\{s_1,\ M_1\}$ must satisfy (1.15). If the government specifies s_1, then M_1 follows from (1.15). No budget constraint says that the government may not leave money M_1 outstanding at the end of the day. If people decide to line their caskets with money or unredeemed debt, if we add $u(M_1)$ to the consumer's utility, no budget constraint forces the government to soak up that money with taxes.

Consumer demand is why $M_1 = 0$, and hence why $B_0 = P_1 s_1$. People don't *want* to hold any money at the end of the day, because they get no utility, purchasing power, tax-paying ability, or pleasure from doing so. Equation (1.14) results from the budget constraint (1.15) *plus* that consumer demand. Equation (1.14) is thus an equilibrium condition, a market-clearing condition, a supply = demand condition, deriving from consumer optimization together with consumer and government budget constraints. Equation (1.14) is not a "government budget constraint."

Budget constraints hold at off-equilibrium prices. Equilibrium conditions need not hold at off-equilibrium prices. Prices adjust to make equilibrium conditions hold. There is no reason that equation (1.14) should hold at a non-equilibrium price, any more than the supply of potatoes should equal their demand at \$10 per potato. When we substitute private sector demands, optimality conditions, or market-clearing conditions into government budget constraints, on our way to finding an equilibrium, we must avoid the temptation to continue to refer to the resulting object as a "budget constraint" for the government.

Why can't you and I demand that the "price level adjust to make our budget constraints hold?" Because we do not issue the currency and nominal debt that define the price level. You and I are like a government that uses another country's currency. We pay debts at the given price level, or we default. Nominal government debt is like corporate equity, whose price adjusts to make a valuation equation hold. Real or foreign currency government debt is like personal or corporate debt, which we must repay or default.

Suppose that the government *chooses* to adjust surpluses s_1 so as to make the equilibrium condition (1.14) $B_0/P_1 = s_1$ hold for any price level P_1. Suppose the government follows a fiscal rule, setting the surplus at time 1 by

$$s_1 = \tau_1 y_1 = \frac{B_0}{P_1}, \tag{1.16}$$

as if it were a budget constraint, lowering the tax rate as the price level rises and raising the tax rate as the price level falls. This is a possible *choice*. This choice is known as a "passive" fiscal policy. If the government follows such a policy, P_1 cancels from left and right, and (1.14) no longer determines the price level. In essence, the government's supply curve lies directly on top of the private sector's demand curve. A government that wishes to let the price level be set by other means, such as a foreign exchange peg, a gold standard, a currency board, use of another government's currency, the equilibrium-selection policies of new-Keynesian models, or $MV = Py$ once we add money demand, follows a passive fiscal policy.

The converse of "passive" is "active." The fiscal theory requires an "active" fiscal policy. Active fiscal policy does not require that surpluses s_T are fixed or exogenous. The surplus may respond to the price level P_1, $s_1(P_1)$ or to other variables including output and employment. The surplus may respond to the quantity of debt, as in $s_1 = B_0/P_1^*$. We just have to exclude the one for one case $s_1 = s_1(P_1) = B_0/P_1$, or multiple crossings, so that there is only one solution to (1.14), one P_1 such that $B_0/P_1 = s_1(P_1)$.

Standard theories of inflation include the government debt valuation equation (1.14), but they add this passive fiscal policy assumption, so that other forces may determine the price level. Specifying the mechanics of fiscal policy that achieves that passive response is an important and neglected part of such models. "Passive" does not mean easy. Coming up with the surpluses to defend the price level involves painful and distorting taxes, or unpopular limitations on government spending. Many papers add a footnote in which they assume the government charges lump-sum taxes to satisfy (1.14), but do not examine or test the resulting fiscal side of their models.

You may now want to follow decades of literature and try to test for active versus passive policy. Such tests are difficult. Both active and passive fiscal regimes include the valuation equation (1.14) as an equilibrium condition. They differ on the direction of its causality, the mechanism by which it comes to hold, and how governments behave for a price level away from the equilibrium which we observe. When the same equation holds in two models, arguing about how it comes to hold brings up subtle identification and observational equivalence issues. Is the price level set somewhere else, and $B_0/P_1 = s_1$ describes how the government sets surpluses? Or does $B_0/P_1 = s_1$ determine the price level given surpluses? All we see in the data is $B_0/P_1 = s_1$. The two views are *observationally equivalent*, at least before we add identification or other restrictions.

We will consider these issues at some length. The important point for now is that the government does not *have to* follow a passive fiscal policy, in the same way that we, and the government, have to follow budget constraints. An active fiscal policy is a logical and economic possibility, one that does not violate any of the rules of Walrasian equilibrium.

A passive fiscal policy is not a natural description of tax and spending policies. With a proportional tax on income, the nominal surplus is $P_1 s_1 = \tau P_1 y_1$. The real surplus $s_1 = \tau y_1$ is then independent of the price level, so fiscal policy is active. Transfer payments and social programs are either explicitly indexed or rise with market prices, so a primary surplus fixed in real terms $s_1 = \tau y_1 - g_1$ is again a natural specification. To engineer a passive policy, the government must change the tax *rate* and real spending in response to the price level, and in the opposite of a natural direction. To achieve $s_1 = B_0/P_1 = \tau_1 y_1$, passive policy requires $\tau_1 = B_0/(P_1 y_1)$. The government must systematically lower the tax rate, or increase real transfers, as the price level rises, and raise the tax rate or cut real transfers as the price level declines. If anything, the tax code generates the opposite sign: Inflation pushes people to higher tax brackets, inflation generates taxable capital gains, and inflation devalues depreciation allowances and past nominal losses carried forward. Inflation reduces the real value of sticky wage payments to government workers, and price-sticky health care payments. Governments facing inflation typically raise taxes and cut spending to fight inflation, while governments facing deflation typically lower taxes and spend more. A passive policy is a deliberate choice, requiring unusual and deliberate action by fiscal authorities.

1.9 Active versus Passive with a Debt Rule

> The policy rule $s_1 = B_0/P_1^*$ clarifies active versus passive policy. It seems that governments which respond to debt by raising surpluses therefore have passive policies. Active policy allows the government to respond to changes in the value of debt that come from changes in nominal debt B_0 or from changes in the price level target P_1^*. Active policy only requires that governments ignore changes in the value of debt that come from unexpected, undesired, multiple-equilibrium inflation.

The active versus passive question is often framed in terms of responses to debt. Interpret the passive surplus policy (1.16) $s_1 = B_0/P_1$ as a fiscal policy rule, in which governments raise surpluses in response to increases in the value of debt. Stated that way, passive policy sounds reasonable, the sort of thing that responsible governments do. (Or at least that they used to do!) One is tempted to run a regression, say $s_1 = a + \gamma(B_0/P_1) + u_1$, and to interpret γ as a test of active versus passive policy.

The active policy example (1.13) $s_1 = B_0/P_1^*$ and equilibrium $P_1 = P_1^*$ clarifies how this idea is mistaken, and how little is actually required of active fiscal policy. This active fiscal government responds one for one to changes in its nominal debt B_0. We observe it to respond one for one to changes in the equilibrium value of its debt, $s_1 = B_0/P_1 = B_0/P_1^*$. The regression would estimate $\gamma = 1$.

There are three sources of variation in the real value of debt: nominal debt B_0 built up from financing previous deficits, unexpected changes in the price level target $(E_1 - E_0)(1/P_1^*)$, and unexpected inflation different from the target $(E_1 - E_0)(1/P_1 - 1/P_1^*)$. Active fiscal policy only requires that the government respond less than one for one to the last component.

It is possible and natural that fiscal policy should respond differently to these three sources of variation in the value of debt. Responding to variation in the nominal value of debt B_0, accumulated by financing past deficits s_0, allows the government to borrow in the first place, to meet a deficit by borrowing rather than time 0 inflation. Responding to changes in an inflation target allows the government to have some inflation or deflation, for example as state-contingent defaults or stimulus in response to wars, pandemics, or crises. Committing not to respond to arbitrary unexpected inflation-induced variation in the value of debt allows the government to produce a stable and determinate price level, avoiding the indeterminacy that (here) would accompany passive policy. And governments do behave this way. They try to pay off debts, conscious of the reputation doing so engenders for future borrowing; they try to coordinate fiscal and monetary policies; yet they respond to undesired inflation with austerity and to undesired deflation with stimulus, not the opposite reactions that passive policy requires.

Thinking in terms of a reaction to debt, we begin to see identification and observational equivalence more clearly. In equilibrium, we see $s_1 = B_0/P_1^* = B_0/P_1$. The regression that attempts to test active versus passive policy is $s_1 = \alpha(B_0/P_1^*) + \gamma(B_0/P_1 - B_0/P_1^*) + u_1$. The coefficient $\gamma = 1$ ($\gamma > 0$ in the later intertemporal model) indicates passive policy. But we never see $P_1 \neq P_1^*$ in equilibrium. The parameter γ that measures active versus passive fiscal policy is not identified. We cannot easily look at time series and distinguish whether price level is set elsewhere and fiscal policy follows passive $\alpha = \gamma = 1$, or whether fiscal policy is active with $\alpha = 1$ and $\gamma = 0$. Testing for active versus passive regimes at least requires identifying assumptions.

Observational equivalence and parameter nonidentification do not mean the enterprise is pointless. It is a crucial guiding theorem, like other observational equivalence theorems throughout economics and finance.

As we look deeper, I will argue that the active versus passive debate has been a dead end. It is a historical theoretical controversy that a fiscal theorist must understand, for now. The active-fiscal passive-money and active-money passive fiscal extremes are useful thought experiments. However, especially as they are observationally equivalent, they are not useful concepts for additional investigation, to understand data, to productively test for one or the other regime, or to analyze policy. In the end, fiscal and monetary policy must be coordinated. The extreme game-of-chicken view that coordination comes about because one is "active" and the other "passive" is not necessary, realistic, or productive.

The "active" and "passive" labels are due to Leeper (1991). The labels are not perfect, as "active" fiscal policy here includes leaving surpluses alone, and "passive" policy means adjusting tax rates and spending according to the price level, which takes a lot of activity. The same possibilities are sometimes called "money-dominant" versus "fiscal-dominant," which has a lot of other meanings, and "Non-Ricardian" versus "Ricardian," which I find terribly confusing. It is not true that active-fiscal regimes fail to display Ricardian equivalence, or that in them government debt is a free lunch. Recognizing the deficiency of good labels, some authors offer symbols such as "Regime F" and "Regime M." Words are better. For this book, I use "active" and "passive" as defined here, and elaborated in context later.

Chapter Two

An Intertemporal Model

THIS CHAPTER introduces a simple intertemporal model. The basic fiscal theory equation quickly generalizes to say that the real value of nominal debt equals an infinite present value of surpluses,

$$\frac{B_{t-1}}{P_t} = E_t \sum_{j=0}^{\infty} \beta^j s_{t+j}.$$

I start by developing this model fully, writing out the economic environment. The ideas sketched in the two-period model of the last chapter gain detail and nuance. We take an important step towards models useful for empirical application.

I then consider again "monetary policy," changes in debt B_t with no change in surpluses, as opposed to "fiscal policy," which changes surpluses, in the context of the intertemporal model. "Monetary" and "fiscal" debt issues are again analogues to share splits versus equity offerings. This insight suggests a reason for the institutional separation between treasury and central bank. We will see that a form of "fiscal stimulus" can cause inflation.

Monetary policy can target the nominal interest rate. Linearizing, a fiscal theory of monetary policy emerges that looks much like standard new-Keynesian models, and resembles current institutions. Therefore, the "fiscal" theory of the price level does not require us to throw out everything we know and our accumulated modeling skills, to ignore central banks, and to think about inflation in terms of debts and surpluses. We can approach data and institutions much as standard monetary modelers do, specifying interest rate targets, and making minor changes in the ingredients and solution methods of standard models.

Distinguishing fiscal theory of monetary policy from new-Keynesian and monetarist alternatives introduces observational equivalence theorems, elaborated in this intertemporal context.

This chapter maintains the other simplifications used so far: one-period debt, flexible prices, an endowment economy with a constant real interest rate and no risk premiums. Later chapters add price stickiness, discount rate variation, risk premiums and other realistic complications.

2.1 The Intertemporal Model

I derive the simplest intertemporal version of the fiscal theory. The government debt valuation equation is

$$\frac{B_{t-1}}{P_t} = E_t \sum_{j=0}^{\infty} \beta^j s_{t+j}.$$

The price level adjusts so that the real value of nominal debt equals the present value of future surpluses.

The two-period model is conceptually useful, but we need a model that describes economies over time. It is also useful to fill out economic foundations to see a complete model. This section describes a full, if still simple, intertemporal model.

The economy starts with bonds B_{-1} outstanding. At the end of each time period $t-1$ the government issues nominal one-period debt B_{t-1}. Each nominal bond promises to pay one dollar at time t. At the beginning of period t, the government prints up new money to pay off the maturing debt. At the end of period t, the government collects taxes net of transfers s_t, and sells new debt B_t at a price Q_t. Both actions soak up money.

Following the money, the government budget constraint is

$$M_{t-1} + B_{t-1} = P_t s_t + M_t + Q_t B_t \tag{2.1}$$

where M_{t-1} denotes non-interest-paying money held overnight from the evening of $t-1$ to the morning of time t, P_t is the price level, $Q_t = 1/(1+i_t)$ is the one-period nominal bond price, and i_t is the nominal interest rate. Interest is paid overnight only, from the end of date t to the beginning of $t+1$, and not during the day.

A representative household maximizes

$$\max E \sum_{t=0}^{\infty} \beta^t u(c_t)$$

in a complete asset market. The household has a constant endowment $y_t = y$.

The household's period budget constraint is almost the mirror of (2.1). The household enters the period with money M_{t-1} and nominal bonds B_{t-1}, receives income $P_t y$, purchases consumption $P_t c_t$, pays net taxes net of transfers $P_t s_t$, buys bonds B_t, and potentially holds money M_t,

$$M_{t-1} + B_{t-1} + P_t y = P_t c_t + P_t s_t + M_t + Q_t B_t. \tag{2.2}$$

Household money and bond holdings must be nonnegative, $B_t \geq 0$, $M_t \geq 0$.

The consumer's first-order conditions and equilibrium $c_t = y$ then imply that the gross real interest rate is $R = 1/\beta$, and the nominal interest rate i_t and bond price Q_t are

$$Q_t = \frac{1}{1+i_t} = \frac{1}{R} E_t \left(\frac{P_t}{P_{t+1}} \right) = \beta E_t \left(\frac{P_t}{P_{t+1}} \right). \tag{2.3}$$

When $i_t > 0$ the household demands no money, $M_t = 0$. When $i_t = 0$ money and bonds are perfect substitutes, so the symbol B_t can stand for their sum. The interest rate cannot be less than zero in this model. Thus, we can eliminate money from (2.1), leading to the flow equilibrium condition

$$B_{t-1} = P_t s_t + Q_t B_t. \tag{2.4}$$

Substituting the bond price (2.3) into (2.4), dividing by P_t, we have

$$\frac{B_{t-1}}{P_t} = s_t + \beta B_t E_t \left(\frac{1}{P_{t+1}} \right). \tag{2.5}$$

Household maximization, budget constraint, and equilibrium $c_t = y$ also imply the household transversality condition

$$\lim_{T \to \infty} E_t \left(\beta^T \frac{B_{T-1}}{P_T} \right) = 0. \tag{2.6}$$

If the term on the left is positive, then the consumer can raise consumption at time t, lower this terminal value, and raise utility. A no-Ponzi condition, that consumers cannot borrow and roll over debt forever, which can be enforced by a bound on debt $B_t \geq B$ with B a large negative number, rules out a negative value.

The transversality condition takes the place of the second day in my two-day model. Transversality conditions lead to many confusing debates, which is why I stopped to show how fiscal theory works in a two-period model, at the cost of some repetition. Online Appendix Section A1.1 covers the transversality condition in more detail.

As a result, we can then iterate (2.5) to

$$\frac{B_{t-1}}{P_t} = E_t \sum_{j=0}^{\infty} \beta^j s_{t+j}. \tag{2.7}$$

The government sets debt and surpluses $\{B_t\}$ and $\{s_t\}$. Debt B_{t-1} is predetermined at time t. The right-hand side of (2.7) does not depend on the price level in this simple model. Therefore, the price level must adjust so that (2.7) holds. The right-hand side of (2.7) is the present value of future primary surpluses. The left-hand side is the real value of nominal debt. So, the fiscal theory says that *the price level adjusts so that the real value of nominal debt is equal to the present value of primary surpluses.*

We have determined the price level, in a completely frictionless intertemporal model.

Another useful approach is to add the transversality condition to the household flow budget constraint (2.2), iterate forward, and express the household present value budget constraint in real terms

$$\frac{B_{t-1}}{P_t} = \sum_{j=0}^{\infty} \beta^j (c_{t+j} - y + s_{t+j}). \tag{2.8}$$

If the value of debt is greater than the present value of surpluses, then the household has extra wealth, which they try to spend on consumption greater than endowment.

Some details and clarifications: The surplus concept denoted by s_t is the real *primary* surplus in government accounting. The usual deficit or surplus includes interest payments on government debt. The surplus in this simple model includes only cash tax receipts less cash transfers. I do not include government purchases of goods and services (roads, tanks), which subtract from produced output to lower private consumption, and may provide benefits in utility. Thus, equilibrium in the goods market is $c_t = y$, not $c_t + g_t = y$, and marginal utility depends on private consumption only. We can easily add those realistic complications.

Rather than a real lump sum, we can also specify that surpluses are a proportional income (endowment) tax less a lump-sum indexed transfer,

$$P_t s_t = \tau_t (P_t y) - P_t x_t,$$

and that the tax rate τ_t and real transfer payments x_t are independent of the price level. This specification ensures that the price level is absent from the right-hand side of (2.7), and taxes do not distort asset or goods prices.

I do not, here or later in this book, write down an objective for the government. That extension is important, and integrates fiscal theory with dynamic public finance. I do not take the next step, and describe government policy as the outcome of a game between players with different objectives. That extension is important too. I simply study the mapping between policy levers and outcomes, with a verbal understanding that governments like low inflation and greater output.

2.2 Dynamic Intuition

> The government debt valuation equation in fiscal theory is an instance of the basic asset pricing valuation equation. Nominal government debt acts as a residual claim to primary surpluses. The price level is like a stock price, and adjusts to bring the real value of nominal debt in line with the present value of primary surpluses, just as the stock price adjusts to bring the value of shares in line with the present value of dividends.

The government debt valuation equation (2.7) is an instance of the basic asset pricing equation: price per share $1/P_t$ times number of shares B_{t-1} equals present value of dividends $\{s_{t+j}\}$. We quote the price level as the price of goods in terms of money, not the price of money in terms of goods, so the price level goes in the denominator not the numerator. Primary surpluses *are* the "dividends" that retire nominal government debt. In an accounting sense, nominal government debt *is* a residual claim to real primary surpluses.

The fact that the price level can vary means that nominal government debt is an equity-like, floating-value, claim. If the present value of surpluses falls, the price level can rise to bring the real value of debt in line, just as a stock price falls to bring market value of equity in line with the expected present value of dividends. Nominal government debt *is* "stock in the government."

Continuing the analogy, suppose that we decided to use Apple stock as numeraire and medium of exchange. When you buy a cup of coffee, Starbucks quotes the price of a venti latte as 1/10 of an Apple share, and to pay you tap your iPhone, which transfers 1/10 of a Apple share in return for your coffee. If that were the case, and we were asked to come up with a theory of the price level, our first stop would be that the value of Apple shares equals the present value of its dividends. Then we would add liquidity and other effects on top of that basic idea. That is exactly what we do with the fiscal theory.

This perspective also makes sense of a lot of financial commentary. Exchange rates go up and inflation goes down when an economy does better, when productivity increases, and when governments get their budgets under control. Well, money is stock in the government.

Backing government debt by the present value of surpluses allows for a more stable price level than the one- or two-period models suggest. In the one-period model any unexpected variation in surplus s_1 translates immediately to inflation. In the dynamic model, examine (2.4):

$$B_{t-1} = P_t s_t + Q_t B_t. \tag{2.9}$$

If the government needs to finance a war or counter a recession or financial crisis, it will want to run a deficit, a lower or negative s_t. In the dynamic model, the government can soak up those dollars by debt sales $Q_t B_t = \beta E_t s_{t+1}$ rather than a current surplus s_t. For that strategy to work, however, the government must persuade investors that more debt today will be matched by higher surpluses in the future.

Surpluses are not "exogenous" in the fiscal theory! Surpluses are a *choice* of the government, via its tax and spending policies and via the fiscal consequences of all its policies. The government debt valuation equation is an equilibrium condition among endogenous variables. Surpluses may react to events. For example, surpluses may rise as tax revenues rise in a boom. Surpluses may also respond to the price level, by choice or by non-neutralities in the tax code and expenditure formulas. We only have to rule out or treat separately the special case of "passive" policy, that the present value of surpluses reacts exactly one for one to changes in the value of nominal debt brought about by changes in the price level, so that equation (2.7) holds for any price level P_t.

It is initially puzzling that this model with one-period debt relates the price level to an infinite present value of future surpluses. One expects one-period assets to lead to a one-period present value, and long-term assets to be valued with a long-term present value. Equation (2.9) tells us why: The government plans to roll over the debt. Most of the payments to today's one-period debtholders, B_{t-1}/P_t, come from new debtholders willing to pay $Q_t B_t / P_t$. If the rollover fails, or if the government retires the debt, not selling new debt, we have $B_{t-1}/P_t = s_t$ only as in the one-period model.

As a result, inflation in the fiscal theory with short-term debt has the feel of a run. If we look at the present value equation (2.7), it seems today's investors dump debt because of bad news about deficits in 30 years. But today's investors really dump debt because they fear tomorrow's investors won't be there to roll over the debt. Directly, consider the flow equation written as

$$\frac{B_{t-1}}{P_t} = s_t + \frac{Q_t B_t}{P_t}.$$

The price level P_t rises because the revenue that debt sales $Q_t B_t / P_t$ generate won't be enough to pay off today's debt B_{t-1} at the originally expected price level. Why are people unwilling to buy bonds? Well, they look at the same situation a period ahead, and worry that investors will not buy bonds B_{t+1}, to pay them off in real terms, and so on. Yes, the indirect cause of inflation can be a worry about surpluses in the far future. But the direct mechanism is a loss of faith that debt will be rolled over. Short-term debt, constantly rolled over, to be retired slowly by a very long-lasting and illiquid asset stream, is the classic ingredient of a bank run or sovereign debt crisis. The main difference is the fiscal theory government in a rollover crisis can devalue via inflation rather than default explicitly.

The fact that inflation can break out based on fear of fiscal events in the far future tells you that inflation can break out with little current news, seemingly out of nowhere, or as an unpredictable apparent overreaction to seemingly small events. This is a helpful analysis because inflation and currency devaluation do often break out with little current news, seemingly out of nowhere. Central bank and private inflation forecasts miss almost as much as stock market forecasts miss. Run mechanics increase this rootless sense. I emphasize rational expectations for simplicity, but one can quickly spy multiple equilibrium variants, a sensitivity to exactly rational expectations, or fear of fear of such events. You may well dump Treasury securities just because you fear others will do so next year, and you want to get out before the flood. Section 7.2.2 investigates these run mechanics in more detail, and analyzes how long-term debt offers governments protection against inflation.

Since the government debt valuation (2.7) looks a lot like a stock valuation equation, we might expect inflation to be as variable as stock prices, and real returns on government bonds as risky as stock returns. However, as we saw briefly in Section 1.5, and will see in more detail later, surpluses typically follow a process with an s-shaped moving average. A deficit, negative s_t in the short run, corresponds to surpluses, positive s_t later on, which at least partially repay the debt issued to finance deficits. As a result, large shocks to near-term deficits may have little impact on the present value of surpluses, and therefore little impact on inflation or the real returns of government bonds. For stocks, we usually think that cash flow shocks are more persistent, and do not substantially reverse. Thus, changes in stocks' cash flows have larger effects on prices. Bonds have s-shaped cash flows: Borrowing is followed by repayment, all or in part. Bonds and stocks are valued by the present value formula. Bond prices also decline when expected future cash flows decline, due to default fears. But bond prices are much less volatile than stock prices. A similar valuation formula with a different cash flow process produces a different result. Government debt has a bond-like surplus process.

What about the first period? If we start with $B_{-1} = 0$, then the price level P_0

must be determined by other means. To tell a story, perhaps the economy uses gold coins, or foreign currency on the day the government first issues nominal bonds. Then, at date 0, the government issues nominal bonds B_0. It could sell these bonds in return for gold coins, to finance a deficit, or in exchange for its outstanding real or foreign currency debt. Then the economy starts in period 1 with maturing government debt B_0, or money printed up to redeem that debt, and a determined price level. The government could also just give people an initial stock of money at the beginning of period zero, which counts as a transfer or negative surplus.

I start here with the simplest possible economic environment, abstracting from monetary frictions, financial frictions, pricing frictions, growth, default, risk and risk aversion, output fluctuations, limited government pre-commitment, distorting taxes, and so forth. We can add all these ingredients and more. But starting the analysis this way emphasizes that no additional complications are *necessary* to determine the price level.

The fiscal theory does rely on specific institutions. The government in this model has its own currency and issues nominal government debt. We use maturing debt, or the currency it promises, as numeraire and unit of account. This is not a theory of clamshell money, or of Bitcoins. It is a theory adapted to our current institutions: government-provided fiat money, rampant financial innovation, interest rate targets, governments that generally inflate rather than explicitly default.

More generally, our monetary and financial system is built around the consensus that short-term government debt is an abundant safe asset, and thus a natural numeraire. This faith may be a weak point in our institutions going forward. If we experience a serious sovereign debt crisis, not only will the result be inflation, it will be an unraveling of our payments, monetary, and financial institutions. Then, we shall have to write an entirely new book, of monetary arrangements that are insulated from sovereign debt. We shall have to construct a numeraire that is backed by something other than the present value of government surpluses. This is a fun bit of free-market financial engineering. I pursue the issue briefly in Section 10.6. But it is so far from current institutions that I do not pursue it at great length. Given the financial and economic calamity that a U.S. or European sovereign debt crisis would be, let us hope that day does not come to pass anytime soon.

2.3 Equilibrium Formation

> What force pushes the price level to its equilibrium value? I tell three stories, corresponding to three consumer optimization conditions. If the price level is too low, money may be left overnight. Consumers try to spend this money, raising aggregate demand. Alternatively, a too-low price level may come because the government soaks up too much money from bond sales. Consumers either consume too little today relative to the future or too little overall, violating intertemporal optimization or the transversality condition. Fixing these, consumers again raise aggregate demand, raising the price level.

What force pushes the price level to its equilibrium value?

The basic intuition is "aggregate demand," just as in the one-period model. If government bonds are worth more than the present value of surpluses, people try to get rid of government bonds. The only way to do so, in the end, is to try to buy more goods and services, thereby bidding up their prices. Aggregate demand is, by budget constraint, always the mirror image of demand for government debt. Equation 2.8 expresses aggregate demand as extra wealth.

People trying to get rid of government bonds might initially try to buy assets. This step would raise the value of assets, and higher asset values induce them to buy more goods and services, the "wealth effect" of consumption.

Technically, if the price level is not at its equilibrium value, the economy is off a supply curve or a demand curve. To tell a story, let us suppose the latter: One of the consumers' optimality conditions is violated. I'll suppose the price level is wrong in the first place because money demand (zero), intertemporal optimization, or the transversality condition are violated. We then ask what actions the consumer takes to improve matters, and how that action brings the price level into equilibrium.

One good story is that if the price level is too low, the government will leave more money outstanding at the end of period t than people want to hold, just as in the one-period model. That money chases goods, driving up the price level, and vice versa. Specifically, the flow budget constraint says that money printed up in the morning to retire debt is soaked up by bond sales or money left outstanding,

$$B_{t-1} = P_t s_t + Q_t B_t + M_t. \tag{2.10}$$

We reasoned from a constant endowment, intertemporal optimization, and the transversality condition, that debt sales generate real revenue equal to the present value of following surpluses, that $Q_t B_t$ in (2.10) comes from

$$\frac{Q_t B_t}{P_t} = E_t \sum_{j=1}^{\infty} \beta^j s_{t+j}. \tag{2.11}$$

Thus, if the price level P_t is too low, the current surplus and the revenue from bond sales in (2.10) do not soak up all the money printed to redeem bonds. Money M_t is left overnight, violating the consumer's money demand $M_t = 0$. As people try to spend the extra money, the price level rises. If you're bothered by negative money in the opposite direction, add a money demand $M_t = M$, which we do explicitly later, so money is insufficient rather than negative.

Alternatively, the price level may be too low because debt sales are soaking up too much money. Debt sales generate more revenue than the present value of surpluses on the right-hand side of (2.11). Consumers try to buy too many bonds, either violating their intertemporal first-order conditions or their transversality condition.

In the first case, consumers save too much now, to dis-save later. That extra saving drives consumption demand below endowment (goods market supply) now, and higher later. When consumers restore a smooth intertemporal allocation of consumption, they provide aggregate demand, raising the price level today. Such intertemporal optimization is the main source of aggregate demand in standard new-Keynesian models.

In the second case, consumers buy too many bonds and hold them forever, letting bond wealth grow at the rate of interest. In this case, via

$$\frac{B_{t-1}}{P_t} = s_t + \frac{Q_t B_t}{P_t} = s_t + E_t \sum_{j=1}^{\infty} \beta^j s_{t+j} + \lim_{T\to\infty} E_t \beta^T \frac{B_{t+T}}{P_{t+T+1}},$$

bonds soak up too much money because consumers are violating the transversality condition. Debt grows at the real interest rate. People could hold less debt, and increase consumption at all dates. When they do so, this wealth effect, as opposed to the previous intertemporal substitution effect, is the source of aggregate demand, pushing up the price level. Contrariwise, a too-high price level pushes debt to negative values, which is ruled out by budget constraint.

Much fiscal theory analysis focuses on the latter possibility. Fiscal price determination is said to rely on a "threat by the government to violate the transversality condition at off-equilibrium prices." But the transversality condition is only one of three sets of consumer optimization conditions: zero money demand, intertemporal optimization, and transversality condition. And there are lots of additional equilibrium formation stories that we can tell in which the transversality condition holds. Violation of the transversality condition is *an* equilibrium formation story, but not the only or most interesting one.

Moreover, the government doesn't *do* anything. It does not take any action that the word "threat" implies. It simply ignores the bubble in government debt and waits for consumers to come to their senses and drive the price level back up. Likewise, if a bubble appears in share prices, a corporation takes no action, it just waits for the bubble to disappear. We do not critique asset pricing as relying on a threat by firms to violate the transversality condition at off-equilibrium prices. Finally, the transversality condition in this model is a combination of consumer optimization and consumer budget constraint, and does not apply to government or firms.

An alternative, and better, perspective on these sorts of exercises starts by recognizing that the equilibrium object is not just today's price level P_t, but the whole sequence of price levels $\{P_t\}$. Rather than say consumers are off an optimality condition, we should say that they optimize, but given a sequence of price levels at which markets do not clear. For example, if the price level is too low today, but will rise later, then the bond price $Q_t = \beta E_t \left(P_t/P_{t+1}\right)$ is too low. Consumers correctly optimize, but the resulting consumption demand today is below endowment while demand in the future is above the endowment. Likewise, we generate the transversality condition or wealth effect story with a price level that is too low forever.

I don't pursue this inquiry too deeply. As in all supply-demand economics, one can tell many stories about out-of-equilibrium behavior. Whether out-of-equilibrium allocations follow a demand curve or a supply curve makes a big difference to the equilibrium formation story. Out of equilibrium, market-clearing conditions do not hold, so don't expect out-of-equilibrium economies to make much sense. As in classic microeconomics, Walrasian equilibrium describes equilibrium conditions compactly with a simple, though unrealistic, description of off-equilibrium behavior, the Walrasian auctioneer. Walrasian equilibrium does not describe well a dynamic equilibrium formation process. Game theoretic treatments of off-equilibrium behavior are more satisfactory though much more complicated. They are also a bit arbitrary, as many dynamic games lead to the same equilibrium conditions. Bassetto (2002) and Atkeson, Chari, and Kehoe (2010) are good examples of game theoretic foundations in this sphere.

Still, it is useful to tell at least one or two equilibrium formation stories behind any model, as part of ensuring the model makes intuitive sense, and in order to use the model as a quantitative parable for describing the world. If you can't tell at least one plausible equilibrium formation story, you don't really understand a model, or the model may be more fragile than you think. Models with multiple equilibria and equilibrium selection criteria are vulnerable to this critique.

Equilibrium formation stories are not common in economics, but reappear throughout this book. I think we should take them more seriously, at least at the verbal level I pursue them. I hope you find that reading them or thinking about them makes the models more believable as quantitative parables. The related concern about supply and demand for the last dollar bill crops up repeatedly as well. It's easy to write down models in which the supply and demand for the last dollar bill uniquely determine the price level. But a quick examination of the utility or financial costs of deviating even slightly from supply and demand curves, or an attempt to write just what would people do out of equilibrium, gives us a sense that the force of this equilibrium condition is slight. Cochrane (1989) argues more generally to think of robust predictions that include a range of behaviors with small utility or financial costs.

2.4 Fiscal and Monetary Policy

I break the basic present value relation into expected and unexpected components:

$$\frac{B_t}{P_t}\Delta E_{t+1}\left(\frac{P_t}{P_{t+1}}\right) = \Delta E_{t+1}\sum_{j=0}^{\infty}\beta^j s_{t+1+j},$$

$$\frac{B_t}{P_t}\frac{1}{1+i_t} = \frac{B_t}{P_t}\frac{1}{R}E_t\left(\frac{P_t}{P_{t+1}}\right) = E_t\sum_{j=1}^{\infty}\beta^j s_{t+j},$$

$$\frac{B_{t-1}}{P_t} = s_t + \frac{B_t}{P_t}\frac{1}{1+i_t} = E_t\sum_{j=0}^{\infty}\beta^j s_{t+j}.$$

In this model, unexpected inflation results entirely from innovations to fiscal policy $\{s_t\}$. A change in debt B_t with no change in surpluses $\{s_t\}$ can determine the nominal interest rate and *expected* inflation. The government can also target nominal interest rates, and thereby expected inflation, by offering to sell any amount of bonds at the fixed interest rate. I call the latter two operations "monetary policy."

Government policy is so far described by two settings, nominal debt $\{B_t\}$ and surpluses $\{s_t\}$. We will spend some time thinking about their separate effects: What if the government changes nominal debt without changing surpluses, or vice versa? Almost all actual policy actions consist of simultaneous changes of both instruments, so beware jumping too quickly from these exercises to the analysis of episodes or policy. But answering these conceptual questions lets us understand the mechanics of the theory more clearly.

We will learn a lot by breaking the basic government debt valuation equation into expected and unexpected components. It will be clearer to move the time index forward and to start with

$$\frac{B_t}{P_{t+1}} = E_{t+1} \sum_{j=0}^{\infty} \beta^j s_{t+1+j}. \tag{2.12}$$

I mostly follow a convention of describing expectations at time t, and news or shocks at time $t+1$.

2.4.1 Fiscal Policy and Unexpected Inflation

Multiply and divide (2.12) by P_t, and take innovations

$$\Delta E_{t+1} \equiv E_{t+1} - E_t$$

of both sides, giving

$$\frac{B_t}{P_t} \Delta E_{t+1} \left(\frac{P_t}{P_{t+1}} \right) = \Delta E_{t+1} \sum_{j=0}^{\infty} \beta^j s_{t+1+j} \, . \tag{2.13}$$

As of time $t+1$, B_t and P_t are predetermined. Therefore, in this simple model,

- *Unexpected inflation is determined entirely by changing expectations of the present value of fiscal surpluses.*

If people expect lower future surpluses, the value of the debt must fall. In this model, unexpected inflation is the only way for that to happen.

In this simple model, bad fiscal news affects inflation for one period only, giving a price level jump. Higher expected inflation cannot devalue short-term debt that has not been sold yet, and you can't expect future unexpected shocks. In reality, we see protracted inflations around fiscal shocks. Long-term debt, varying discount rates, and sticky prices will give us a more drawn-out response.

2.4.2 Monetary Policy and Expected Inflation

Next, multiply and divide (2.12) by P_t, and take the expected value E_t of both sides, giving

$$\frac{B_t}{P_t} E_t \left(\frac{P_t}{P_{t+1}} \right) = E_t \sum_{j=0}^{\infty} \beta^j s_{t+1+j}.$$

Multiplying by β, and recognizing the one-period bond price and interest rate in

$$Q_t = \frac{1}{1+i_t} = \beta E_t\left(\frac{P_t}{P_{t+1}}\right), \tag{2.14}$$

we can then write

$$\frac{B_t}{P_t}\frac{1}{1+i_t} = \frac{B_t}{P_t}\frac{1}{R}E_t\left(\frac{P_t}{P_{t+1}}\right) = E_t\sum_{j=1}^{\infty}\beta^j s_{t+j}. \tag{2.15}$$

The first term in (2.15) is the real revenue the government raises from selling bonds at the end of period t. The last term expresses the fact that this revenue equals the present value of surpluses from time $t+1$ on. The outer terms thus express the idea that the real value of debt equals the present value of surpluses, evaluated at the *end* of period t. The inner equality tells us about *expected* inflation, the counterpart of the unexpected-inflation relation (2.13).

Now, examine equation (2.15), and consider what happens if the government sells more debt B_t at the end of period t, without changing surpluses $\{s_{t+j}\}$. The price level P_t is already determined by the version of (2.12) that holds at time t. In particular from (2.13) at time t, bond sales B_t, though they may change unexpectedly at time t, do not change the price level at time t. If surpluses do not change in (2.15), then the bond price, interest rate, and expected future inflation must move one for one with the debt sale B_t.

- *The government can control interest rates i_t, bond prices Q_t and expected inflation $E_t(P_t/P_{t+1})$, by changing the amount of debt sold, B_t, with no change in current or future surpluses.*

If the government does not change surpluses as it changes debt sales B_t, then it always raises the same revenue Q_tB_t/P_t by bond sales. Equation (2.15) with unchanged surpluses describes a unit elastic demand curve for nominal debt: Each 1% rise in quantity gives a 1% decline in bond price, since the real resources that pay off the debt are constant. The analysis is just like that of the two-period model with the present value of surpluses in place of the time 1 surplus s_1. Selling bonds without changing surpluses is again like a share split.

This fact explains why only surplus innovations $\Delta E_{t+1}s_{t+j}$ change unexpected inflation in (2.13), and why changing expectations of future bond sales $\Delta E_{t+1}B_{t+j}$, $j \geq 1$ make no difference at all to either formula. Given the surplus path, selling more bonds, $\Delta E_{t+1}B_{t+1}$ in particular, raises no additional revenue.

2.4.3 Interest Rate Targets

Rather than announce an amount of debt B_t to be sold, the government can also announce the bond price or interest rate i_t and then offer people all the debt B_t they want to buy at that price, with no change in surpluses. A horizontal rather than vertical supply curve of debt can intersect the unit elastic demand for government debt. In that case, equation (2.15) describes how many bonds the government will sell at the fixed price or interest rate.

- *The government can target nominal interest rates by offering debt for sale with no change in surpluses.*

This is an initially surprising conclusion. You may be used to stories in which targeting the nominal rate requires a money demand curve, and reducing money supply raises the interest rate. That story needs a friction: a demand for money, which pays less than bonds. We have no frictions.

You might have thought that trying to peg the interest rate in a frictionless economy would lead to infinite, zero, negative, or otherwise pathological demands; or other problems. Equation (2.15) denies these worries. The debt quantities are not unreasonably large either. If the government raises the interest rate target by one percentage point, it will sell one percent more nominal debt.

Contrary intuition comes from different implicit assumptions. The proposition only states that the government can fix the *nominal* interest rate. An attempt to fix the real rate in this model would lead to infinite demands.

From (2.14), $1/(1+i_t) = \beta E_t\left(P_t/P_{t+1}\right)$,

- *The nominal interest rate target determines expected inflation.*

I use the word "monetary policy" to describe setting a nominal interest rate target or changing the quantity of debt without directly changing fiscal policy. Central banks buy and sell government debt in return for money. Central banks cannot, at least directly, change fiscal policy. They must always trade one asset for another. They may not write checks to voters. They may not drop money from helicopters. Those are fiscal policies. I will spend some time later mapping these ideas to current institutions.

The definition of "monetary policy" will generalize in other contexts and require some thought. For example, rather than specify surpluses directly, I will later characterize fiscal policy by a rule, in which surpluses respond systematically to inflation, output, debt, interest costs, or other variables. In the two-period model, we already saw how a fiscal rule that targets the price level alters the effects of monetary policy. With such rules in place, it can be interesting to define "monetary policy" as a change in interest rates that does not change the fiscal policy *rule*, though surpluses themselves may change. We will also add non-interest-paying money, in which case central bank actions can directly produce one source of surplus: seigniorage. In the end, no single clean definition of "monetary policy" independent of fiscal policy emerges. The most general direction is to be aware of monetary–fiscal interactions and to make sure you ask an interesting question. Still, it is useful first to explore this simple conceptual experiment of interest rate targets with no change in surpluses, and to add various mechanisms for fiscal–monetary interactions later.

Terminology: "Monetary policy" is a somewhat antiquated term. Central banks now set interest rate targets directly, by simply offering to borrow (pay interest on reserves) and lend at specified rates. "Monetary policy" in this model has nothing to do with the quantity of money, an interest spread for liquid assets, and so forth. However, I follow convention and continue to call setting an interest rate target "monetary policy" with this disclaimer.

An interest rate *peg* means an interest rate that is constant over time and does not respond to other variables. A peg can also mean a commitment to buy and sell freely at a fixed price, as in a gold standard or foreign exchange rate peg. A *time-varying peg* moves the interest rate over time but does not respond systematically to other endogenous variables like inflation and unemployment. An interest rate *target* means that the government sets the nominal interest rate, but may change that rate over time and also in response to endogenous variables such as inflation and unemployment, as in a Taylor rule. A "target" can also mean an aspiration, a goal that a central bank tries to move toward slowly while controlling another variable. A 2% "inflation target" works this way. I do not distinguish between "target" and "instrument," as Poole (1970) suggests. A more precise language would say that the central bank uses an interest rate instrument to achieve an inflation target.

I refer to "the government" uniting treasury and central bank balance sheets, and treating government decisions as those of a unitary actor. In this model, the separation between treasury and central bank balance sheets is irrelevant, and will remain so until we start to think about considerations that revive its relevance.

2.4.4 Fiscal Theory With an Interest Rate Target

In sum,

- *Monetary policy can target the nominal interest rate, and determine expected inflation, even in a completely frictionless model. Fiscal policy determines unexpected inflation.*

You might have thought "fiscal theory" would lead us to think about inflation entirely in terms of debt and deficits. We learn that this is not the case. "Monetary policy," choosing interest rates $\{i_t\}$ without changing fiscal policy, can fully control expected inflation in this simple model. Fiscal policy fills in the gap, determining unexpected inflation and thus fully determining inflation.

It is a classic doctrine that the government cannot peg the nominal interest rate. An attempt to do so leads to inflation that is unstable (Friedman (1968)) or indeterminate (Sargent and Wallace (1975)). The fiscal theory overturns these classic doctrines.

- *Inflation can be stable and determinate under an interest rate target, or even an interest rate peg.*

The classic propositions are not wrong, they just assume passive fiscal policy. Details follow.

In a perfect foresight version of this economy, monetary policy generates a family of price level paths, while fiscal policy only determines the first "shock," the time-zero price level given preexisting debt B_{-1}, thereby choosing which of the many price level paths is unique. With that sort of model in mind, one might complain that we have a theory of the price level, not a theory of inflation; a theory of equilibrium selection, not of inflation dynamics. However, in a stochastic economy there is a new shock every period, so fiscal policy matters continually. Inflation is the change in the price level, so if fiscal concerns determine the price level each period they are a necessary part of a theory of inflation. More deeply, when we add sticky prices in continuous time, we will see the price level jump disappear entirely. Fiscal policy chooses one of many inflation paths, each of which starts from the same price level. And "equilibrium selection" is a central part of any theory, indispensable to generating its predictions. If we remove any one of the equilibrium conditions, the others generate multiple equilibria and the removed condition is reduced to "equilibrium selection," so the disparaging view really does not make sense.

The neat separation that "monetary policy" determines expected inflation and "fiscal policy" determines unexpected inflation does not generalize directly. Typically, we can read the equilibrium conditions that monetary policy along with the rest of the model generates a family of equilibria and the government debt valuation equation selects among them, choosing the innovation in one combination of state variables. That combination of state variables may not even include an unexpected change in inflation. In some examples, only expected future inflation changes.

2.5 The Fiscal Theory of Monetary Policy

We linearize the model with an interest rate target, to

$$i_t = r + E_t \pi_{t+1}$$

$$\Delta E_{t+1}\pi_{t+1} = -\Delta E_{t+1} \sum_{j=0}^{\infty} \beta^j \tilde{s}_{t+1+j} \equiv -\varepsilon_{\Sigma s,t+1}.$$

This is the simplest example of a fiscal theory of monetary policy. The interest rate target sets expected inflation, and fiscal news sets unexpected inflation.

Figure 2.1 presents the response of this model to an interest rate shock with no fiscal change, and a fiscal shock with no interest rate change. The interest rate shock is Fisherian—inflation rises one period later—as it should be in this completely frictionless model.

By "fiscal theory of monetary policy," I mean models that incorporate fiscal theory, yet in their other ingredients incorporate standard DSGE (dynamic stochastic general equilibrium) models, including price stickiness or other non-neutralities of new-Keynesian models that are most commonly used to analyze monetary policy. In particular, a central bank follows an interest rate target, and we want to understand how movements of that target spread to the larger economy, or offset other shocks to the economy.

I start here with an interest rate target in the simple model we are studying so far, with one-period debt and no monetary or pricing frictions. I do so in a conscious parallel to the similar and beautifully clarifying development of new-Keynesian models in Woodford (2003) Chapter 2. Later, I add long-term debt, pricing frictions, and the other elements of contemporary models. We obtain more realistic responses.

Here and later, I stay within a textbook new-Keynesian framework, with simple forward-looking IS and Phillips curves. Like everyone else, I recognize the limitations of those ingredients. But it's best to modify one ingredient at a time, to understand the effect of changing fiscal assumptions in well-known standard models, before innovating other ingredients. And in this case, there is not yet a better, simple, well-accepted alternative.

The connection to standard models is clearer by linearizing the equations of the last section, as standard models do. Monetary policy sets an interest rate target i_t, and expected inflation follows from

$$\frac{1}{1+i_t} = E_t\left(\frac{1}{R}\frac{P_t}{P_{t+1}}\right)$$
$$i_t \approx r + E_t\pi_{t+1}. \tag{2.16}$$

When we think of variables as deviations from steady state, we drop r. Fiscal policy determines unexpected inflation via (2.13). Linearizing, denoting the real value of nominal debt by

$$V_t \equiv B_t/P_t,$$

and denoting the surplus scaled by steady-state debt with a tilde,

$$\tilde{s}_t = s_t/V,$$

we can write (2.13) at time $t+1$ as

$$\Delta E_{t+1}\pi_{t+1} = -\Delta E_{t+1}\sum_{j=0}^{\infty}\beta^j\tilde{s}_{t+1+j} \equiv -\varepsilon_{\Sigma s,t+1} \tag{2.17}$$

The final equality of equation (2.17) defines the notation $\varepsilon_{\Sigma s,t+1}$ for the shock to the present value of surpluses, scaled by the value of debt. I add the Σ to distinguish this shock from the shock to the period $t+1$ surplus itself, $\varepsilon_{s,t+1} = \Delta E_{t+1}\tilde{s}_{t+1}$.

Debt B_t now follows from the interest rate target and other variables. We can recover the quantity of debt from the expected valuation equation, (2.15),

$$\frac{B_t}{P_t}\frac{1}{1+i_t} = \frac{B_t}{P_t}\beta E_t\left(\frac{P_t}{P_{t+1}}\right) = E_t\sum_{j=1}^{\infty}\beta^j s_{t+j}. \tag{2.18}$$

It has no further implications for inflation or anything else. I linearize this equation later. The value of debt will be useful as it directly measures the present value of surpluses. We also typically express models in VAR(1) form, and the value of debt is an important state variable. Including debt is useful to solve the model numerically. But for solving the model analytically, we can pretend we see the surplus shock $\varepsilon_{\Sigma s,t+1}$ and ignore the value of debt.

The combination (2.16) and (2.17),

$$i_t = E_t\pi_{t+1}$$
$$\Delta E_{t+1}\pi_{t+1} = -\varepsilon_{\Sigma s,t+1}$$

now form the simplest example of a fiscal theory of monetary policy. Here I drop r

and interpret variables as deviations from steady state.

Using

$$\pi_{t+1} = E_t \pi_{t+1} + \Delta E_{t+1} \pi_{t+1},$$

then, the full solution of the model—the path of inflation as a function of monetary and fiscal shocks—is

$$\pi_{t+1} = i_t - \varepsilon_{\Sigma s,t+1}. \tag{2.19}$$

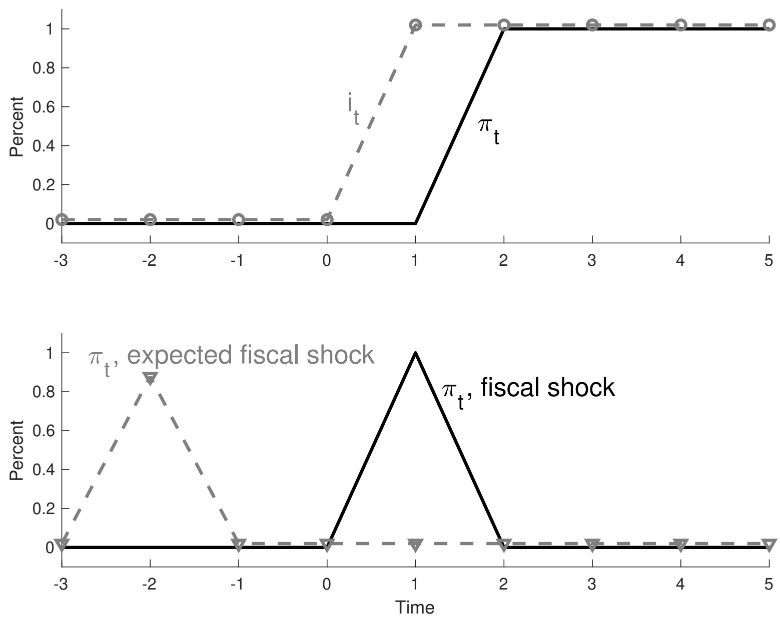

Figure 2.1: Impulse Response Functions, Simple Model. Top: Responses to a permanent interest rate shock, with no fiscal response, both expected and unexpected. Bottom: Responses to a fiscal shock, with no interest rate response. The "expected" fiscal shock is a decline in surpluses that starts at time 1, but is announced at time -2. The model solutions are (2.19), $\pi_{t+1} = i_t - \varepsilon_{\Sigma s,t+1}$.

Using (2.19), Figure 2.1 plots the response of this model to a permanent interest rate shock at time 1 with no fiscal shock $\varepsilon_{\Sigma s,1} = 0$, and the response to a fiscal shock $\varepsilon_{\Sigma s,1} = -1$ at time 1 with no interest rate movement.

In response to the interest rate shock, inflation moves up one period later. The Fisher relation says $i_t = E_t \pi_{t+1}$ and there is no unexpected time-t inflation without a fiscal shock.

The response is the same if the interest rate movement is announced ahead of time, so I don't draw a second line for that case. If $E_{t-k} i_t$ rises, then $E_{t-k} \pi_{t+1}$ rises. Many models offer different predictions for expected versus unexpected policy, and in many models announcements of future policy can affect the economy on the date of the announcement. Not in this case. An announcement only affects long-term nominal bond prices. Since so many interest rate changes are announced long ahead of time, we should spend more effort evaluating the response to expected policy changes.

In response to the negative fiscal shock $\varepsilon_{\Sigma s,1} = -1$ with no change in interest rates, there is a one-time price level jump, corresponding to a one-period inflation. The fiscal shock is a shock to the present value of surpluses, so when surpluses actually change does not matter. The "expected fiscal shock" line makes this point. This is the same change in surpluses, but the shock is the announcement, that occurs at time -2. Actual surpluses do not change until after time 1. Inflation happens when the shock is announced, not when surpluses appear.

These are unrealistic responses. They are, on reflection, exactly what one expects of a completely frictionless model. That's good news. A model with no pricing, monetary, or expectational frictions *should* be neutral. The model shows us that we *can* rather easily construct a fiscal theory of monetary policy, even in a completely frictionless model. It verifies that in a frictionless model, monetary policy is neutral, and makes specific just what "neutral" means. To get realistic and interesting dynamics, we should expect that we have to add monetary–fiscal interactions, sticky prices, long-term debt, cross-correlated and persistent policy responses, or dynamic economic mechanisms in preferences, production, and capital accumulation, or other ingredients.

In particular, these graphs give a perfectly "Fisherian" monetary policy response. An interest rate rise leads to *higher* inflation, one period later. Since in the long run higher nominal interest rates must come with higher inflation, an immediate jump to this long-run equilibrium is again natural behavior of a frictionless, neutral model.

These simple plots are best, then, for showing exactly how a neutral and frictionless fiscal theory of monetary policy model with one-period debt works. It's not realistic, but it's *possible*. It also shows us how simple and transparent the basic theory is, before we add elaborations. Yes, there is something as simple as money demand and supply, epitomized by $MV = Py$, and flexible prices, on which to build realistic dynamics.

You don't *have to* apply fiscal theory via a fiscal theory of monetary policy. In later chapters I step away from interest rate targets. We analyze quantitative easing, fiscal stimulus, and money supply rules. But you *can* apply fiscal theory by making technically small modifications to standard new-Keynesian models based on interest rate targets. And it is interesting to do so. Central banks set nominal interest rates and want to know what happens in response to changes in interest rate targets. We have a lot of investment in new-Keynesian DSGE interest rate models, and those models have accomplished a lot. It is useful, in exploring a new idea, initially to preserve as much of past progress as possible.

In this section I take another important fork in the road: I marry fiscal theory with a rational expectations model of the rest of the economy. That entire model is, here, represented by the Fisher equation $i_t = r + E_t\pi_{t+1}$ (and $r = 0$ taking deviations from a steady state). Later I fill in details to make that a complete model of the economy. Right there, however, you see rational expectations at work. Rational expectations turns out to have deep implications, even as we add many frictions to the model. Most of all, rational expectations with or without fiscal theory means that the economy is stable under an interest rate target, and that higher interest rates eventually *raise* inflation, as they do quickly in this simple model. We will spend a lot of time dealing with those implications. One can also marry fiscal theory to nonrational expectations models which do not have that stability property. I don't do so largely because they are less well developed and a lot more complicated. As with everything else, I try the simple model first, and add complications only when really forced to do so by evidence. I highlight the point here, however: Many of the properties that will consume us for many chapters come fundamentally from marrying fiscal theory with rational expectations models of the rest of the economy. If the total edifice proves wanting, fiscal theory itself may well survive, but married to different models of the rest of the economy.

2.5.1 Monetary–Fiscal Interactions

A fiscal policy rule that sets surpluses to attain a price level target produces a response to monetary policy in which higher interest rates lower inflation.

We can produce an inflation decline even in this frictionless model by combining the interest rate rise with an unexpected fiscal contraction. In that case, the joint monetary–fiscal shock produces one period of lower inflation $\Delta E_{t+1}\pi_{t+1} = -\varepsilon_{\Sigma s,t+1}$. Sticky prices will smear out this negative response, producing more realistic dynamics. But is such a pairing of monetary and fiscal shocks interesting, or realistic as a description of policy or events? Why might monetary and fiscal shocks come together?

The two-period model of Section 1.3 presents one such specification, which will reappear in several guises. The surplus at time 1 responds to nominal debt at time 0, whether issued by treasury or central bank, via $s_1 = B_0/P_1^*$. The equilibrium price level at time 1 is $P_1 = P_1^*$. We saw that with this fiscal policy specification, a rise in the interest rate target i_0 lowers the price level P_0, leaving P_1 alone, rather than raising P_1 leaving P_0 alone, as was the case with a fixed s_1.

The same idea works in our linearized intertemporal model,

$$i_t = E_t\pi_{t+1}$$
$$\Delta E_{t+1}\pi_{t+1} = -\varepsilon_{\Sigma s,t+1}.$$

Suppose that the fiscal authority again will raise or lower surpluses as necessary to attain price level targets $\{p_{t+1}^*,\ p_{t+2}^*, \ldots\}$. The central bank raises the interest rate i_t at time t. Then the price level at time t, p_t, must decline so that $i_t = E_t(p_{t+1}^* - p_t)$. A higher interest rate now immediately lowers inflation. The higher interest rate spurs greater bond sales. To defend the price level targets $\{p_{t+1}^*,\ p_{t+2}^*, \ldots\}$, the fiscal authority will be induced to raise future surpluses, producing the fiscal contraction that lowers inflation $\Delta E_t\pi_t = \Delta E_t p_t - p_{t-1} = -\varepsilon_{\Sigma s,t}$.

This dynamic extension emphasizes the perpetual need for fiscal–monetary coordination. If the fiscal authority is also committed at date t to do what it takes to set $p_t = p_t^*$, then we are at a loggerhead. To describe this regime in a symmetric way for all time periods, we need to specify that fiscal policy allows p_t^* to decline when monetary policy wishes it to do so, but not otherwise. I take up this fuller description below.

This is not a realistic example, just as fixed surpluses are not realistic. I present it to show how a different fiscal policy rule can result in dramatically different conclusions about the effects of monetary policy, and how fiscal–monetary interactions offer one route to understanding lower inflation with higher interest rates.

Other mechanisms can also provoke a fiscal contraction coincident with a monetary policy shock, without imagining that the central bank directly controls fiscal policy.

Higher interest rates that provoke higher long-term inflation can raise long-term surpluses through a variety of mechanisms, including imperfect indexing, sticky prices and wages for the things government buys, seigniorage revenue, imperfect tax indexation, and fiscal rules or habits by which fiscal authorities fight inflation with austerity. With any of these mechanisms, a higher nominal interest rate can produce a rise in the present value of surpluses, and thus lower inflation immediately.

A correlation between fiscal and monetary shocks may also describe historical episodes. Monetary and fiscal authorities respond to the same underlying shocks, so we see a decline in inflation coincident with an interest rate rise just because of that correlation of actions. Monetary stabilizations frequently involve coincident monetary tightening and fiscal reforms. VARs to measure the effects of monetary policy shocks do not (yet) try to find interest rate shocks uncorrelated with changes to the present value of fiscal surpluses. These thoughts offer a contrary warning that history may include correlated shocks that would not be present should the central bank use that historical evidence and move interest rates without the typical coincident fiscal shock.

The new-Keynesian approach to this simple economic model, as in Woodford (2003), produces a negative inflation response to an interest rate shock by creating a contemporaneous fiscal tightening. In that model, the central bank has an "equilibrium selection" policy on top of an interest rate policy. The bank threatens hyperinflation for any but one value of unexpected inflation. That threat gets the private sector to jump to the bank's desired value of unexpected inflation. Fiscal policy is "passive," setting $\varepsilon_{\Sigma s,t+1} = -\Delta E_{t+1}\pi_{t+1}$ in response to whatever inflation happens. This passive fiscal policy produces the necessary coincident fiscal shock. In Section 16.1, I judge this not to be a compelling story, but you can see it here as a possibility in which a joint monetary–fiscal regime produces a negative response of inflation to an interest rate shock.

2.6 Interest Rate Rules

I add a Taylor-type rule

$$i_t = \theta \pi_t + u_t$$
$$u_t = \eta u_{t-1} + \varepsilon_{i,t}$$

to find the equilibrium inflation process

$$\pi_{t+1} = \theta \pi_t + u_t - \varepsilon_{\Sigma s,t+1}.$$

Figure 2.2 plots responses to monetary and fiscal policy shocks in this model. The persistence of the monetary policy disturbance and the endogenous response of the interest rate rule introduce interesting dynamics, and show how monetary policy affects the dynamic response to the fiscal shock.

The standard analysis of monetary policy specifies a Taylor-type interest rate rule rather than directly specifying the equilibrium interest rate process, as I did in the last section. The model becomes

$$
\begin{aligned}
i_t &= E_t \pi_{t+1} && (2.20)\\
\Delta E_{t+1}\pi_{t+1} &= -\varepsilon_{\Sigma s,t+1} && (2.21)\\
i_t &= \theta \pi_t + u_t && (2.22)\\
u_t &= \eta u_{t-1} + \varepsilon_{i,t}. && (2.23)
\end{aligned}
$$

The variable u_t is a serially correlated monetary policy disturbance: If the Fed deviates from a rule this period, it is likely to continue deviating in the future as well. Rules are often written with a lagged interest rate,

$$
i_t = \eta_i i_{t-1} + \theta \pi_t + \varepsilon_{i,t},
$$

which has much the same effect. The variables are deviations from steady state, or $r = 0$ in (2.20).

Terminology: I use the word "disturbance" and the symbol u for deviations from structural equations. Disturbances may be serially correlated or predictable from other variables. I reserve the word "shock" and the letter ε for variables that only move unexpectedly, like $\varepsilon_{i,t+1}$ with $E_t \varepsilon_{i,t+1} = 0$. I use "shock" and "structural" somewhat loosely, to refer to forces external to the simplified model at hand. For example, the fiscal policy "shock" $\varepsilon_{\Sigma s,1}$ reflects news about future surpluses, which in turn has truly structural roots in productivity, tax law, politics, and so forth. A full general equilibrium model would reserve the "structural" word for the latter.

Eliminating the interest rate i_t, the equilibria of this model are now inflation paths that satisfy

$$
\begin{aligned}
E_t \pi_{t+1} &= \theta \pi_t + u_t && (2.24)\\
\Delta E_{t+1}\pi_{t+1} &= -\varepsilon_{\Sigma s,t+1}
\end{aligned}
$$

and thus

$$
\pi_{t+1} = \theta \pi_t + u_t - \varepsilon_{\Sigma s,t+1}. \tag{2.25}
$$

The top panel of Figure 2.2 plots the response of inflation and interest rates to a unit monetary policy shock $\varepsilon_{i,1}$ in this model, and the line labeled u_t plots the associated monetary policy disturbance in (2.22).

The combination of two AR(1)s – the shock persistence η and the interest rate rule θ – generates a pretty hump-shaped inflation response. Inflation still follows the interest rate with a one-period lag, following $i_t = E_t \pi_{t+1}$, and with no time 1 fiscal shock, π_1 cannot jump either way.

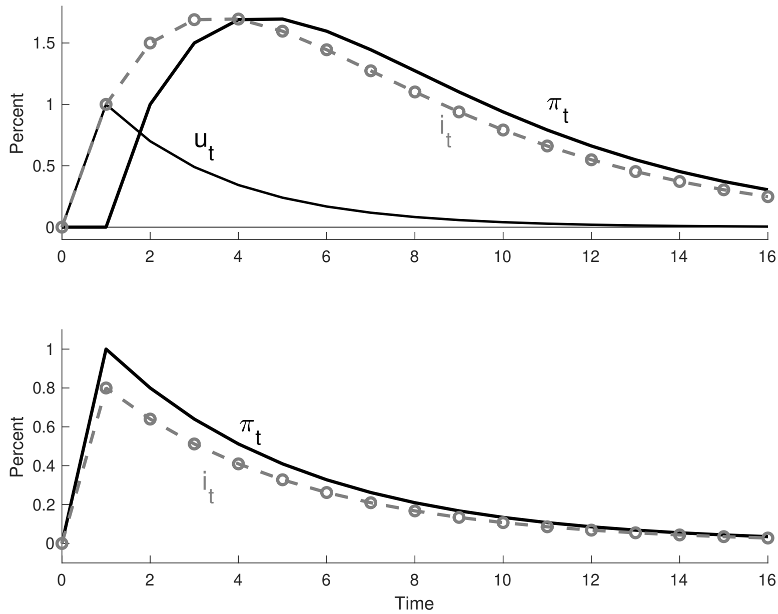

Figure 2.2: Responses to Monetary and Fiscal Shocks. The top panel graphs the response of inflation π_t and interest rate i_t to a unit monetary policy shock $\varepsilon_{i,1} = 1$. The monetary policy disturbance is u_t. The parameters are $\eta = 0.7$, $\theta = 0.8$. The bottom panel plots the response of inflation and interest rate to a unit fiscal shock $\varepsilon_{\Sigma s,1} = -1$.

Comparing the top panels of Figure 2.1 and Figure 2.2, you can see the same economic model at work. Since $i_t = E_t \pi_{t+1}$, if we had fed the equilibrium $\{i_t\}$ response of Figure 2.2 into the calculation (2.19) behind Figure 2.1, as if that path were an exogenous time-varying peg, we would have obtained the same result as in Figure 2.2. The monetary policy rule is a mechanism to endogenously produce an interest rate path with interesting dynamics, and for us to ask questions of the economy in which we envision monetary policy reacting systematically to inflation. But inflation follows the interest rate in the same way, whether we model the interest rate as following a rule or whether we specify the resulting equilibrium interest rate directly.

The lower panel of Figure 2.2 plots the response to a unit fiscal shock $\varepsilon_{\Sigma s,1} = -1$. By definition, this disturbance is not persistent. The fiscal loosening produces an instant inflation; that is, a price level jump, just as in Figure 2.1. The endogenous $i_t = \theta \pi_t$ monetary policy response now produces more interesting dynamics.

As (2.21) reminds us, fiscal policy alone sets the initial unexpected inflation of this response function, $\Delta E_1 \pi_1$. But what happens after that, $\Delta E_1 \pi_2$ and beyond, is a change in expected inflation that depends on monetary policy, via either the interest rate rule $\theta \pi_t$ or a persistent disturbance u_t. Monetary policy could return the price level to its previous value. Monetary policy could turn the event into a one-time price level shock, with no further inflation. Or monetary policy could let the inflation continue for a while, as it does here with $\theta > 0$. When we add long-term debt and sticky prices, these future responses will have additional effects on the instantaneous inflation response $\Delta E_1 \pi_1$. Monetary policy matters a lot in this fiscal model, to the dynamic path of expected inflation after the shock.

These responses are still not realistic. The important lesson here is that we *can* use fiscal and monetary policy rules that react to endogenous variables, and we can produce impulse-response functions including policy rules, just as we do with standard models of interest rate targets.

We also learn that monetary policy rules are an important source of dynamics. Impulse-response functions do not just measure the economy's response to shocks. Policy rules are particularly useful for defining interesting conceptual experiments: What if there is a fiscal shock, and the Fed responds by raising interest rates in response to any subsequent inflation? A drawn-out inflation results.

2.7 Fiscal Policy and Debt

> A rise in debt that is accompanied by larger future surpluses raises revenue, that can fund a deficit or lower inflation. Normal fiscal policy consists of deficits, funded by increased debt, that corresponds to higher subsequent surpluses. The value of debt measures how much expected surpluses have risen.

Monetary policy as I have defined it here consists of changing debt B_t, without changing surpluses. Fiscal policy may change debt B_t while also changing surpluses.

To gain a picture of fiscal policy operations in this intertemporal context, write the debt valuation equation (2.15)

$$\frac{B_{t-1}}{P_t} = s_t + \frac{1}{1+i_t}\frac{B_t}{P_t} = s_t + E_t \sum_{j=1}^{\infty} \beta^j s_{t+j}. \tag{2.26}$$

and take innovations,

$$\frac{B_{t-1}}{P_{t-1}} \Delta E_t \left(\frac{P_{t-1}}{P_t}\right) = \Delta E_t \left(s_t + \frac{1}{1+i_t}\frac{B_t}{P_t}\right) = \Delta E_t \left(s_t + E_t \sum_{j=1}^{\infty} \beta^j s_{t+j}\right). \tag{2.27}$$

Suppose that the government raises debt B_t and raises expected subsequent surpluses. The real value of debt $1/(1+i_t) B_t/P_t = E_t \sum_{j=1}^{\infty} \beta^j s_{t+j}$ rises. The bond sale soaks up extra money. This extra money can finance a deficit, a lower s_t, with no unexpected inflation.

As in the two-period model, this "fiscal policy" increase in debt B_t *with* higher expected subsequent surpluses is like an equity issue, as contrasted with the "monetary policy" increase in debt without higher expected surpluses, which acts like a share split. In the intertemporal context, the analogy to stock pricing is clearer, and we see that the corresponding surpluses can be long delayed.

This bond sale can generate a disinflation, $\Delta E_t (P_{t-1}/P_t) > 0$ rather than fund a deficit. Inflations are often successfully fought by getting the fiscal house in order. But it does not matter whether the government produces a *current* surplus s_t, or even an immediate future surplus s_{t+1}. What matters is generating a long-lasting credible stream of surpluses $\{s_{t+j}\}$. Doing so often requires an institutional reform; solving the underlying structural problem causing deficits, rather than just acts of today's politicians. Such a credible fiscal reform can coexist with ongoing or even larger short-term deficits, yet produce a disinflation.

The case that future surpluses just balance the current deficit, so there is no unexpected inflation, $\Delta E_t \left(P_{t-1}/P_t\right) = 0$, is particularly important. To generate this case, the change in future surpluses balances the near-term deficits, so there is no innovation to the present value $\Delta E_t \sum_{j=0}^{\infty} \beta^j s_{t+j} = 0$. To generate such a pattern, the surplus process must have an s-shaped moving average; that is, one which changes sign.

If we think of responsible governments in "normal times" adapting to fiscal needs without state-contingent devaluations via inflation, and instead maintaining a steady price level, this is "normal" fiscal policy.

- *Normal fiscal policy consists of debt sales that finance current deficits. Such sales promise higher future surpluses, and do not change interest rates or the price level.*

In our intertemporal context, the higher surpluses may be delayed, and may last decades, rather than showing up immediately in s_1.

Equation (2.27) offers a breakdown of how a deficit $\Delta E_t s_t < 0$ may be financed in this intertemporal context. The government may borrow, promising future surpluses, with no inflation. The government may inflate, with no change in future surpluses, in which case the value of debt does not rise despite the deficit. And if the surplus follows an AR(1) $s_t = \eta s_{t-1} + \varepsilon_{s,t}$ or similar process, in which the deficit is followed by additional deficits, then unexpected inflation is larger than the deficit shock, $\Delta E_t \left(P_{t-1}/P_t\right) = \Delta E_t \sum_{j=0}^{\infty} \beta^j s_{t+j} = 1/(1-\beta\eta)\varepsilon_{s,t}$. The AR(1) generates a value of debt $\Delta E_t \sum_{j=1}^{\infty} \beta^j s_{t+j} = \beta\eta/(1-\beta\eta)\varepsilon_{s,t}$ that goes *down* when there is a deficit $\varepsilon_{s,t} < 0$.

In the postwar data for the United States and other advanced countries, the value of debt increases with deficits and falls with surpluses, debt sales raise revenue, and surprise inflation is small relative to surplus and deficit shocks. The borrowing mechanism predominates, and the AR(1) is a particularly bad model.

The second terms of (2.26) and (2.27), $1/\left(1+i_t\right) B_t/P_t = E_t \sum_{j=1}^{\infty} \beta^j s_{t+j}$ make an important point: The revenue raised from bond sales is a direct measure of how much bond investors believe future surpluses will rise to pay off the debt. Survey expectations, CBO projections, regression forecasts, and economists' intuitions about future surpluses, including my own, may doubt that surpluses are coming. The investors who are buying bonds have more faith, and measurably believe that each increase in debt corresponds to an increase in eventual surpluses.

The fact that we can *observe* market expectations of future surpluses by observing changes in the value of debt is often overlooked. In the discussion of fiscal stimulus, one faces the baseline prediction of Ricardian equivalence: When the government runs deficits, people anticipate future taxes to pay back the debt, and so fiscal stimulus has no effect (Barro (1974)). The counterargument is that people ignore future taxes. The value of debt allows us to *measure* changes in expectations of future surpluses and resolve this controversy. If a deficit raises the value of debt, then at least the people who hold the debt expect higher taxes or lower spending. If the value of debt does not rise, the deficit stimulates. With no price stickiness, here, the deficit results instantly in inflation. In fact, deficits clearly raise the value of debt, and surpluses lower the value of debt in U.S. time series. Now, the Ricardian case is not completely closed: One could argue that households ignore the surpluses that bond markets foresee. And discount rates may also change, affecting the value of debt. But measurement of the amount by which the value of debt rises as a result of a deficit is a powerful tool for addressing the Ricardian debate.

2.8 The Central Bank and the Treasury

> The institutional division that the treasury conducts fiscal policy and the central bank conducts monetary policy works like the institutional division between share splits and secondary offerings. Treasury issues come with promises of subsequent surpluses. Central bank open market operations do not.
>
> To create a fiscal inflation, the treasury must persuade people that increased debt will not be paid back by higher future surpluses. That has proved difficult to accomplish. It is more difficult still to accomplish while preserving a reputation that allows later borrowing.

The "monetary policy" debt sale and the "fiscal policy" debt sale of the last section look disturbingly similar. The visible government action in each case is identical: the government as a whole sells more debt. One debt sale engenders expectations that future surpluses will not change. That sale changes interest rates and expected inflation, and raises no revenue. The other debt sale engenders expectations that future surpluses will rise to pay off the larger debt. That sale raises revenue with no change in interest rates or prices. How does the government achieve these miracles of expectations management?

Answering this question is important to solidify our understanding of the simple frictionless model as a sensible abstraction of current institutions. It also stresses the importance of monetary *institutions*. A government, like any asset issuer, must form people's expectations about how it will behave in distant, state-contingent, and infrequently or even never-observed circumstances. Monetary and fiscal institutions serve the role of communicating plans, and committing government to those plans.

Stock splits and equity issues also look disturbingly similar. The visible corporate action in each case is identical: More shares are outstanding. A split engenders expectations that overall dividends will not change, so a 2:1 split cuts the stock price per share in half. A share issue engenders expectations that total dividends will rise, so the price per share is unaffected and the company gets new funds for investment. (Yes, a long literature in finance studies small price effects of splits and offerings, as these corporate actions may reveal information about the company. Such wrinkles operate on top of the clear first-order effect.)

Companies achieve this miracle of expectations management by issuing shares in carefully differentiated institutional settings, along with specific announcements, disclosures, and legal environments that commit them to different paths. Companies do not just increase shares and let investors puzzle out their own expectations.

This parallel helps us to understand the institutional separation between central banks and treasuries. The Treasury conducts "fiscal policy" debt sales. Before the 1940s, many U.S. federal debt issues were passed by Congress for specific and transitory purposes, and backed by specific tax streams (see Hall and Sargent (2018)). That legal structure is an obvious aid to assuring repayment; that is, to promising higher future surpluses. Many state and municipal bonds continue these practices: They issue bonds to finance a toll bridge, say, and promise that the tolls will repay the bonds. The gold standard also gave a promise to repay rather than inflate. That commitment was not ironclad as governments could and did suspend convertibility or devalue, but it was helpful. U.S. federal debt now has no explicit promises, but the Treasury and Congress have earned a reputation for largely paying back debts incurred by Treasury issues, going back to Alexander Hamilton's famous assumption of Revolutionary War debt, and lasting at least through the surpluses of the late 1990s. Large debts, produced by borrowing, produce political pressure to raise taxes or cut spending to pay off the debts, part of Hamilton's point, rather than default explicitly or routinely devalue via inflation. The implicit promise to repay debt has not always been ironclad, and one can read it to include escape clauses, state-contingent defaults in certain emergencies. But it has helped.

Hall and Sargent (2014) note a less celebrated fact: Following Hamilton's plan, the U.S. government did not repay colonial *currency*, which largely inflated away. The experience emphasizes different promises implicit in currency versus debt, which we may trace to central banks versus treasuries today. Devaluation of paper currency by inflation did not have the same reputational cost as default on the debt would have had. The U.S. government did default on a large part of the Revolutionary War debt, via inflation, but still acquired a reputation that allowed it to borrow when it later needed to do so.

The idea that Treasury debt sales raise revenue rather than just raise nominal interest rates and expected inflation is now so ingrained, that the possibility of a share-split-like outcome may seem weird. Outside of a currency reform, who even imagines an increase in Treasury debt that does not raise revenue, and instead just pushes up nominal interest rates? The requirement that the debt sale engender expectations of higher subsequent surpluses is less well recognized, but the outcome requires that expectation (absent concurrent changes in real interest rates). Other governments are not so lucky, and have lost investors' confidence and a reputation for repayment. Their debt issues fail or just push up interest rates. You can only signal so much, and reputations are finite.

"Monetary policy" is conducted by a different *institution*. Central banks are, to a first approximation, legally *forbidden* from fiscal policies. They cannot alter tax rates or expenditures directly. At most, central bankers can give speeches advocating fiscal stimulus or fiscal responsibility, though even these are often seen as exceeding their mandates. Though central banks are mandated to control inflation, central banks are legally forbidden from "helicopter drops," perhaps the most effective means of inflating. Central banks cannot send cash or write checks to people or businesses. They must always buy something in return for issuing cash or reserves; or lend, counting the promise to repay as an asset. Central banks doubly cannot conduct a helicopter vacuuming, confiscating money from people and businesses without issuing a corresponding asset, though that would surely be an effective way to stop inflation! Only the Treasury may write checks to voters or confiscate their money, and for many good political as well as economic reasons. Independence, in a democracy, must come with limited authority. Central banks are limited in the securities they may buy, typically government securities, high-quality fixed income securities, or securities with government guarantees, to avoid central banks holding risk that eventually floats back to the Treasury. Federal Reserve asset purchases and lending in the financial and COVID-19 crises were largely conducted by lending to special purpose vehicles, in which the Treasury took an equity and risk-absorbing share.

The separation between fiscal and monetary policy is not perfect. In the presence of non-interest-paying currency, inflation produces seigniorage revenue, which the central bank remits to the Treasury. We will model this interaction. Liquidity spreads on government debt offer similar opportunities. Some central bank profits from crisis lending likewise flowed to the Treasury, as the losses would have done had asset prices not recovered.

Central bank actions have many *indirect* fiscal implications, which will be a central modeling concern. Inflation raises surpluses through an imperfectly indexed tax code. Monetary policy affects output and employment, with large budgetary consequences. With sticky prices and short-term debt, interest rate rises also raise the Treasury's real interest expense. Many central banks are charged to keep government interest expense low, as was the U.S. Fed through WWII and into the 1950s. With debt-to-GDP ratios now over 100%, interest expense will certainly weigh on the Fed should it need to raise rates in the future. We can and will model many of these indirect fiscal effects.

Still, a central bank open-market operation is a clearly distinct action from a Treasury issue, though in both cases the government as a whole exchanges money (reserves) for government debt. Treasury issues typically fund deficits, raise revenue, and are therefore expected to be repaid from subsequent surpluses. Open-market operations do not fund deficits or raise revenue. The restriction against central bank fiscal policy is closer to holding than not.

Our legal and institutional structures have many additional provisions against inflationary finance, adding to the separation between treasuries and central banks, and helping to guide expectations. The Treasury cannot sell bonds directly to the Fed. The Fed must buy any Treasury bonds on the open market, ensuring some price transparency and reducing the temptation to inflationary finance. The legal separation and tradition of central bank independence adds precommitments against inflationary finance. These limitations make sense if people regard central bank debt sales as inflationary.

In sum, the separation between Treasury and central bank is useful. One institution sells debt that raises revenue, implicitly promising future surpluses, and does not affect interest rates and inflation. A distinct institution sells debt without raising revenue, without changing expected surpluses, and in order to affect interest rates and inflation. This separation mirrors the different structures for equity offerings versus share splits.

There are many additional reasons for the institutional separation of the Treasury and Congress from a central bank, and strong limitations on central bank actions, including a force against loose monetary policy around every election, and a force to limit central banks from subsidizing credit or directing bank credit to businesses or constituencies that central bankers or politicians may favor.

However, these observations should not stop us from institutional innovation. A government under fiscal theory that wishes to stabilize the price level faces a central problem: If we just think of surpluses as an exogenous stochastic process, as we often model corporate dividends, then the price level as present value of those surpluses is likely to be quite volatile, like that of stocks. The government would like to offer some commitments: It would like to commit that the present value of surpluses will not change much, that deficits will be repaid by surpluses rather than cause inflation, and that surpluses will just pay down debt rather than cause deflation. The separation between treasury and central bank helps to make and communicate such a commitment. But the current structure evolved by trial and error, and it certainly was not designed with this understanding in mind.

To stabilize the price level, how can the government minimize variation in the present value of surpluses, and commit to those surpluses? When the government wishes to inflate or to stop deflation, how can it better commit *not* to repay debts, but in a defined amount, and preserving its reputation for future borrowing?

Our institutions evolved in response to centuries of experience with the need to fight inflation, to commit *to* back debt issues with surpluses. Fighting deflation or persistent below-target inflation became a central policy concern in the 2010s. Fighting deflation, modifying institutions to commit *not to* back some debt issues, but in a limited and defined amount, is new territory.

Our institutional structures also did not evolve to mitigate a potential sovereign debt crisis, which large short-maturity debts and unfunded promises leave as an enduring possibility. The Euro debt crisis could be the first example of others to come.

Can we construct something better than implicit, reputation-based Treasury commitments, along with implicit state-contingent defaults, devaluation via inflation? Can we construct something better than nominal interest rate targets following something like a Taylor rule? We'll come back to think about these issues. For now, the point is merely to make my parable about debts with and without future surplus expectations come alive.

2.9 The Flat Supply Curve

> In our simple model, the government fixes interest rates and offers nominal debt in a flat supply curve. In reality, the Treasury auctions a fixed quantity of debt, which seems to contradict this assumption. But the Treasury sets the quantity of debt *after* seeing the interest rate, raising the quantity of debt if the bond price is lower. The Treasury and central bank acting together generate a flat supply curve.

The above description of monetary policy, in which a government sets interest rates by offering any amount of debt B_t at a fixed interest rate i_t, while holding surpluses constant, seems unrealistic. The U.S. Treasury and most other treasuries auction a fixed quantity of debt. However, on closer look, the horizontal supply mechanism can be read as a model of our central banks and treasuries operating together, taken to the frictionless limit.

The Fed currently sets the short-term rate by setting the interest rate it pays to banks on reserves. It also sets the discount rate at which banks may borrow reserves, and the rates it offers on repo and reverse repo transactions for non-bank financial institutions. Reserves are just overnight, floating-rate government debt. Central banks allow free conversion of cash to interest-paying reserves. Thus, paying interest on reserves and allowing free conversion to cash really is already a fixed interest rate and a horizontal supply of overnight debt. In reality, people still also hold cash overnight, but that makes little difference to the model, as we will shortly see by adding such cash.

Historically, the Federal Reserve controlled interest rates by open-market operations rather than by paying interest on reserves. The Fed rationed non-interest bearing reserves, affecting i via M in $MV(i) = Py$. But the Fed reset the quantity limit daily, forecasting daily demand for reserves that would result in the interest rate hitting the target. So on a horizon longer than a day, reserve supply was flat at the interest rate target.

One could stop here, and declare that Treasury auctions involve longer maturity debt, which we have not yet included. But there is another answer, which remains valid with longer maturities: If the central bank sets the interest rate, and the Treasury then auctions a fixed quantity of debt, the central bank and Treasury together produce a flat supply curve for that debt.

The central bank sets the interest rate, by setting interest on reserves. The Treasury decides how many bonds B_t to sell *after* it observes the interest rate and bond price. Given the bond price Q_t, the flow condition (2.15),

$$\frac{B_{t-1}}{P_t} = s_t + Q_t \frac{B_t}{P_t}, \tag{2.28}$$

then describes how much nominal debt B_t the Treasury must sell to roll over debt and to finance the surplus or deficit s_t. It describes the process that the Treasury accountants go through to figure out how much face value of debt B_t to auction. If the central bank raises interest rates one percentage point, the Treasury sees 1% lower bond prices. The Treasury then raises the face value of debt it sells by 1% to obtain the same revenue. In this two-step process, the central bank plus Treasury thus really do sell any quantity of debt at the fixed interest rate, though neither Treasury nor central bank may be aware of that fact.

Treasury auctions do change interest rates by a few basis points, because the Treasury auctions longer-term bonds and there are small financial frictions separating reserves from Treasury bonds. But if the resulting bond price is unexpectedly low, and revenue unexpectedly low, the Treasury must still fund the deficit s_t. The Treasury goes back to the market and sells some more debt. In the end only the small spread between short-term Treasury and bank rates can change as the result of Treasury auctions, and that spread disappears in our model with no financial frictions.

It is a bit of a puzzle that the central bank can set market interest rates by setting interest on reserves, while also limiting the supply of reserves. It can. If the central bank offers more interest on reserves, competitive banks will offer more interest on deposits to try to attract depositors from each other, and they will require higher interest rates on loans to try to divert investments to reserves. That they cannot do so in aggregate does not mean that they do not try individually, so the higher interest on reserves leaks out to market prices. Banks aren't that competitive, so in reality the process may be slow. Offering an unlimited supply curve, open to nonbank financial institutions, may be more effective. But the theoretical possibility is valid. Cochrane (2014b) offers a more extended analysis on these points.

When the Fed wishes to lower interest rates, it may have to spruce up the discount window, to allow freer borrowing at a lower rate than the market wishes. Just why the Fed keeps a relatively large band between its borrowing and lending rates, discourages borrowing, and pays different rates to different borrowers—a lower rate on repos from money market funds than it pays to banks on reserves—is all a bit of a puzzle from a monetary policy point of view.

Just *how* the central bank sets the short-term interest rate is important, and usually swept under the rug. Most papers do not mention the question. Woodford (2003) invokes a cashless limit: The Fed manipulates a vanishingly small quantity of money, which via $MV(i) = Py$ sets the nominal interest rate. This proposal undercuts the idea that the interest rate target alone is a full theory of the price level, though Woodford is not as concerned with that purity as I am here.

Woodford wrote before 2008, when the U.S. Fed began paying interest on reserves. The New York Fed did actually each morning try to guess the quantity of reserves for that day that would lead to an equilibrium federal funds rate equal to the Fed's target. Reserves were small, on the order of \$10 billion. (Hamilton (1996) is an excellent description of the procedure and its flaws.) The financial and banking system did plausibly approximate Woodford's cashless limit. However, most other countries had already moved to a corridor system, lending freely at the interest rate target plus a small spread, and borrowing freely at that target minus a small spread. After 2008, the United States moved to immense reserves, in the trillions of dollars, which are only adjusted slowly, and pay essentially the same interest rates as those on other short-term government debt. So the standard new-Keynesian tradition is missing a story roughly conformable to current institutions on just how the central bank sets the nominal interest rate. The analysis of this section might be adapted to new-Keynesian models, if anybody cares to do so.

2.10 Fiscal Stimulus

A deliberate fiscal loosening creates inflation in the fiscal theory. However, to create inflation one must convince people that future surpluses will be lower. Current deficits per se matter little. The U.S. and Japanese fiscal stimulus programs contained if anything the opposite promises, and did not overcome their long traditions of debt repayment.

In the great recession following 2008, many countries turned to fiscal stimulus, in part as a deliberate attempt to create inflation. Japan tried these policies earlier. This simple fiscal theory can offer perspectives on this attempt.

There are two ways to think of fiscal inflation, or "unbacked fiscal expansion," in our framework. First, equation (2.13),

$$\frac{B_{t-1}}{P_{t-1}}\Delta E_t\left(\frac{P_{t-1}}{P_t}\right) = \Delta E_t \sum_{j=0}^{\infty}\beta^j s_{t+j},$$

describes how lower surpluses can create immediate unexpected inflation. Second, we might think of fiscal stimulus as an increase in nominal debt B_t that does not correspond to future surpluses, designed to raise nominal interest rates and, in equilibrium, to raise expected future inflation,

$$\beta\frac{B_t}{P_t}E_t\left(\frac{P_t}{P_{t+1}}\right) = \frac{1}{1+i_t}\frac{B_t}{P_t} = E_t\sum_{j=1}^{\infty}\beta^j s_{t+j}.$$

Now, the point of stimulus is to raise output. To see that we need a model in which inflation raises output. Such models distinguish expected and unexpected inflation, and the time path of expected inflation. For now let's just ask how the government might create inflation, either expected or unexpected.

Both equations point to the vital importance of *future* deficits in creating inflation via fiscal stimulus. Current debt and deficits matched by future surpluses won't create any inflation.

The U.S. fiscal stimulus of the 2008 recession and following years of deficits, and the long-standing Japanese fiscal stimulus programs that added more than 100% of GDP to its debt, failed at the goal of increasing inflation. This observation suggests an explanation. The U.S. administration promised debt reduction to follow once the recession was over; that is, that the new debt would be paid back. That is what a Treasury does that wants to finance current expenditure *without* creating current or expected future inflation. To create inflation, the key is to promise that future surpluses will *not* follow current debts. Even in a traditional Keynesian multiplier framework, which is how the U.S. administration analyzed its stimulus, one wishes people to ignore future surpluses in order to break Ricardian equivalence. One wishes that people do not see future taxes to pay off the debt and do not save more to pay those taxes. Calling attention to the future surpluses is counterproductive. Japan was similarly criticized for never being clear that it would not repay debt, instead raising taxes on several occasions to signal the opposite.

The debt issues of fiscal stimulus did not raise interest rates, did raise revenue, and did raise the total market value of debt. These facts speak directly to investors' expectations that subsequent surpluses would rise, or at least that real discount rates fell. From the perspective of this simple model, such conventional fiscal stimulus –borrow money, don't drive up interest rates, spend the money, repay the debt—has no effect at all on current, unexpected, or expected future inflation.

It is simply a rearrangement of the path of surpluses: less now and more later.

Even if the U.S. administration had tried to say that the debt would not be paid back, reputations and institutional constraints on inflationary finance are often hard to break. Once people are accustomed to the reputation that Treasury issues, used to finance current deficits, will be paid back in the future by higher surpluses, and the idea that the central bank is fully in charge of inflation, it is hard to break that expectation. Institutions, especially regarding debt repayment, long outlast politicians and their promises. That is the point of institutions.

The expectations involved in a small inflation are harder yet to create. A government might be able to persuade bondholders that a fiscal collapse is on its way, debt will not be repaid, and create a hyperinflation. But how do you persuade bondholders that the government will devalue debt by 2%, and only by 2%? If you can do that, how do you later convince them that new debts, when the government wishes to raise revenue, will be fully repaid? A partial and temporary unbacked fiscal expansion is tricky to communicate on the fly. It needs institutional commitment, not ephemeral promises by the political leaders of a moment. We will see some ideas later.

The $5 trillion fiscal expansion in 2020-2021 did result in substantial inflation. Chapter 21 covers this episode and explores how it is different from the 2008-2020 efforts at fiscal stimulus.

Part II

Assets, Rules, and Institutions

Chapter Eight

Assets and Choices

Societies can choose a wide range of assets and institutions with which to run their fiscal and monetary affairs. In this chapter, I examine some possibilities, how the fiscal theory generalizes to include these possibilities, and some thoughts on which choices might be better than others in different circumstances.

Fiscal and monetary policy face many trade-offs. A government facing a fiscal shock may choose inflation, explicit partial default, partial defaults on different classes of debt held by different investors, distorting taxes, capital levies, spending cuts, or other measures. Each of these options has welfare and political costs. Each decision is also dynamic, as actions taken this time influence expectations of what will happen next time, and constraints on later actions. Expectations of rarely observed, or "off-equilibrium" behavior matter. Precommitment, time-consistency, reputation, moral hazard, and asymmetric information are central considerations in a monetary and fiscal regime. For this reason, fiscal and monetary policy is deeply mediated by laws, constraints, rules, norms, customs, and institutions.

A theme recurs throughout this part: How can the government commit to surpluses that underlie a stable price level, and communicate that commitment? The expectation on the right-hand side of the valuation equation is otherwise nebulous and potentially volatile. Most governments would like to precommit and communicate that they will manage surpluses to defend a stable price level or inflation rate—no more, and no less. That inflation is much less volatile than stock prices suggests that our governments have been able to make such commitments, at least partially and implicitly. Examining and improving the institutions that allow such commitment is an important task.

The government might like a more sophisticated commitment: that it will manage surpluses to defend stable inflation, but with escape clauses in war, deep recession, and so forth when it might like to implement a state-contingent default, or redistribution from savers to borrowers, via inflation. In the 2010s, our governments struggled to deliberately create modest inflation. Institutions designed to contain inflation struggled to make the necessary commitment to a limited reduction in backing.

These chapters pull together ideas from monetary theory, corporate finance, dynamic public finance, and sovereign debt and default, in a fiscal theory context.

8.1 Indexed Debt, Foreign Debt

I extend fiscal theory to include real debt — indexed debt, or debt issued in foreign currency. Such debt acts as *debt*, where nominal debt acts as *equity*. If the government is to avoid explicit default, it must raise surpluses sufficient to pay off real debt. With only real debt outstanding and surpluses independent of the price level, the price level is not determined by the valuation equation.

Governments often issue indexed debt, debt issued in another country's currency or, historically, debt redeemable in gold. Such debt acts like corporate or individual debt. It must be repaid or default. Government-issued nominal debt functions like corporate equity. Its price—the price level — can adjust in response to lower surpluses, just as corporate equity prices can adjust in response to lower dividends. As a corporation does not have to adjust its dividends upward to match an increase in its stock price, neither does a government that has issued nominal debt have to adjust surpluses to follow changes in the price level.

Indexed debt pays $\$P_t$ rather than \$1 when it comes due at time t. If the price level rises from 100 to 110, an indexed bond pays \$110. Denote the quantity of one-period indexed debt issued at time $t-1$ and coming due at time t by b_{t-1}. Suppose the government finances itself entirely with indexed debt. The government must then pay $b_{t-1}P_t$ dollars at time t. It collects $P_t s_t$ dollars from surpluses. Likewise, each bond sold at the end of t promises P_{t+1} dollars at time $t+1$. With a constant real rate, risk-neutral pricing, and discount factor β, the flow condition becomes

$$\begin{aligned} b_{t-1}P_t &= P_t s_t + E_t\left[\beta \frac{P_t}{P_{t+1}} \times P_{t+1}\right] b_t \\ b_{t-1} &= s_t + \beta b_t. \end{aligned}$$

The term in square brackets in the first equation is the nominal price of indexed bonds. Iterating forward, we obtain

$$b_{t-1} = \sum_{j=0}^{\infty} \beta^j s_{t+j}. \tag{8.1}$$

If real surpluses s_t are independent of the price level, the price level disappears from the valuation equation. The fiscal theory is not an always and everywhere theory. For the fiscal theory to determine a price level, we need an equation with something nominal and something real in it. However, surpluses that respond to the price level $s(P)$ can lead to a determinate price level even with real debt.

(The timing of this equation is a bit unusual, to preserve the analogy with the B_{t-1} notation for nominal debt. The quantity b_{t-1} is the real value of debt at the beginning of time t. It is known at time $t-1$, and hence the subscript is not inaccurate. Its real value at time $t-1$ is βb_{t-1}.)

If the government is to avoid default, equation (8.1) now describes a restriction on surpluses, essentially that surpluses must rise to fully pay off past deficits with interest; $a(\rho) = 0$ in our earlier moving average notation or $a_s(\rho) - a_r(\rho) = 0$ with varying real interest rates. Though equation (8.1) is often called "the government's intertemporal budget constraint," it is in reality a no-default condition, at least in models as well as reality in which default can happen.

Cash still exists in this indexed debt story, and indexed debt is settled with cash. Write the nominal equilibrium flow condition

$$P_t b_{t-1} = P_t s_t + P_t \beta b_t.$$

The government prints up cash to pay $\$P_t$ to each maturing indexed bond. It soaks up those dollars with primary surpluses, and by selling indexed debt. If surpluses obey (8.1), then this flow condition holds for any price level. If not, then we see instant hyperinflation or hyperdeflation.

Which kind of debt *comes due* is the key question. The amount raised by the debt sale equals the present value of subsequent surpluses, whether the government sells real or nominal bonds. Thus, if real debt is outstanding, but the government issues nominal debt, the price level is still undetermined. If nominal debt is outstanding and the government issues real debt,

$$\frac{B_{t-1}}{P_t} = s_t + \beta b_t = s_t + \beta \sum_{j=0}^{\infty} \beta^j s_{t+1+j}$$

then the price level is determined, at least for this one time period.

Foreign currency debt works in a similar way. Suppose the government dollarizes, or proclaims a permanent foreign exchange peg. This case can be handled with the usual valuation equation, denominating everything in foreign currency:

$$\frac{B_{t-1}}{P_t^*} = E_t \sum_{j=0}^{\infty} \beta^j s_{t+j}. \tag{8.2}$$

Now, P_t^* represents the price of goods in terms of the foreign currency, and s_t is the surplus measured in the same units. Equation (8.2) is again a constraint on surpluses that the government must run in order to avoid default.

The same logic applies to a country in an idealized currency union. Greece uses euros, and agrees to pay its debts in euros. Therefore, (8.2) requires that Greece either run surpluses to pay its debts, or default. The European price level need not adjust in response to Greece's debts.

The situation is the same as the debt of a company, household, or state and local government denominated in dollars. These borrowers must repay or default. If the dollar price level falls, they must raise additional real resources to avoid default.

Many real-world arrangements occupy a muddy middle. Bailouts to people, banks, companies, EU member states, U.S. state and local governments link public and private debts. For now the point is just to accommodate idealized indexed and foreign debt into fiscal theory.

8.2 Currency Pegs and Gold Standard

> Exchange rate pegs and the gold standard are really fiscal commitments. Foreign currency or gold reserves don't matter to first order, as governments do not have enough reserves to back all of their nominal debt. If people demand foreign currency or gold, the government must eventually raise taxes, cut spending, or promise future taxes to obtain or borrow reserves. The peg says "We promise to manage surpluses to pay off the debt at this price level, no more and also no less." The peg makes nominal debt (equity) act like real debt (debt). Unlike full dollarization, a peg gives the country the right to devalue without the costs of explicit default. But the country pays the price for that lesser precommitment. Likewise, a gold standard offers the option of temporary suspension of convertibility and permanent devaluation or revaluation. Both gold and foreign exchange rate pegs suffer that the relative price of goods and gold, or foreign currency, may vary.

In an exchange rate peg or under the gold standard, the country issues its own currency, and borrows in its own currency. But the government promises to freely exchange its currency for foreign currency or for gold, at a set value.

These arrangements suggest that money gains its value from the conversion promise. But exchange rate pegs and the gold standard are in fact *fiscal* commitments. The value of the currency comes ultimately from that fiscal commitment. They are instances of, not alternatives to, fiscal theory. To peg to gold, the government must have or be able to get the gold. To peg to foreign currency, the government must have or be able to get the foreign currency.

Analysis of the gold standard and exchange rate pegs often focuses on whether the government has enough reserves to stand behind its conversion promise. Enough has not always been enough, though, and gold promises and foreign exchange rate pegs have seen "speculative attacks" and devaluations. (Switzerland in 2015 experienced the rare opposite possibility, a speculative attack leading to an undesired *rise* in currency value, and challenging the country's ability to run fiscal *deficits*.) A currency board takes the reserves logic to its limit: It insists that all domestic currency must be backed 100% by foreign currency assets. One hundred percent gold reserves against currency issue are a similar and common idea.

But reserves are, to first order, irrelevant. It is the ability to *get* reserves when needed that counts. Countries, even those on currency boards, do not back all of their *debts* with foreign assets or gold. If a country could do so, it wouldn't have needed to borrow in the first place. When those debts come due, if the government cannot raise surpluses to pay them off or roll them over, the government must print unbacked money or default. When the government runs into fiscal trouble, abandoning the gold standard or currency board and seizing its reserves will always be tempting. Argentina's currency board fell apart this way in a time of fiscal stress. (Edwards (2002) includes a good history.) Moreover, if people see that grab coming, they run immediately, leading to inflation and devaluation.

Conversely, if the government has ample ability to tax or borrow reserves as needed, credibly promising future taxes or spending cuts, then it can maintain convertibility with few reserves. Sims (1999) cites a nice example:

> From 1890 to 1894 in the U.S., gold reserves shrank rapidly. U.S. paper currency supposedly backed by gold was being presented at the Treasury and gold was being requested in return. Grover Cleveland, then the president, repeatedly issued bonds for the purpose of buying gold to replenish reserves. This strategy eventually succeeded.

Cleveland persuaded bond buyers that the United States would run larger future fiscal surpluses. The United States' final abandonment of gold promises in 1971 followed a similar outflow of gold to foreign central banks, presenting dollars for gold. The Nixon administration was unable or unwilling to take the fiscal steps necessary to buy or borrow gold.

Reserves may matter to second order, if financial frictions or other constraints make it difficult for the government to tax or borrow needed gold or foreign exchange quickly. But they only matter for that short window. Likewise, solvent banks do not need lots of reserves because they can always borrow reserves or issue equity if needed. Insolvent banks quickly run out of even ample reserves.

With reserves, we can write the government debt valuation equation as

$$\frac{B_{t-1}}{P_t} = G_t + E_t \sum_{j=0}^{\infty} \beta^j s_{t+j}.$$

Here, P_t is the price of goods in terms of gold or foreign currency, and G_t is the quantity of gold or foreign currency reserves. Reserves per se are irrelevant. They are one fiscal resource to back the issue of currency and nominal debt, but they enter in parallel with the usually larger present value of surpluses.

Foreign exchange pegs or the gold standard, when successful, are thus primarily *fiscal commitments* and *communication devices*. If P_t is going to be constant, then the government must adjust surpluses s_t on the right-hand side as needed, not too little (inflation) but not too much (deflation) either. The peg says, "We will manage our taxes and spending so that we can always pay back our debts in foreign currency or gold at this fixed exchange rate, no more and no less." When that promise is credible, it removes the uncertainty of a present value of surpluses and stabilizes the price level. In the language of Section 4.2, they are a precommitment to an active surplus process with $a(\rho) = 0$. In the language of Section 5.4 they are a v versus v^* fiscal policy that precommits to repay debts but not to respond to unanticipated inflation or deflation of the currency relative to gold. Free conversion helps to enforce and make visible this commitment.

This sort of fiscal commitment and communication is valuable. If the government left the price level to the vagaries of investor's expectations about long-run surpluses, inflation could be as volatile as stock prices. But if governments offer and communicate a commitment that surpluses will be adjusted to defend a given price level, and debt will be paid off at that price level, no more as well as no less, the price level is stabilized. A successful currency peg or gold standard produces what looks like a passive fiscal policy, but is in fact an active fiscal policy arranged to determine a steady price level.

Conversely, abandoning the gold standard or revaluing an exchange rate peg can create a defined amount of inflation or deflation, by defining the change in surpluses. If the government says, rather than $20 per ounce, the dollar will now be pegged at $32 per ounce, that means that the government will only run enough surpluses to pay off existing debt at $32 per ounce, not $20. But people may lend new money to the government with confidence that the dollar will not quickly fall to $50 an ounce. A devaluation is a way of announcing a partial default via inflation, and its exact amount. Like all capital levies, of course, the trick is to convince people that sinning once does not portend a dissolute life; that this is a once-and-never-again devaluation or at best a rare state-contingent default, not the beginning of a bad habit.

The gold standard or exchange rate pegs thus offer a fiscal commitment with an escape clause. The government can enjoy in normal times the advantages of a fiscal precommitment, giving a steady price level and anchored long-term expectations, while leaving open the option of state contingent default achieved through devaluation in emergencies. The government also pays the price of an interest rate premium when people think it likely to use the escape clause too lightly.

A large disadvantage of the gold standard and exchange rate peg is that the relative price of goods and gold varies, and the relative price of domestic and foreign goods, the real exchange rate, varies. Pegging the currency in terms of gold, there was still unpleasant inflation and deflation in the price of goods and services. Exchange rate pegs turn real exchange rate variation into inflation or deflation.

This variation has fiscal consequences. Define the price level in terms of a price index for all goods and services, as we normally do. Specifically, define the price level in the formulas as the relative price of currency versus goods and services, not the relative price of currency versus gold. That's the price level we care about. If the price of gold and currency together relative to goods and services rises, if there is a deflation under the gold standard, the government must raise the present value of surpluses in terms of goods and services to accommodate that deflation. If the relative price of domestic relative to foreign goods declines—if demand for a country's commodity exports declines, for example—a government on an exchange rate peg must pay off debt with surpluses that are more valuable in terms of domestic goods and services.

A gold standard is an *active* fiscal policy with respect to deviations of the value of currency from gold, but a *passive* fiscal policy with respect to deviations of the price level from the joint value of currency and gold. A foreign-currency peg is active with respect to deviation of the value of domestic from foreign currency, but passive with respect to deviations of the price level from that joint value.

That is pretty much what happened to the gold standard in the 1930s. The price level fell, the value of gold rose, and the value of the currency relative to goods and services rose with it. If the government was going to maintain the gold standard, it would have to cut spending or raise taxes to pay a real windfall to bondholders.

Countries either devalued or abandoned the gold standard. The result, and to us the key mechanism, is that they thereby abandoned a fiscal commitment to repay nominal debt at the now more valuable gold price. This step occasioned lawsuits in the United States, which went to the Supreme Court. The court said, in essence: Yes, the United States is defaulting on gold clauses; yes, this means the government does not have to raise taxes to pay bondholders in gold; and yes, the U.S. government has the constitutional right to default (Kroszner (2003), Edwards (2018)). The government also abrogated gold clauses in private contracts, to avoid a transfer from borrowers to lenders, which the court also affirmed as constitutional. Jacobson, Leeper, and Preston (2019) describe the 1933 revaluation in this way, as a device to allow a defined fiscal devaluation when the gold standard demanded fiscal austerity.

The gold standard is well designed to prevent long-term fiscal inflation. It is much less well designed to prevent deflation.

This episode is also important for forming the expectations underlying today's formally unbacked regime. If a 1933 deflation were to have broken out in 2008, standard passive-fiscal analysis, explicit in new-Keynesian models and IS-LM stories, states the government would dutifully raise taxes and cut spending to pay an unexpected real windfall to bondholders, just as it would have had to do under the gold standard. Obviously, expectations were strong that the government would respond instead exactly as it did: Ignore the "temporary" price level drop, and run a large fiscal expansion under the guise of stimulus until the emergency ended. The memory of 1933 certainly did not hurt in forming that expectation. Consequently, the deflation of 1933 did not repeat.

The shackles of the gold standard can be useful when loosened. When a country devalues, it makes clear the *fiscal* loosening will happen, and its amount. Attempts at unbacked fiscal expansion during the recent zero bound era were not able to communicate that debts would not be repaid. Tying yourself to a mast has the advantage that it is clear when you tie yourself to a shorter mast, and just how much shorter the mast is.

This analysis is simplistic, emphasizing the fiscal points. Thorough analysis of the gold standard takes into account its many frictions: the costs of gold shipment, the way coins often traded above their metallic content (Sargent and Velde (2003)), limits on convertibility, trade frictions, financial frictions, multiple goods, price stickiness, the fee to turn gold into coins, and so forth. Gold standard governments also ran interest rate policies, and raised interest rates to attract gold flows. That combination is initially puzzling. Doesn't the promise to convert gold to money describe monetary policy completely? It merits analysis in the same way we added interest rate targets to the fiscal theory.

A foreign exchange peg begs the question, what determines the value of the foreign currency? Not everyone can peg. The obvious answer is, fiscal theory in the primary country, and we have to investigate fiscal commitments in the primary country or the institutions of the currency union.

The parallel question arises regarding gold: What determines the value of gold in the first place? We often tell a story that the value of gold is determined by industrial uses or jewelry independent of monetary policy. But this story is clearly false. Almost all gold was used for money and is now stored underground, in vaults rather than mines. Based only on industrial use, its value would be much lower.

The gold standard was built on economies that used gold coins. Gold coins are best analyzed, in my view, as a case of $MV = Py$, rather than a case in which money has value because it carries its own backing as an independently valuable commodity. Gold is in sharply limited supply, with few substitutes, especially for large denomination coins. A transactions and precautionary demand for gold, in a world in which gold coins were widely traded, gave gold its value. A gold standard piggybacks on *that* value to generate a value of currency. Think of gold-standard currency then as inside money.

The gold standard has many faults. I do not advocate its return, despite its enduring popularity as a way to run a transparent rules-oriented monetary policy that forswears fiscal inflation, at least inflation of the currency relative to gold.

Most of all, a gold or commodity standard requires an economic force that brings the price level we *do* want to control into line with the commodity that can be pegged. In the gold standard era, gold and gold coins continued to circulate. If the price of gold and currency relative to other goods rose, if there was deflation, then people had more money than they needed. In their effort to spend it on a wide variety of assets, goods, and services, the price level would return. The $MV = Py$ of gold coins made gold a complement to all goods and services. But if the price of gold relative to other goods rises now, this mechanism to bring the relative price of gold to goods back in line is absent. Gold is just one tiny commodity. Tying down its nominal price will stabilize the overall price level about as well as if the New York Fed operated an ice-cream store on Maiden Lane and decreed that a scoop of chocolate fudge ice cream shall always cost a dollar. Well, yes, a network of general equilibrium relationships ties that price to the CPI. But not very tightly. One may predict that ice cream on Maiden Lane will be $1 but the overall CPI will wander around largely unaffected by the peg.

Conventional analysis predicts that if we moved back to a gold standard, the CPI would inherit the current volatility of gold prices. But if the Treasury returned to pegging the price of gold, it is instead possible that it, well, pegs the price of gold, but the CPI wanders around unaffected. The relative price of gold to CPI would lose its current high frequency volatility, but the CPI would wander off.

Foreign exchange rate pegs suffer some of the same disadvantage. The economic force that pulls real exchange rates back, purchasing power parity, is weak. At a minimum, that's why countries peg to their trading partners, and pegs are more attractive for small open economies.

There is evidence that as I hypothesized for gold, the real relative price of foreign and domestic goods depends on the regime. Mussa (1986) pointed out a fact that's pretty clear just looking out the window: Real exchange rates are much more stable at high frequency under a nominal exchange rate peg than under floating rates. The real relative price of a loaf of bread in Windsor, Ontario versus Detroit is more volatile when the U.S. and Canadian dollars float than when they are pegged. This stabilization of real exchange rates is an important argument in favor of exchange rate pegs and common currencies. But it undermines the argument for an exchange rate peg for an individual country's price level control when countries are not well integrated. Countries face a tradeoff between inflation volatility and real exchange rate volatility.

8.3 The Corporate Finance of Government Debt

I import concepts from corporate finance of equity versus debt to think about when governments should issue real (indexed or foreign currency) debt, when they should have their own currencies and nominal debt, and when they might choose structures in between, like an exchange rate peg or gold standard that can be revalued without formal default.

Governments issue more real debt when they cannot precommit by other means not to inflate or devalue, and when their institutions and government finances are more opaque. To issue nominal debt, governments must offer something like control rights of equity. In modern economies, many voters are mad about inflation, which helps to explain that stable democracies have the most successful currencies.

Should a government choose indexed or nominal debt? Or should it construct contracts and institutions that are somewhat in between, such as the gold standard, foreign exchange peg, or price level target, which are like debt with a less costly default option? Corporations also fund themselves with a combination of debt, equity, and intermediate securities, so we can import much of that analysis.

The government faces shocks to its finances and trade-offs between (at least) three ways of addressing those shocks: formal default (b, B), default via inflation (B/P), and raising taxes or cutting spending (s). Formal default is costly. Unexpected inflation and deflation is also destructive with sticky prices, nominal rigidities or unpleasant effects of surprise redistributions between lenders to borrowers. Distorting taxes are costly, and governments regard "austerity" spending cuts as costly too. Each option has dynamic and moral-hazard implications. Lucas and Stokey (1983) argue for state-contingent partial defaults to minimize tax distortions. Schmitt-Grohé and Uribe (2007) add price stickiness and argue for more tax variation and less inflation variation. But clearly the optimum is an interior combination depending on the costs.

Governments may issue a combination of real and nominal debt. With such a combination, the valuation equation becomes

$$b_{t-1} + \frac{B_{t-1}}{P_t} = E_t \sum_{j=0}^{\infty} \beta^j s_{t+j}. \tag{8.3}$$

The price level is determined by the ability to devalue the nominal debt only.

A corporation that finances itself by more debt and less equity increases the volatility of its stock returns, and is at greater risk of default. Likewise, a government that issues more real debt and less nominal debt, other things constant (they never are), increases the volatility of inflation and raises the chance of explicit default.

That consideration suggests that governments issue more nominal debt and less real debt. Even a country such as Norway that has a substantial sovereign wealth fund may wish to continue to issue extra nominal debt and buy additional real assets, as it has done.

On this basis, Sims (2001) argues against Mexico adopting the dollar or issuing dollar-denominated debt. Dollarization means that fiscal problems must be met with distorting taxes, spending cuts, or costly explicit default. A floating peso and peso-denominated debt allows for subtle devaluation via inflation. More peso debt allows Mexico to adapt to adverse fiscal shocks with less inflation, and lower-still costs of explicit default or devaluation, just like a corporation that finances itself with equity rather than debt.

The same argument lies behind a fiscal theoretic interpretation of the widespread view that countries like Greece should not be on the euro. Currency devaluation and inflation implements state contingent default, perhaps less painfully than explicit default or "austerity" policies to raise surpluses.

On the other hand, corporate finance also teaches us that debt helps to solve moral hazard, asymmetric information, and time consistency or precommitment problems. An entrepreneur may not put in the required effort, may be tempted to divert some of the cash flow due to equity investors, or may not be able to credibly report what the cash flow is. Debt leaves the risk and incentive in the entrepreneur's hands, helping to resolve the agency problems. So, despite the risk-sharing and default-cost reductions of equity financing, the theory of corporate finance predicts and recommends widespread use of debt. Equity only works when the issuers can certify performance, through accounting and other monitoring, and by offering shareholders control rights.

Real government debt is a precommitment device. The legal structure of real debt, and the actual and reputational cost of default, help the government to produce surpluses that repay debt, even if doing so involves unpleasant taxation or spending cuts.

Default also has costs. If it did not, real debt would not offer any precommitment. Those costs are regretted ex post. Greece is a good example: By joining the euro, so its bonds were supposed to default if Greece could not repay them, Greece precommitted against default. That precommitment allowed Greece to borrow a lot of euros at low interest rates, and to avoid the regular bouts of inflation and devaluation that it had suffered previously. Alas, when Greece finally did run into a rollover crisis, it discovered just how large those costs might be.

Sims's (2001) argument, like that for the drachma, does not consider the possibility of mismanagement, the difficulty of fiscal probity, and the need for fiscal precommitments, evident in decades of deficits, crises, devaluations, wasted spending, and inflation around the world. It neglects that surpluses are a choice, not an exogenous shock. The properties of the surplus process $\{s_t\}$ are not independent of real versus nominal financing. Evaluating Sims's advice for Mexico, one might consider the comparative fiscal and monetary history of Ecuador and Panama, fully dollarized; Argentina, with bouts of dollar pegs; and Venezuela, with its own currency (Buera and Nicolini (2021), Restuccia (2021)). I'm cherry-picking of course, and repeated crises of exchange rate pegs, and formal defaults on dollar debt are also painful. But a precommitment value of dollarization exists; a weaker but substantial precommitment value of a peg exists as well; and issuing and borrowing in a national currency, and then quickly inflating to solve every fiscal shock is not always optimal.

Nominal government debt, like corporate equity, works better when government accounts are more trustworthy and transparent. Nominal debt works better when the country has other means to commit to an s-shaped surplus process.

Corporate equity requires some mechanism to guarantee dividend payments in place of the explicit promises offered by debt, backed by law, collateral, or other penalties and punishments. For corporate equity, control rights are that mechanism. If the managers don't pay dividends or seem to be running the company badly, the shareholders can vote them out and get new management. What are the equivalent of control rights for nominal debt, which behaves like government equity? Most naturally in the modern world, *voters*. If nominal government debt gets inflated away, a whole class of voters is really mad. Inflation is even more powerful than explicit default in this way. If the government defaults, only bondholders lose, and a democracy with a universal franchise may not care. Or the bondholders may be foreigners. If the government *inflates*, every private contract is affected. The government's debt devaluation triggers a widespread private devaluation, and everyone on the losing end of that devaluation suffers. The chaos of inflation hurts everyone. Alexander Hamilton is justly famous for the insight that a democracy needs widespread ownership of government debt, by people with the political power to force repayment. Widespread pain of inflation is even more powerful. Why do we use government debt as our numeraire, thus exposing private contracts to the risks of government finances? Well, the fact that we do, and we vote, means that there is a large group of voters who don't like inflation.

The standard ideas of corporate finance thus suggest that countries with precommitment problems, poor fiscal institutions, and untrustworthy government accounts, who tend to issue and then default or inflate, should or have to issue real or foreign currency debt. To borrow at all they may even have to offer collateral or other terms that make explicit default additionally painful. Countries that have alternative precommitment mechanisms, strong institutions, and stable democracies with a widespread class of people who prefer less inflation, are able to issue government equity. Such countries have their own currencies and borrow in those currencies.

Confirming this view, dollarization, currency pegs, and indexed and foreign debt are common in the developing and undemocratic world. Nominal debt and local currencies here often come with stringent capital controls, financial repression, wage and price controls, and frequent inflation. Successful non-inflating currencies and large domestic currency debts seem to be the province of stable democracies.

Though we appear to determine the price level via (8.3) with an arbitrarily small amount of nominal debt, we should as always be cautious about such limits. Consider smaller and smaller amounts of nominal debt in (8.3), with more and more real debt, and coupled with a surplus process that steadily pays back more debt, approaching $a(\rho) = 0$, so that inflation remains the same as we make this change in debt. The price level is determinate all the way along at the limit, but not the limit point. This is a fiscal theory version of the cashless limit puzzle. Yes, when debt is down to the \$10 in pennies in your sock drawer plus \$20 trillion of indexed debt, and the expected surpluses decline by \$1, there should be a 10% inflation. But the economic force for that inflation is clearly weak. You might just leave the pennies in your sock drawer. The wealth effect of nominal government bonds is weaker as the size of nominal debt declines. If we wish to think about a backing theory of money for small amounts of nominal debt, backing that debt with a visible set of real assets and an explicit redemption promise offers a greater force to value adjustment. That may be a reason for gold standards in the previous era of small government debt. I take up below how one might construct such an institution today.

The sovereign debt literature studies the extent to which reputation and other punishments can induce repayment, since governments are difficult to sue and sovereign debt typically does not offer collateral. This theory is useful for us to import, thinking about inflation in place of default.

In the history of government finance, it took centuries for governments to somewhat credibly promise repayment, and thereby to borrow in large quantities at low rates. The parallel development of paper currencies that did not quickly inflate took hundreds of years as well. Government debt is full of institutions that help to precommit to repayment and limit ex post inflation and default. The Bank of England and parliamentary approval for borrowing, taxation, and expenditures were seventeenth-century institutions that limited the sovereign's authority to default. That limit allowed the U.K. government to borrow more ex ante. The French absolute monarch, being more powerful, could not precommit to repay, so he could not borrow as much. This deficiency has long been regarded an important factor in France losing the wars of the eighteenth century and eventually the French revolution itself (Sargent and Velde (1995)). Imperial Britain used force to get other sovereigns to repay. Today, sovereign debt includes many institutions beyond reputation to try to force repayment, including third-country adjudication, the right of creditors to seize international assets, and threats by international institutions to cut defaulters off. All have partial success, but also partial failures given the repeated foreign debt crises of the last several decades.

8.4 Maturity, Pegs, Promises, and Runs

> Long-term debt can offer a buffer against surplus shocks and real interest rate shocks. Long-term debt opens the door to policies that resemble quantitative easing. Long-term debt insulates the government, and inflation, from the run-like dynamics of short-term debt. However, all insurance invites moral hazard. Short-term debt and pegs may impose a commitment to fiscal discipline.

Should governments choose long-term or short-term financing? This choice has varied a great deal over time. The Victorian United Kingdom was largely financed by perpetuities, preceded by centuries in which perpetual debt was a common instrument. The current U.S. government has, as above, a quite short maturity structure, rolling over about half the debt every two to three years. Governments in fiscal trouble find themselves pushed to shorter and shorter maturities by higher and higher interest rates for longer-term debt. Markets think default or inflation more likely than the government wishes, and attempts to buy lots of insurance in the form of long-term debt just raise suspicions further.

Why governments chose long-term debt and why they have moved to short-term debt is an interesting open question. In part, the less developed financial and communication technology of the time may have played a part. Rolling over a large principal payment might have been difficult. In part, long-term price stability under the gold standard made long-term financing relatively cheaper. In part, we may simply have lost some wisdom of the of our ancestors.

Sections 3.5.2, 7.2.1, and 7.6 showed ways in which long-term debt can offer a buffer against fiscal shocks. The linearization (3.22) shows some mechanisms compactly,

$$\sum_{j=0}^{\infty}\omega^j \Delta E_{t+1}\pi_{t+1+j} = -\sum_{j=0}^{\infty}\rho^j \Delta E_{t+1}s_{t+1+j} + \sum_{j=1}^{\infty}(\rho^j - \omega^j)\Delta E_{t+1}r_{t+j}. \tag{8.4}$$

If $\omega = 0$, short-term debt, then the entire revision in the present value of surpluses must be met by immediate inflation $\Delta E_{t+1}\pi_{t+1}$. The longer the maturity of debt ω, the more the revision in present value of surpluses can be spread to future inflation, though at the cost of more total inflation. In many views of price stickiness, a protracted small inflation is better than a short large inflation. Long-term debt empowers monetary policy to reduce current inflation when it spreads inflation forward.

On a flow basis, long-term debt leaves the budget and hence the price level less exposed to real interest rate variability. If the government borrows short term, then a rise in the interest rate raises real interest costs in the budget and necessitates tax increases or spending decreases, or results in inflation. If the government borrows long term, then the increase in interest cost only affects the government slowly. The tradeoff is familiar to any homeowner choosing between a fixed and floating rate mortgage. If interest rates rise, the floating-rate borrower has to pay more immediately. The fixed rate borrower pays the same amount no matter what happens to interest rates.

We can see this effect in identity (8.4) as well. An increase in real interest rate is an increase in the expected real bond return on the right hand side. The larger ω, the smaller the weights $\rho^j - \omega^j$. In the limit $\omega = \rho$, a real-rate increase has no inflationary effect. The rate rise still makes unexpected future deficits more costly to finance, but it means the government can pay off current debt with the currently planned surpluses, ignoring interest costs.

In this case, the linearization is a bit misleading. It values discount rate effects at the average surplus, and surplus effects at the average discount rate. The obvious proposition, that the government is insulated from real rate shocks when the maturity of debt matches the maturity of surpluses, requires the interaction term. We can see the effect more clearly with the continuous time present value relation

$$\frac{\int_{\tau=0}^{\infty} Q_t^{(t+\tau)} B_t^{(t+\tau)} d\tau}{P_t} = E_t \int_{\tau=0}^{\infty} e^{-\int_{j=0}^{\tau} r_{t+j}dj} s_{t+\tau} d\tau.$$

Using the expectations hypothesis for bond prices,

$$Q_t^{(t+\tau)} = E_t\left(e^{-\int_{j=0}^{\tau} r_{t+j}dj}\frac{P_t}{P_{t+\tau}}\right),$$

we have

$$\int_{\tau=0}^{\infty} E_t e^{-\int_{j=0}^{\tau} r_{t+j}dj} \frac{B_t^{(t+\tau)}}{P_{t+\tau}} d\tau = E_t \int_{\tau=0}^{\infty} e^{-\int_{j=0}^{\tau} r_{t+j}dj} s_{t+\tau} d\tau.$$

Now, if today's debt maturity $B_t^{(t+\tau)} E_t \left(1/P_{t+\tau}\right)$ matches the path of expected real surpluses $s_{t+\tau}$ as a function of τ, then real interest rate changes r_{t+j} cancel from both sides. The stream of assets matches the stream of liabilities. Otherwise, the mismatch between the maturity of debt and the usually much longer maturity of the surplus determines how the price level reacts to real interest rates.

Section 7.2.2 emphasized how the intertemporal linkages of the present value relation come from rolling over short-term debt. Short-term investors hold government debt because they believe other short-term investors will buy their debt. A rollover crisis or run on nominal debt causes a sudden inflation or devaluation. Long-term debt cuts off this crisis or run-like mechanism entirely. Sovereign debt crises too are (almost) always and everywhere crises of short-term debt.

All of these considerations point to long-term debt for its buffering properties. But again they take the surplus process as given. Corporate finance also points us to *incentive* properties of short-term debt. Making things worse ex post gives an incentive for, and precommitment to, more careful behavior ex ante. Governments that issue short-term debt will, the theory goes, be more attentive to fiscal policies, to maintaining their ability to borrow, and will be forced to take painful fiscal adjustments sooner. In return, markets will offer better rates to governments who bind themselves via short-term debt in this way, unless the governments have other commitment devices. Diamond and Rajan (2012) argue that run-prone short-term debt disciplines bankers. Run-prone short-term debt might discipline governments as well.

Long-term debt offers insurance. All insurance risks moral hazard. The more long-term debt, the easier it is for the government to put off a fiscal reckoning, letting it fall on long-term bond prices rather than current budgets, refinancing, or interest costs. In turn, that expectation leads to higher interest rates for long-term debt, so that a sober government feels it pays too much. Greenwood et al. (2015), for example, advocate that the U.S. Treasury borrow short to save interest costs. Like not buying insurance, if the event does not happen the premium is a waste. If markets look at who is buying insurance and charge higher rates still, insurance is doubly expensive. And if the absence of insurance prods one to more careful behavior, insurance can be additionally expensive.

The conversion promise of a gold standard and foreign exchange peg, rather than the more elastic guidepost of a gold price or foreign exchange target, which can be missed temporarily, adds an additional invitation to run, and thereby another precommitment to sober fiscal policy. Offering that anyone can bring in a dollar and receive gold, or everyone can bring in a peso and get a dollar, immediately, invites an instantaneous run when, as always, governments do not back currency 100% with reserves, or when they have additional debt or a temptation to grab the reserves. In turn, a government that offers such a right ties itself even more strongly to the mast to always maintain plenty of fiscal space.

I offer benefits and costs on both sides, to frame the long versus short discussion, not to answer it. As I judge the maturity issue, a U.S. or global advanced-country sovereign-debt rollover crisis, though unlikely, would be an immense economic and financial catastrophe. The U.S. Congress, though not unable to reform, needs time to do it and not to bungle the process. A small insurance premium seems worth it. Long-term nominal interest rates of 1.5%, slightly negative in real terms as I write in late 2021, seem a low premium for the insurance they provide. True, if 0% short rates continue for 30 years, the interest costs of short-term debt will turn out to have been even lower. Also true, I offered the same advice 10 years ago, and short rates have been lower that whole time. I have also, by this ex post logic, wasted 35 years of fire insurance premiums on my house. It's a judgment, and the probability of the event and risk aversion must matter. I note however, that terrorist attacks, housing price collapses, and a global pandemic were all thought to have lower probability ex ante than they do now.

Whether markets would still offer 1.5% long rates should the U.S. federal government wish to buy a lot more of that insurance is also an interesting question. It would be enlightening to find out, as it would measure just how insatiable demand for U.S. debt is, and how ironclad markets' faith in U.S. finances really is.

Whether the additional precommitment of run-inviting short-term debt or pegs is useful is also debatable. I judge not, but that too is a judgment, awaiting more careful research. Just how strong is the fiscal precommitment value of government debt structure that is prone to inflation runs or rollover runs? Are there not other precommitment devices that are not so dangerous when they fail? Peltzman (1975) famously argues for spikes on the dashboard to encourage safer driving. But we chose seatbelts and other incentives instead.

In this context, the Diamond and Rajan (2012) analysis of bank capital structure, advocating run-prone debt for its incentive properties, is controversial. In fact, equity holders can and do monitor and punish bank managers as they do for all other corporations. In fact, most holders of short-term debt do no fundamental analysis of borrowers' cash flows. Short-term debt is an "information insensitive" security designed so that its holders *do not* do any monitoring, in the contrary Gorton and Metrick (2012) view of banking, until all of a sudden they wake up and run. To be fair, Diamond and Rajan emphasize short-term financing offered by big investment banks and broker-dealers, rather than bank deposits or the commercial paper market. Monitoring is more plausible in that case. But that story then applies even less to government debt. Short-term government debt is the paradigmatic "risk-free" security, held as cash, by investors doing no "fundamental" analysis of long-run government fiscal affairs. Until they wake up and run.

As equity-financed banking has a good point (Cochrane (2014c)), despite the need for equity rather than short-term debt holders to monitor management, so government finance based on long-term nominal debt and targets rather than pegs, monitored by grumpy voters, may have a point as well.

Greece not only signed up for euro-denominated debt, it rolled over short-term debt. It failed in a rollover crisis. Apparently, the discipline of run-prone debt was not large enough for Greece to mend its fiscal affairs.

The history of the gold standard and foreign exchange pegs is replete with crisis after crisis, as the history of banking funded by immediate service, run-prone deposits is one of crisis after crisis, in which the disciplinary forces failed.

The end of the Bretton Woods era in 1971 offers an example of a peg gone awry.[1] In the Bretton Woods era, foreign central banks could demand gold for dollars, though people and financial institutions could not do so. Exchange rates were fixed, and capital markets were not open as they are today. A persistent trade deficit could not easily result in devaluation, or be financed by a capital account surplus; by foreigners using dollars to buy U.S. stocks, bonds, or even government debt. Trade deficits had to be financed by paper dollars, and gold if foreign central banks did not want those dollars. Bretton Woods was simply not designed for a world with large persistent trade deficits and surpluses and capital flows. Instead, the persistent trade deficit, fueled by persistent fiscal deficits, resulted in foreign central banks accumulating dollars. The banks grew wary of dollars and started demanding gold. The resulting run on the dollar precipitated the U.S. abandoning Bretton Woods and the gold standard entirely, allowing the dollar to devalue, and inaugurating the inflation of the 1970s. It was a classic sovereign debt crisis. With the remaining tie of dollars to gold at a fixed price, Bretton Woods was fundamentally incompatible with steady, even small, inflation.

The fiscal deficits of the Vietnam War and Great Society and the era's trade deficits were large by the standards of the time. Both are smaller than we have grown accustomed to today. Why did those deficits cause a great crisis and inflation, while post-2000 immense trade and fiscal deficits resulted in nothing, at least until 2021? Well, the institutional framework matters. The combination of a gold promise to foreign central banks, fixed exchange rates, and largely closed capital markets shut off today's adjustment mechanisms.

In one sense our mechanisms are much better. Our government can now borrow immense amounts of money, and our economy can run immense trade deficits, financed in capital markets, not by gold flows. In another sense, our mechanisms expose us to a much bigger and more violent reckoning if and when the reckoning comes.

Just why did the Johnson and Nixon administrations not borrow, and buy gold, as Grover Cleveland did, to stem the gold flows? Sure, they were already borrowing a lot, but it's hard to argue that the United States was unable to borrow more, and pledge higher future surpluses in so doing. Or were the restrictions in international capital markets tight enough to turn off this saving mechanism?

The gold standard retains an allure. The government freely exchanges money for gold, thereby transparently and mechanically determining the value of money, without the need for central banker clairvoyance. As we have seen, it is at heart a fiscal commitment, which is both good and bad. It rules out the option to devalue via inflation, which helps the government to borrow ex ante and resist inflationary temptation ex post. But at times inflation is a better option than sharp tax increases, or spending cuts. It also signals that surpluses will *only* be

[1] Shlaes (2019) tells the history well, as do Bordo (2018) and Bordo and Levy (2020) with more economic analysis. Rueff (1972), writing at the time, emphasizes the link between the trade deficit and inflation in the Bretton Woods system:

> The trouble that I denounced in 1961 has brought about all the consequences I had foreseen: a perennial deficit in America's balance of payments, inflation in creditor countries, and in the end, disruption of the monetary system by requests for reimbursement of the dollar balances so imprudently accumulated. (p. 179.)

The dollar nonetheless remained the reserve currency despite the end of Bretton Woods.

large enough to pay off debt at the promised gold peg, thus precommitting against deflation of currency relative to gold.

But in its failures, more frequent than usually remembered, the gold standard leads to explicit default, chaotic devaluation, speculative attack when devaluation looms, or a suspension of convertibility with uncertain outcomes. A gold standard, as opposed to a gold price target, introduces run-like commitments. These further bind the government to fiscal probity to avoid runs, but make crises worse when runs do break out. The gold standard was not as mechanical as advocates remember, including in the United Kingdom an active central bank also setting interest rates. And, most of all, the gold standard allows inflation and deflation when the price of gold and currency rise or fall together relative to goods and services. The gold standard imposes a passive fiscal commitment to tighten fiscal policy in the event of such deflation, or to loosen fiscal policy to validate such inflation. Such volatility is more likely now that gold is disconnected from the financial system and hence other prices.

8.5 Default

> Fiscal theory can incorporate default. An unexpected partial default substitutes for inflation in adapting to a fiscal shock. A preannounced partial default creates a defined fiscal inflation. It is analogous to a gold parity devaluation, or devaluing a currency peg.

Fiscal theory can easily incorporate default. We do not need to assume that governments always print money to devalue debt via inflation rather than default.

Suppose that the government at date t writes down its debt: It says, for each dollar of promised debt, we pay only $D_t < 1$ dollars. Now, we have

$$\frac{B_{t-1}D_t}{P_t} = E_t \sum_{j=0}^{\infty} \beta^j s_{t+j}. \tag{8.5}$$

The price level is still determined. An unexpected partial default allows the government to adapt to a negative surplus shock with less or no inflation. Fiscal theory does *not* require that governments always inflate rather than default.

With short-term debt and no change in surpluses, a pure *expected* partial default has no effect on the current price level, but it can influence future inflation. It works analogously to bond issues. With the possibility of future partial default, and simplifying with no current default $D_t = 0$, the flow condition remains

$$\frac{B_{t-1}}{P_t} = s_t + \frac{Q_t B_t}{P_t} = s_t + E_t \sum_{j=1}^{\infty} \beta^j s_{t+j}. \tag{8.6}$$

The bond price becomes

$$Q_t = \frac{1}{1+i_t} = E_t\left(\beta \frac{P_t}{P_{t+1}} D_{t+1}\right). \tag{8.7}$$

If at time t people expect a partial default $D_{t+1} < 1$, with no change in surpluses, this change has no effect on the current price level P_t, by (8.6). An expected partial default just lowers bond prices, and thus lowers the revenue the government raises from a given amount of nominal bonds. With the same surpluses, the government must sell more nominal bonds to generate the same real revenue.

The effect of an expected partial default on the future price level P_{t+1} and expected inflation $E_t(P_t/P_{t+1})$ depends on monetary policy— how much nominal debt B_t the government sells, or how it sets the interest rate target i_t. If the government allows the interest rate to rise, fully reflecting the higher default-risk probability, then neither P_t nor P_{t+1} is affected by the announced partial default. The government just sells more nominal debt B_t. Selling two bonds when people expect a 50% haircut is exactly the same as selling one bond when people expect no haircut, except nominal bond prices fall by half. It generates the same revenue, and results in the same future issue of $1 to pay off the debt. However, if the government sticks to the nominal interest rate target, requiring that i_t and Q_t are unchanged, then the expected future price level P_{t+1} declines. It's equivalent to selling less nominal debt or lowering the interest rate target.

But an announced partial default with no surplus news is a strange and unrealistic intervention. When we think of a default, we think that the government is *not* going to raise surpluses to repay some debt. Thus, a more realistic story pairs an expected future default with bad news about future surpluses.

So, suppose at time t, the government announces a 10% haircut for $t+1$, $D_{t+1} = 0.90$. People infer that surpluses from $t+1$ onwards $E_t \sum_{j=1}^{\infty} \beta^j s_{t+j}$ will be 10% lower; not, as in our last example, that surpluses are unchanged. This expected future default then raises the price level today by 10%.

- *Expected future default can trigger current inflation in the fiscal theory.*

Monetary policy still determines the expected future price level. If the government allows the interest rate to rise, to follow the increased default premium, then by (8.7), the expected price level at $t+1$ is also 10% higher.

Really the *point* of this announced default is a commitment and communication device that the government really will lower future surpluses, and will not, as customary, repay debts without causing inflation. A preannounced partial default, along with monetary policy that allows nominal interest rates to rise, is analogous to a 10% devaluation under a gold standard or foreign exchange peg. Those are likewise good devices to communicate a fiscal commitment and produce 10% cumulative inflation, and only 10% cumulative inflation.

Many governments at the zero bound and with inflation stubbornly below their central banks' 2% inflation targets have *wanted* to inflate, but to inflate only a little and in a controlled fashion. They turned to fiscal stimulus with little effect. Evidently, bond markets did not lightly abandon government's hard-won reputations for repaying debt. "Unbacked fiscal expansion" is easy to say, but hard to do, and hard to do in a limited way. A preannounced partial default could raise inflation by reducing fiscal backing, and by a clear and calibrated amount. Like all devaluations, just how to communicate that this is a one-time policy and not the beginning of a regular pattern is the hard part.

8.6 Central Bank Independence

> An independent central bank is a fiscal commitment. Independent central banks make it difficult to finance deficits with non-interest-bearing money, thereby forcing the fiscal authorities to raise current or future surpluses, or to default.

So far, I have integrated central bank and treasury. In the end, central bank and treasury are part of the same government. Their finances are united, as are the finances of members of a family. If the treasury issues debt and the central bank buys that debt and issues reserves or cash, it is the same as the treasury issuing reserves or cash directly. In this section, I consider some ways in which the separation between central bank and treasury matters.

Central bank independence is usually thought of in terms of monetary control, with the price level set by $MV = Py$ or by interest rate policy. Giving an independent central bank control of M or interest rates helps the government to precommit against goosing the economy or lowering interest rates for short-term political reasons; for example, ahead of elections.

An independent central bank can act as an important *fiscal* commitment, in a fiscal theory of the price level. It is part of a larger institutional structure that tries to precommit the government against inflationary finance, and more generally commit to debt repayment and s-shaped surpluses so the government can borrow in the first place.

Most of all, since the central bank alone controls the issuance of currency and reserves, the treasury may not print currency or issue reserves to repay debt or to finance deficits. To do that, the treasury must issue debt, and the central bank must buy that debt. When a debt limit default loomed in 2009, it surprised many commentators that the U.S. Fed cannot monetize deficits to continue spending, even to avoid default, as much as the mantra is repeated that a country that defines its own currency need never default.

The Federal Reserve may not buy Treasury securities directly from the Treasury. Treasury securities promise reserves, not more Treasury securities. The Treasury may not legally grab assets from the Fed's balance sheet. Additionally, central bankers have a certain amount of political independence, both in law and custom.

We should not assume that central bankers always dislike inflation. Many, viewing themselves as macroeconomic planners, would choose more inflation in order to achieve other goals. Many would like to use the central bank's great financial and regulatory power to pursue policy objectives unrelated to inflation. Removing the inflation punch bowl just as the party gets going, as William McChesney Martin put it, doesn't get anyone cheers at Davos or more glamorous political positions. Part of the institutional structure therefore also precludes the *central bank* from fiscal policy and fiscal inflation: Central bank helicopter drops are illegal; central banks must always lend or buy assets. Central banks cannot give out money, and they are often limited in the range of assets they can buy; including, traditionally, not providing subsidized credit to favored businesses and industries. Central banks are given limited mandates. Mandates are as important for what they omit and forbid as for what they include and command. Mandate limitations and political autonomy give central bankers the authority to resist political pressure for inflation or to finance deficits. They also limit central bankers who might prefer inflation, or to use the central bank's tools for their own political and policy priorities, and might misuse independence to do so.

Independence and mandates are limited in their anti-inflationary effect. The Federal Reserve's official mandate also includes "maximum employment," and some sort of "financial stability" mandate is widely agreed on. No agency can or should be totally independent in a democracy. Federal Reserve officials are appointed by the President and confirmed by Congress. Central bankers who wish to be reappointed tend to bend to the desires of the administration and Senate. Mandate limitations only matter if Congress and the administration object when the Federal Reserve goes boldly where it has not gone before.

The point, for us: An independent central bank, with monopoly on the issuance of reserves and currency, that cares about inflation either from its own preferences or by paying attention to a mandate, that can refuse to purchase more than a certain amount of government debt, can force a recalcitrant treasury and government to pay back its debt with surpluses, to credibly promise future surpluses to roll over the debt, or to default, rather than inflate. Even in my simple frictionless models, inflation in the end comes from too much cash. The "government" prints money to redeem debt and soaks money up with surpluses and more debt. If the central bank controls the printing press, the government can be precluded from the first step, and thereby from fiscal inflation.

Independent central banks and rules against deficit finance have been important parts of inflation control, and more generally for ensuring that governments repay debt, for centuries. The Bank of England was founded in 1694, as a private company with a monopoly on the issuance of bank notes, precisely to buy government debt and ensure its repayment. Restoring central bank independence has been an important element of fiscal–monetary stabilizations from the Fed–Treasury accord of 1951, to inflation-targeting episodes, to the end of inflations and hyperinflations (Section 14.2), and to stabilizations under inflation-targeting regimes (Section 9.1).

Central bank independence is important for many additional reasons unrelated to the price level and fiscal issues of this book. Central banks have enormous regulatory power to tell banks who to lend to and who not to lend to, how banks operate, and how much to pay depositors. Central banks can directly lend or buy assets at inflated prices. Precommiting not to use central banks' tools for such inherently political acts, and keeping independent central bankers from following their own policy preferences with such acts, have historically been important for sound economic performance where successful, and damaging when unsuccessful.

I do not emphasize a separate balance sheet, independence, and other aspects of central bank versus treasury tensions in this book. There is a huge literature on the subject. Most of the analysis applies quickly to fiscal theory, and I don't have much novel to say about it. Once central bank and treasury play whatever the game between them is, we can draw again a circle around the government, and model the government's decisions and its unified balance sheet. It's really a part of the larger political economy question *how* the government makes decisions about surpluses and debt or interest rates, and thus a theory of what sorts of decisions it is likely to make. Like other theories of preference formation, we don't *have* to look under the hood in order to analyze how the effects of government policy—treasury and central bank together—affect the economy.

Chapter Nine

Better Rules

Leaving surpluses to expectations or implicit commitments is clearly not the best institutional structure for setting a monetary standard. If only the government could commit and communicate that the present value of surpluses shall be this much, neither more nor less, then it could produce a more stable price level. It could also quickly produce inflation or deflation when it wishes to do so. Historically, committing and communicating against inflationary finance was the main problem. In the Great Depression and the 2010s, committing and communicating that surpluses will not rise, to combat deflation, became important as well.

The gold standard offers this kind of commitment: The present value of future surpluses shall be just enough to pay back the current debt at the gold peg, neither more nor less. Alas, the gold standard suffers the above list of problems that make it unsuitable for the modern world. I examine here alternative institutions that may analogously communicate and commit the government to a present value of surpluses.

The fiscal theory of monetary policy combines fiscal policy that determines surpluses and monetary policy that implicitly sets the path of nominal debt. We have grown accustomed to monetary policy that consists of nominal interest rate targets, set by hopefully wise central bankers. I investigate alternative monetary rules, including a target for the spread between nominal and indexed debt.

9.1 Inflation Targets

> Inflation targets have been remarkably successful. I interpret the inflation target as a fiscal commitment. The target commits the *legislature and treasury* to pay off debt at the targeted inflation rate, and to adjust fiscal policy as needed, as much as it commits and empowers the central bank. This interpretation explains why the adoption of inflation targets led to nearly instant disinflation, and that central banks have not been tested to exercise the toughness that conventional analysis of inflation targets says they must occasionally display.

Inflation targets have been remarkably successful. Figures 9.1 and 9.2 show inflation around the introduction of inflation targets in New Zealand and Canada. On the announcement of the targets, inflation fell to the targets quickly, and stayed there, with no large recession, and no period of high interest rates or other monetary stringency, such as occurred during the painful U.S. and U.K. stabilizations of the early 1980s. Sweden had a similar experience. Just how were these miracles achieved?

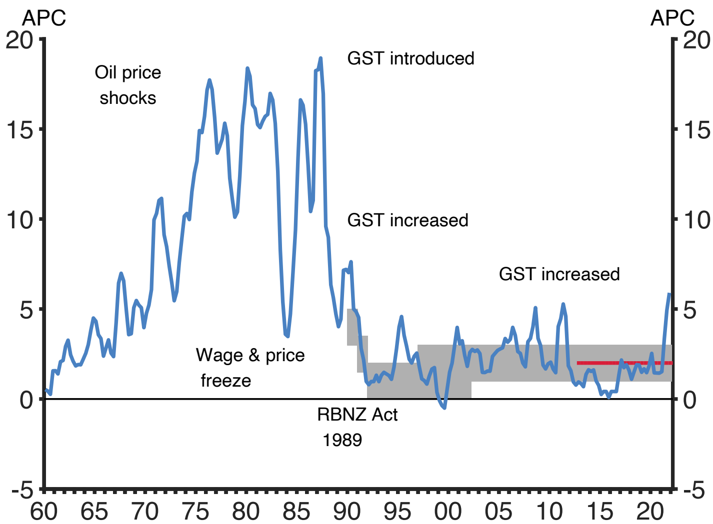

Figure 9.1: Inflation Surrounding the Introduction of a Target in New Zealand. Shading indicates the inflation target range. Source: McDermott and Williams (2018). Figure courtesy of Rebecca Williams.

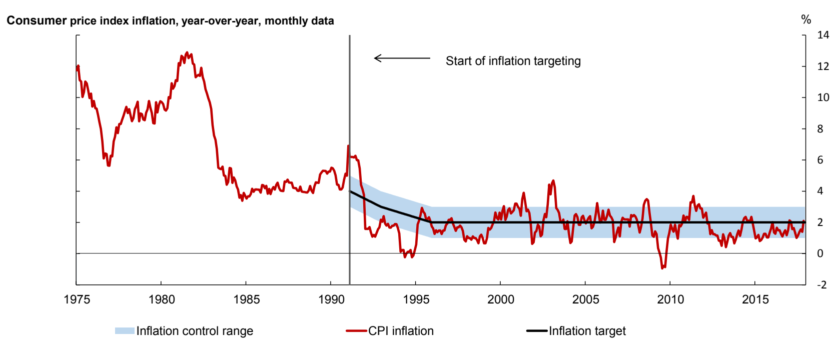

Figure 9.2: Inflation Surrounding Canada's Introduction of an Inflation Target. Source: Murray (2018).

Inflation targets consist of more than just promises by central banks. Central banks make announcements and promises all the time, and people regard such statements with skepticism well-seasoned by experience.

Inflation targets are an agreement between central bank, treasury, and government. The conventional story of their effect revolves around central banks: The inflation target agreement requires and empowers the central bank to focus only on inflation. The agreement gives the bank greater independence. It typically does not spell out a rule, rather giving the bank relatively free rein in how it is to achieve the inflation target. Central bankers are evaluated by their performance in achieving the inflation target, which is a commitment by the government *not* to complain about other objectives.

These stories are wanting. Did previous central banks just lack the guts to do what's right, in the face of political pressure to inflate? Did they wander away from their clear institutional missions and need reining in? Moreover, just what does the central bank *do* to produce low inflation after the inflation target is announced? One would have thought, and most people did think, that the point of an inflation-targeting agreement is to insulate the bank from political pressure during a long period of monetary stringency. To fight inflation, the central bank would have to produce high real interest rates and a severe recession such as accompanied the U.S. disinflation during the early 1980s. And the central bank would have to repeat such unwelcome medicine regularly. For example, that is the diagnosis repeated by McDermott and Williams (2018) of the 1970s and 1980s.

Nothing of the sort occurred. Inflation simply fell like a stone on the announcement of the target, and the central banks were never tested in their resolve to raise interest rates, cause recessions, or otherwise squeeze out inflation. Well, "expectations shifted" when the target was announced, and became "anchored" by the target, people say, but why? The long history of inflation certainly did not lack for speeches from politicians and central bankers promising future toughness on inflation. Why were *these* speeches so effective? Why did they produce anchors and not sails?

Figure 9.1 provides a hint, with the annotation "GST [goods and services tax] introduced" and "GST increased." Each of these inflation targets emerged as a part of a package of reforms including fiscal reforms, spending reforms, financial market liberalizations, and pro-growth regulatory reforms. McDermott and Williams (2018), though focusing on central bank actions, write "A key driver of high inflation in New Zealand over this period [before the introduction of the inflation target] was government spending, accommodated by generally loose monetary policy." It follows that a key driver of non-inflation afterwards was a reversal of these policies, not just a tough central bank.

I therefore read the inflation target as a bilateral commitment. It includes a commitment by the *legislature and treasury* to 2% (or whatever the target is) inflation. They commit to run *fiscal* and economic affairs to pay off debt at 2% inflation, no more, and no less. People expect the legislature and treasury to back debt at the price level *target*, but not to respond to changes in the real value of debt due to changes in the price level away from the target. Above-target inflation will lead to fiscal stringency. Below-target inflation will lead to inaction or stimulus. The inflation target captures the π^* and v^* of my simple fiscal models.

The inflation target functions as a gold price or exchange rate target, which commit the legislature and treasury to pay off debt at a gold or foreign currency value, no more and no less. But the inflation target aims at the CPI directly, not the price of gold or exchange rate, eliminating that source of relative price variation. And the inflation target avoids the run-inducing promise to freely trade cash for gold or foreign currency.

One can read the success of inflation targets as an instance of the Sargent (1982b) analysis of the ends of inflations, which I review in Section 14.3. When the long-run *fiscal* problem is credibly solved, inflation drops on its own, almost immediately. There is no period of monetary stringency, no high real interest rates moderating aggregate demand, no recession. Interest rates fall, money supply may rise, and deficits may rise temporarily as well, with the government newly able to pledge surpluses. As such, inflation targeting episodes are as revealing about lack of mechanical stickiness in expectations, specifically in the Phillips curve, as they are about the fiscal foundations of those inflation expectations.

Berg and Jonung (1999) discuss another example, from an earlier period in history, Sweden's price level target of the 1930s. It called for systematic interest rate increases if the price level increased and vice versa, answering the question of what action the central bank was expected to take. Like the modern experience, the central bank never had to do it, and actually pegged the exchange rate against the pound during the period.

An inflation target failed instructively in Argentina 2015-2019. In the analysis of Cachanosky and Mazza (2021) and Sturzenegger (2019), the basic problem was that the necessary fiscal commitment was absent. The latter also points out interesting fiscal dynamics:

> After an initial success, each program was discontinued because of a distinct form of fiscal dominance: as pensions are indexed with a lag and represent a large portion of spending, quick disinflations jeopardize fiscal consolidation.

Argentina's failure reinforces my point that a successful inflation target is as much a commitment by treasury as a commitment by and commandment to the central bank.

This sort of fiscal commitment is not written in official inflation targeting agreements, nor is it (yet) much discussed in the surrounding literature. But it surely seems like a reasonable expectation of what the commitments to fiscal reform in inflation-targeting legislation mean. And writing a model of an inflation target that ignores the fiscal reforms, and the financial and regulatory reforms that grow the tax base, putting in a footnote about passive fiscal policy and lump sum taxes, seems to miss a central part of the historical lesson.

Still, that commitment is implicit. If my reading of the inflation target is right, a more formal fiscal rule, announcing how fiscal policy will and won't react to inflation, would make sense.

9.2 A Simple Model of an Inflation Target

I construct a model of an inflation target. The surplus responds to pay off higher debts at the price level target, $s_t = s_{0,t} + \alpha V_t^*$, where V_t^* accumulates deficits at the price level target P_t^*. Together with an interest rate target $Q_t = \beta E_t(P_t^*/P_{t+1}^*)$, the price level is determined and equal to $P_t = P_t^*$. The government commits to pay back any debt incurred by deficits $s_{0,t}$, at the price level target. But the government commits not to respond to off-target inflation or deflation.

To construct a simple dynamic model of the inflation target, I use the same π^*, v^* idea as in Section 5.4 and Section 5.5, which generalize the $s_1 = B_0/P_1^*$ idea from the two-period model in Section 1.6.

Define a state variable V_t^* by

$$V_1^* = \frac{B_0}{P_1^*}, \tag{9.1}$$

$$V_{t+1}^* = \frac{1}{Q_t}\frac{P_t^*}{P_{t+1}^*}\left(V_t^* - s_t\right). \tag{9.2}$$

One-period debt follows the flow condition

$$\frac{B_{t-1}}{P_t} = s_t + Q_t \frac{P_{t+1}}{P_t}\frac{B_t}{P_{t+1}}$$

and hence

$$\frac{B_t}{P_{t+1}} = \frac{1}{Q_t}\frac{P_t}{P_{t+1}}\left(\frac{B_{t-1}}{P_t} - s_t\right). \tag{9.3}$$

Comparing (9.2) and (9.3), the state variable V_t^* represents what the real value of debt would be if the price level were always equal to the target. It accumulates past deficits, but does not respond to arbitrary unexpected inflation and deflation.

Fiscal policy follows a rule that responds to the state variable V_t^*, thereby ignoring changes in the value of the debt that come from off-target inflation,

$$s_t = s_{0,t} + \alpha V_t^*. \tag{9.4}$$

In this example, consider an exogenous process for $\{s_{0,t}\}$.

Monetary policy sets an interest rate consistent with the price level target,

$$Q_t = \frac{1}{1+i_t} = \beta E_t\left(\frac{P_t^*}{P_{t+1}^*}\right). \tag{9.5}$$

With a time-varying real rate, the central bank has a nontrivial job to do. It must try to figure out the correct real rate, and adjust the nominal rate to mirror that real rate plus the inflation target. As above, the interest rate must obey the equilibrium conditions of the model given the price level target, a nontrivial job. Errors result in monetary policy shocks.

In this setup $P_t = P_t^*$ is the unique equilibrium price level. To show that, I first establish that V_t^* is the present value of surpluses. Substituting (9.5) in (9.2) and taking expectations,

$$\beta E_t\left(V_{t+1}^*\right) = V_t^* - s_t. \tag{9.6}$$

Using (9.4),

$$\beta E_t\left(V_{t+1}^*\right) = (1-\alpha)V_t^* - s_{0,t}.$$

For bounded $\{s_{0,t}\}$, the V_t^* variable converges,

$$\lim_{T\to\infty} \beta^T E_t\left(V_{t+T}^*\right) = 0.$$

Thus, we can iterate (9.6) forward, and the limiting term drops out, leaving us

$$V_t^* = E_t \sum_{j=0}^{\infty} \beta^j s_{t+j}.$$

Since the real value of nominal debt is also the present value of surpluses, we have $B_{t-1}/P_t = V_t^*$ at all dates. From (9.1) we have $P_0 = P_0^*$. With (9.2) and (9.3), $P_t = P_t^*$ then implies $P_{t+1} = P_{t+1}^*$ so we have $P_t = P_t^*$ at all dates.

This result requires both the surplus rule (9.4) and the interest rate target (9.5), which is the main point of this section. *An inflation target consists of a monetary policy rule and a fiscal commitment.* The interest rate target alone is insufficient. The surplus rule alone is also insufficient. Even with the rule (9.4), a decision to double nominal debt without changing surpluses will have the usual effect of doubling the future price level. So the surplus rule needs to be paired with some rule for setting the quantity of nominal debt, which sets expected inflation. Here I write the more conventional interest rate target. Other monetary policy rules could also work.

9.3 Fiscal Rules

The government could systematically raise and lower surpluses in response to inflation and deflation, in a sort of fiscal Taylor rule, $s_t = s_t(P_t)$. Such fiscal rules can determine the price level with purely indexed debt. They can also be a helpful part of a monetary–fiscal regime with nominal debt.

9.3.1 Indexed Debt in a One-Period Model

> In a one-period model, $b_{T-1} = s(P_T)$ can determine the price level P_T. Better, $s_T(P_T) = b_{T-1} + \theta_{sp}(P_T - P_T^*)$ determines $P_T = P_T^*$. Fiscal theory does not require nominal debt.

A fiscal rule can determine the price level even with fully indexed debt.

In a one-period model, suppose indexed debt b_{T-1} is outstanding at time T, but the government follows a rule or systematic policy in which the surplus rises with the price level, $s_t(P_t)$ with $s_t'(P_t) > 0$. Then, the equilibrium condition at time T is

$$b_{T-1} = s_T(P_T).$$

This condition can determine the price level P_T, although the debt is fully indexed. Better, suppose the government commits to repay real debts, but also reacts to the price level,

$$s_T(P_T) = b_{T-1} + \theta_{sp}(P_T - P_T^*). \tag{9.7}$$

Then the equilibrium price level is $P_T = P_T^*$.

Continuing the usual story, in the morning of time T, the government prints up $P_T b_{T-1}$ dollars to pay off the outstanding indexed debt. The government then commits to raising sufficient taxes to soak up the money, thereby paying off the debt, and additionally that any spending at time T is also financed by taxes at time T, so there is no additional primary surplus or deficit. But if the price level is below P_T^*, the government commits to money-financed expenditures or tax cuts, an unbacked fiscal expansion, while if the price level is too high the government commits to a fiscal austerity, raising taxes and cutting spending to soak up money.

The fiscal theory only needs something real and something nominal in the same equation. The fiscal rule can be the something nominal. *Fiscal theory does not require nominal debt*, as this example shows.

I started this book with a simple example of a constant tax rate and no spending, $P_t s_t = \tau P_t y_t$, to establish that the real surplus does not naturally *have* to depend on the price level. But surpluses can and do depend on the price level. Tax brackets, capital gains, and depreciation allowances are not indexed. Government salaries, defined-benefit pensions, and medical payments are at least somewhat nominally sticky. All of these forces should result in somewhat higher surpluses with inflation $s_t'(P_t) > 0$. So this mechanism should already be part of an empirical investigation of price level determination.

More importantly, the government can *intentionally* vary surpluses with inflation or the price level to improve price level control, as central banks following a Taylor rule or inflation target intentionally vary the interest rate with inflation or the price level to improve their control. And governments do routinely tighten fiscal policy as part of inflation-fighting efforts, and loosen fiscal policy when fighting deflation.

In a dynamic model, the main issue is to convince people that the latter fiscal changes are really unbacked, that today's inflation-fighting surpluses or deflation-fighting deficits will not be repaid.

9.3.2 A Dynamic Fiscal Rule with Indexed Debt

In a dynamic model with surplus rule $s_t = s_{0,t} + \theta_{sp}(P_t - P_t^*)$, the valuation equation is

$$b_{t-1} = \sum_{j=0}^{\infty} \beta^j s_{0,t+j} + \theta_{sp} \sum_{j=0}^{\infty} \beta^j (P_{t+j} - P_{t+j}^*).$$

With $s_{0,t} = a(L)\varepsilon_{s,t}$ and $a(\beta) = 0$, this rule determines the weighted sum $\sum_{j=0}^{\infty} \beta^j P_{t+j}$. A debt policy $\beta b_t = b_{t-1} - s_{0,t}$ or an interest rate target $1/(1+i_t) = \beta E_t \left(P_t^*/P_{t+1}^*\right)$ can complete the regime and determine $P_t = P_t^*$.

Next, think about this fiscal rule in a dynamic context. As usual, since the quantity of debt must come from somewhere, we need both fiscal and monetary policy.

Continuing our flexible-price, constant real rate model, the flow equilibrium condition with indexed debt states that old debt is paid off by surpluses or new debt,

$$b_{t-1} = s_t(P_t) + \beta b_t. \tag{9.8}$$

Iterating forward and imposing the transversality condition,

$$b_{t-1} = \sum_{j=0}^{\infty} \beta^j s_{t+j}(P_{t+j}).$$

This expression holds ex post. Real debt must be repaid or default. Any shocks to surpluses must be met by subsequent movement in the opposite direction. Now the price level can vary to cause that surplus movement, and in so doing we help to determine the price level.

Now consider the surplus rule

$$s_t = s_{0,t} + \theta_{sp}(P_t - P_t^*)$$

in this dynamic context. The valuation equation is

$$b_{t-1} = \sum_{j=0}^{\infty} \beta^j s_{0,t+j} + \theta_{sp} \sum_{j=0}^{\infty} \beta^j (P_{t+j} - P_{t+j}^*). \tag{9.9}$$

The valuation equation and the surplus rule determine the value of the sum $\sum_{j=0}^{\infty} \beta^j P_{t+j}$ but not the shape of that path and hence the price level at each date.

To keep the example simple, assume that $\{s_{0,t+j}\}$ follows a process with moving average representation $s_{0,t} = a(L)\varepsilon_{s,t}$ and $a(\beta) = 0$. Debt incurred to finance this component of primary deficit is paid off by following surpluses. The next section writes a more realistic model with this feature. With real debt and no defaults, the overall surplus $\{s_t\}$ must follow the analogous restriction. Now we *can* have $P_t = P_t^*$ at every date. As there are no innovations to the first term on the right-hand side of (9.9), there are no innovations to the second term as well. But that term is the weighted sum of deviations from target. Individual price levels may deviate from their targets.

Write the flow budget constraint

$$b_{t-1} = s_{0,t} + \theta_{sp}(P_t - P_t^*) + \beta b_t + \frac{M_t}{P_t}. \tag{9.10}$$

In equilibrium, $M_t = 0$. Now if the price level P_t is below the target P_t^*, the government sells additional debt b_t. The following path of prices has $P_{t+j} > P_{t+j}^*$, to generate surpluses that pay off this additional debt.

We can determine the price level at each date by enlarging the regime to cut off the latter possibility. If the government holds real debt sales fixed at the value needed to roll over real debt and to finance the underlying real deficit,

$$\beta b_t = b_{t-1} - s_{0,t}, \tag{9.11}$$

then from the flow equilibrium condition (9.8), we must have $P_t = P_t^*$. In the flow budget constraint (9.10), $P_t < P_t^*$ must now result in $M_t > 0$, the familiar mechanism that produces fiscal inflation. In words, *the government commits that in the event of a too-low price level it will embark on printed-money fiscal expansion. It will not soak up extra money with sales of (indexed) debt,* and vice versa. The "fiscal rule" for debt sales b_t can be the residual of a money-printing rule.

It should not be surprising that in order to fully determine the price level, we need a surplus policy and a debt sale policy. The lesson applies to real as it does to nominal debt.

As before, we do not have to interpret this model as precise adherence to an inflexible target, exactly 2% per year inflation for example. We can interpret the stochastic P_t^* target to allow inflation to rise and fall, and as what the government is willing to put up with rather than what it aspires to. We can interpret the stated 2% per year target as a long-run value of P^*, not its precise value on each date.

I have pushed the example hard, to show that a fully fiscal model of price level determination is possible with indexed debt. Rather than select the price level path with the fiscal rule (9.11), however, or its equivalent money printing rule, we can rely as usual on an interest rate target to set the price level path. An interest rate target i_t requires

$$\frac{1}{1+i_t} = \beta E_t \left(\frac{P_t}{P_{t+1}} \right). \tag{9.12}$$

Thus, if the interest rate target is set by

$$\frac{1}{1+i_t} = \beta E_t \left(\frac{P_t^*}{P_{t+1}^*} \right),$$

then only the sequence $P_t = P_t^*$ satisfies both (9.9) and (9.12).

9.3.3 A Better Fiscal Rule

> I write the fiscal rule as a combination of a regular, backed budget, and an emergency or price level stabilization unbacked budget. The two budgets follow $s_{r,t} = s_{0,t} + \alpha_r b_{r,t}$ and $s_{p,t} = \theta_{sp}(P_t - P_t^*)$, respectively. This separation allows the government to communicate how much debt is backed and unbacked, and to deliberately inflate while also retaining its commitment to repay regular debts and thereby borrow when needed. The α_r term generates repayment of the $s_{r,t}$ component without requiring an exogenous s-shaped moving average.

The model in the last section is a little strained. I specify an exogenous surplus process $\{s_{0,t}\}$ with $a(\beta) = 0$. It is prettier, more intuitive, more practical, and more realistic to produce this feature with a fiscal rule rather than a direct s-shaped moving average. This section presents such a rule.

I phrase this model in the language of Jacobson, Leeper, and Preston (2019), who describe the Roosevelt administration's separation of finances into a "regular" budget whose debts are repaid and an "emergency" budget that is unbacked. The Roosevelt administration was battling deflation. They first devalued the dollar relative to gold. This step already changes the backing of nominal debt, which should create inflation—it changes expected surpluses. But they wanted to do more: They wanted to undertake an unbacked fiscal expansion to create additional inflation. They wanted to increase nominal debt without promise of additional surpluses. At the same time, they did not want to turn the United States into a hyperinflationary basket case. They wanted to maintain the U.S. government's reputation, so that if it wished to borrow in the future, when the Depression was over, it could pledge surpluses to that future borrowing. That reputation would soon be needed, in large measure. How do you run a little bit of state contingent default and unbacked fiscal expansion, yet retain a reputation for backing your future fiscal expansions after the threat of deflation has ended?

To accomplish this feat of expectations management, the Roosevelt administration separated the budget into a "regular" budget whose debts are repaid and an "emergency" budget that is unbacked. The administration proposed to fund the emergency budget entirely by borrowing until the Depression ended, but then to end the practice. Separating the items on the regular versus emergency budget, and tying the emergency budget to visible economic conditions neatly unties the Gordian knot.

This brilliant idea (or, more accurately, this brilliant reinterpretation of the Roosevelt administration's actions) forms the basis not just of a deflation-fighting scheme, but of a broader fiscal rule that works under indexed debt or gold standard debt. Let the "regular" budget surplus be

$$s_{r,t} = s_{0,t} + \alpha_r b_{r,t},$$

and the corresponding portion of the debt $b_{r,t}$. Let the price stabilization surplus be

$$s_{p,t} = \theta_{sp}(P_t - P_t^*), \tag{9.13}$$

with corresponding portion of the debt $b_{p,t}$. The total surplus and debt are

$$\begin{aligned} s_t &= s_{r,t} + s_{p,t} \\ b_t &= b_{r,t} + b_{p,t}. \end{aligned}$$

Each debt accumulates separately,

$$\begin{aligned} b_{r,t} &= R\left(b_{r,t-1} - s_{r,t}\right) \\ b_{p,t} &= R\left(b_{p,t-1} - s_{p,t}\right). \end{aligned}$$

One might implement this idea with distinct debt issues, as public debt is distinct from debt sold to the Social Security trust fund.

With $\alpha_r > 0$, the regular surplus repays its debts automatically, without an extra $a(\beta) = 0$ assumption,

$$b_{r,t-1} = \sum_{j=0}^{\infty} \beta^j s_{r,t+j},$$

and ignoring the price level completely. The regular part of the deficit and its repayment drop completely out of price level determination.

The price level stabilization budget separately obeys

$$b_{p,t-1} = \sum_{j=0}^{\infty} \beta^j s_{p,t+j} = \theta_{sp} \sum_{j=0}^{\infty} \beta^j (P_{t+j} - P_{t+j}^*). \tag{9.14}$$

The price level control part of the surplus does not feature automatic repayment.

There is no α_p term in (9.13). The whole point of this budget is to threaten unbacked fiscal expansion or contraction, or money left outstanding, and to force the price level sequence to adjust.

As before, the price level budget (9.14) only sets this weighted sum of the price level path, but not each element. To continue in a realistic way, as above, we can pair this fiscal policy with a debt target, here $b_{p,t} = 0$, an equivalent money printing policy, or, better, with a monetary policy that controls the nominal interest rate and therefore the price level path.

These examples need elaboration. We need to include nominal as well as indexed debt, sticky prices or other important frictions, and a realistic distinction between aspirations—a steady 2% inflation, or steady price level—and the equilibrium inflation that the rule is willing to tolerate in the presence of shocks.

9.3.4 A Fiscal Rule with Inflation and Interest Rates

I introduce a model with an interest rate rule $i_t = \theta_{i\pi}\pi_t + u_t$ and a surplus that reacts to inflation, $s_t = s_{0,t} + \theta_{s\pi}\pi_t$. We have

$$b_{t-1} = \frac{\theta_{s\pi}}{1-\beta\theta_{i\pi}}\pi_t + \frac{\beta\theta_{s\pi}}{(1-\beta\theta_{i\pi})(1-\beta\eta)}u_t + E_t\sum_{j=0}^{\infty}\beta^j s_{0,t+j},$$

so inflation is determined.

I pursue a little model that merges fiscal and monetary policy to determine inflation, rather than the price level, with indexed debt. Monetary policy picks the inflation path, while the fiscal policy rule sets the level of inflation. The model is expressed in a form that more easily invites adaptation to linearized sticky price models.

Monetary policy follows an interest rate target,

$$i_t = \theta_{i\pi}\pi_t + u_t$$
$$u_t = \eta u_{t-1} + \varepsilon_t$$

with $\theta_{i\pi} < 1$. The economy has flexible prices and a constant real rate so

$$i_t = E_t\pi_{t+1}.$$

Inflation therefore follows

$$E_t\pi_{t+1} = \theta_{i\pi}\pi_t + u_t. \tag{9.15}$$

Fiscal policy follows a rule that responds to inflation,

$$s_t = s_{0,t} + \theta_{s\pi}\pi_t.$$

Only indexed debt is outstanding. The government debt valuation equation reads

$$b_{t-1} = E_t \sum_{j=0}^{\infty} \beta^j \left(s_{0,t+j} + \theta_{s\pi}\pi_{t+j}\right).$$

Using (9.15) to eliminate expected future inflation, we have

$$b_{t-1} = \frac{\theta_{s\pi}}{1-\beta\theta_{i\pi}}\pi_t + \frac{\beta\theta_{s\pi}}{(1-\beta\theta_{i\pi})(1-\beta\eta)}u_t + E_t\sum_{j=0}^{\infty}\beta^j s_{0,t+j}. \tag{9.16}$$

This equation now determines inflation at each date π_t, despite completely indexed debt.

The surplus response to inflation $\theta_{s\pi}$ is key to the result. Without this response, π_t drops from the equation. The monetary response to inflation $\theta_{i\pi}$ is not essential. With $\theta_{i\pi} = 0$ we have $i_t = u_t$ and

$$b_t = \theta_{s\pi}\pi_t + \frac{\beta\theta_{s\pi}}{1-\beta\eta}i_t + E_t\sum_{j=0}^{\infty}\beta^j s_{0,t+j}.$$

An interest rate target is essential. Without it, the expected future inflation that drives expected future surpluses is not pinned down. (This example modifies Sims (2013). Section 16.10.8 summarizes Sims's point, which is different.)

9.3.5 Fiscal Rules with Nominal Debt

> Surplus rules that respond to the price level can be useful parts of a fiscal regime that also includes nominal debt. They add to the force of price level determination, valuable if nominal (own-country) debt is small. They offer fiscal austerity/stimulus in place of the outstanding debt devaluation as a fundamental mechanism.

Now, consider mixed real and nominal debt with a fiscal rule. As usual, the basic ideas are easiest to see in the simple one-period model. With mixed real and nominal debt, we have

$$b_{T-1} + \frac{B_{T-1}}{P_T} = s(P_T).$$

Ruling out the passive possibility, which requires $s'(P) < 0$, the price level is determined.

With any nominal debt, a surplus rule is not strictly needed for determinacy. But $s'(P) > 0$ helps. The stronger the divergence in price level dependence between the left- and right-hand sides of the valuation equation, the better price level determination must be.

In Section 8.3, we worried about the cashless limit problem, in which a vanishingly small amount of nominal debt tries to determine the price level. A surplus rule addresses that problem. If nominal debt is only 10% of all debt, then a 10%

change in the price level only affects the value of government bonds by 1%. A larger change in $s(P)$ on the right-hand side can make up for that weakness.

The fiscal rule also changes the nature of price determination. The stronger $s'(P)$, the more that inflationary fiscal shocks are met by an induced fiscal tightening, and the less they are met by inflating away outstanding nominal government debt.

A little bit of nominal debt, or money, also is useful to allow monetary policy to set the nominal interest rate, an issue I glossed over above.

9.4 Targeting the Spread

> Rather than target the nominal interest rate, the central bank can target the *spread* between indexed and nonindexed debt. This policy determines expected inflation, while letting the level of interest rates rise and fall according to market forces. The policy can be implemented by allowing people to trade indexed for nominal debt, or by offering inflation swaps, at fixed rates. A spread target, like a nominal interest rate target, only nails down expected inflation. Actual inflation also depends on fiscal policy.

Rather than target the level of the nominal interest rate, suppose the central bank targets the *spread* between indexed and nonindexed debt. The nominal rate equals the indexed (real) rate plus expected inflation, $i_t = r_t + E_t\pi_{t+1}$. So, by targeting $i_t - r_t$, the central bank can target expected inflation directly.

This target can also be implemented as a peg, like an exchange rate peg or gold standard, by offering to freely trade indexed for nonindexed debt. Bring in a one-year, zero-coupon indexed bond, which promises to pay $\$1 \times \Pi_{t+1}$ at maturity, where Π_{t+1} is the gross inflation rate. You get in return Π^* zero-coupon nominal bonds, each of which pays \$1 at maturity, where Π^* is the inflation target. If inflation comes out to $\Pi_{t+1} = \Pi^*$, the two bonds pay the same amount. This policy will drive the spread between real and nominal debt to Π^*, so inflation expectations settle down to Π^*. We have to check the latter statement—that the economy is stable and determinate under a spread target. That analysis follows.

The central bank could also target rather than peg the real–nominal spread by conventional instruments of monetary policy. It could adjust the level of nominal interest rates in order to achieve its desired value for the real–nominal spread, as some central banks adjust nominal interest rates to target the exchange rate without actually pegging or buying and selling foreign currency, or as historically monetary policy chased a gold price target without offering to buy and sell gold.

Why target the spread? I have simplified the discussion by leaving out real interest rate variation, and treating the real interest rate as known. To target expected inflation by targeting nominal interest rates, the central bank just adds the real rate r_t to its inflation target $E_t(\pi^*_{t+1})$, and sets the nominal interest rate at that value $i_t = r_t + E_t(\pi^*_{t+1})$. But in reality, the real rate varies over time. The real rate is naturally lower in recessions: People want to save more but invest less; consumption growth is low; the marginal product of capital is low. The real rate is naturally higher in expansions, for all the opposite reasons. But there is no straightforward way to measure the natural, neutral, correct, or proper real rate. With sticky prices, the real rate varies as the central bank varies the nominal rate, so the bank partially controls the thing it wants to measure. There is currently a big discussion over lower frequency variation in the natural real rate, whether "r^*" is lower. Even with complex models, the Fed struggles to measure r^* (see Holston, Laubach, and Williams (2017)), as the Fed struggles to define and measure the "natural" rate of unemployment. Measuring business cycle or higher frequency in the "natural" rate is an order of magnitude harder. Yet that is, essentially, what central banks try to do in order to figure out what nominal interest rate to set.

Economic planners have had a tough time setting prices. Economic philosophers have had a tough time proclaiming the just price for centuries. Real interest rates are no exception. If the underlying or natural interest rate is like all other prices, especially asset prices and exchange rates, it moves a lot in response to myriad information that planners do not see, befuddling even ex post rationalization.

In this context, then, if the central bank targets the *spread* between indexed and nonindexed debt, and thereby targets expected inflation directly, it can leave the *level* of real and nominal interest rates entirely to market forces. This policy leaves the central bank in charge of the nominal price level only, and can get it out of the business of trying to set the most important real price in the economy.

The spread target can also vary over time or in response to the state of the economy, if one wishes to accommodate, rather than eschew, central banks' macroeconomic planning tendencies. Rather than view its "stimulus" or cooling efforts through desired nominal or real interest rates, a central bank could stimulate by raising expected inflation directly, and vice versa. Such efforts might also be more effective at raising or lowering expected inflation than moving nominal rates, or making forward guidance promises about such movements.

The idea can extend throughout the yield curve. The central bank can target expected inflation at any horizon, and it can implement that target by offering to trade indexed for nonindexed debt at any maturity. Thus, the spread target also offers a way to directly "anchor" long-run inflation expectations. The central bank could operate a short-run interest rate target, QE, and other interventions, while also targeting the spread between indexed and nonindexed long-term debt to better anchor long-run expectations. Since prices are sticky and short-run inflation is hard to control, such a separation between conventional short-run policy and long-term expectations management may prove useful.

The practical effect on monetary policy of this change may not be great, in equilibrium, and in response to the usual shocks. If the central bank follows a Taylor rule, $i_t = (r + \pi^*) + \theta_\pi \pi_t + \theta_x x_t$, and if the real interest rate tracks inflation and output, $r_t = r + \theta_\pi \pi_t + \theta_x x_t$, then the spread target produces the same result as the Taylor rule. But targeting the spread is clearer, and helps better to set inflation expectations. A spread target may perform better when the economy is hit by a different set of shocks, so the historical correlation of the real rate with the Taylor rule changes. Rules developed from history and experience have a certain wisdom, but that wisdom often reflects correlations that change over time.

In my storytelling, I offer a year or more horizon. Why not a day, you might ask, and let the central bank target daily expected inflation? Well, prices are sticky, so one should not expect that the central bank can control daily expected inflation. A year seems to me about the shortest horizon at which one might expect inflation to be able to move in response to the spread rather than vice versa. But this intuition needs to be spelled out.

A spread peg can be implemented via CPI futures or swaps rather than, or in addition to, trading underlying bonds. In an inflation swap, parties agree to pay or receive the difference between realized inflation and a reference rate set at the beginning of the contract period. They pay or receive $P_{t+1}/P_t - \Pi_t^*$. No money changes hands today. The reference rate Π_t^* adjusts to clear the market, and is equal to the risk-neutral expected inflation rate. If the central bank targets or pegs the inflation swap rate Π_t^*, expected inflation adjusts. Entering an inflation swap is the same thing as buying one indexed bond that pays P_{t+1}/P_t in one period, and selling nominal bonds that pay Π_t^*. (Dowd (1994) describes a peg to a contract similar to CPI futures.)

Indexed debt (TIPS) in the United States is currently rather illiquid, and it suffers a complex tax treatment. Simplifying the security would make it far more liquid, transparent, and reflective of inflation expectations. Cochrane (2015b) contains a detailed proposal for simplified and more liquid federal debt, consisting of tax-free indexed and nonindexed perpetuities and swaps between these simple securities. Fleckenstein, Longstaff, and Lustig (2014) document arbitrage between TIPS and CPI swaps, a sure sign of an ill-functioning market. Central banks should work with treasuries more broadly to modernize and simplify the latter's offerings, and of indexed debt in particular. The absence of significant inflation up to 2021 may have removed the incentive for institutional change, but that incentive may reappear.

Central banks can also create and offer more extensive real and nominal term liabilities, which is a good idea for many reasons. Banks offer certificates of deposit, why not the central bank? Central bank liabilities are really liquid. And, at least initially, CPI swaps or futures may end up being the most liquid and forceful implementation of these ideas.

Obviously, central banks would inch their way to such a proposal. Start by paying a lot more attention to the spread. Work to get the markets more liquid and implement better securities. Start gently pushing the spread to where the central bank wants the spread to go with QE-like purchases in fixed amounts. Get to a flat supply curve at the spread target slowly. And allow time and experience so people understand the regime.

Targeting the spread is really only a small step from the analysis so far. If the government can target the nominal interest rate i_t, and then expected inflation will adjust in equilibrium to $E_t\pi_{t+1} = i_t - r_t$ with r_t the real interest rate determined eventually elsewhere, then the economics of a spread target are really not fundamentally different from those of an interest rate target. This statement needs to be verified, and the next two sections do so.

9.4.1 Fiscal Theory with a Spread Target

I write the spread target in the sticky price fiscal theory of monetary policy model to verify that it works, and how it works. A spread target determines expected inflation, while the government debt valuation equation determines unexpected inflation. The spread target works just as the interest rate target works in the sticky price model. The spread target leads to i.i.d. inflation around the target, and leads to endogenous real interest rate variation that offsets IS shocks. We can also support a spread target with active monetary policy—the idea is not intrinsically tied to fiscal theory.

Writing $i_t - r_t = E_t\pi_{t+1}$, and concluding that if the central bank pegs the left side the right side will adjust, may seem straightforward. The condition $i_t - r_t = E_t\pi_{t+1}$ is a steady state of practically every model. But one may worry that this steady state may be unstable, that pegging the spread between real and nominal bonds may lead to spiraling inflation or deflation rather than inflation or deflation that converges to the spread. *Can* the government even force the spread to be 2% without trading infinite quantities? The spread between indexed and nominal bonds *measures* inflation expectations, but silencing the canary does not make the mine safe. Which way is it? That's what we need models for.

With flexible prices, the real interest rate is independent of inflation, so the spread target is stable and determinate when an interest rate target is stable and determinate, and vice versa. In old-Keynesian adaptive expectations models, an interest rate peg leads to unstable inflation, and a spread target has the same outcome. In new-Keynesian models, an interest rate peg leads to indeterminate inflation, and one can anticipate the same result of a spread target. But in fiscal theory of monetary policy, an interest rate peg can be stable and determinate. If that is true, a spread peg is also stable and determinate.

Let us put a spread target in the standard sticky price fiscal theory of monetary policy model, in place of a nominal interest rate target. I start with an even simpler version of the model,

$$x_t = -\sigma(i_t - E_t\pi_{t+1}) \tag{9.17}$$

$$\pi_t = E_t\pi_{t+1} + \kappa x_t. \tag{9.18}$$

Here I delete the $E_t x_{t+1}$ term in the first equation, so it becomes a static IS curve, in which output is lower for a higher real interest rate. This simplification turns out not to matter for the main point, which I verify by going through the same exercise with the full model. But it shows the logic with much less algebra. Section 17.1 uses this simplified model extensively to cleanly exposit new-Keynesian and old-Keynesian versus fiscal theory of monetary policy approaches.

Denote the real interest rate

$$r_t = i_t - E_t\pi_{t+1}. \tag{9.19}$$

We can view the spread target as a nominal interest rate rule that reacts to the real interest rate,

$$i_t = \theta r_t + \pi^{e*}, \tag{9.20}$$

rather than react to inflation. (I add e for expected and $*$ for target to π.) The spread target happens at $\theta = 1$, but the logic will be clearer and the connection of an interest rate peg and interest spread peg clearer if we allow $\theta \in [0, 1]$ to connect the possibilities and track the limit as $\theta \to 1$.

Eliminating all variables but inflation from (9.17)-(9.20), we obtain

$$E_t \left(\pi_{t+1} - \pi^{e*}\right) = \frac{1-\theta}{1-\theta+\sigma\kappa} \left(\pi_t - \pi^{e*}\right). \tag{9.21}$$

When the coefficient on the right-hand side is less than 1, inflation is stable but indeterminate, as $\Delta E_{t+1}\pi_{t+1}$ can be anything. We complete the model with the government debt valuation equation, in linearized form

$$\Delta E_{t+1}\pi_{t+1} = -\Delta E_{t+1} \sum_{j=0}^{\infty} \rho^j \tilde{s}_{t+1+j} + \Delta E_{t+1} \sum_{j=1}^{\infty} \rho^j (i_{t+j} - \pi_{t+1+j}), \tag{9.22}$$

which determines unexpected inflation.

We can substitute (9.19) and (9.20) into (9.21), iterate forward, and solve (9.22) to find unexpected inflation,[1]

$$\Delta E_{t+1}\pi_{t+1} = -\frac{(1-\rho)(1-\theta)+\sigma\kappa}{(1-\rho)(1-\theta)+\sigma\kappa+\rho} \varepsilon_{\Sigma s,t+1}. \tag{9.23}$$

Equations (9.21) and (9.23) now completely describe the solution—expected and unexpected inflation.

Starting at the familiar $\theta = 0$, an interest rate peg, we have

$$E_t \left(\pi_{t+1} - \pi^{e*}\right) = \frac{1}{1+\sigma\kappa} \left(\pi_t - \pi^{e*}\right)$$

$$\Delta E_{t+1}\pi_{t+1} = -\frac{1-\rho+\sigma\kappa}{1+\sigma\kappa} \varepsilon_{\Sigma s,t+1}.$$

[1] Algebra: Uniting (9.19) and (9.20),

$$r_t = \frac{1}{1-\theta} \left(\pi^{e*} - E_t \pi_{t+1}\right).$$

From (9.21)

$$\Delta E_{t+1}\pi_{t+1+j} = \left(\frac{1-\theta}{1-\theta+\sigma\kappa}\right)^j \Delta E_{t+1}\pi_{t+1}.$$

We can then write (9.22)

$$\Delta E_{t+1}\pi_{t+1} = -\varepsilon_{\Sigma s,t+1} + \sum_{j=1}^{\infty} \rho^j \Delta E_{t+1} \left[\frac{1}{1-\theta} \left(\pi^{e*} - \pi_{t+j+1}\right)\right]$$

$$\Delta E_{t+1}\pi_{t+1} = -\varepsilon_{\Sigma s,t+1} - \frac{1}{1-\theta} \sum_{j=1}^{\infty} \rho^j \left(\frac{1-\theta}{1-\theta+\sigma\kappa}\right)^j \Delta E_{t+1}\pi_{t+1}$$

and solving, we get (9.23).

There are only fiscal shocks, which cause unexpected inflation. That inflation settles down to the inflation target with an AR(1) response driven by price stickiness.

As we raise the real interest rate response $0 < \theta < 1$, the solution (9.21) and (9.23) remains qualitatively the same. As θ rises, the dynamics of (9.21) happen faster, and expected inflation converges more quickly to the target.

At $\theta = 1$, the spread target $i_t - r_t = \pi^{e*}$ nails down expected inflation. Equivalently, expected inflation settles down to the target infinitely fast. Equation (9.21) becomes

$$E_t \pi_{t+1} = \pi^{e*}.$$

Equation (9.23) becomes

$$\Delta E_{t+1} \pi_{t+1} = -\frac{\sigma\kappa}{\sigma\kappa + \rho} \varepsilon_{\Sigma s, t+1}.$$

In sum, the model obeys

$$\pi_{t+1} = \pi^{e*} - \frac{\sigma\kappa}{\sigma\kappa + \rho} \varepsilon_{\Sigma s, t+1}.$$

Inflation is not equal to the target period by period. But inflation is uncorrelated over time, which is as close as we can get with an expected inflation target. Output and real and nominal rates then follow

$$\begin{aligned} x_t &= \frac{1}{\kappa} \left(\pi_t - \pi^{e*}\right) \\ r_t &= -\frac{1}{\sigma\kappa} \left(\pi_t - \pi^{e*}\right) \\ i_t &= \pi^{e*} - \frac{1}{\sigma\kappa} \left(\pi_t - \pi^{e*}\right). \end{aligned}$$

A fiscal shock leads to a one-period inflation, and thus a one-period output increase. Higher output means a lower interest rate in the IS curve, and thus a lower nominal interest rate. Real and nominal interest rates vary due to market forces, while the central bank does nothing more than target the spread.

We may wish for more variable expected inflation, and central banks may wish for something to do. Many models find that it is desirable to let a long smooth inflation accommodate a shock. Both desires can be accommodated by varying the expected inflation target. The central bank could follow $\pi_t^{e*} = E_t \pi_{t+1} = \theta_\pi \pi_t$ to produce persistent inflation. Or, the central bank could follow $\pi_t^{e*} = p^* - p_t$ to implement an expected price level target p^* with one-period reversion to that target. Or the bank could follow $\pi_t^{e*} = \theta_\pi \pi_t + \theta_x x_t + u_{\pi t}$ in Taylor rule tradition, including discretionary responses to other events in the $u_{\pi t}$ term. The point is not tied to a desire for a constant expected inflation peg, nor to require central bank inaction. The point is only that a spread target is possible and will not explode.

One may be a bit surprised that expected inflation is exactly equal to the spread target, even though prices are sticky. But the definition $r_t = i_t - E_t \pi_{t+1}$ guarantees that unless the model blows up, expected inflation must instantly equal the spread target. When prices cannot move, the real interest rate moves. The danger is that the real interest rate might explode in the attempt. It does not do so.

The same behavior occurs in the full new-Keynesian model. I simultaneously allow shocks to the equations and a time-varying spread target. The model is

$$x_t = E_t x_{t+1} - \sigma(i_t - E_t \pi_{t+1}) + u_{x,t} \tag{9.24}$$

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t + u_{\pi,t}. \tag{9.25}$$

Write the spread target as $i_t - r_t = \pi_t^{e*}$. With the definition $r_t = i_t - E_t \pi_{t+1}$, we simply have $E_t \pi_{t+1} = \pi_t^{e*}$. As in the simple model, the spread target directly controls equilibrium expected inflation. Unexpected inflation is set by the same government debt valuation equation (9.22), now with discount rate terms. I won't write out the solution for unexpected inflation, as it is algebraically large and unenlightening.

Given inflation, output and the real rate follow from (9.25) and (9.24),

$$x_t = \frac{1}{\kappa}\left(\pi_t - \beta\pi^{e*} - u_{\pi,t}\right) = \frac{1}{\kappa}\left[(1-\beta)\pi^{e*} + \Delta E_t \pi_t - u_{\pi,t}\right]$$

$$r_t = \frac{1}{\sigma\kappa}\left(-\Delta E_t \pi_t + \kappa u_{x,t} + u_{\pi,t} - E_t u_{\pi,t+1}\right). \tag{9.26}$$

These are the case of a constant target, $\pi_t^{e*} = \pi^{e*}$. Changes in that target add additional dynamics, and are now the response to monetary policy, so worth pursuing. In this simple case, output again follows inflation with serially uncorrelated movements, plus Phillips curve disturbances. The real rate and nominal interest rate also follow inflation with a serially uncorrelated movement, plus both IS and Phillips curve shocks. The IS shock does not appear in equilibrium output. Endogenous real rate variation $\sigma r_t = u_{x,t}$ offsets the IS shock's effect on output in the IS equation $x_t = E_t x_{t+1} - \sigma r_t + u_{x,t}$. This is an instance of desirable real-rate variation that the spread target accomplishes automatically.

This discussion is obviously only the beginning. We need to see the spread target at work in more realistic models. The sense in which it is desirable, adapting automatically to shocks that the central bank cannot directly observe, needs to be expressed formally. Optimal monetary policy sets the interest rate, in a conventional policy, and the spread target, in this policy, as a function of the underlying shocks. But the central bank cannot see those shocks. How does the spread target compare to other rules in approximating the ideal response to shocks that the central bank cannot see? Clearly something about the Phillips curve makes this a sensible idea for targeting long-run inflation expectations, but not at a monthly or daily horizon. What is that something?

I phrase the spread target in the context of the fiscal theory of the price level, choosing unexpected inflation from the government debt valuation equation (9.22), because that is the point of this book. However, targeting the spread rather than the level of interest rates does not hinge on active fiscal versus active monetary policy. In place of (9.22), one could determine unexpected inflation by adding a monetary equilibrium-selection policy instead. The central bank can threaten to let the spread diverge explosively for all but one value of unexpected inflation, in classic new-Keynesian style. In place of $i_t = i_t^* + \phi(\pi_t - \pi_t^*)$, write $\pi_t^{e*} = i_t - r_t = \pi_t^* + \phi(\pi_t - \pi_t^*)$, where π_t^* is the full inflation target, i.e. obeying $\pi_t^{e*} = E_t \pi_{t+1}^*$.

Hetzel (1991) is the earliest suggestion of a spread target I am aware of. Milton Friedman mentions a spread target approvingly in Friedman (1992) (p. 229), as a way to accommodate the hands-off philosophy of money growth rules in an interest rate targeting environment. Holden (2020) presents the spread target idea, in the latter new-Keynesian context, showing that the rule $i_t = r_t + \phi\pi_t$ with $\phi > 1$ achieves a determinate price level.

9.4.2 Debt Sales with a Spread Target

Would the offer to trade real for nominal debt at fixed prices lead to explosive demands? The mechanics are a straightforward generalization of the effect that selling additional nominal debt raises the future price level. If the government offers more nominal bonds per real bond than the market offers, people will take the government's offer, thereby creating the change in debt that raises the expected price level. The offer to exchange indexed for nominal debt at a fixed rate is stable, with finite demands, and drives expected inflation to the target.

A second worry one might have about a spread peg, which implements a spread target by offering to sell real for nominal bonds at a fixed rate, is that the bond demands might explode. We need to verify that this is not the case; that the bond demands which support a spread target are well defined.

The argument is analogous to the case of an interest rate peg. We saw that by selling nominal bonds without changing the surplus, the government raises the expected future price level. We then saw that by offering bonds at a fixed nominal rate, again holding surpluses constant, people would buy just enough bonds so that the expected future price level is consistent with that nominal rate. Bond demand is well defined with a flat nominal supply curve. The mechanics of targeting expected inflation via a real for nominal debt swap is a simple extension of the same idea. In both cases, the caveat "holding surpluses constant" is key, and the hard work of institutional implementation. If people read changes in future surpluses into today's nominal bond sales, when offered in exchange for real bonds, reactions are different and an offered arbitrage opportunity could indeed explode. As in the case of nominal debt and an interest rate target, this observation offers a reason for the central bank, rather than treasury, to operate the spread target.

Start with the government debt valuation relation with both indexed and nominal debt,

$$b_{t-1} + \frac{B_{t-1}}{P_t} = E_t \sum_{j=0}^{\infty} \beta^j s_{t+j}.$$

An indexed bond pays $\$P_{t+1}$ at time $t+1$ and is worth βP_t dollars at time t. The real interest rate is constant, which hides the usefulness of the idea, but clarifies the mechanics. Express the valuation equation in terms of end of period values, after bonds are sold,

$$\beta b_t + \beta B_t E_t \left(\frac{1}{P_{t+1}} \right) = \beta b_t + Q_t \frac{B_t}{P_t} = E_t \sum_{j=1}^{\infty} \beta^j s_{t+j}. \tag{9.27}$$

If the government offers to exchange each real bond for $E_t(1/P_{t+1})$ nominal bonds, or if it exchanges real for nominal bonds at market prices, the left-hand side does not change, so the real versus nominal structure of the debt is irrelevant to the expected price level. People are indifferent at market prices.

Now let us see that selling more real and fewer nominal bonds with a trade-off different from market prices affects the future price level. Suppose the government sells $b_{0,t}$ and $B_{0,t}$ real and nominal debt, and then modifies its plan, selling P^* additional nominal bonds in return for each real bond,

$$-\left(B_t - B_{0,t}\right) = \left(b_t - b_{0,t}\right) P^*.$$

Plug into (9.27),

$$\beta\left(b_{0,t} - \frac{B_t - B_{0,t}}{P^*}\right) + \beta\left[B_{0,t} + \left(B_t - B_{0,t}\right)\right] E_t\left(\frac{1}{P_{t+1}}\right) = E_t \sum_{j=1}^{\infty} \beta^j s_{t+j}$$

$$\beta b_{0,t} + \beta B_{0,t} E_t\left(\frac{1}{P_{t+1}}\right) + \beta\left(B_t - B_{0,t}\right)\left[E_t\left(\frac{1}{P_{t+1}}\right) - \frac{1}{P^*}\right] = E_t \sum_{j=1}^{\infty} \beta^j s_{t+j}.$$

It's easiest to see the effect of exchanging real for nominal debt by taking derivatives,

$$dB_t\left[E_t\left(\frac{1}{P_{t+1}}\right) - \frac{1}{P^*}\right] + B_t d\left[E_t\left(\frac{1}{P_{t+1}}\right)\right] = 0$$

$$d\left[E_t\left(\frac{1}{P_{t+1}}\right)\right] = -\left[E_t\left(\frac{1}{P_{t+1}}\right) - \frac{1}{P^*}\right]\frac{dB_t}{B_t}.$$

If $1/P^* = E_t\left(1/P_{t+1}\right)$, then the expected price level is independent of the real/nominal split. If $1/P^* < E_t\left(1/P_{t+1}\right)$—if the government offers more nominal bonds per real bond than the market offers—then as nominal debt B_t rises, $E_t\left(1/P_{t+1}\right)$ falls, and, roughly, the expected future price level rises. The previous description of monetary policy was in effect $P^* = \infty$; the government simply increased nominal debt with no decline in real debt, and that change resulted in next-period inflation. This case is a generalization. The government sells more nominal debt, but undoes some of the dilution by taking back real debt. But if it takes back less real debt than the current market price trade-off, then increasing B_t nominal debt still lowers $E_t\left(1/P_{t+1}\right)$; that is, raises the future price level.

Now, what happens if the government offers people the option to trade real for nominal bonds at a fixed relative price? If $1/P^* < E_t\left(1/P_{t+1}\right)$, if the government gives more nominal bonds per real bond than offered by the market, it's worth exchanging a real bond for a nominal bond. But as people exchange real bonds for nominal bonds, they drive down $E_t\left(1/P_{t+1}\right)$, until $1/P^* = E_t\left(1/P_{t+1}\right)$ and the opportunity disappears. Likewise, if $1/P^* > E_t\left(1/P_{t+1}\right)$, then people will exchange nominal bonds for real bonds, driving up $E_t\left(1/P_{t+1}\right)$ until $1/P^* = E_t\left(1/P_{t+1}\right)$ again.

In sum,

- *Offering to freely exchange real debt for nominal debt at the rate P^*, while not changing surpluses, drives the expected price level to $E_t(1/P_{t+1}) = 1/P^*$.*

This operation simply generalizes offering nominal debt at a fixed nominal interest rate, without any real debt in return.

The real versus nominal debt split, even at market prices, still matters for how future unexpected inflation reacts to future shocks. The government can retain control of the real versus nominal split of its debt in equilibrium. Trades of real for nominal debt *at* the market price have no effect on the price level. In our equations, the value of $B_{0,t}$ versus $b_{0,t}$ has no effect on the price level. That the treasury sells debt via auctions, but the central bank offers fixed interest rates, makes additional sense.

9.5 A Price Level Target via Indexed Debt

If the government targets the nominal price of indexed debt, then the price level is fully determined. This target can be accomplished by a peg: Offer to freely buy and sell indexed debt at a fixed nominal price. Its operation is analogous to a gold standard, commodity standard, or foreign exchange rate peg. It offers free conversion of dollars into a valuable commodity, next period's consumption. This peg determines the price level, but real interest rate variation induces price level volatility.

Suppose the government targets the nominal price of indexed debt. Indeed, suppose the government pegs that price, committing to trade any quantity of cash or reserves for indexed debt at a fixed price. This policy can nail down the price level. It combines fiscal and monetary policy into one rule. In essence, the government runs a commodity standard, with next-period consumption being the commodity.

Here, the government pegs the *level* of the indexed bond nominal price, rather than the *spread* between indexed and nonindexed debt. The advantage is that this peg determines the price level rather than the expected future price level, and includes the fiscal commitment that fully determines that level. The disadvantage is that real interest rate variation now adds to price level volatility, unless the central bank artfully adjusts the peg; whereas the spread target nails down expected inflation, allowing the real rate to vary according to market forces.

To be concrete, a one-period indexed bond pays $\$P_{t+1}$ at time $t+1$. Maintaining the constant real rate and flexible prices, indexed bonds have real time t value β and nominal time t value βP_t. Suppose the government pegs the nominal value of such bonds at βP_t^*; that is, it says you can buy or sell indexed bonds for βP_t^* dollars at time t. Then we must have an equilibrium price level $P_t = P_t^*$. We fully determine the time t price level, not just expected inflation.

As one way to see the mechanism, note that with the peg in place buying bonds gives a real return $1/(1 + r_t) = \beta P_t^*/P_t$. If $P_t < P_t^*$, then the real interest rate is too low and the bond price is too high. At a too-low interest rate, people want to substitute from future to present consumption. More demand for consumption today is more aggregate demand, which pushes the price level up.

Specifically, suppose that the government only issues real debt. The government sells bonds b_t at nominal price βP_t^*, real bond price $q_t = \beta P_t^*/P_t$, and soaking up $\beta P_t^* b_t$ dollars. The real flow condition is

$$b_{t-1} = s_t + q_t b_t = s_t + \beta \frac{P_t^*}{P_t} b_t. \tag{9.28}$$

In our frictionless model with a constant endowment, with the opportunity to buy and sell indexed debt at the fixed nominal price, people's demands for consumption and government debt follow the first-order condition and budget constraint (with $M_t = 0$),

$$\frac{\beta P^*}{P_t} u'(c_t) = \beta E_t u'(c_{t+1}) \tag{9.29}$$

$$y + b_{t-1} = c_t + s_t + \beta \frac{P_t^*}{P_t} b_t.$$

Consider a one-period deviation from the equilibrium price level path, with $P_t \neq P_t^*$ but $P_{t+j} = P_{t+j}^*$ for all $j > 0$. Then the first-order condition (9.29) for all future time periods gives $c_{t+j} = c_{t+j+1}$, so any extra or lesser wealth is spread evenly across all future consumption. As the price level P_t falls, consumption demand c_t rises, and demand to invest in bonds $\beta\,(P_t^*/P_t)\, b_t$ and therefore bonds themselves b_t and future consumption smoothly decrease. With demand c_t greater than supply y_t, the price level must rise. Price level determination comes by equilibrium—aggregate demand equals aggregate supply—not by arbitrage. With concave utility, consumption and bond demands do not explode at off-equilibrium prices.

Thus the nominal peg of a real bond is like a gold or commodity standard, or foreign exchange peg. It pegs the dollar in terms of an imperfect substitute for the general consumption basket: gold, foreign goods, or in this case, next-period consumption. A marginal real bond gives the consumer a marginal unit of next period consumption. By selling a real bond at nominal price βP_t^*, the government allows the consumer to trade one unit of future consumption for $\beta = 1/R$ units of consumption today, at the equilibrium price $P_t = P_t^*$. If the actual price level is lower, consumption today is more attractive. In a commodity standard, the government allows the consumer to trade one unit of the commodities for P_t^* dollars and hence P_t^*/P_t units of consumption immediately. If the actual price level is lower, total consumption is more attractive, and the consumer substitutes away from the commodities in the standard to the other goods in the total basket. Doing so drives down the commodity price and up the price level until equilibrium is reestablished. Only if the commodity standard includes all consumption goods, or perfect substitutes, is it an arbitrage.

This is still fiscal theory. When the government issues additional real debt, it must promise additional surpluses to repay that debt. Money today, used to pay off today's maturing indexed debt, and soaked up by today's indexed debt issues, is automatically backed by the present value of surpluses.

The indexed debt peg continues to determine the price level in the presence of nominal debt. However, nominal debt functions differently in this regime than before. Since $P_{t+1} = P_{t+1}^*$ is set, selling more nominal debt B_t cannot raise P_{t+1}. And selling more nominal debt cannot change the current price level P_t. Thus, as the government sells more nominal debt B_t, it simply ends up selling less real debt

b_t. The split between real and nominal debt remains in the government's control.

Nominal debt still functions as a buffer, and can play an important part in price level determination. Suppose the government unexpectedly devalues at time t, raising the price level target P_t^* and therefore raising the price level P_t. This action devalues outstanding nominal debt B_{t-1}, and the present value of surpluses still declines by the amount of that devaluation. In the absence of nominal debt, the higher price level has no fiscal consequences. In return, without nominal debt outstanding, the government cannot react to an adverse fiscal shock by raising the price level target, and thereby inflating away some outstanding debt. Thus, though the decision to sell more nominal rather than real debt does not affect the current or expected future price levels, it sets up a state variable that affects the response of inflation to future shocks.

The central bank need not vanish. The government may wish to devolve debt management to the central bank. The central bank can manage the indexed debt peg, exchanging reserves for indexed debt according to the peg, as a corridor central bank pegs a nominal interest rate or as a gold standard bank exchanges cash for gold. The bank could buy and sell nominal treasury debt B_t to accommodate maturity and liquidity demands for nominal debt versus reserves. The central bank could be in charge of setting a time-varying bond price target. And when we introduce frictions to the model, the central bank may set interest rates or quantities of various kinds of debt, as central banks also set interest rates in the gold standard era. Whether all this activity is desirable is another issue, but it is certainly possible.

As we add realism to the model, this policy will not in practice completely fix the price level, for several reasons. First, the real interest rate varies over time, in ways the government is not likely to understand. (In this frictionless model, imagine variation in the endowment $\{y_t\}$.) This variation motivated the spread target above. With a fixed nominal bond price target Q^* we have

$$Q^* = \frac{1}{1+r_t} P_t^*,$$

so real interest rate variation will result in price level variation, unless the government or central bank knows the correct real interest rate and artfully changes its bond price target. Another reason for a central bank appears, the same one that holds with a nominal interest rate target. Likewise, a gold standard, commodity standard, or foreign exchange rate peg induces price level variation, unless the government artfully changes the conversion price to match the market-clearing relative price. No government tried to do so on a regular basis, leaving devaluation and revaluation for rare extreme circumstances, a fact that may reflect precommitment problems and the value of a stated peg as a commitment device. Tomorrow's consumption is likely more closely linked to today's consumption basket than are gold, foreign goods, or the sorts of baskets of commodities of such proposals, so this proposal improves on those standards. But it remains imperfect as a means for exactly targeting the price level.

Second, prices are sticky. One might think of stabilizing the actual price level by using this proposal at the highest possible frequency. Real interest rate variation from today to tomorrow is next to nothing. But obviously targeting the overnight indexed debt rate will not cause the price level today to change, because prices are sticky for a day. Intuitively, this proposal must act on a time scale in which prices are free to move. Like the spread target, that horizon is at least a year and potentially more. Thus, this proposal may end up being a long-term fiscal rule and commitment coexisting with shorter-term interest rate, spread, or other targets. But this conclusion is speculative, and needs analysis within explicit models with sticky prices. One may expect that just how prices are sticky will matter.

Third, the fiscal underpinnings are vital, as always. To see this and the last point, imagine we speed up the process to a five minute horizon. Suppose the CPI is 250, but the government wishes to hit a price level target of 200. So, for $200 you can buy a contract that pays $250 in five minutes. Buy! Now, to buy bonds you have to reduce consumption. But a five minute reduction in consumption demand is, in our world, not likely to reduce the price level from 250 to 200 in five minutes. So, the government maintains the offer. You can use the $250 to buy more bonds, that pay $312.50 in five more minutes, and so on. Something seems to be going wrong. The indexed debt peg was supposed to be soaking up money, causing disinflation, but instead money is exploding.

What's wrong? Well, in the first five minutes, the policy does soak up money in exchange for indexed debt, and that may even give some downward price level pressure in the first five minutes. Cancel dinner reservations, we're buying bonds. Each five minutes that one keeps holding and rolling over the indexed debt, one consumes less and drives down the price level. This process does in the end soak up money and keep it soaked up into indexed debt.

When the government issues more indexed debt, it also promises larger subsequent surpluses to pay off that debt. Each step in this story raises expected surpluses by just as much as the additional issuance of long-term debt. Eventually, when the price level reaches 200, the merry go round stops, and government gets to work steadily paying off the astronomical accumulated debt with astronomical surpluses. People have a lot of government debt, but also a lot of taxes to pay, so the day does not end with a bonanza in which people spend the money on nonexistent goods and services. But like any promise to deliver something real in exchange for money, like any rule promising future surpluses to retire debt, the scheme works only so long as that fiscal promise remains credible. The five minute promise would break down long before the price level declines, as the debt issue and promised surpluses would be immense.

Thus, this story at a longer horizon may describe how an indexed debt target would work with sticky prices. The price level could be persistently above target; during that period people persistently accumulate indexed debt, forcing a fiscal contraction, and slowly drive the price level back to target.

The example also suggests why one might wish to target longer-term debt in the presence of sticky prices. At a one year horizon, the offer to buy indexed debt at $200 when the price level is 250 is a $100 \times (250/200 - 1) = 25\%$ real interest rate. That's a good incentive to consume less and drive down aggregate demand. At a one day horizon, the offer is a 25% overnight return; that is, a $100 \times [(250/200)^{365} - 1] = 2.3 \times 10^{37}\%$ annualized interest rate. That offer, especially if persistent, sends consumption demand essentially to zero. Well, all the better for getting the price level down to 200 in the next five minutes. But when prices cannot move in the next five minutes, there is no point to doing so, or to force a $2.3 \times 10^{37}\%$ rise in indexed debt via intermediate payments on indexed debt.

The last few paragraphs are clearly speculative. One should develop this idea in the context of explicit price stickiness, as well as in the context of an inflation target rather than a price level target.

9.6 A CPI Standard?

> A CPI standard that mimics the gold standard, by offering instant exchange of cash for some financial contract linked to the CPI, is an intriguing idea. The spread target and indexed debt target take us halfway there.

A gold standard remains attractive in many respects: It represents a mechanical rule, embodying both fiscal and monetary commitments, that determines the price level without requiring prescient central bankers. Nostalgia for the gold standard, and even advocacy for its return, remains active in many quarters. Yet, as we have seen, the actual gold standard will not work well for a modern economy. The relative price of gold and everything else varies over time, so the gold standard leaves substantial inflation volatility. More importantly, in my view, the price of gold is poorly connected to the price of goods and services. The relative price of gold will change once it is pegged, so a new gold standard may settle down the price of gold but leave the price of goods and services unmoored. A gold price peg leads to runs and crashes.

Is there a way to have the advantages of a gold standard or currency peg, without unwanted inflation or deflation when the relative price of gold or foreign currency moves, and in a way that actually will control the price level, not just the price of the commodity? How can a government peg the consumer price index?

Most of the components of the CPI are not tradable, so the government cannot just open a huge Walmart and trade the components of the CPI for money, though it is fun to think of such a scheme. We must design a commitment that trades a dollar for some cash-settled financial contract. I use the term "CPI standard" to refer to such a scheme.

Many authors have suggested commodity standards: In return for one dollar you get a basket of short-dated, cash-settled commodity futures—wheat, pork bellies, oil, metals, and so on, or commodities that are physically traded. But the value of any commodity basket is also volatile relative to other goods and services, and they are only a bit more connected to the general price level than is the value of gold. Crude oil futures and health insurance premiums may diverge for quite a while. Given that loose connection, like gold and foreign exchange pegs, targeting commodity values might stabilize the prices of those commodities, but not have much effect on the overall price level.

One might consider an adaptation of the Modern Monetary Theory proposal for a federal jobs guarantee: Peg the price of unskilled labor at $15 per hour, by offering a job to anyone who wants it at $15 an hour and, on the margin, printing money to do so. But unskilled labor is also a small part of the economy, not well linked to the general level of prices and wages. And such a program presents obvious practical difficulties. From an inflation control point of view, the government must leave the wage at \$15 an hour in the event of stagflation, having low-skilled labor lead other prices and wages down, where the government will naturally wish to raise the wage, to help struggling people on the bottom end of the labor market. Gold mining provided a similar channel: When the price level declines, the value of time spent mining gold rises, encouraging people to trade time for creating money. But that too is an imperfect and slow mechanism, and all that work is a social waste.

One might peg the dollar to a basket of real assets, including stocks, corporate bonds, and real estate, as well as commodities. But then variation in the relative price of real assets to consumption, so-called "asset price inflation," would show up in the price level. When the real interest rate declines, long-term real asset prices rise relative to the consumption basket, much more so than the price of a one-year indexed bond.

The spread peg and indexed debt peg can be thought of as improvements in this scheme. The spread peg ties down the expected future price level, the expected future rate of exchange between dollars and the entire basket of goods. The indexed debt peg ties down today's price of next year's basket of goods. The value of next year's CPI basket is more tightly tied to the value of today's CPI basket than gold, commodities, foreign goods, and so forth.

The question remains, is there a way to peg the dollar to *today's* basket of goods via a cash-settled financial contract? I don't have the answer, but as I ponder the question I believe it has to be answered in the context of somewhat sticky prices. The CPI standard must allow actual prices to deviate from the target and move slowly toward it, without offering arbitrage opportunities that imply infinite fiscal commitments. If the government offers to trade \$250 for a CPI-linked bet that pays off today, and the CPI is 260, people will trade infinite amounts; buying at \$250, getting \$260, reinvesting, and so forth; my example of the last section speeded up to infinity.

The basic structure of the fiscal theory, and its interpretation of our current institutions, already addresses much of the commodity standard desideratum. Taxes are based on the entire bundle of goods and services, not one or a few specific goods. Thus the essential promise of the fiscal theory—bring us a dollar and we relieve you of a dollar's worth of tax liability—functions as a commodity standard weighted by the whole bundle of goods, not one particular good such as gold, and without requiring delivery of that bundle of goods.

Still, a CPI standard would be an important addition to our understanding of theoretical possibilities. Perhaps there is a better structure than the indexed debt peg or spread target.

Part III

Monetary Doctrines, Institutions, and Some Histories

Monetary theory is often characterized by doctrines, statements about the effects of policy interventions or the operation of monetary and fiscal arrangements and institutions. Examples include "Interest rate pegs are unstable," "The central bank must control the money supply to control inflation." These propositions are not tied to particular models, though many models embody standard doctrines. The doctrines pass on in a largely verbal tradition, much like military or foreign policy "doctrines," more durably than the models that embody them.

Reconsidering classic doctrines helps us to understand how fiscal theory works and matters, how fiscal theory is different from other theories, and which might be the right theory. By observational equivalence, fiscal and conventional theories can each give an account of events. But fiscal theory suggests different results of policy interventions, different sets of preconditions for different outcomes, different results of changed institutional arrangements, different views of fiscal–monetary institutions, different "doctrines." As we see the results of different policy institutions, as we organize experience on the doctrines, we can also learn which theory is right. (Ljungqvist and Sargent (2018), Section 27.3, also list 10 related monetary "doctrines.")

Experience is ripping out the underpinnings of classic doctrines, and thereby putting them to the test. The distinction between "money" and "bonds" is vanishing, undermined by rampant financial innovation. Money pays interest. Central banks target interest rates, not monetary aggregates. Interest rates were stuck near zero for most of a decade in the United States, more than a decade in the European Union, and nearly a quarter century in Japan, yet inflation remained quiet. Under Quantiative Easing, central banks undertook open market operations thousands of times larger than ever contemplated before, with no effect on inflation. The clash of doctrines in such events can provide nearly experimental, or cross-regime, evidence on fiscal versus classic theories of inflation.

This part contrasts core doctrines under the fiscal theory with their nature under classic monetary theory, in which the price level is determined by money demand $MV = Py$ and control of the money supply, and under interest rate targeting theory, in which the price level is determined by an active interest rate policy. I develop those alternative theories in detail in later chapters. However, since the point now is to understand what the fiscal theory says rather than to understand those alternative theories in detail, and since these doctrines are likely familiar to most readers and stand apart from specific models, we can proceed now to discuss classic doctrines and later fill in details of models that capture alternative theories.

Chapter Eleven

Monetary Policies

I START with doctrines surrounding monetary policies, in the traditional sense of the word: operations that affect the supply of money.

11.1 The Composition versus the Level of Government Debt

> Monetarism states that $MV = Py$ and control of money M sets the price level. Surpluses must then adjust to satisfy the government debt valuation equation. The split of government liabilities between debt B and money M determines the price level, and must be controlled.
>
> Fiscal theory states that the overall quantity of government liabilities relative to surpluses sets the price level, and the split between M and B is irrelevant to a first approximation. The split must passively accommodate money demand. Fiscal theory rehabilitates a wide swath of passive money policies and institutions, which we observe along with quiet inflation.
>
> An exception to the rule: If, as hypothesized, the central bank is set up to issue debt that will not be repaid, and the treasury debt is repaid, then the split between central bank and treasury issuance is central to the price level. However, this split has nothing to do with the monetary nature of central bank liabilities.

The monetarist tradition states that $MV = Py$ sets the price level P. The *split* of government liabilities M versus B determines the price level, because only the M part causes inflation. This theory requires a money demand—an inventory demand for special liquid assets, a reason M is different from B—and also a restricted supply of money. Monetarist tradition emphasizes that this split must not be passive, responding to the price level, or the price level becomes unmoored.

In this view, fiscal policy must be passive, adjusting surpluses to pay off unexpected inflation- or deflation-induced changes in the value of government debt. "Passive" fiscal policy is not always easy. Many inflations occur when governments cannot raise surpluses and instead print money to repay debts or to finance deficits. Monetarist thought recognizes that monetary–fiscal coordination is important, and that monetary authorities must have the fiscal space to abstain from repaying debts or financing deficits by printing money. The "passive" word comes from the fiscal theory tradition; monetarists use words like monetary–fiscal coordination, or fiscal support for monetary policy.

In the fiscal theory, the total quantity of government liabilities $M + B$ matters for the price level. The split of government liabilities between M and B, to first order, is irrelevant. If people have a demand for money as distinct from bonds due to liquidity, transactions, or other reasons, supply of that money must be passive. Fiscal policy must be active, refusing to adjust surpluses to changes in the value of government debt that flow from any arbitrary change in the price level.

So, fiscal theory rehabilitates passive money policies. Passive money comes in many guises. The following sections illustrate a variety of passive money policies and institutions that have been followed in the past, or are followed or considered now, and that are critiqued as undermining price level stability. The fact that inflation has often been quiet under passive monetary policies and institutions is a point in favor of fiscal theory.

A few equations help to make this and the following discussion concrete. The simple fiscal theory with one period debt, interest-paying money and a constant discount rate from Section 3.4 states

$$\frac{B_{t-1}+M_{t-1}}{P_t} = E_t \sum_{j=0}^{\infty} \beta^j \left[s_{t+j} + \frac{i_{t+j}-i^m_{t+j}}{\left(1+i_{t+j}\right)\left(1+i^m_{t+j}\right)} \frac{M_{t+j}}{P_{t+j}} \right]. \tag{11.1}$$

Add a money demand function,

$$M_t V(i_t - i^m_t, \cdot) = P_t y_t. \tag{11.2}$$

To first order, ignore seigniorage, with $i - i^m$ small, or imagine a fiscal policy that changes surpluses to account for seigniorage. In monetarist thought, control of M and $MV = Py$ determines P in (11.2), and then surpluses s must adjust to validate any changes in the price level in (11.1). In fiscal theory, the government debt valuation equation (11.1) sets the price level, and then monetary policy must "passively" accommodate the money demand requirement in (11.2).

11.2 Open Market Operations

Classic doctrine: Open market purchases lower interest rates and then raise inflation. The composition, not quantity, of government debt matters for inflation.

Fiscal theory: Open market operations have no first-order effect on the price level or interest rates. The composition of government debt (B versus M) is irrelevant to first order. Differences in liquidity lead to interest rate spreads between various kinds of debt.

Seigniorage and liquidity demands, and the effects of changing the maturity structure of debt, add second-order effects to fiscal theory. An open market operation with no other change in policy is not a well-posed question. Observational equivalence reminds us that empirical evaluation of open market operations will not settle the issue.

The open market operation is the textbook instrument of classical monetary policy. The central bank buys government bonds, issuing new money in return, or vice versa. It is a change in the *composition* of government debt, which does not change the overall quantity of government debt. By increasing the supply of money M, an open market operation is inflationary in standard monetarist thought.

Since $M + B$ appears on the left-hand side of the government debt valuation equation, to first order, an open market operation swapping M for B has no effect in fiscal theory. Think of money as green M&Ms, and debt as red M&Ms. If the Fed takes some red M&Ms and gives you green M&Ms in return, this has no effect on your diet. To monetarists, only the green M&Ms have calories.

Now, there is potentially an important exception to this rule. Early on, I hypothesized that central banks are set up distinctly from treasuries, to signal "share split" debt issuance, without change in surpluses versus "equity issue" debt issuance that comes with the expectation and reputation of future surpluses, if not an explicit promise. Below, I speculate that the 2021 fiscal expansion was inflationary in part because so much of it came from newly created reserves.

If this is the case, a change in the composition of government debt from treasury debt to central bank reserves is inflationary. However, that effect comes entirely from different *fiscal* foundations and commitments of central bank reserves versus treasury debt, not from their liquidity, money-like nature, status in the banking system, convertibility to cash, and so forth. Central bank versus treasury debt becomes regular versus emergency budget debt, unbacked versus backed debt. In this analysis, an open market operation, buying treasury debt and issuing reserves, converts some debt from backed to unbacked. However, as in our analysis of QE, the inflationary effect of such reserves depends on whether people expect the central bank to undo reserve issuance, by selling treasurys in its portfolio and converting reserves back to backed debt. Pointing to 2021 is too easy, as we need a story why 2021 was inflationary and QE was not.

In the monetarist view, any effect of monetary policy comes entirely from the quantity of money. The fact that the bond supply B decreases in an open market operation is irrelevant. In particular, if an open market sale (less M, more B) raises interest rates, that rise comes from an interest-elastic money demand $MV(i) = Py$, not from greater bond supply. A helicopter drop of more M with no decline in B has the same effect as an open market operation in which B declines.

Bond supply ideas are often used to analyze quantitative easing. That view is centered on frictions such as segmented bond markets, which are not part of the traditional monetarist view. The bond supply channel turns monetarism on its head, viewing the increase in interest-paying reserves as irrelevant. The central bank is thought to lower long-term interest rates by "removing duration" from the bond market. This bond supply view also requires purchases that are a nonnegligible fraction of the bond supply. Open market operations were traditionally tiny. Before 2008, total reserves were on the order of \$10 billion dollars, open market purchases an order of magnitude smaller, and all of this a drop in the ocean of bond supply. Quantitative easing operations are thousands of times larger.

The simple fiscal theory of monetary policy has a bond supply channel, but it is entirely a *nominal* bond supply channel. The central bank and treasury together sell bonds with no change in surpluses, to raise expected inflation. This is a frictionless model with no segmentation, only changing real rates if there is some stickiness to prices.

I hedge these statements with “first order” to acknowledge several second-order possibilities and other caveats.

These statements are clearest when money pays full interest $i_t = i_t^m$, or interest rates are zero. When there is an interest spread, an open market operation creates seigniorage on the right-hand side of the valuation equation (11.1), which can affect the price level. I argued that seigniorage is tiny for advanced economies in normal times. Fiscal policy may also adapt, offsetting the $i - i^m$ term in (11.1), without becoming passive. But seigniorage is not small when governments are financing large deficits by printing non-interest-bearing money, and we should include this channel when thinking about large fiscal inflations.

Seigniorage effects can go either way. If the interest elasticity of money demand is low, as in the monetarist tradition, raising future M adds future seigniorage revenue, which *lowers* the initial price level. It trades future inflation for current deflation. But once past the hyperinflationary point on the inflation-tax Laffer curve, raising M lowers seigniorage revenue and creates both current and future inflation.

An open market operation also changes the maturity structure of government debt. The analysis of maturity structure rearrangements from Chapter 7 applies. This consideration was minor with the small open market operations of the small reserves regime, but the trillions of dollars of quantitative easing asset purchases substantially shorten the maturity structure of government debt. The Treasury could offset these changes by issuing longer debt, or engaging in swap contracts. The Treasury and Federal Reserve need to come to a new accord about who is in charge of the maturity structure and hence interest rate risk exposure of the debt. It seems a bit dysfunctional that the Treasury issues long-term debt to lengthen the maturity structure, and the Fed quickly snaps that debt up and turns in into overnight debt.

Today, almost all money other than cash pays interest. With $MV(i-i^m) = Py$, an exchange of B for M can result in a change in the interest rate paid to money i^m, with little effect on anything else. Then velocity takes up the slack of an open-market operation. In the old days with $i^m = 0$, the interest rate on everything else had to change in order to satisfy money demand.

With $MV(i - i^m) = Py$ describing money demand, we can satisfy the money demand curve by letting the interest rate spread $i - i^m$ vary; we do not need “passive” supply of money itself. This statement includes a substantial generalization of what “passive” monetary policy means for fiscal theory. The price of money can be passive rather than the quantity of money.

More generally, variation in the composition of government debt of varying liquidity, including reserves, on the run versus off the run, treasury versus agency, high or low coupon issues, and so forth, can just result in a change in interest rate spreads between the various flavors of government debt, with no effect on the underlying interest rate i that governs intertemporal substitution and is connected to inflation. The central bank can control the relative quantities of money, liquid, and illiquid bonds, if it wishes to do so, even in fiscal theory.

Short-run endogenous velocity and a fuzziness to the money demand function is a more general possibility. Even a die-hard monetarist would not predict from $M_tV = P_ty_t$ that if the money supply increases at noon on Monday, nominal GDP must rise proportionally Monday afternoon. There are "long and variable lags." Velocity is only "stable" in a "long run." Short-run elasticities are different than long-run elasticities. It takes a while for people to adjust their cash management habits in response to changes in the interest costs of holding money. If the Fed buys bonds, even in a fiscal theory world, it's sensible that people just hold the extra money for a while, and velocity (a residual) moves. The pressures from money supply greater than money demand can take months or even years to appear. This endogenous velocity result is even more likely when the interest cost of holding money is small, and when money and bonds become nearly perfect substitutes. The requirement to satisfy money demand under fiscal theory can be elastic. (These thoughts are formalized in Akerlof and Milbourne (1980) and Cochrane (1989).)

We cannot easily settle which theory holds by estimating the effect of open market operations, however. Observational equivalence tells us that must be the case. Specifically, if a rigid money demand relation $M_tV = P_ty_t$ applies, an open market exchange of M_t for B_t that changes nothing else—neither surpluses nor overall quantity of debt—is not a well-posed policy. In the monetarist view, pay attention to the footnote about passive fiscal policy or fiscal coordination. An open market operation that reduces inflation must come with a fiscal contraction to pay off the larger value of debt. If that monetary fiscal coordination does not happen, we have uncoordinated or overdetermined policy, equations that contradict each other. In a fiscal theory view, the government must adopt a passive monetary policy to ensure $MV = Py$ is satisfied, so changing M versus B without changing anything else likewise makes no sense. To accomplish the open market operation, the government must tighten fiscal policy. Then the "passive" monetary policy exchanges some M for B. Both theories describe an identical fall in money M, rise in debt B, and a rise in surpluses s. Time series tests will not tell them apart.

11.3 An Elastic Currency

> Classic doctrine: Elastic money supply leaves an indeterminate price level, so it leads to unstable inflation or deflation.
>
> Fiscal theory doctrine: Elastic money supply is consistent with and indeed necessary for a determinate price level.

Suppose monetary policy offers the split between bonds and money passively: The central bank assesses Py, and issues the appropriate M in response. It responds to perceived "tightness" in money and credit markets, or to its perception of how much money people and businesses demand. It provides an "elastic currency" to "meet the needs of trade."

From a monetarist perspective, you can see the flaw. If the price level starts to rise, the central bank issues more money, the price level keeps rising, and so forth. Any P is consistent with this policy. The central bank must control the quantity of monetary aggregates.

Yet even the title of the 1913 Federal Reserve Act states that the Fed's first purpose is to "furnish an elastic currency." Congress mandated passive money supply. The price level was considered to be determined at least in the long run by the gold standard, not by the Fed. The Act does not task the Fed with controlling inflation or the price level at all. It was viewed that banks, private debt markets, and the Treasury's currency issues did not sufficiently adjust money supply to match demand. There were strong seasonal fluctuations in interest rates (Mankiw and Miron (1991)) as around harvest time, and a perceived periodic and regional scarcity of money. Financial crises smelled of a lack of money then as now. The Fed was founded largely in response to the 1907 financial crisis. So, the Fed's main directive was to supply money as needed.

Monetarists acknowledge that money supply should accommodate supply-based changes in real income y, so that higher output need not cause deflation. Money supply should also accommodate shifts in money demand—shifts in velocity V—rather than force those to cause inflation, deflation or output fluctuation. The central bank should and does accommodate seasonal variation in money demand around Christmas and April 15. The trouble is as always to distinguish just where a rise in money demand comes from; for the Fed to react to the "right" shifts deriving from real income, seasonals, and panics, but not to the "wrong" shifts in money demand that result from higher inflation or expected inflation, or, in the conventional view, "excess" aggregate demand, "inflationary pressures," and so forth. Milton Friedman argued for a 4% money growth rule not because it is full-information optimal, but because he thought the Fed could not distinguish shocks in this way, or implement an activist strategy in real time.

Fiscal theory frees us from this conundrum. The price level is fixed by fiscal surpluses and the overall supply of government debt, the latter either directly or via an interest rate target. A passive policy regarding the split of the *composition* of government debt between reserves and treasury securities does not lead to inflation.

11.4 Balance Sheet Control

Should central banks control the size of their balance sheets? Or should they allow banks and other financial institutions to trade securities versus reserves at will?

Conventional doctrine: The central bank must control the size of its balance sheet, or it will lose control of the price level.

Fiscal theory doctrine: The central bank may offer a flat supply of reserves, and any size balance sheet, with no danger of inflation. Such a policy can be desirable, as it implements passive money without conscious intervention.

Contemporary central bank doctrine: Central banks think that balance sheet size matters, though not through traditional monetary channels. The Fed controls that size, and nature of assets, as well as target interest rates by the rate it pays on excess reserves.

The Federal Reserve balance sheet contains Treasury and other securities (mostly mortgage-backed securities) as assets, and the monetary base equal to reserves plus cash, as liabilities. Open market and quantitative easing operations increase the size of the balance sheet. The "size of the balance sheet" is often used as a synonym for the stimulative stance of monetary policy. The word choice is interesting for focusing attention on how many and which assets the central bank holds, rather than just the liability side, i.e. monetary base, or the broader money supply.

Should central banks control the size of their balance sheets, offering a vertical supply of reserves, holding a fixed quantity of assets for long periods of time, and using the size of the balance sheet as a policy instrument, distinct from the level of the nominal rate? Or should central banks offer a horizontal supply of reserves, letting people freely trade Treasury or other qualifying debt for reserves, borrow reserves against specified collateral, or lend to the central banks, holding reserves, each at fixed rates?

The conventional monetarist answer is that the central bank must control the size of its balance sheet, or risk inflation. If anyone can bring a Treasury security in and get money, then the money supply—the split between M and B—is not controlled.

In fiscal theory, the central bank can open its balance sheet completely. The split of government liabilities between reserves and treasurys in private hands has no effect on the price level. A flat reserve supply easily achieves the passive money that a fiscal regime requires.

A flat reserve supply and a passive balance sheet solve the primary practical problem with my description of elastic currency: How does the central bank know it should supply more or less money? By allowing people (financial institutions) to get money any time they need it, in exchange for Treasury debt, the central bank accomplishes mechanically the passive money that must accompany the fiscal theory: It "provides an elastic currency," to "meet the needs of trade," without itself having to measure the sources of velocity, the split of nominal income between real and inflation, or to decide on open market operations.

Balance sheet control has been a central part of Federal Reserve policy for a long time. Before the 2008 move to interest on reserves, the New York Fed's trading desk tried to forecast each morning how many reserves were needed to hit the interest rate target for that day, supplied those via open market operations, and then closed up shop for the day. There were often interest rate spikes later in the day if banks turned out to need more reserves than had been supplied (Hamilton (1996)). During the day, at least, the supply curve was vertical. Over horizons longer than a day, however, hitting an interest rate target required adjusting the size of the balance sheet, effectively providing a flat supply curve. Just why the Fed thought this necessary, and why many economists yearn for a return to these operating procedures, is unclear to me. There is something faintly monetarist about it; controlling the money supply, if only for a day.

Other central banks have followed a corridor system since the 1990s, lending and borrowing throughout the day at fixed rates and thus leaving the size of the balance sheet open. I see no evidence that the corridor system led to less control over interest rates or the economy.

In 2008 the Fed started paying interest on reserves, and using interest on reserves as its main tool for setting interest rates. The Fed soon exploded reserves from $10 billion to trillions of dollars in quantitative easing operations. Yet, though the size of the balance sheet no longer has anything to do with controlling interest rates, the Fed has also maintained strict control over the total size of the balance sheet, with assets that do not change at all for long periods of time, and grow or decline linearly when they do change. The Fed raises and lowers the balance sheet by trillions of dollars in the belief, echoed widely on Wall Street, that such changes stimulate or cool the economy. Some of the feeling is related to the idea that the Fed thereby affects financial market risk premiums.

Fixing both a price (interest rate) and a quantity is tricky. Unlike most price pegs, it's possible. See Section 2.9 and Cochrane (2014b). Why try? As with the earlier operating procedures, some Cheshire-cat residual monetarism remains, I think, in central bankers' doctrines: a view that a large balance sheet is permanently stimulative by itself, for the same level of interest rates, even if reserves are held in superabundance compared to reserve requirements and other regulations, and even if reserves pay *more* interest than short-term Treasurys, as they frequently did. The Fed's view of "stimulus" seems to combine the interest on reserves and federal funds, a direct effect of the quantity and the nature (maturity, Treasury versus mortgage versus commercial paper etc.) of balance-sheet assets, and "forward guidance" speeches about future intentions with regard to all of the above. From 2018 to 2020, the Fed deliberately reduced the size of the balance sheet and reserves, eventually provoking a resurgence of spikes in overnight rates, a characteristic of the earlier daily fixed supply regime (Hamilton (1996)), as new liquidity regulations started to bite (Copeland, Duffie, and Yang (2021), Gagnon and Sack (2019)). Opening up the discount window, or a standing repo facility that would allow banks to immediately get reserves, would quiet those spikes. Fiscal theory says this sort of policy poses no danger for price level control.

11.5 Real Bills

> The real bills doctrine states that central banks should lend freely against high-quality private credit.
>
> Classic doctrine: A real bills policy leads to an uncontrolled price level.
>
> Fiscal theory doctrine: A real bills policy is consistent with a determinate price level.

The real bills doctrine states that central banks should lend money freely against high-quality private credit. Bring in a "real bill," private short-term debt, either as collateral or to sell to the central bank, and the central bank will give you a new dollar in return, expanding the money supply. The Federal Reserve Act's second clause says "to afford means of rediscounting commercial paper," essentially commanding a real bills policy, though the Fed does not now follow such a policy. The classic doctrine specified private rather than government debt. Thus it combines two separate ideas, backing by real assets, and a flat supply curve or open balance sheet. The classic doctrine also distinguished between bills that finance "real" production versus bills that finance "speculation."

A real bills doctrine endogenizes the money supply as well, so in classic monetarist thought it therefore destabilizes the price level. As P rises, people need more M. They bring in more real bills to get it, and M chases Py.

Under the fiscal theory, the price level is determined with a real bills doctrine. If the central bank accepts private "real bills" in return for new M, that action expands total government liabilities on the left side of the valuation equation, but it equally expands assets on the right-hand side of the valuation equation, either directly or in the stream of dividends such assets provide.

Real bills' force for price stability is strong, because real bills are salable assets. If people don't want the money any more, they can have the real bills back. The government need not tax or borrow against future surpluses to soak up extra money. "Real" bills are not typically indexed, so this is not a "pot of assets" regime in which assets = liabilities determines the price level. But real bills insulate the backing of money for liquidity provision from government finances. The real bills mechanism is usually thought of as a way to trade something less liquid, the real bill, for something more liquid, government money. But it is also a way to replace a government liability, backed by the government's willingness and ability to tax or abstain from spending, for a private liability, backed by real assets, a stream of private cash flows, and legal contract enforcement.

In the fiscal theory, a real bills doctrine can be a desirable policy, as it is one way to automatically provide the passive money that fiscal price determination requires. It is especially useful in a situation in which there is little treasury debt outstanding, so that providing needed monetary base is difficult by a similar promise to exchange treasury debt for money. That is not our current situation, but government debt was not so large in the early 1900s and not so liquid. Perhaps someday we will return to a small amount of government debt, as appeared briefly possible in the late 1990s. Real bills may also be useful to isolate money from government finance if government debt threatens default.

The real bills doctrine raises many issues beyond inflation control. Private debt has credit risk, which raises financial stability, political, and economic questions. Whether the central bank or treasury takes the credit risk is unimportant for the rest of the economy but important for the political independence of the central bank. Today, the Fed typically buys or lends against private securities in a special purpose vehicle in which the Treasury takes the first losses.

Much motivation for real bills purchases or direct central bank lending to private institutions and people concerns the supply of credit and avoiding financial panics, flights from risky securities to government debt. Since 2008, the Federal Reserve and other central banks have expanded their assets beyond Treasurys to include agency securities, mortgage-backed securities, state and local government debt in the United States, member state debt in Europe, commercial paper, corporate bonds, stocks, "toxic assets," and "green" bonds. The Fed typically offers to buy or to lend against collateral a limited though large amount. Up to that amount, it looks a lot like a real-bills policy.

Central bank purchases are aimed to prop up the prices of those assets, and to encourage borrowers to issue such assets so those borrowers can make real investments, not to increase the supply of reserves, which today could easily be accomplished by buying or lending against some of the immense supply of Treasurys. Such central bank purchases of private and nonfederal government securities can also easily cross the line to bailouts, price guarantees, and subsidized central bank financing of low-value and politically favored investments. This only risks *inflation* if the central bank overpays, but the practice has obvious risks and benefits from other points of view. A central bank may well wish to insulate itself against moral hazard and malfeasance by announcing a fixed quantity of such operations rather than an unlimited flat supply curve.

Chapter Twelve

Interest Rate Targets

Central banks today do not stimulate or cool the economy by increasing or decreasing the monetary base or monetary aggregates. Most central banks follow interest rate targets. Interest rate pegs or targets that vary less than one for one with inflation are criticized by traditional doctrine, as letting inflation get out of control. The fiscal theory allows pegs or sluggish targets. That fact opens the door to analyzing many periods in which we observe poorly reactive interest rate targets, including zero bound periods.

12.1 Interest Rate Pegs

> Classical doctrine: An interest rate peg is either *unstable*, leading to spiraling inflation or deflation, or *indeterminate*, leading to multiple equilibria and excessively volatile inflation.
>
> Fiscal theory: An interest rate peg can be stable, determinate, and quiet (the opposite of volatile).

An interest rate peg is another form of passive money supply, that standard monetary theory has long held leads to a loss of price level control.

First, as crystallized by Friedman (1968), an interest rate peg is thought to lead to *unstable* inflation. In "What Monetary Policy Cannot Do," the first item on Friedman's list is "It cannot peg interest rates for more than very limited periods." (By "peg" in this context, Friedman means a target that is constant over time, not necessarily an offer to buy and sell bonds at a fixed price or to borrow and lend at a fixed interest rate.)

Friedman starts from the Fisher relationship $i_t = r_t + \pi_t^e$, where π_t^e represents expected inflation. One of the two great neutrality propositions of his paper is that the real interest rate is independent of inflation in the long run. (The other proposition is that the unemployment rate is also independent of inflation in the long run.) Thus, higher nominal interest rates must eventually correspond to higher inflation.

But to Friedman, this Fisher equation describes an unstable steady state. The central bank cannot fix the nominal interest rate i_t and expect inflation to follow. Instead, if, say, the interest rate peg i_t is a bit too low, the central bank will need to expand the money supply to keep the interest rate low. More money will lead to more inflation, and more expected inflation. Now the peg will demand an even lower real interest rate. The central bank will need to print even more money to keep down the nominal rate. In Friedman's description, this chain does not spiral out of control only because the central bank is not that pig-headed. Eventually, the central bank abandons the low interest rate peg, bringing back the Fisher equation at a higher level of interest rate and inflation. Yes, inflation and interest rates move together in the long run, but like balancing a broom upside down, the central bank cannot just move interest rates and count on inflation to follow.

Friedman's prediction comes clearly from adaptive expectations:

> Let the higher rate of monetary growth produce rising prices, and let the public come to expect that prices will continue to rise. Borrowers will then be willing to pay and lenders will then demand higher interest rates—as Irving Fisher pointed out decades ago. This price expectation effect is slow to develop and also slow to disappear. (Friedman (1968), p. 5–6.)

Standard IS-LM thinking with adaptive expectations gives the same result, though through a different mechanism that de-emphasizes the money supply. In that view, the real interest rate directly affects aggregate demand. So a too-low nominal rate implies a too-low real rate. This low rate spurs aggregate demand, which produces more inflation. When expectations catch up, the real rate is lower still, and off we go. Section 17.3.2 models these views with simple equations and a graph.

These views predict an uncontrollable deflation spiral when interest rates are effectively pegged by the zero bound. Such a spiral was widely predicted and widely feared in 2008 and following years, correctly following the logic of these views. The spiral did not happen.

When rational expectations came along, a different problem with interest rate pegs emerged, as crystallized by Sargent and Wallace (1975). An interest rate peg leads to *indeterminate* inflation. Under rational expectations, expected inflation is $\pi_t^e = E_t\pi_{t+1}$. The Fisher equation $i_t = E_t\pi_{t+1}$ is stable: If the central bank pegs the interest rate i, then $E_t\pi_{t+1}$ settles down to $i - r$ all on its own. With sticky prices the real interest rate r may move for a while, but the real interest rate eventually reverts, and inflation follows the nominal interest rate. However, unexpected inflation $\Delta E_{t+1}\pi_{t+1}$ can be anything; it is indeterminate.

Though indeterminacy means that the model has nothing to say about unexpected inflation, most authors writing about such policies such as Clarida, Galí, and Gertler (2000) and Benhabib, Schmitt-Grohé, and Uribe (2002) equate indeterminacy with excess inflation *volatility*, as unexpected inflation jumps around following sunspots or some other economically irrelevant coordination mechanism. The difference between stability, volatility, and determinacy is subtle, and not all authors use the words as I do here.

Indeterminacy counts as a "doctrine" as it is a robust characterization of many models. But most central bankers and commenters continue to think in old-Keynesian or monetarist adaptive-expectations terms, and don't worry about multiple equilibria and sunspots. So indeterminacy can only be said to be a doctrine among modelers who really understand rational expectations.

The fiscal theory of monetary policy contradicts these doctrines. An interest rate peg can leave the price level and inflation stable, determinate, and quiet (the opposite of volatile). Even a peg at zero interest rate can work. If the economy demands a positive real rate of interest, a slight deflation would emerge to produce it.

The classic doctrines are not logically wrong, they just make an opposite assumption. They explicitly or implicitly assume passive fiscal policy—that the government will adapt surpluses to unexpected re-valuations of nominal debt due to inflation or deflation. Active fiscal policy cuts off this possibility.

Stability and determinacy at an interest rate peg is clearest when we marry fiscal theory with a rational expectations model of the economy, as I mostly do in this book. The rational expectations part gives stability, and fiscal theory adds determinacy. Yet stability at an interest rate peg is a troublesome doctrine, contradicting much intuition. It leads to the Fisherian proposition that raising nominal interest rates must eventually raise inflation. All that comes from the rational expectations part of the model, not from fiscal theory per se.

One can also marry fiscal theory with adaptive expectations models. Fiscal theory can help to cut off inflation and deflation spirals in those models. A deflation spiral requires fiscal austerity to pay a real windfall to bondholders in those models as well. No austerity, no spiral. I don't pursue fiscal theory with adaptive expectations in this book. I stop here to emphasize the "can" in my restatement of the doctrine. It is clearest with rational expectations, and worked out here. It is suggestive with adaptive expectations, but needs working out.

That inflation has been quiet despite long periods of constant near-zero interest rates in the United States, Europe, and Japan is a feather in the fiscal cap, and the sort of observation that helps us to surmount observational equivalence questions.

I emphasize "can" here, because a stable, determinate, and quiet peg requires fiscal policy as well as the interest rate peg. Countries with unsustainable deficits cannot just lower interest rates and expect inflation to follow! Countries with volatile fiscal policies, or who suffer volatile discount rates, will see volatile unexpected inflation under a peg.

Though a peg is *possible*, a peg is not necessarily *optimal*. Under a peg, variation in the real rate of interest r_t, due to variation in the marginal product of capital, for example, must express itself in varying expected inflation. When prices are sticky, such variation in expected and therefore actual inflation will produce unnecessary output and employment volatility. A central bank that could assess variation in the natural rate r_t, and raise and lower the nominal interest rate in response to such real interest rate variation could produce quieter inflation and by consequence quieter output. We have also seen that varying the nominal interest rate in response to output and inflation can help to smooth fiscal and other shocks. Of course, a central bank that is not very good at measuring variation in the natural rate may induce extra volatility by mistimed stabilization efforts. So the case for a peg is something like Milton Friedman's case for a 4% money growth rule—not full-information optimal, but a robust strategy for a controller with limited information or decision-making ability. My previous suggestion to peg the spread rather than the level of nominal rates addresses some of these concerns.

12.2 Taylor Rules

> The Taylor principle states that interest rates should vary more than one for one with inflation.
>
> Conventional doctrine: Interest rates must follow the Taylor principle, or inflation will become unstable or indeterminate, and therefore volatile.
>
> Fiscal theory: Inflation can be stable and determinate when interest rates violate the Taylor principle, as they are under a peg.
>
> A strong reaction of interest rates to inflation may nonetheless remain wise policy, exploiting a negative short-run inflationary effect.

Beginning in the 1980s, academics started to take seriously the fact that central banks control interest rates, not money supplies, and theories about desirable interest rate targets emerged. The Taylor principle is the most central doctrine to emerge from the experience of the early 1980s and this investigation: Interest rates should vary more than one for one with inflation. An interest rate rule that follows the Taylor principle cures instability in adaptive expectation, IS-LM, old-Keynesian models and it is thought to cure indeterminacy in rational expectation new-Keynesian models. (I write "thought to" because I take issue with that claim below.)

So, standard doctrine now states that interest rate targets should vary more than one for one with inflation. If it does not do so, instability (adaptive expectations) or indeterminacy (rational expectations) will result, and inflation will be volatile.

Fiscal theory contradicts this doctrine. Insufficiently reactive interest rates, like a peg, can leave stable and determinate, hence quiet, inflation.

The fiscal theory doctrine is helpful for us to address the many times in which interest rate targets evidently did not move more than one for one with inflation, including the recent zero bound period, the 1970s, the postwar interest rate pegs, and interest rate pegs or passive policy under the gold standard.

Still, an interest rate target that follows something like a Taylor rule, raising interest rates when inflation or output rise, can be a good policy even in an active fiscal, passive money regime. A Taylor-type rule can implement the idea that the central bank should raise the nominal rate when the "natural rate" is higher. As the natural interest rate, output, and inflation all move together, we are likely to see nominal interest rates that rise with output and inflation. Taylor-like responses to output and inflation smooth shocks, leading to smaller output and inflation variance than we would otherwise see.

In the simple models we have examined, under fiscal theory the response of interest rate to inflation should be less than one for one, "passive." That seems to contradict the view that sometimes, as in the early 1980s, interest rates rise more than one for one with inflation. In the simple fiscal theory model, with $i_t = \phi\pi_t$ and $i_t = E_t\pi_{t+1}$, equilibrium is $E_t\pi_{t+1} = \phi\pi_t$. If $\phi > 1$, inflation is determinate but unstable.

First, regression evidence is actually not that clear. Second, the greater than one-for-one response of the Taylor principle is not visible in equilibrium of new-Keynesian models, as it represents an off-equilibrium threat. Regression estimates, which relate observed equilibrium quantities, should show a coefficient less than one. (Equations are in Section 16.5.) Third, interest rates may rise more than one for one with inflation, even in a well-run, active fiscal, passive money regime. The key property is the number of system eigenvalues greater or less than one. What matters, here, is the ϕ in $E_t\pi_{t+1} = \phi\pi_t$, not the ϕ in $i_t = \phi\pi_t$. The response of interest rates to inflation does not generically control that eigenvalue. It does so in the simple model of the last paragraph. The simple sticky price model we have studied is also Fisherian. But in more complex models, the eigenvalue can be stable while the coefficient of interest rate on inflation is greater than one. What matters in the new-Keynesian view is that the central bank commits to explode *inflation* for all but one initial value of inflation. The Taylor principle is a means to that end in some models, but not always.

One of Taylor's central and robust points is the advantage of rules—any rules—over the shoot-from-the-hip discretion that characterizes much monetary policy. Rules help to stabilize expectations, reducing economic volatility.

Chapter Thirteen

Monetary Institutions

If the price level is determined ultimately by the intersection of money supply and demand, the government must engage in a certain amount of financial repression. It must ensure a substantial demand for money, and it must limit money supply. It must control the creation of inside money, it must regulate the use of substitutes including foreign currency or cryptocurrency, it must restrict financial innovation that would otherwise reduce or destabilize the demand for money, it must maintain an artificial illiquidity of bonds and other financial assets lest they become money, it must forbid the payment of interest on money, and it must stay away from zero interest rates. None of these restrictions are necessary with fiscal price determination.

It must ensure a substantial demand for money, and it must limit money supply. It must control the creation of inside money, regulate the use of substitutes including foreign currency or cryptocurrency, restrict financial innovation that would otherwise reduce or destabilize the demand for money, maintain an artificial illiquidity of bonds and other financial assets lest they become money, forbid the payment of interest on money, and it must stay away from zero interest rates. None of these restrictions are necessary with fiscal price determination.

13.1 Inside Money

> Classic doctrine: The government must control the quantity of inside money or the price level becomes indeterminate.
>
> Fiscal theory doctrine: The price level can remain determinate with arbitrary creation of inside moneys. Reserve requirements and limitations on the creation of liquid inside assets are not needed.
>
> Reserve requirements and restrictions on the private issuance of liquid short-term debt remain useful for the separate question of financial stability, preventing runs.

Currency and reserves are not the only assets that people can use for transactions and other money-related activities. Checking accounts are the easiest example of inside money. When a bank makes a loan, it flips a switch and creates money in a checking account.

More generally, short-term debt can circulate as money. If I write an IOU, say "I'll pay you back $5 next Friday," you might be able to trade that IOU to a friend for a beer this afternoon, and your friend collects from me. Nineteenth century banks issued notes. Commercial paper and other short-term debts have long been used in this way, essentially writing a tradable IOU. Money market funds offer money-like assets, backed by portfolios of securities. Inside money can help to satisfy the transactions, precautionary, liquidity, etc. demands that make "money" a special asset.

Recognizing this fact, we should write money demand as

$$(Mb + Mi)\, V = Py,$$

distinguishing between the monetary base Mb and inside money Mi. More sophisticated treatments recognize that liquid assets are imperfect substitutes for money rather than simply add them together.

Again, the monetarist view determines the price level from the intersection of such a money demand with a limited supply. To that end, it is not enough to limit the supply of the monetary base Mb. The government must also limit the supply of inside money Mi. Reserve requirements are a classic supply-limiting device. To create a dollar in a checking account, the bank must have a certain amount of base money. If the reserve requirement is 10%, then checking account supply is limited to 10 times the amount of reserves. Other kinds of inside money are regulated to limit their liquidity or quantity, or illegal. Bank notes are now illegal.

In sum,

- Classic doctrine: The government must control the quantity of inside money.

In the fiscal theory, the price level is fundamentally determined by the value of government liabilities. Hence there is no need, on price level determinacy grounds, to limit inside money at all.

- Fiscal theory doctrine: The price level can remain determinate with arbitrary creation of inside moneys. Reserve requirements and limitations on the creation of liquid inside assets are not needed for price level determination.

This doctrine is fortunate. Inside moneys have exploded. In January 2020, reserves were \$1,645,384 million, though required reserves were only \$158,765 million. (Federal Reserve H.3.) The reserve requirement, which is the classic constraint on the supply of demand deposit money, was slack by one and a half trillion dollars. In March 2020, the Fed eliminated reserve requirements altogether. Commercial paper, repurchase agreements, money market funds, and other highly liquid financial instruments, whose supply is not controlled at all, dominate the "cash" holdings of financial institutions, along with Treasury securities.

My point here is narrow, about price level determination. There are excellent financial-stability reasons to limit inside moneys. A financial institution that issues short-run liquid debt against illiquid assets is prone to a run. In the financial stability context, I argue for much stronger regulation against inside money than we have now (Cochrane (2014c)). I argue that the government should indeed take over entirely the business of providing fixed value, run-prone electronic money. The government analogously took over the business of note issuance in the 19th century, thereby ending runs on privately issued bank notes. Interest-paying central bank digital currency, or its equivalent provided by the Treasury are under discussion as modern equivalents. In my view, it would be better for the government to allow 100% backed narrow banks, as private institutions are likely to operate customer-facing payments software more efficiently. Traditional banks should finance risky investments with equity and long-term debt. Such a system would eliminate private financial crises forever, leaving us only with sovereign debt to worry about.

Reserve requirements were instituted to forestall runs, and may retain that role in fiscal theory. They were only repurposed to have a money supply and inflation control function much later.

The inside money question illuminates a key distinction between fiscal theory and a fiat money theory based on transactions demand. One might look at $MV = Py$ and $B = P \times EPV(s)$, where $PV(s)$ means present value of s, and conclude they are basically the same. In place of money we have all government debt, and in place of a transactions demand related to the level of output, we have the present value of surpluses. But here we see a big difference: Only direct government liabilities appear on the left-hand side of the fiscal theory, while private liabilities also appear in M.

By analogy, consider the question, whether short sales, futures, and options affect the value of a stock. By uniting a put option with a call option, you can buy or sell a synthetic share of the stock. These are "inside shares" in that they net to zero. For every synthetic purchaser there is a synthetic issuer. They impose no liability on the corporation. Do these "inside stock shares" compete with "real stock shares" to drive down the value of stocks? Well, in the baseline frictionless theory of finance, no. The company splits its earnings among its real owners only, and doesn't owe anything to the owners of inside shares. Therefore, we begin the theory of valuation with price times company-issued shares = present value of company-paid dividends, ignoring inside shares.

Likewise, primary surpluses repay only holders of actual government debt, not those who have bought private assets that promise to pay government debt, such as checking accounts. So, to first order, the value of government debt is not affected by inside claims. For every private buyer of inside money, there is a private issuer, so the number of such claims, or their valuation, has no aggregate wealth effect.

There can be secondary effects. In finance, scarcity of share supply can affect asset prices, and supply of inside shares due to short-selling, futures, or options can satisfy a demand for shares (Lamont and Thaler (2003), Cochrane (2003)). In monetary affairs, liquidity demands can potentially affect the price level. Gold and silver coins often circulated at values above their metallic content (Sargent and Velde (2003)). So too can government debt, or equivalently the discount rate in the valuation formula can be low. In such a situation, the provision of inside-money substitutes can reduce that valuation difference, and affect the price level. My statement that only government liabilities appear on the left-hand side also is not so crisp in a world of deposit insurance, bailouts, and implicit and explicit credit guarantees. Private debt does compete a bit for government surpluses.

Money substitutes, though not promises to deliver dollars, function in much the same way. Money substitutes, including foreign currency, help to facilitate transactions and compete with government money. Financial repression is often used to reduce that competing demand. In times of monetary shortages, stamps, bus tokens, cigarettes and other commodities have circulated as money. That even persistent monetary shortages have usually not led to deflation, which would restore the real value of money and end its shortage, is a fact suggestive of a fiscal or backing value of money.

13.2 Financial Innovation

Classic doctrine: Regulation must limit financial innovation and transactions technologies to maintain price level control.

Fiscal theory doctrine: The price level is determined with arbitrary financial innovation, and even if no transactions are accomplished using the exchange of government liabilities.

For monetary price level determination to work, it must remain costly to hold money. Money must pay less interest than other assets. But the cost of holding money gives people an incentive to economize on money holding, by financial innovation, which increases and destabilizes velocity. Even when money needs to be used for transactions, the key to money demand and $MV = Py$ is that one must hold that money for a discrete amount of time before making transactions. If one could obtain money a few milliseconds before buying, and the seller could redeposit that money in a few additional milliseconds, money demand would vanish; velocity would explode.

Thus monetary price level determination needs constraints on financial innovation. Yet our economy is evolving with rampant financial innovation, which reduces the need to hold money. Better electronic payment systems are obvious cases. Interest-bearing inside money and repo can be seen as such a money-saving, transactions-facilitating innovation rather than a competing form of private money. If we write $Mb \times V = Py$, checking accounts raise the velocity of base money.

We already live in a surprisingly money-free system. If I write you a \$100 check, and we use the same bank, the bank just raises your account by \$100 and lowers mine by the same. No actual money ever changes hands. If we have different banks, our banks are most likely to also net our \$100 payment against someone else's \$100 payment going the other way. The banks transfer the remainder by asking the Fed to increase one bank's reserve account by \$100 and decrease the other's. That operation still requires banks to hold some reserves. But by 2008, banks were able to accomplish the transactions in the (then) \$10 trillion economy, including the massive volume of financial transactions, with only \$10 billion or so of non-interest paying reserves, an impressive velocity indeed.

Credit cards, debit cards, and electronic funds transfers allow us to accomplish the same transactions, as well as to enjoy the other features of "money," without holding government money, and without suffering the lost interest that an inventory of money represents. Electronic payments systems in many countries are ahead of those in the United States, and avoid the holding and exchange of government money. Cryptocurrency enthusiasts think they will provide payments systems that leave the government out altogether.

As a first abstraction, our economy looks a lot more like an electronic accounting system, an electronic barter economy, than it looks like an economy with transactions media consisting of cash and checking accounts, which suffer an important interest cost, are provided in limited supply, and are rigidly distinguished from illiquid savings assets such as bonds and savings accounts.

But it is a logical consequence of monetarism that all this must be stopped. If V goes through the roof, then $MV = Py$ can no longer determine P. If V becomes unstable, so does Py. Chicago monetarists were pretty free market, and in favor of an efficient and innovative financial system as in other parts of the economy. This circumstance posed a conundrum. The fiscal theory liberates us to consider financial innovation on its merits, without worrying about price level control.

Sure, one might think that as V increases, M can decrease, from \$10 billion to \$1 billion, and finally to an economy of quickly circulating electronic claims to the last \$1 bill—the puzzle that started me on this whole quest. But as velocity explodes, the power of money to control the price level must surely also disappear. If you hold still the last hair on the end of the dog's tail, it is unlikely that the tail will stay still and the dog will wag back and forth instead. When suboptimal behavior has trivial costs, don't expect quick adjustments. Surely, velocity becomes endogenous instead. When the whole economy is operating at the 1 cent interest cost of holding one dollar bill, it will happily just pay 2 cents if the Fed wishes the economy to hold two dollars, at least for quite some time.

Such endogenous velocity likely holds on the way to this limit. At a velocity of 10, typical of the pre-zero-bound era, and at a 2% nominal interest rate, the cost of holding money is 0.2% of income. If money increases 10%, which ought to lead to a substantial 10% inflation, the interest cost of not maximizing is 0.02% of income. And since money has benefits too, the overall cost of not maximizing is an order of magnitude lower.

A theory that works at the limit *point*, zero money demand, not just in the limit with one last dollar of money demanded and supplied, is better adapted to an economy that is moving toward that limit.

The money demand story and Baumol-Tobin model are still repeated to undergraduates. You go to the bank once a week to get cash, and then use cash for all transactions. That model may reasonably describe the 1960s. But it must sound like ancient history to those undergraduates. If the people have no cash, let them use a credit card or Apple Pay, the undergraduate might say. If you ask an economist from Mars to choose a simple model to describe today's financial system, and the choice is Baumol-Tobin versus Apple Pay, linked to a cashless electronic netting system based on short-term government debt, I bet on Apple Pay.

The money supply and demand story falls apart if people can use assets they hold entirely for savings or portfolio reasons, without suffering any loss of rate of return, to accomplish transactions, precautionary, and other motivations for money demand. If you could costlessly wire around claims to the stocks in your retirement portfolio, or if you could sell stocks and refill your checking account one second before using it to wire out a transaction, you wouldn't need to hold money at all. Monetary price level determination falls apart. We are rapidly approaching that world too.

I argued against inside money on financial stability grounds, though inside money does not undermine the price level. The instant exchange of *floating* value securities can give us the best of both worlds—immense liquidity, and no more private sector financial crises.

Yes, a great deal of cash remains. But more than 70% of U.S. cash is in the form of $100 bills, and most is held abroad. Cash supports the illegal economy, tax evasion, undocumented workers, illegal drugs, sanctions evasion, and U.S. financing of various groups abroad. Cash, and U.S. cash especially, is a store of value around the world where governments tax rapaciously and limit capital movement. (Cryptocurrency may undermine these demands.) One could, I suppose, found a theory of the price level on the illegal demand for non-interest-bearing cash, but I doubt this approach would go far. Federal Reserve writings and testimony arguing for continued illegal activity to bolster money demand and allow inflation control are a humorous idea to contemplate. Perhaps most importantly, monetary price level control requires limited supply. But the U.S. Fed and other central banks freely exchange of cash for reserves. So if we base a theory of the price level on illegal cash demand, we instantly face a flat supply curve.

In sum,

- Classic doctrine: For the price level to be determined, regulation must stop the introduction of new transactions technologies, which threaten to explode V.
- Fiscal theory doctrine: The price level is determined with arbitrary transactions technologies, and even if no transactions are accomplished using the exchange of government liabilities.

13.3 Interest-Paying Money and the Friedman Rule

Classic doctrine: Money must not pay interest, or at least it must pay substantially less interest than risk-free short-term bonds. If the interest rate is zero, or if money pays the same interest as bonds, the price level becomes undetermined. We cannot have the Friedman optimal quantity of money. Money and competing liquid assets must be artificially scarce to control the price level.

Fiscal theory doctrine: The price level is determined even if money pays exactly the same interest as bonds, either directly or if the interest rate is zero. We can have the Friedman optimal quantity of money; we can be satiated in liquidity, using assets held and valued only for savings purposes to make transactions and fulfill other liquidity demands.

The possibility of zero interest rates, or that money pays the same interest as bonds, undermines $MV = Py$ price level determination. When there is no interest cost to holding money, money and bonds become perfect substitutes. Now V is Py divided by whatever M happens to be. A switch of M for B has no effect at all. When money pays the same interest as other assets, money demand ceases to be a function, but is instead a correspondence, with any amount of money consistent with a zero interest cost. With no interest costs, money becomes an asset held as part of an investment portfolio. One gets the liquidity services of an asset held for other reasons, for free. Monetary price determination fails.

The fiscal theory offers the opposite conclusion. If money M_t pays the same interest as B_t, if M_t and B_t are perfect substitutes, we're simply back to $(B_t + M_t)/P_t = E_t \sum_{j=0}^{\infty} \beta^j s_{t+j}$ with no money, no seigniorage, and no other change. The price level is easily determined.

The famous Friedman (1969) optimal quantity of money states that zero nominal interest rates, so non-interest-bearing money and bonds have the same rate of return, is optimal. Slight deflation gives a positive real rate of interest. Since printing more money costs society nothing, we should have as much of it as we want. At a minimum, we save on needless trips to the cash machine. Zero also means no hurry to collect on bills or other contracts that do not include interest clauses, and no need to write interest clauses into such contracts. Cash management to hold less money, and thereby save on interest costs, is a social waste. Money is like oil in the car. We don't slow down a car by deliberately starving it of oil, especially if we can print oil for free.

As money becomes interest-bearing checking accounts, money market funds, or similar securities, and as transactions become electronic using such funds, we can generalize the Friedman optimum to say that the supply of money-like liquid, transactions-facilitating, assets should be so large that they pay the same return as illiquid assets. They should also be allowed to pay such rates contra decades of regulation forbidding interest payments on checking accounts. In particular, reserves should be abundant and pay the same interest as short-term Treasurys.

But Friedman did not argue for an interest rate peg at zero or for interest-paying money. He never took the optimal quantity of money seriously as a policy proposal. He argued for 4% money growth. Why not? I speculate because, if the price level comes from money supply and money demand, it becomes unmoored by interest-paying money or a peg at zero. Society must endure the costs of an artificial scarcity of liquid assets, in order to keep inflation under control. If the gas pedal is stuck to the floor, and the brakes don't work, you have to slow the car by draining oil.

The fiscal theory denies this doctrine. Summing up,

- Classic doctrine: Money must not pay interest, or at least it must pay substantially less interest than risk-free short-term bonds. We must stay away from the zero-interest liquidity trap. We cannot have the Friedman optimal quantity of money.
- Fiscal theory doctrine: The price level is determined even if money pays exactly the same interest as bonds. That interest rate can be zero, or money may pay the same interest as bonds. We can have the Friedman optimal quantity of money, we can be satiated in liquidity, using assets held and valued only for savings purposes to make transactions and fulfill other liquidity demands.

Again, this is a fortunate prediction because our world looks less and less like one that meets the classical requirements. Reserves pay interest, at times larger than short-term Treasurys, and are thousands of times larger than required. Checking accounts can pay interest. Money market funds, repos, and other interest-paying money abounds. Treasurys themselves are liquid and a money-like store of value for financial institutions. To the extent that these instruments do not pay interest, it is because the United States, Europe, and Japan were stuck at the zero bound for so long, equally troubling to classic doctrine.

The monetarist position is more nuanced. Zero nominal rates, as observed in the Great Depression, sparked a central and classic controversy. Keynesians view the situation as a "liquidity trap" in which monetary policy loses its power. Money and bonds are perfect substitutes, so trading M for B does nothing, and interest rates cannot be lowered below zero.

Monetarists often counter with a view that more money M can still stimulate nominal income Py at the zero bound. The issue comes down to the behavior of velocity V. If money and bonds are truly perfect substitutes, then V is meaningless. It adapts to whatever split of M versus B that the government chooses, with no effect on Py. But monetarists argue that velocity V is not so infinitely adaptable, even at the zero bound. Velocity is "stable" or stable in some "long run," so more money and less bonds will still encourage more spending.

This view that more M for less B does any good at the zero bound requires some *upper* limit on money demand, not a lower limit or some residual transactions value. It requires some reason people would want to get rid of "too much money" in favor of bonds that pay exactly the same amount. It neglects that zero interest rates are a *consequence* of liquidity satiation: If, unlike car oil, arbitrarily large money holdings still provide marginal liquidity services, then it would have taken an infinite amount of money to drive the interest rate on money and bonds to the same value. That we observe equal rates between money and bonds, or even lower rates on bonds (Treasurys have paid less than reserves in the United States, bonds paid negative rates in Europe for long periods) means directly that, like a car with oil, the economy can be satiated with liquidity.

The intuition remains strong that "helicopter drops" of money can stimulate inflation at the zero bound. Fiscal theory agrees: More M with no change in B and no change in future s creates inflation. But that fact and intuition does not tell us that open market operations, more M and less B, do any good.

The last section concluded with a vision of a nearly cashless economy, in which we handle all transactions by wiring around claims to a stock portfolio. This section concludes with a vision of an abundant cash economy, in which interest-bearing money is the same as short-term bonds, held also for investment purposes, and we handle transactions by wiring around claims to that portfolio. Either one works under fiscal theory. Ether one undermines the price level for a monetary theory.

13.4 Separating Debt from Money

Classic doctrine: Bonds must be kept deliberately illiquid, or the price level will not be determined. Bonds may not be issued in small denomination, discount, bearer, fixed value, or cheaply transferable form.

Fiscal theory doctrine: An artificial separation between "bonds" and

> "money" is not necessary for price level determination. The Treasury can issue fixed value, floating rate, electronically transferable debt. Savings vehicles may be allowed to be as liquid as technology can make them.

In $MV = Py$, we need to have a definite separation between "liquid," or transactions-facilitating assets M and "illiquid" savings vehicles B. Control of M and the split between M and B gives control over the price level. This is the reason for banning interest-paying money, so that money does not become a savings vehicle like bonds. Here, I discuss the complementary doctrine: Bonds should not become a transactions vehicle like money. It is important to deliberately limit the liquidity of public and private debt issues.

Bank notes are illegal, though they are just zero maturity, zero interest, small denomination bearer bonds issued by banks. They are illegal for good financial stability reasons, but the doctrine states that they must remain illegal, or supply limited, for price control reasons. Corporations and state and local governments must not issue small-denomination or bearer bonds that might circulate. (Bearer bond principal and coupons are paid to whoever shows up with them, and are not registered. They gradually fell out of favor and are now illegal for a host of reasons.) Even the U.S. Treasury must deliberately hobble the liquidity of its securities, the doctrine says, despite the lower interest rates that doing so could produce. It must sell illiquid securities and let the Fed buy them to issue a limited quantity of liquid debt—cash and reserves—in its place.

Indeed, the U.S. Treasury does not issue bonds in denomination less than \$1,000—only recently reduced from \$10,000—and not in anonymous (bearer) form. The shortest Treasury maturity is a month, and the Treasury does not issue fixed value, floating rate debt. The Treasury sells securities via its website treasurydirect.gov but does not buy them or allow transfers from one account to another as banks can do with reserves at the Fed. Treasury sells hundreds of distinct securities rather than bundle its debt into two or three issues that would be vastly more liquid.

This deliberate illiquidity, keeping "bonds" separate from "money," is crucial in the $MV = Py$ world:

- Classic doctrine: Bonds must be deliberately illiquid, and separate from money, or the price level will not be determined. Bonds may not be issued in small denomination, discount, bearer, fixed value, or cheaply transferable form that might be used for transactions demand.

The fiscal theory denies this proposition. The maturity, denomination, transaction cost, bearer form, or other liquidity characteristics of private or government debt make no difference to price level determination. To the extent that such features lower Treasury interest costs overall, so much the better for government finances and liquidity provision to the economy.

- Fiscal theory doctrine: An artificial separation between "bonds" and "money" is not necessary for price level determination.

In a more detailed proposal (Cochrane (2015b)), I argue that the Treasury should offer to all of us the same security the Fed offers to banks: fixed value, floating rate, electronically transferable debt, in arbitrary denominations. This is the same security that the Fed offers to banks as reserves. Treasury electronic money might be a good name for it. I also argue that the Treasury should supply as much of this security as people demand. (Roughly the same structure is advocated as central bank digital currency, opening up reserves to all, see for example Duffie and Economy (2022). But central banks are set up to serve banks, and already reluctant to allow nonbank institutions to hold reserves, as well as resistant to financial innovation that would undermine bank profits. The Treasury already sells debt to the public.) The Treasury can manage its duration and interest rate risk exposure with longer maturities or swaps. I also argue that the Treasury and Fed should allow narrow banks and private nonbank payment processing companies to operate, using this security as 100% reserves, since private institutions are likely better at operating low-cost transactions and intermediation services, interacting with retail customers. Finally, the Treasury should offer nominal and indexed perpetuities, and thereby dramatically reduce the number of its distinct long-term debt issues. This move would increase the liquidity of long-term debt.

The Federal money market fund—a private mutual fund that offers fixed value, floating rate, electronically transferable investments, backed by a portfolio of short-term Treasurys—should be a threat to price level control. After all, the Federal Reserve itself is such a fund. Such funds are already widespread. They don't yet have immediate electronic transfers, ATM machines, and link to a debit card, in part to avoid being regulated as "banks," and that regulation serves to reduce competition for bank deposits. But that is a legal limitation, not a technological limitation. Add that feature and we have already completely circumvented the Federal Reserve's intermediation of Treasury debt to electronic money. If control of the size of the Fed's balance sheet matters, money market funds undermine that control.

Such a proposal is anathema in a monetarist view, as the price level would be unmoored. The relative quantity of B and M would become endogenous, and the character of B and M (reserves) would become identical.

13.5 A Frictionless Benchmark

Classic doctrine: We must have monetary frictions to determine the price level.

Fiscal theory doctrine: The price level is well defined in an economy devoid of monetary or pricing frictions, and in which no dollars exist. The dollar can be a unit of account even if it is not the medium of exchange or store of value. The right to be relieved of a dollar's taxes is valuable even if there are no dollars.

The frictionless model is a benchmark on which we build models with frictions as necessary. But unlike standard monetary economics, frictions are not *necessary* to describe an economy with a determinate price level. And the simple frictionless model can provide a first approximation to reality.

In classical monetary theory, some monetary friction is necessary to determine the price level. In a completely frictionless economy, with no money demand, money can have no value.

As we have seen, fiscal theory can determine the price level even in a completely frictionless economy. We do not need liquidity demands, transactions demands, speculative demands, precautionary demands, incomplete markets, dynamic inefficiency (OLG models), price stickiness, wage stickiness, irrational expectations, and so forth. Such ingredients make macroeconomics fun, and realistic. We can and do add them to better match dynamics, as I added price stickiness in previous chapters. But the fiscal theory does not *need* these ingredients to determine the price level.

We can even get rid of the "money" in my stories. People can make transactions with maturing government bonds, in Bitcoin, with foreign currency, by transferring shares of stock, or by an accounting and netting system. The "dollar" can be a pure unit of account. Government debt can promise to pay a "dollar," even if nobody ever holds any dollars at all. The right to be relieved of one dollar's worth of tax liability establishes its value as numeraire and unit of account.

This frictionless view describes the frictionless *limit point*, not just a frictionless limit. A theory that holds at the limit point is more reliable to describe economies that are near the limit, avoiding the tail wags the dog problem.

Frictionless valuation is a property of a backing theory of money. If dollars promised to pay gold coins, then we could establish the value of a dollar equal to one gold coin, also if nobody used dollars in transactions. Money may gain an *additional* value if it is specially liquid and limited in supply, or it may pay a lower rate of return. But in a backing theory, a fundamental value remains when the liquidity value or supply limits disappear. Entirely fiat money loses all value in that circumstance.

To summarize, continuing my list of doctrines,

- Classic (fiat-money) doctrine: We must have some monetary frictions to determine the price level.
- Fiscal theory doctrine: We can have a well-defined price level in an economy devoid of monetary or pricing frictions, and in which no dollars exist. The dollar can be a unit of account even if it not medium of exchange or store of value. The right to be relieved of a dollar's taxes is valuable even if there are no dollars.

This observation really sums up previous ones – interest-paying money, abundant inside money not constrained by reserve requirements, debt that can function as money, and financial innovations that allow us to make transactions and satisfy other demands for money without holding money are all different aspects of the march to a frictionless financial system, which we now need not fear.

Chapter Fourteen

Stories and Histories

A FEW SIMPLE stories, histories, and conceptual experiments quickly come up when we think of any monetary theory. It's important to see how fiscal theory is consistent with and interprets these monetary stories.

14.1 Helicopters

> Dropping money from helicopters surely raises inflation. Does this story establish that money causes inflation? No. Fiscal theory also predicts that prices rise under a helicopter drop. The drop is an expansion of nominal debt with no change in surpluses. It does not follow that more M with less B creates inflation. A helicopter drop is a brilliant device for communicating a fiscal commitment, that surpluses will not be raised to pay off the new debt.

Milton Friedman famously proposed that if the government wishes inflation, it should drop money from helicopters. People will run out and spend the money, driving prices up. Doesn't this, one of the most famous gedanken experiments in economics, prove that in the end money causes inflation?

No. The government debt valuation equation has money and bonds $M + B$ on the left-hand side. Dropping money M from helicopters with no change in surpluses s and no change in debt B raises the price level P in the fiscal theory too. The sign of the response to this conceptual experiment does nothing to distinguish monetary from fiscal theories of inflation.

The helicopter drop remains a key conceptual experiment. But first of all, recognize this is not what central banks do. Central banks do not print money (create reserves) and hand it out. They always *exchange* money for something else. A helicopter drop is fiscal policy, or at least a joint fiscal–monetary policy operation. To accomplish a helicopter drop in our economy, the Treasury must borrow money, hand it out, record it as a transfer payment, and the central bank must buy the Treasury debt. Even when the Fed simply prints money and hands it out, it must make a loan not a gift, and book a promise to repay. Central banks do not print M and hand it out, they exchange M for B.

Yes, the central bank, charged with controlling inflation, is forbidden this one most obvious tool for creating inflation. It is even more forbidden the single most obvious tool for stopping inflation: helicopter vacuums, confiscating money. There are excellent reasons for this institutional limitation. An independent agency in a democracy should not print money and give it to voters, or to chosen businesses and asset holders. It certainly should not confiscate or tax wealth. Those are the jobs of the politically accountable Treasury, administration, and Congress. Even in the extreme measures of the financial crisis and COVID-19 recession, the U.S. Fed carefully structured its massive interventions as plausibly risk-free lending, with the Treasury taking credit risk.

Suppose that while the Fed helicopter-drops \$1,000 of cash in your backyard, the Fed burglars come and take \$1,000 of Treasury bills from your safe. How much would that combined operation make you spend? The answer is not so obvious, and "nothing" is reasonable. You, and the economy as a whole, have more transactions balances than you need. But you're also no better off. You have correspondingly less savings than you had before, so you feel no wealthier, no drive to spend to increase consumption.

The helicopter drop story artfully leads you to jump from intuition about a wealth effect, increasing the overall amount of government liabilities with no promise of future surpluses, to a composition effect, more money relative to bonds. This conflation is not dishonest. In monetarist thinking, only $MV = Py$ matters to the price level. Whether the money supply increases because the Fed buys bonds, buys stocks, lends it to banks, or simply drops it from helicopters makes no difference at all to inflation. The wealth effect is tiny or irrelevant in monetarist thinking. But your intuition may be guided by the wealth effect, not by an excess of transactions balances. If so, you're thinking in fiscal theory terms. Likewise, many monetary models specify money "injections" or "transfers" in which the central bank just hands out or confiscates money, and then draw implications for open market operations.

Imagine that the government drops cash from the sky, with a note that says: "Good news: We have dropped \$1 trillion from the sky. Bad news: Next week taxes go up \$1 trillion. See you in a week!" Now how much will people spend? In the fiscal theory, this is a parallel rise in M_t and s_{t+1}, which has no effect on the price level.

Dropping cash from helicopters is a brilliant way to communicate a *fiscal* expectation: We're dropping this cash on you, and we will *not* raise taxes to soak it up. Go spend it.

Imagine that the government drops newly printed one-month Treasury bills from the sky. Would that have a much different effect than dropping the corresponding cash? The monetarist interpretation says that this operation would have no effect on inflation. The frictionless fiscal theory would say that the Treasury bill drop could well have the same effect as the money drop, if people think the debt, like the money, will not be repaid. (In both cases, the people initially receiving the bounty may spend it. The question is whether the private sector as a whole does so; what do the ultimate bond buyers or money holders believe? The representative agent sums over a lot of heterogeneity.)

The latter is the key question. Is dropping securities, whether cash or bonds, from helicopters, the key innovation that changes people's expectations? Or is it the nature of the dropped security, and it doesn't matter how people get it? Tobin (1980) (p. 53) considers this "bond rain." In his view, the monetarist says it has no effect, because the monetarist is Ricardian: People expect that bonds are repaid by subsequent surpluses, and money is not. The nature of the security, not the mechanism of its dropping, is what matters. But perhaps your intuition says yes: Bonds dropped from helicopters would be interpreted just like money dropped from helicopters, as unbacked. Perhaps the same rhetorical reason that Friedman chose to say helicopters, not stimulus checks, applies to bonds.

We commonly think that bonds, sold to investors, even if used to finance cash transfers to consumers, are less likely to be inflationary, because the institutional structure of bond sales is set up to communicate the expectation of surpluses. Both the security (bonds) and the method (sales, transfers via legislated social programs, not helicopters) engenders that expectation. That view helps to explain why supposed "helicopter drops" of the zero bound, fiscal stimulus, QE era did not do much to inflate. The main difference above between Treasury issues, like stock issues, and central bank actions, like share splits, is that institutional setting.

Tobin wrote that the bond rain would be stimulative, and eventually inflationary, because people ignore Ricardian equivalence. Fiscal theory, with nominal rather than real debt, allows such a "non-Ricardian" bond sale without change in future surpluses, without needing to invoke short-sighted investors. Or the opposite. Expectations of future surpluses are not always the same for issues of the same security. Tobin went on to write that the open market operation would have little effect because money and bonds have roughly the same fiscal backing; but Friedman would have said he ignored how money is special.

In sum, a good reason for the power of Friedman's parable is that helicopters signal fiscal expectations differently. We have struggled with institutions to communicate fiscal expectations and our governments struggled during the 2010s to create inflation. Alas, literal helicopters (or, today, drones) are not a practical idea. However, in 2020 the Federal Reserve and Treasury got together, created about $2.5 trillion of new reserves, and sent people checks. This move arguably set off the 2021 inflation, surveyed below. So perhaps we have, unintentionally, found our helicopter.

Magnitudes may distinguish the monetarist and fiscal theory answers to the helicopter drop experiment. Suppose each family has $100,000 savings in Treasury bonds, and earns $100,000 a year. They hold an additional $1,000 in cash to make transactions. The government drops $1,000 per household in cash. How much does the price level rise? A fiscal theory answer is, 1%. Overall Treasury debt just got diluted 1%. The monetarist answer is 100%. The money supply doubles, so the price level must double. You may spend your extra $1,000, but then someone else has $2,000. People only want 1% of their income in cash. People collectively keep trying to spend their extra cash until they have doubled the price level, doubled nominal income, and the $2,000 in cash per person is 1% of the now $200,000 per household nominal GDP. The fact that this doubling of the price level wipes out half of the real value of $100,000 of Treasury savings has no effect on the price level. Passive fiscal policy means that the present value of taxes has declined $50,000.

14.2 Hyperinflations and Currency Crashes

> Governments print a lot of money in hyperinflations, but this fact does not prove that money causes inflation. Hyperinflations involve intractable fiscal problems. A central bank that refuses to print money would not likely stop a fiscal hyperinflation.

Governments print huge amounts of money in hyperinflations. Doesn't that fact prove that money causes inflation?

No. Every hyperinflation has indeed occurred when governments print money. But the governments printed money to finance intractable deficits, expanding the amount of total government debt, with no surpluses in sight. No hyperinflation has occurred from central bank policy errors in a government with healthy finances.

Imagine that a central bank of a hyperinflation-ridden country refuses to print more money, and the government funds its deficits by issuing one-month bonds instead, paying suppliers with such bonds, and rolling over old bonds with new bonds directly. Would that stop the inflation? Likely not. People would still try to unload government debt by buying real assets, foreign assets, and goods and services.

If the central bank creates a means of payment shortage in this situation, people will use foreign currency, barter, credit, government bonds, put in more effort to hold money for the least possible time, and so forth. At best, the central bank can try to force a fiscal reform by its refusal to print more money. This refusal can put pressure on fiscal authorities. A commitment not to monetize debt can help a new regime to get going. But if the fiscal problem is not cured, the bank can at best force a default. Thus, stronger central bank commitments seem to be most useful with explicit fiscal reform, and seldom successful on their own. Without fiscal reform, changing the composition of government debt has little effect.

A similar situation occurs when the currencies of countries having fiscal and balance of payments crises start to collapse. The central bank may try to fight the crisis by soaking up domestic currency in return for nominal bonds. But nobody wants the nominal bonds either, and high interest costs worsen the deficit. Governments in both cases try financial repression and capital controls to force people to hold their debt. That too eventually fails.

Monetarist analyses have long recognized fiscal limits, and that successful control of the money supply requires a solvent fiscal policy, monetary–fiscal coordination. But the fact that hyperinflating countries do typically print up a lot of money does not tell us that money printing alone causes inflation, that inflation could be stopped by more spine at central banks, or that an *exchange* of money for bonds has the same effect as printing money to finance deficits.

14.3 Ends of Inflations

Inflations have been ended by solving the underlying long-run fiscal problem, and by changing the fiscal and monetary *regime*. Ends of inflations have included printing *more* money, *lower* interest rates, continued deficits and little or no output loss. I review Tom Sargent's classic studies and their place in history. Many inflations have not ended in response to monetary tightening alone or temporary fiscal measures.

Hyperinflations end when the underlying fiscal problem is solved. Monetary reforms are often involved, so we should call them joint fiscal–monetary reforms.

The ends of large inflations typically involve printing *more* money. Real money demand expands when the interest costs of holding money decline. People start holding money for weeks, not hours, so the economy needs more of it. The nominal interest rate *declines* when the fiscal problem is solved. There is no period of monetary stringency. Near-term fiscal deficits may stay the same or increase. Fixing the long-run problems allows the government to borrow more. Inflations have ended with no rise in unemployment or decline in output, and quickly improving economies. Inflation, attendant demonetization, financial repression and the difficulty of finance during inflation, and fiscal chaos all drag down the economy, and are quickly improved when the stabilization package arrives.

Sargent (1982b) "The ends of four big inflations" is the pathbreaking study of the ends of hyperinflations and their fiscal roots. It set the sails of fiscal theory. It also shows by example how historical analysis of regime changes lets us surmount observational equivalence and Lucas critique concerns. It insists that good economics should describe the big events first and foundationally, not as outliers to be treated with different economics from more sedate times.

Sargent studied the immense hyperinflations of Austria, Germany, Poland, and Hungary in the early 1920s, and their abrupt ends, along with the placebo test of Czechoslovakia, which avoided inflation despite being surrounded by inflation.

Start with Austria, displayed in Figure 14.1. The inflation is dramatic and its end instantaneous. What happened? I quote from Sargent, in part to document the role of this foundational work in developing fiscal theory:

> The hyperinflations were each ended by restoring or virtually restoring convertibility to the dollar or equivalently to gold.

This sounds like a monetary policy change, but it is not. Sargent states as I have that the gold standard is primarily a fiscal commitment:

> since usually a government did not hold 100% reserves of gold, a government's notes and debts were backed by the commitment of the government to levy taxes in sufficient amounts, given its expenditures, to make good on its debt. [Note debt, not just money.] In effect, the notes were backed by the government's pursuit of an appropriate budget policy . . . what mat-

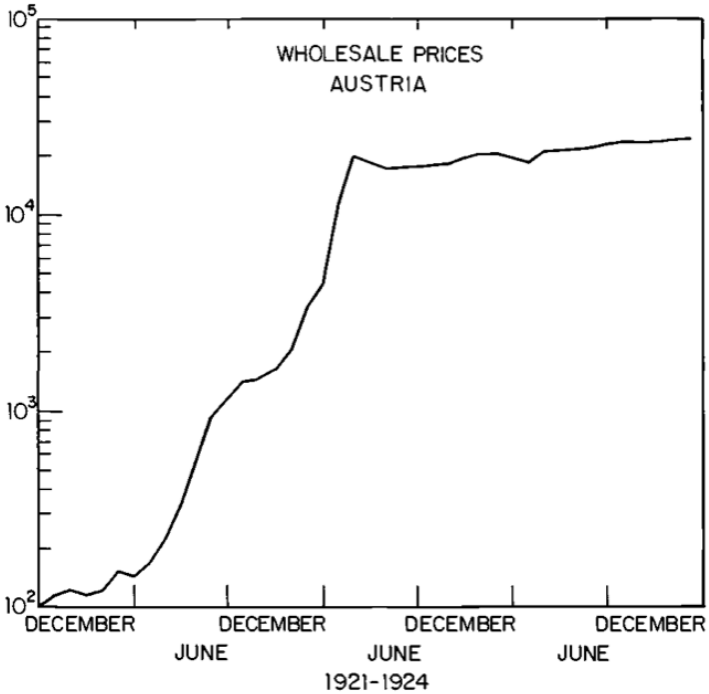

Fig. 2.1 Wholesale prices in Austria.

Figure 14.1: Wholesale Prices in Austria. Source: Sargent (1982b).

> tered was not the current government deficit but the present value of current and prospective future government deficits. The government was like a firm whose prospective receipts were its future tax collections. The value of the government's debt was, to a first approximation, equal to the present value of current and future government surpluses . . . In order to assign a value to the government's debt, it was necessary to have a view about the fiscal policy regime in effect, that is, the rule determining the government deficit as a function of the state of the economy now and in the future. The public's perception of the fiscal regime influenced the value of government debt through private agents' expectations about the present value of the revenue streams backing that debt.

Sargent emphasizes the importance of a change in *regime*. To believe that the present value of surpluses has changed, people need to see that fiscal and monetary affairs have changed in a durable way. Announcements, decisions, and reversible policies by today's politicians don't often budge long-term expectations. I have used the word "institutions" that guide expectations in much the same spirit.

The new fiscal regime *allowed* the countries to restore convertibility:

> The depreciation of the Austrian crown was suddenly stopped by the intervention of the Council of the League of Nations and the resulting binding

> commitment of the government of Austria to reorder Austrian fiscal and monetary strategies dramatically.

This event included internal fiscal reform:

> Expenditures were reduced by discharging thousands of government employees . . . Deficits in government enterprises were reduced by raising prices of government-sold goods and services. New taxes and more efficient means of collecting tax and custom revenues were instituted . . . Within two years the government was able to balance the budget.

But a larger issue hung over Austria: whether it would continue as a nation, and repay its debts, and how much reparations the Allies would demand.

> The first protocol was a declaration signed by Great Britain, France, Italy, Czechoslovakia, and Austria that reaffirmed the political independence and sovereignty of Austria.
>
> At the same time, it was understood that the Reparation Commission would give up or modify its claim on the resources of the government of Austria.

This did the trick, and instantly stopped the inflation. Indeed,

> even before the precise details of the protocols were publicly announced, the fact of the serious deliberations of the Council brought relief to the situation.

Monetary policy alone did little. Yes,

> The Austrian government promised to establish a new independent central bank, to cease running large deficits, and to bind itself not to finance deficits with advances of notes from the central bank.

But such promises have been made hundreds of times in failed stabilizations. Unless you solve the structural problem, change the regime, swearing not to finance deficits is a pie-crust promise (easily made, easily broken). On the other hand, central bank reforms are a useful part of a joint monetary–fiscal stabilization; they help to prevent future inflationary finance, and they help the fiscal reform to stick.

Money supply expanded, and money-financed deficits continued. Neither monetary stringency nor an immediate end to deficit spending mattered. Curing the expectation of future deficits mattered.

> The Austrian crown abruptly stabilized in August 1922, . . . prices abruptly stabilized a month later. This occurred despite the fact that the central bank's note circulation continued to increase rapidly . . . from August 1922, when the exchange rate suddenly stabilized, to December 1924, the circulating notes of the Austrian central bank increased by a factor of over 6.

The key difference:

> Before the protocols, the liabilities of the central bank were backed mainly by government treasury bills; that is, they were not backed at all, since treasury bills signified no commitment to raise revenues through future tax collections. After the execution of the protocols, the liabilities of the central bank became backed by gold, foreign assets, and commercial paper, and ultimately by the power of the government to collect taxes . . . The value of the crown was backed by the commitment of the government to run a fiscal policy compatible with maintaining the convertibility of its liabilities into dollars. Given such a fiscal regime, to a first approximation, the intermediating activities of the central bank did not affect the value of the crown . . .

It is striking that "backing" by treasury bills was regarded as no backing at all, relative to our current economies and financial systems that seem to regard backing by treasury bills as the safest kind of backing. That is not written in stone for us either. The calming effect of a real "pot of assets" in the central bank is also interesting.

Germany presents an even starker case. Figure 14.2 presents the German price level during its post-WWI hyperinflation. Notice the exponents on the vertical axis.

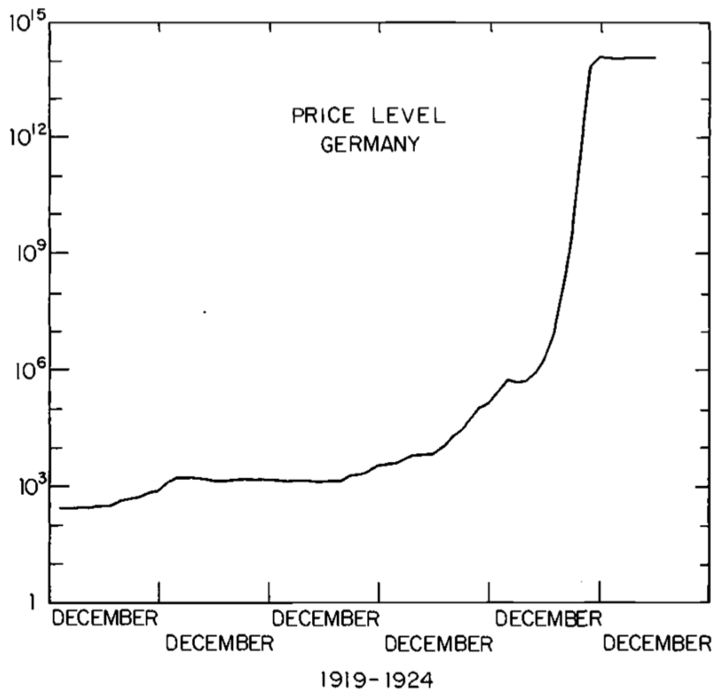

Figure 14.2: Wholesale Prices in Germany. Source: Sargent (1982b).

> After World War I, Germany owed staggering reparations to the Allied countries. This fact dominated Germany's public finance from 1919 until 1923 and was a most important force for hyperinflation . . . except for 1923, the

> budget would not have been badly out of balance except for the massive reparations payments made.
>
> For one thing, considerably larger sums were initially expected of Germany than it ever was eventually able to pay. For another thing, the extent of Germany's total obligation and the required schedule of payments was for a long time uncertain and under negotiation. From the viewpoint that the value of a state's currency and other debt depends intimately on the fiscal policy it intends to run, the uncertainty about the reparations owed by the German government necessarily cast a long shadow over its prospects for a stable currency.

Germany's hyperinflation stopped just as suddenly as Austria's, when the long-term fiscal problem was solved.

> Simultaneously and abruptly three things happened: additional government borrowing from the central bank stopped, the government budget swung into balance, and inflation stopped.

The fiscal trouble was not all reparations:

> The government moved to balance the budget by taking a series of deliberate, permanent actions to raise taxes and eliminate expenditure . . . the number of government employees was cut by 25 percent; all temporary employees were to be discharged; all above the age of 65 years were to be retired . . . The railways, overstaffed as a result of post-war demobilization, discharged 120,000 men during 1923 and 60,000 more during 1924. The postal administration reduced its staff by 65,000 men; the Reichsbank itself which had increased the number of its employees from 13,316 at the close of 1922 to 22,909 at the close of 1923, began the discharge of its superfluous force in December . . .

But reparations were a central component:

> Substantially aiding the fiscal situation, Germany also obtained relief from her reparation obligations. Reparations payments were temporarily suspended, and the Dawes plan assigned Germany a much more manageable schedule of payments.

Again, the stabilization did not involve monetary stringency. The opposite occurred. While the inflation was going on, the usual substitution away from real money holdings took hold:

> In response to the inflationary public finance and despite the efforts of the government to impose exchange controls, there occurred a "flight from the German mark" in which the real value of Reichsmark notes decreased dramatically. The fact that prices increased proportionately many times more than did the Reichsbank note circulation is symptomatic of the efforts of Germans to economize on their holdings of rapidly depreciating German marks. Toward the end of the hyperinflation, Germans made every effort to avoid

> holding marks and held large quantities of foreign exchange for purposes of conducting transactions.

When the inflation stopped, Germany printed *more* money.

> a pattern that we have seen in the three other hyperinflations: the substantial growth of central bank note and demand deposit liabilities in the months after the currency was stabilized.

There was also no Phillips curve:

> By all available measures, the stabilization of the German mark was accompanied by increases in output and employment and decreases in unemployment.

"Stopping Moderate Inflations: The Methods of Poincaré and Thatcher," Chapter 4 in Sargent (2013), covers the end of the much smaller French inflation of the 1920s. The same principles apply, which is important: Fiscal theory and fiscal–monetary interactions are often grudgingly acknowledged for hyperinflations and crashes, but said to be unimportant in less extreme events.

France had borrowed a large amount to fight WWI, and was hoping to repay that debt from German reparations. When it became clear that Germany would not pay, in 1924, the Franc started depreciating quickly. The period was volatile. Sargent's data includes years of surprising deflation as well. The period was

> characterized by a massive flight of French capital abroad, partly an anxiety reaction to some of the tax proposals under discussion, such as a capital levy.

The *dénoument:*

> Poincaré was a fiscal conservative, . . . As soon as he assumed control of the government . . . the Franc recovered and inflation stopped.

Sargent details the subsequent tax changes (both increases and decreases), a return to the gold standard at a low and thus more easily sustainable value, implying an 80% devaluation of wartime nominal debt, and limits on central bank finance. Sargent emphasizes again the gold standard as a fiscal commitment. But the gold standard must be backed up by fiscal possibility, or it is an empty promise, not a commitment. Most importantly, people believed that the change was permanent:

> there had been broad consensus both about the principal economic factors that had caused the depreciation of the Franc—persistent government deficits and the consequent pressure to monetize government debt—and the general features required to stabilize the Franc—increased taxes and reduced government expenditures sufficient to balance the budget, together with firm limits on the amount of government debt monetized by the Bank of France . . . a political struggled had been waged over *whose* taxes would be raised.

But now,

> all political parties except the socialists and communists gathered behind Poincaré. Five former premiers joined his government.

Sargent's point in this work was only half about the fiscal foundations of inflation. Much of his point is about the Phillips curve. Sargent wrote in the early 1980s, in the context of the U.S. and U.K. inflation stabilization, which at the time had only begun. At the time, the conventional Keynesian consensus held that expectations are mechanically and slowly adaptive, prices and wages are mechanically sticky, so it would take a prolonged and costly depression to get rid of inflation. Sargent cites contemporary estimates that a 1 percentage point reduction in inflation would cost 8% of GNP. And U.S. inflation peaked above 14%. Conventional wisdom argued it was better to live with inflation, or pursue (again) wage and price controls and jawboning, pressuring companies and unions not to raise prices and wages, rather than to suffer such a fate.

In this context, Sargent argued for the possibility that if people see a new fiscal and monetary regime in place, expectations of inflation and hence actual inflation can decline quickly, with little output loss or even a gain. In the context of the Phillips curve we have written down, $\pi_t = E_t\pi_{t+1} + \kappa x_t$, getting $E_t\pi_{t+1}$ to fall is the key to a recession-free inflation reduction.

Most of Sargent's point is about how *hard* this outcome is to achieve and how perilous the U.S. and U.K. stabilizations actually were. Today, economists tend to breezily assert that central banks just need to give speeches to "manage" inflation expectations: Announce a different inflation target, talk about "anchoring," give "forward guidance." Driving down (or up) expected inflation is not nearly so easy, especially with three failed attempts in the rearview mirror as was the case in 1980.

Sargent's point is that a swift and relatively painless end of inflation, a credible stick-to-it reform, a change in Phillips curve expectations, can happen, but it will only happen with a credible change of *regime*. The central lesson of intertemporal economics is that we cannot think about policy actions in isolation, as standard Keynesian economics does. Instead, we must think of regimes, policy rules and traditions, institutions and commitments, and through these expectations of future policy. Speeches, promises, and one-time policies do not reliably change expectations. In turn such a change in regime often needs a political realignment, a change in institutions and commitments they embody, or change of external circumstance. Per Sargent, "the change in the rule ... [must be] widely understood, uncontroversial and unlikely to be reversed."

Sargent took a skeptical view of the U.S. and U.K. stabilization attempts, as of the time he wrote. Both United States and United Kingdom had large deficits. The early part of the U.S. monetary tightening did not come with an immediate change in fiscal policy. Though we now see debts and primary deficits that are neither large by later standards, nor unusual given the size of the recession, the "Reagan deficits" were big and contentious in economic discussion at the time.

The central point of Sargent and Wallace's (1981) "Unpleasant Monetarist Arithmetic" (covered in detail in Section 19.6), as well as these historical writings, was to point out the fiscal underpinnings of inflation, and to argue that the United States and United Kingdom needed quickly to undertake fiscal reforms, or the stabilizations would fail. Being expected to fail, or at least with great uncertainty about whether the governments would or could stick with it, expectations in the Phillips curve would not shift, and the attempt would be unnecessarily costly in terms of output and employment.

Sargent's "Methods of Poincaré and Thatcher" was even harsher on likely success in the U.K. He wrote

> Mrs. Thatcher comes to power against the background of over twenty years of 'stop-go' or reversible government policy actions. Her economic policy actions are vigorously opposed both by members of the Labour party and by a strong new party, the Social Democrats . . . Mrs. Thatcher's party now runs third in the political opinion polls . . . speculation has waxed and waned about whether Mrs. Thatcher herself would be driven to implement a "U-turn" . . . there is widespread dissent from Thatcher's actions among British macroeconomic scholars [an understatement, and equally true of President Reagan among U.S. scholars] . . . for all these reasons it is difficult to interpret Thatcher's policy actions in terms of the kind of once-and-for-all, widely believed, uncontroversial and irreversible regime change that rational expectations equilibrium theories assert can cure inflation at little or no cost in terms of real output . . . and employment.

Sargent writes at greater depth about the "gradualism" of U.K. policy. Gradualism is always an invitation to renegotiation.

The U.S. and U.K. stabilizations did not fail. The Reagan administration did not choose short-term fiscal austerity, raising taxes and cutting spending in the middle of the 1982 recession. But in 1982 and especially 1986, the United States passed a profound fiscal reform, lowering marginal rates and broadening the tax base. The United States and United Kingdom left behind the malaise of the 1970s, at least in part due to less distorting taxes and microeconomic deregulation, and embarked on two decades of strong growth. By the late 1990s, the United States was running large primary surpluses, and economists were debating what to do when the federal government had paid down all the debt. Surpluses did rise, and their present value bore out the disinflation. Surpluses repaid the Reagan deficits, the larger real value of 1980 nominal debt, and the high real interest costs of the 1980s. Surpluses rose with greater delay and a different mechanism—more growth and wider base, not higher tax rates or programmatic spending cuts. Sargent did not clairvoyantly foresee just what an Iron Lady Mrs. Thatcher turned out to be, persisting through the disinflation despite political and economic storms, or that Reagan and Volcker would similarly persist. Indeed, one might view the election of Reagan, or his acceptance of the remarkable Schultz et al. (1980) stick-to-it memorandum, as an event like the election of Poincaré. It made the outcome clear, if not the path by which the country would get there.

Sargent was right that the 1980s disinflations were not painless, however. The recessions of 1980 and 1982 were severe. The United States and United Kingdom experienced high ex post real interest rates for a further decade, arguably in part reflecting continued doubt that the countries would once again give up and return to inflation. But Sargent was also right that the disinflations were nothing like the dire predictions offered by contemporary Keynesians. From 14.4% in May 1980, inflation fell to 2.35% in July 1983. Inflation and its expectation did drop, in the end quite suddenly, without decades of pain. Unemployment was severe, but recovered with remarkable speed, especially relative to the subsequent "jobless recoveries." The disinflations could well have been swifter still, less costly, had they come with a clear, contemporaneous and permanent change in fiscal, monetary, and (importantly) microeconomic and regulatory regime. However, profound reforms like that of 1986 do not happen overnight, as evidenced by the United States' inability to do anything like it in the following 35 years. Later inflation stabilizations involving inflation targets and explicit coordination between fiscal monetary and microeconomic reforms, covered in Section 9.1, did achieve nearly painless disinflation.

This discussion is also tainted by selection bias. We study stabilizations that worked. A study of *failed* stabilizations might be more informative. And there are plenty of them. Latin American history is sadly full of such attempts (Kehoe and Nicolini (2021)). Typically, there is a monetary tightening or reform, together with promises of fiscal and microeconomic reforms. Inflation quiets down for a year or two. The fiscal and microeconomic reforms evaporate, and inflation takes off again. Viewed through this prism, the 1986 tax reform, and the regulatory reforms of the Reagan era, continuing through Bush and Clinton, play a larger role. Had they not occurred, a purely monetary tightening might have failed in North America as well.

The lesson that only policy *regimes* durably change expectations remains foreign to most central bankers and many economists. The inconvenient lesson that only by constraining one's freedom to act ex post can one offer reliable promises ex ante is just as frequently ignored. Central bankers seem to think expectations are "anchored" by their speeches, not by repeated, credible actions and precommitments. "Forward guidance," in which the central bank promises today to take actions it will not want to follow next year—for example, keeping interest rates low despite a resurgence of inflation and strong output—is now considered by the Federal Reserve to be one of its most important "tools." Many economists advocate that central bankers announce new policies such as a higher inflation target, and expect people to immediately believe such promises. Indeed, the whole notion of rules as precommitments or regimes, not as descriptions of discretion, is foreign to the operation of most central banks, who simply wake up each day and make decisions. (Readers at the Fed may bristle at this characterization. Read the Federal Open Market Committee (2020) "strategy," however. Is there any decision the Fed could make that it could not justify as following this elastic description of its strategy?) We who must form expectations somehow are left with guessing the reputations and habits of central bankers.

As I write in Winter 2021, inflation is surging. It may pass, a one-time price level rise occasioned by stimulus payments and a negative, pandemic induced, transitory, aggregate supply or Phillips curve shock. Or those payments, evidently unbacked fiscal expansion, may lead to continued inflation, if people see our continued structural deficits as more unbacked expansion. If the United States has to contain inflation once again, Sargent's message will resonate, especially with the larger debts and clearer fiscal roots of this inflation. Containing inflation will require a joint fiscal, monetary, and microeconomic reform, putting in place new commitments and institutions of sound fiscal policy, debt repayment, and higher economic growth that last a generation. Whether the U.S. political system is capable of such a reform will be a crucial test.

14.4 Episodes of War and Parity

> Countries at war under the gold standard typically suspended convertibility and borrowed and printed money to finance the war. They promised to restore convertibility after the war, though whether they would do so remained uncertain and dependent on the outcome of the war. Fiscal backing is the obvious way to think about inflation and deflation in these episodes.

Countries under the gold standard financed war by suspending convertibility, issuing currency and nominal debt. There was an implicit promise that sometime after the war was over, the country would restore convertibility at the prewar level. Doing so is a promise to pay back rather than inflate away the debt. Whether that would happen, or what conversion rate would hold, was uncertain, and naturally depended on the outcome of the war, so there was often inflation and a fall in bond prices during the war, requiring deflation if parity were to be restored afterwards. But the reputation for returning to parity, for repaying currency as well as debt allows the government to borrow and issue currency next time. The United Kingdom through the wars with France ending with victory over Napoleon is perhaps the paradigmatic example. Bordo and Levy (2020) give a good capsule account of inflation and war finance, including the Swedish Seven Years' war, the American Revolution, the Civil War, and World Wars I and II.

Rather obviously, inflation in such episodes reflects expected fiscal backing of nominal government debt, not supply versus demand of the medium of exchange or central bank management of currency versus debt.

The United States, though it famously followed Alexander Hamilton's advice to repay interest-bearing debt from the Revolutionary War, left the paper continental dollars inflated and ultimately redeemed them at one cent on the dollar. Hall and Sargent (2014) analyze this episode as a clever combination of a one-time capital levy on money and a reputation-buying investment by repaying debt. It offers a similar lesson to the Jacobson, Leeper, and Preston (2019) story of the Roosevelt administration, which also inflated while preserving a reputation for future borrowing.

In the Civil War, the United States issued paper greenbacks, which inflated and lost value relative to gold coin dollars, perhaps in part from the example of continental dollars. But the United States after the Civil War eventually returned to par, repaying both greenback dollars and Civil War debt in full, though after a long debate only settled by President Grant. The "one-time" capital levy always beckons, especially ex post. The debate whether to return to parity or devalue after wartime inflation, reignited in the United Kingdom and France after WWI. Understanding the inflation and deflation of greenbacks clearly starts with money and bond holder's evaluation of the U.S. fiscal commitment to repay Civil War debts.

Fiscal backing is even clearer in the correlation of currency value with battlefield outcomes. In their Figure 11, Hall and Sargent (2014) plot the discount of greenbacks versus gold in the Civil War. They write "after a string of Union defeats in the Spring of 1863, 60 gold dollars bought $100 in greenbacks. The price rebounded to 80 after victories at Gettysburg and Vicksburg but fell again reaching its nadir in June 1864 at a price below 40 gold dollars." McCandless (1996) provides background and detail. McCandless quotes Mitchell (1903),

> While the war continued there could be no thought of redeeming the government's notes. Hence every victory that made the end of the hostilities seem nearer raised the value of the currency, and every defeat depressed it. The failures and successes of the Union armies were recorded by the indicator in the gold room more rapidly than by the daily press...

A nice comment on efficient markets. Mitchell continues, perfectly stating my main point:

> fluctuations in the premium on gold were so much more rapid and violent than the changes in the volume of the circulating medium that not even academic economists could regard the quantity theory as an adequate explanation of all the phenomena. (p. 188.)

He opined that these fluctuations

> followed the varying estimates which the community was all the time making of the government's present and prospective ability to meet its obligations. (p. 199.)

Mitchell describes the fiscal theory in a nutshell. Its essence has indeed been with us a long time.

McCandless investigates the value of Confederate currency. Confederate dollars also rose and fell with battlefield success and loss. The chance of Confederate currency being repaid if the South lost the Civil War was pretty clearly zero. But money supply versus transactions demand does not change the day after a lost battle.

Like all interesting episodes, Civil War inflation remains open to debate. Lerner (1956) and Friedman and Schwartz (1963) provide classic accounts, emphasizing the immense printing of paper money. But money creation was also deficit finance, so they don't really address our crucial question, how much money demand was limited by quantity theory versus fiscal backing.

Burdekin and Weidenmier (2001) examine the consequences of the 1864 Confederate currency reform, in which the quantity of money differed dramatically between the eastern and western Confederacy. The price level differed, indicating an important liquidity value on top of a presumably uniform set of expectations about eventual redemption. Weidenmier (2002) is a nice review of the literature. Hall and Sargent (2014) artfully place civil war inflation in the context of the modern theory of public finance.

The post-WWI history is more famous. The conventional view credits France, which went back on gold at 20% of the prewar parity, with wisdom for avoiding the deflation and recession suffered by the United Kingdom, which went back fully to the prewar parity. Fiscal affairs are complicated by the status of large international loans, especially from the United States, by prospective reparations from Germany, and by the British gold exchange system. Still, to our point, we would not begin to understand the price level in this era based on transactions demand and money supplies, or interest rate manipulations and a Phillips curve; rather than think about the gold standard, its fiscal backing, a nation's ability and will to establish one or another parity to gold, and the tradeoff between fiscal austerity and deflation against the reputation that repaying debt brings the next time a government wishes to borrow.

In which circumstances deflation or disinflation matters to output is another interesting question of these and other episodes. The post–Civil War United States had a steady deflation, especially of greenback values, with no obvious aggregate consequences. (Bryan's "cross of gold" consequences were distributional, borrowers versus lenders, not a Phillips curve of low aggregate output.) Hall and Sargent (2019) contrast the price level and output history of post–Civil War and post–WWI episodes. We add to our list of times when the Phillips curve seems to operate, and times including currency reforms, the ends of hyperinflations, and the introduction of inflation targets with fiscal reforms, when it seems completely absent.

Perhaps the fact that gold currency circulated in the United States during the civil war and its aftermath helped people to adjust to the much larger greenback deflation. The numeraire matters. In the other direction, Velde (2009) gives a fascinating account of seventeenth century France. There were two currencies, a numeraire and unit of account (livres) in which prices were quoted, and a distinct medium of exchange (ecus) held and used for all transactions. A revaluation of the unseen unit of account, needing a decline in quoted prices, led to a severe recession. Velde's article is also a testament that unit of account and medium of exchange may be completely separate, as in my stories of economies in which a "dollar" is valued, though people never hold any dollars.

Was the United Kingdom really unwise to restore gold parity, as Keynes so famously argued? Was there a way to do so and avoid a recession, as so many other stabilizations have done? Why was the post–WWI Phillips curve so severe in the United Kingdom? By restoring parity, the United Kingdom purchased a lot of debt repayment reputation. That hard-won reputation might have been valuable to finance World War II with more money or debt and lower taxes, had the United Kingdom not abandoned the gold standard in the 1930s. France might have needed such a reputation had it not lost the second war so quickly. Keynes might have been wrong. Perhaps "don't buy a reputation you won't keep," is the lesson, and "don't waffle about whether you are going to buy that reputation."

Chapter Fifteen

Esthetics and Philosophy

> Keynesianism, new-Keynesianism, and monetarism were each useful theories, to then-current political debates or to the concerns of central bankers. Fiscal theory is currently less useful to those concerns but that may change. The fiscal theory, by allowing free financial innovation, may replace some of the usefulness of monetarism. It may rescue many useful properties of new-Keynesianism, by fixing the latter's foundations. Fiscal theory is simple and elegant. Simpler and more elegant theories are often correct.

THE OPPORTUNITY to base a theory of the price level on a perfectly frictionless supply and demand model, on which we build frictions as necessary, is also esthetically pleasing. Everywhere else in economics, we start with simple supply and demand, and then add frictions as needed. Monetary economics has not been able to do so. Now it can.

In this way, fiscal theory fills a philosophical hole. It is initially puzzling that Chicago championed both monetarism and free markets. The traditional Chicago philosophy generally pushes toward a simple supply and demand explanation of economic phenomena, and generally tries where possible to arrive at solutions to social problems based on private exchange and property rights. Yet Chicago started its macroeconomics with one big inescapable friction separating money from bonds. Though Friedman advocated floating exchange rates and other financial innovations, monetarism cannot withstand financial innovation that makes bonds fully liquid, or money that pays full interest. This financial innovation only started in earnest in the 1980s.

That philosophy makes sense in historical context. The Chicago view was a lot less interventionist than the Keynesian view of the time. And at the time, there was no alternative for macroeconomic affairs. Fiscal theory as presented here did not exist. Fiscal theory needs intertemporal tools that had not been developed. The quantity theory tradition from Irving Fisher was well developed and ready to be put to use.

But now there is an alternative. The fiscal theory can offer a monetary theory that is more Chicago than Chicago. A monetary theory that allows a free market financial system, and allows us interest-paying money, liquidity satiation, and the Friedman-optimal quantity of money might have been attractive to the Chicago monetarists.

Theories prosper when they are logically coherent and describe data. But empirically, theories also prosper when they are useful to understand important events, or a larger debate or political cause. Keynesianism in the 1930s has been praised for saving capitalism. Against the common view at that time that only Soviet central planning, fascist great-leader direction, or Rooseveltian NRA micromanagement could save the economy, Keynesians said no: If we just fix a single fault, "aggregate demand," with a single elixir, fiscal stimulus, the economy will recover, without requiring a government takeover of microeconomics, and abolition of private property and markets. Even if one regards that Keynesian economics as a fairy tale, embodying in one place dozens of classic economic fallacies, it may have been a *useful* fairy tale as it emboldened resistance to total nationalization in the 1930s.

The tables turned in the postwar era. Now Keynesianism continued to be useful to the left of the U.S. political spectrum. Communist central planning was no longer on the table. But Keynesianism remained potent in U.S. economic debates, part of a softer paternalistic dirigisme. The Keynesians' vision of continual aggregate-demand management fit well with their advocacy of banking, financial and exchange controls, industrial policies, and wage and price controls, as well as extensive microeconomic government management in the postwar era.

In this context, monetarism was likewise *useful* to the free market resurgence in the 1960s. In the face of the then-dominant static Keynesian paradigm, Friedman and the Chicago school could not hope to prevail by asserting that recessions are the normal work of a frictionless market. The *possibility* of this view embodied in Kydland and Prescott (1982) was a long way away. Nobody had the technical skills to build that model, and the verbal general equilibrium assertions of the 1920s were generally dismissed with derision. Something, obviously went very wrong in the Great Depression. Views of the 1930s driven by financial frictions following bank runs (see the influential literature starting with Bernanke (1983)) and views emphasizing the microeconomic distortions of misbegotten policies (see, for example, Cole and Ohanian (2004)) were simply not yet available by theory, historical analysis, or empirical work. The intellectual and political climate demanded that the government do *something* about recessions, that government should have done something about the Great Depression, and demanded a simple, understandable, uni-causal theory without the subtleties of modern intertemporal economics. Intertemporal general equilibrium thinking is hard, harder still with frictions, and has little impact on policy to this day, which remains guided by the embers of hydraulic Keynesianism. When in trouble, reach for stimulus. The argument could be phrased as monetary versus fiscal stimulus, which at least removed the question of just what the government would spend money on from the imprimatur of macroeconomics. Monetarism was perfect to the purpose.

But as the set of facts we must confront has changed since the 1960s, the policy and intellectual environment has changed too. We don't *need* monetarism any more. Fiscal theory fits today's facts, it is adapted to our much-changed institutional and financial structure, and it's ready for the evident challenge: price level control in the shadow of debt and deficits. And the fiscal theory fits much of Friedman's philosophical and intellectual purposes in today's environment, even if it turns many monetarist propositions on their heads. So, I hope that even Friedman might change his mind if he were around today.

I'm beating a dead horse. Monetarism is not a current force, though money supply equals demand shows a surprising resilience in economic theory articles that need to determine the price level somehow, and in commentary. Adaptive expectations IS-LM thinking dominates policy, untied from the quantitative models that gave it some rigor in the 1970s. New-Keynesian models featuring dynamic general equilibrium, explicit frictions, and explicit if not rational expectations, dominate in academia, combined with a Taylor-rule description of interest rate setting. These theories too grew out of empirical and practical necessity. Inflation surged under interest rate targets in the 1970s and declined under the same targets in the 1980s. Monetary aggregate based policy fell apart in the early 1980s. We have to talk about interest rate targets. These are *useful* theories.

Moreover, they connect with the concerns of central bankers. If a central banker asks, "Should we raise or lower the interest rate?," and you answer, "You should control the money supply," you won't be invited back. If you answer, "Recessions are dominated by supply, credit, and other shocks with interesting dynamics, and monetary policy doesn't have that much to do with them," you won't be invited back. If you answer "The price level is dominated by fiscal policy," you won't be invited back. If you answer "Let's talk about the interest rate rule and regime," you might be invited back to a technical conference, but the banker will surely press, "Yes, yes, but what should we do *now*?" Central banks follow interest rate targets, and central banks are the central consumers of macroeconomic advice. A useful theory of monetary policy, that any central banker will pay any attention to, must model interest rate targets—even if, as here, it ends up suggesting there are better ways to run monetary policy.

The economic conceptual framework used by people in policy positions is often fundamentally wrong, of course. And one should say that. But if we want to understand why theories around us prosper, usefulness as well as pure scientific merit has strong explanatory power. And where possible without sacrificing scientific merit, trying to find common ground or speak to issues of the day is not a totally undesirable characteristic of an economic theory. Moreover, listening isn't a bad habit either. Sometimes the practical knowledge of people in the thick of things reveals facts and economic logic we have not considered.

This book takes its long tour of interest rate targets and central bank actions to offer supply to that demand as well. I have worked to show how fiscal theory can fill the gaping holes of new-Keynesian models, allowing at least continuity of methodology if not necessarily of results, and thereby making fiscal theory *useful* to researchers who want to improve new-Keynesian style models of monetary policy and to central bankers and treasury officials who wish to guide inflation. There are many other ways fiscal theory suggests that we might set up a monetary system in the future. But these considerations are not terribly useful right now, so I have spent less time on them in this book.

New-Keynesian economists are explicit in an intellectual goal, equally esthetic, philosophical, and useful to the larger debate: to revive the general flavor of IS-LM in a framework that survives the devastating Lucas (1976) critique of IS-LM theory, Sims's 1980 "Macroeconomics and Reality" critique of its empirical practice, and the evident grand failure of IS-LM in the inflationary 1970s and disinflationary 1980s. It is designed to be a theory of monetary policy, based on interest rate targets, more than it is designed to be a theory of recessions, which come from the same ephemeral "shocks" as in other theories. It gives the Fed something to do and a framework to think about the effects of its decisions, which the real business cycle theory of recessions does not offer. New-Keynesian economics is designed to be a theory that is both beautiful and useful. Fiscal theory of monetary policy is not likely to offer justification for IS-LM thinking, but it turns out the actual equations of new-Keynesian models don't do so either.

Fiscal theory is not immediately useful to one side or another of today's economic, methodological, ideological, or political debates. Indeed, I have intentionally broadened its appeal to indicate how a wide variety of modeling philosophies can incorporate fiscal theory.

In my framing, fiscal theory takes on some of the mantle of monetarism. Fiscal theory offers a theory of inflation based on simple explainable supply and demand foundations. It stresses the underlying importance of stable monetary and fiscal institutions. Nothing is more forward-looking than a present value formula. I spend time using the fiscal theory to think about proposals and possibilities for a pure inflation target, a gold-standard like, interest spread operating rule, and the possibility of private institutions taking over. I use it to embrace the possibilities that current communication, computation, and financial technology offer us today. But this pursuit reflects my economic philosophy, and one could use fiscal theory in much different ways. One can use fiscal theory to patch up new-Keynesian theoretical holes, and proceed in a much more interventionist direction. One can more swiftly add layers of frictions for policy to exploit.

Esthetic and philosophical considerations don't make a theory right. Usually we pretend such concerns don't exist, so we do not write about them. But they shouldn't be ignored. Though economics is often criticized for playing with pretty theories rather than the "real world," the most successful theories of the past have been simple and elegant in economics as in the rest of science. Epicycles seldom survive, even if, as in Copernicus' case, they temporarily fit the data better than the simpler and eventually victorious theory (Kuhn (1962)). Supply and demand, comparative advantage, the burden of taxation, the great neutrality results—all have a decisive simplicity.

At least in the eyes of this beholder, the fiscal theory is truly beautiful. I hope by now to have infected you with that view as well. Fiscal theory can be expressed in a simple model, with a simple story. Nothing like the simplicity and clarity of the first chapter of this book underlies new or IS-LM Keynesian models, or even monetarism.

These are secondary concerns. The primary case for fiscal theory is that it holds together logically, it is consistent with the facts of our monetary and financial institutions, and it describes events such as the zero bound period in a way that monetarism or new and old Keynesianism do not do. If a surge of clearly fiscal inflation breaks out after this book is published, and if its insights help to address that surge, its need will be even clearer. These are the true tests of a useful theory.

Chapter Twenty Two

Observational Equivalence

With the new-Keynesian and monetarist models before us, and with their equilibrium selection rules spelled out, I return to summarize and extend observational equivalence and nonidentification, and their implications.

22.1 Equivalence and Regimes

I state observational equivalence and the nonidentification theorems in the simplest models,

$$
\begin{aligned}
i_t &= i_t^* + \phi\left(\pi_t - \pi_t^*\right) + u_{i,t} \\
\tilde{s}_{t+1} &= \alpha v_t^* + \gamma(v_t - v_t^*) + u_{s,t+1}.
\end{aligned}
$$

In equilibrium, where variables equal the starred values, equilibrium time series do not distinguish the active-money passive-fiscal $\phi > 1$, $\gamma > 0$ regime from the active-fiscal passive-money $\phi < 1$, $\gamma = 0$ regime. The parameters ϕ and γ are not identified.

The clearest simple example of observational equivalence for interest rate regimes comes from Section 16.6. We wrote monetary and fiscal policy rules

$$
\begin{aligned}
i_t &= i_t^* + \phi\left(\pi_t - \pi_t^*\right) + u_{i,t} \qquad (22.1)\\
\tilde{s}_{t+1} &= \alpha v_t^* + \gamma(v_t - v_t^*) + u_{s,t+1} \\
\rho v_{t+1}^* &= v_t^* - \Delta E_{t+1}\pi_{t+1}^* - \tilde{s}_{t+1},
\end{aligned}
$$

in the context of a frictionless model with

$$
\begin{aligned}
i_t &= E_t \pi_{t+1} \\
\rho v_{t+1} &= v_t + i_t - \pi_{t+1} - \tilde{s}_{t+1}. \qquad (22.2)
\end{aligned}
$$

Parameters $\phi > 1$, $\gamma > 0$ generate the active-money, passive-fiscal regime, and most clearly $\alpha = \gamma$. Parameters $\phi < 1$, $\gamma = 0$ generate the active-fiscal, passive-money regime, and most clearly $\phi = 0$. The government debt valuation formula results from iterating 22.2 forward and imposing the transversality condition.

The clearest simple example for monetary-control regimes comes from Section 19.1. Simplifying further to the case that money pays interest, or that surpluses react one for one to seigniorage, we have

$$M_t V_t = P_t y_t,$$
$$\frac{B_{t-1} + M_{t-1}}{P_t} = E_t \sum_{j=0}^{\infty} \beta^j s_{t+j}.$$

In an active-money passive-fiscal regime, the quantity of money M determines the price level P, and surpluses s follow. In an active-fiscal passive-money regime, fiscal surprises control unexpected inflation, and then the central bank provides the needed money passively. We can characterize these behaviors in the policy-rule tradition: In an active-money regime, the money supply does not react one for one with the price level to validate any inflation or deflation. In an active fiscal regime, surpluses do not react one for one with the price level to validate any inflation or deflation.

As this reminder makes clear, the "regimes" are observationally equivalent:

- *The equilibrium conditions are the same in each regime. Any time series produced by an active-money passive-fiscal regime can be produced by an active-fiscal/passive-money regime and vice versa.*

The regimes differ in how we imagine the government behaves away from equilibrium, when variables do not equal their starred counterparts, how monetary and fiscal authorities hash out a coordinated policy. We can't observe that behavior in data drawn from the equilibrium.

Observational equivalence is the same as nonidentification:

- *Without additional identifying assumptions, the parameters that separate regimes such as γ and ϕ are not identified from time series of observable equilibrium variables.*

22.2 Implications Overview

Observational equivalence goes both ways. Any rejection of fiscal theory from equilibrium time series also rejects other theories of equilibrium formation. Observational equivalence opens the door to understanding any sample equally via fiscal theory as via new-Keynesian models. It guides us to find and examine the identifying assumptions of any proposed test. It guides us to look to institutions, regimes, commitments, and statements by fiscal and monetary authorities about how they operate, commentary on how people expect them to operate, narrative approaches to historical events, and times of regime change or construction.

22.2.1 Equivalence Goes Both Ways; A Feature Not a Bug

On first glance, observational equivalence seems like a show-stopper. Why bother investigating a theory that doesn't seem to have rejectable predictions? On further reflection, however, observational equivalence is a feature, not a bug. It is an important guide to productive and unproductive investigation, like the observational equivalence and neutrality theorems of many other areas of economics.

Equivalence means equivalence. It goes both ways. It says that one cannot reject new-Keynesian or monetarist equilibrium selection stories in favor of the fiscal theory story. But it says that new-Keynesian or monetarist models cannot reject fiscal theory either. For the new kid on the block, proving that the door is open is good news. If an observation dooms fiscal theory, then it equally dooms new-Keynesian or monetarist theories. There is no scientific burden of proof based on who came along first.

In particular, a strand of fiscal theory evaluation looks to puzzles of the government debt valuation equation—why isn't there inflation in Japan?—and proclaims such puzzles as a rejection of fiscal theory. But, an instance of the last bullet point,

- *The government debt valuation equation is an equilibrium condition for all of these models.*

Any puzzle of the government debt valuation equation is equally a puzzle for interest rate and monetarist models. It is a puzzle of debt sustainability in equilibrium, not an indication of how that equilibrium is formed. It does not reject fiscal theory in favor of the others, which also include this condition.

Observational equivalence opens the door to casting out the other theories entirely, to looking at a whole sample in fiscal theory terms. One is not limited to looking for periods of fiscal versus monetary dominance, as has been common in fiscal theory literature. Observational equivalence provides a recipe by which one can transform *any* monetarist or new-Keynesian model into a fiscal theory model, without changing any of its implications for observable time series. One can only improve on them, by being led to better specifications of the equilibrium conditions and more comprehensive evaluation, and by including fiscal implications of those theories in their evaluation.

Nonidentification is related to observational equivalence. From equivalence, it follow that equilibrium-selection parameters which distinguish theories cannot be identified. But nonidentification doesn't require the existence of multiple theories. It just says that equilibrium-selection parameters can't be identified from equilibrium time series, without additional assumptions.

22.2.2 Tests and Assumptions

There are many observational equivalence results in economics. We often surmount them with identifying assumptions. Observational equivalence points one to write models in ways that express observational equivalence, and then to state and evaluate identifying assumptions. They are not technical details, they are the whole game, and they are often unstated or implicit.

I review the many identifying assumptions that have been used to try to make such tests. I come to a rather negative assessment of progress so far, and not much hope for the productivity of future effort. Observational equivalence thus warns us at least to be wary of formal time series tests that to try to estimate or test regimes, like tests to distinguish broad classes of models. I conclude that it will be more productive to use evidence other than that provided by tests based on equilibrium quantities. But observational equivalence per se is not the central case against extant tests for fiscal versus monetary regimes. The central case is that identifying restrictions don't make sense.

22.2.3 Beyond Tests

Observational equivalence only says that time series of observables may be produced by either class of models. It does not rule out troves of other types of evidence. We can look at the historical, institutional, and economic plausibility of equilibrium-selection stories, in general and in the context of specific episodes.

By looking deeply at the foundations of monetarist and new-Keynesian regimes in the last few chapters, we see that their equilibrium selection stories don't conform with lots of information we have about how governments behave. No central bank says it operates as the $\phi > 1$ equilibrium selection policy describes, and nobody expects it to do so. Active fiscal policy, in which surpluses do not respond to arbitrary inflation and deflation, is plausible and consistent with episodes and how people expect governments to react. Central banks don't control money supplies.

If the equilibrium selection theories are contradicted by evidence we have about how governments behave, if there is no complete, coherent, and plausible alternative to fiscal theory, I conclude that tests and estimates of fiscal versus these other theories are doubly pointless. There is no point to adding identifying restrictions, which will hurt the ability of the model to fit data, in order to attempt time series tests on top of this other information.

We can go on: Read the Federal Open Market Committee (2020) official description of its strategy, the minutes of Federal reserve meetings, the commentary of the financial press describing how people expect the Fed to behave if inflation should rise or fall. Narrative evidence in the tradition following Romer and Romer (1989) can help us to see shocks and disturbance $\varepsilon_{i,t}$ and $u_{i,t}$, from which one can infer ϕ. We can study episodes, such as the zero bound. We can study moments of regime change, institutional reform, and government choices in terms of objectives and constraints. We can measure, as I did above, the pattern of surpluses and discount rates that accounts for inflation, rather than try to proclaim no such patterns exist. We can *use* fiscal theory as we use other theories and see which proves more useful.

22.2.4 The Same Situation Elsewhere

Observational equivalence theorems abound in economics and finance. Supply versus demand shifts, behavioral versus rational finance, and money versus income causality all present observational equivalence theorems. Those theorems do not mean that the theories are empty. Observational equivalence theorems are simply fundamental guiding principles for logical critical thought.

In microeconomics, data alone do not tell us the slopes of supply and demand curves—what buyers or sellers would do if the price came out differently from the equilibrium price. Well, we write models and make identifying assumptions. We look for instruments, we think hard about their plausibility.

Finance has a similar observational equivalence theorem. Marginal utility and probability always enter together in asset pricing formulas, $p = \sum_s \pi_s u'(c_s) x_s$, where p is price, s indexes states of nature, π are probabilities, u' is marginal utility and x is payoff. "Rational" (u') and "behavioral" (misperceived probabilities π) finance are observationally equivalent. This is a modern version of the Fama (1970) "joint hypothesis" theorem, formalized in the Harrison and Kreps (1979) martingale measure theorem and the Hansen and Richard (1987) discount factor existence theorem. Attempts to show that all "rational" or efficient market asset pricing is wrong with a statistical test are empty.

Observational equivalence has not stopped these and subsequent branches of finance from productive investigation, nor does it prove that the debate is empty. But it usefully pours cold water on attempts to construct a statistical test using data on prices, payoffs, and economic variables, which will prove one or the other *class* of theory wrong. There is no interesting test of the present value relation per se—not volatility tests, not regression tests, not the hundreds of anomalies and alphas.

Instead, asset pricing now gets to work on writing an economic or psychological model of the discount factor, and the stochastic process of dividends. So, the heart of asset pricing, just like the heart of monetary economics, is to think hard about what is reasonable, and to evaluate what is useful. A lot of acrimony in finance could have been saved by paying attention to this basic theorem. We can save a lot of time and effort by not repeating for fiscal theory the difficult history of empirical asset pricing.

Applied to government debt, the discount factor theorem states that

- *Absent arbitrage, there is a discount factor that reconciles the value of debt to surpluses, a* $\{\Lambda_t\}$ *such that*

$$\frac{B_{t-1}}{P_t} = E_t \sum_{j=0}^{\infty} \frac{\Lambda_{t+j}}{\Lambda_t} s_{t+j}.$$

Thus, present value puzzles are doubly irrelevant as tests of the fiscal theory versus conventional theories. Present value puzzles are entirely puzzles of a discount factor or probability model.

Behavioral versus rational finance, Keynesian versus monetarist versus rational expectations versus new-Keynesian versus real business cycle macroeconomics were never settled by formal tests of equilibrium time series that reject a whole class of model. Even without observational equivalence, since the models study different equilibrium conditions, there was always a patch, a way to carry on. Theories gradually gain or lose steam as their foundations, explanations, and policy analysis seem more or less reasonable and useful. Fiscal theory versus interest rate equilibrium selection or money supplies will be settled the same way. This is the normal nature of all economics. Many tests were tried in all these fields. It is natural to have tried for such tests for fiscal versus other theories of inflation. That they reached a dead end here as in other fields just tells us to get on comparing theories as we always have in other fields.

Observational equivalence points us to what will and will not be a fruitful way to proceed. It starts by largely telling us not to waste more time on formal tests.

22.3 Regime Tests and Model-Based Estimates

Observational equivalence forces us to find and analyze identifying assumptions underlying tests. Once examined, common implicit identifying assumptions aren't sensible: Off-equilibrium responses need not be the same as responses in equilibrium. Disturbances need not be AR(1) or otherwise limited. Inflation and debt should Granger-cause surpluses, and larger surpluses should forecast declines in debt, even in a fiscal regime. Regime tests have stacked the deck against a fiscal regime in the quest for identification, leaving the impression that fiscal theory only describes an unfortunate "fiscal dominant" situation in which monetary policy loses control and inflation breaks out. Removing the identification restrictions opens the door to fiscal theory that describes a whole sample, including periods of low inflation, and acts to stabilize inflation.

The most natural desire, when playing with a new theory, is to find a test, either of the theory or of its competitors. It was completely natural, with the fiscal theory freshly in hand, to try to estimate whether we live in an active fiscal or an active money regime, to test one versus the other. It was natural to want to see periods of better or worse inflation performance as switches between a "money dominant" and a "fiscal dominant" regime.

Leeper (1991) kicked off fiscal theory with models of the form

$$i_t = E_t \pi_{t+1} \tag{22.3}$$

$$i_t = \phi \pi_t + u_{i,t} \tag{22.4}$$

$$\tilde{s}_{t+1} = \gamma v_t + u_{s,t+1} \tag{22.5}$$

$$v_{t+1} = v_t + i_t - \pi_{t+1} - \tilde{s}_{t+1}. \tag{22.6}$$

Active monetary and passive fiscal policy is $\phi > 1$, $\gamma > 0$. Leeper pointed out the possibility of active fiscal and passive monetary policy $\phi < 1$, $\gamma = 0$.

Faced with a model like this, the most natural thing in the world to do is to run regressions of (22.4) or (22.5) to estimate ϕ or γ, and thereby see which regime we are in. Run more realistic versions of $i_t = \phi \pi_t + u_{i,t}$, as Clarida, Galí, and Gertler (2000) do. Run more realistic versions of $s_t = \gamma v_{t-1} + u_{s,t}$ as in Bohn (1998a) or Table 4.1. (Bohn runs the regression, but does not interpret it as a test of regimes.)

Better, estimate the full model by maximum likelihood or Bayesian methods, including parameters ϕ and γ. Use the associated distribution theory to test regimes. Next, think of time-varying coefficients γ and ϕ, perhaps governed by Markov-switching models, and estimate subperiods of "monetary dominance" or "fiscal dominance," perhaps formalizing a different expression of the idea that the 1980 switch in policy rules was a good thing. The extensive fiscal theory model-building exercise surveyed in Section 24.2 below has largely followed this direction.

Observational equivalence tells us that any such estimate or test of the active/passive regime must be *entirely* based on the auxiliary assumptions and model restrictions one introduces to gain identification. It tells us to write the model in a form that exhibits observational equivalence and to study the identifying assumptions. Models throughout economics and finance include basic principles plus auxiliary assumptions. Observational equivalence per se is not a problem. The question is whether, when we dig in to state them, the identifying assumptions are believable.

Repeating (22.1)-(22.2) for convenience and rewriting slightly,

$$i_t = E_t \pi_{t+1} \tag{22.7}$$
$$i_t = \theta \pi_t^* + \phi(\pi_t - \pi_t^*) + u_{i,t} \tag{22.8}$$
$$\tilde{s}_{t+1} = \alpha v_t^* + \gamma\left(v_t - v_t^*\right) + u_{s,t+1}. \tag{22.9}$$
$$\rho v_{t+1}^* = v_t^* - \Delta E_{t+1} \pi_{t+1}^* - \tilde{s}_{t+1} \tag{22.10}$$
$$\rho v_{t+1} = v_t + i_t - \pi_{t+1} - \tilde{s}_{t+1}. \tag{22.11}$$

Remember that in equilibrium, starred variables equal their unstarred observable counterparts, so the two theories are equivalent in equilibrium.

The parameter ϕ is not identified. The interest rate policy rule parameter θ can, in principle, be measured. But written this way, you see that information about θ tells you nothing about ϕ. The specification $i_t = \phi\pi_t + u_{i,t}$ thus adds an implicit assumption, $\phi = \theta$. That assumption overcomes nonidentification and observational equivalence. It is a separate and crucial assumption.

The parameters θ and ϕ have distinct economic functions. The parameter θ governs the relation between inflation and interest rates in equilibrium, devoted to smoothing fluctuations. The parameter ϕ is an equilibrium-selection threat, devoted to making multiple equilibria unpleasant. In this simple model, we should have $\theta < 1$ if we wish stationary solutions, and $\phi > 1$ if we wish determinate solutions. There is no reason the parameters should be the same, and many reasons they should be different.

An analogous point applies to the fiscal rule. In the form (22.11), we see that the identifying assumption in (22.5) $s_{t+1} = \gamma v_t + u_{s,t+1}$ is $\alpha = \gamma$, that the government raises surpluses to validate any inflation or deflation that comes along, in the same way as it raises surpluses to pay off previous borrowing. There is no reason that governments must equate these responses, and there are excellent reasons for governments to respond differently to the different sources of variation in value of the debt, given that governments wish to borrow to finance deficits and wish to control inflation. We may easily see a positive coefficient in a regression of surpluses on debt from a fiscal-theory equilibrium. Section 5.5 constructs an example. Leeper and Li (2017) also show that regressions of surplus on debt do not establish passive fiscal policy.

Since the question is the fundamental "cause" of inflation, one is tempted to run Granger causality[1] tests between debt or deficits and inflation or surpluses. Observational equivalence warns us to be wary. Without the assumption that people in the economy see no more information than we do, Granger causality tests are not causality tests. And that assumption is not plausible. If people learn from reading the news that surpluses will be poor, they rush to sell government bonds and drive up the price level. Inflation helps us who observe the economy to forecast deficits. Analogously, asset prices help to predict, and hence Granger-cause, subsequent dividends and returns. That doesn't mean that price changes *cause* dividend and return changes. People have information about good future dividends, say, and then bid up asset prices. We, studying the economy with less information, see an unexpectedly higher price, and then the higher dividends. Consumption Granger-causes income. You learn of a raise next year and go out to dinner. The dinner helps an econometrician to forecast larger income. Going out to dinner does not cause a raise (alas). Section 19.8 makes the same point regarding the correlation of money with nominal income.

Causality tests suffer from a deeper problem in this application. The active fiscal versus active money question is which off-equilibrium expectation supports an equilibrium. Unlike the money versus nominal income question that generated Granger causality tests, off-equilibrium expectations leave no signature in the temporal ordering of equilibrium variables. The equilibrium conditions are the same in both regimes. So the joint dynamic process of surplus, debt, discount rate, and inflation tells us nothing about which equilibrium selection regime produces the inflation. Equilibrium selection adjustments do not happen with a delay.

A second source of identification restrictions comes by restricting the stochastic process of the disturbances and $\{u_{s,t}\}$ in particular. Indeed, any identification of the parameters ϕ and γ must include restrictions on the disturbances, since they can soak up or offset any behavior of the $\phi(\pi_t - \pi_t^*)$ and $\gamma(v_t - v_t^*)$ terms. The surplus process can have an s-shaped response by virtue of $\alpha > 0$ and $\Delta E_t \pi_{t+1}$ uncorrelated with shocks to $u_{s,t+1}$, or it can have an s-shaped response with $\alpha = 0$ by virtue of an s-shaped $\{u_{s,t+1}\}$ process. The parametric form is convenient but not necessary. In his identification critique of Keynesian models, Sims (1980) cites identification by disturbance lag-length restrictions or by exclusion restrictions (leaving v_t out of the VAR) as assumptions with particularly weak foundations.

For example, Canzoneri, Cumby, and Diba (2001) test whether a shock to surpluses reduces subsequent debts, as analyzed in Sections 4.2.2 and 4.2.6, and interpret that finding as refutation of fiscal theory. But we saw that test comes down to the identifying restriction $a(\rho) > 1$ in $\tilde{s}_t = a(L)\varepsilon_{s,t}$.

Unusually, Cumby, Canzoneri, and Diba recognize that active money and active fiscal regimes are observationally equivalent, writing

> it is quite difficult (and perhaps impossible) to develop formal tests that discriminate between R [active money] and NR [active fiscal] regimes, since (as Cochrane, 1998, points out) both regimes use exactly the same equations

---

[1] A variable x_t is said to Granger-cause y_t if surprises to x_t, $\Delta E_{t+1} x_{t+1}$, forecast surprises to subsequent y_t, $\Delta E_{t+1} y_{t+1+j}$, where $\Delta E_{t+1}(\cdot) \equiv E(\cdot|z_{t+1}) - E(\cdot|z_t)$ and z_t is a vector of VAR variables including x_t and y_t. Equivalently, x_t does not Granger-cause y_t if the impulse-response function of y_{1+j} to x_1 shocks is zero. One handles contemporaneous correlation of the x and y shocks by assumptions.

to explain a given data set.

They acknowledge that a long-run negative autocorrelation of surpluses, $a(\rho) < 1$ is possible and solves their puzzle. They opine that $a(\rho) < 1$ is not *plausible:*

> NR [fiscal-theory] regimes offer a rather convoluted explanation that requires the correlation between today's surplus innovation and future surpluses to eventually turn negative. We will argue that this correlation structure seems rather implausible in the context of an NR regime...

If one wishes to test fiscal theory, this is just the right sort of argument to have. Recognize observational equivalence, state identifying assumptions used to overcome it, and think about whether those assumptions are plausible.

Twenty years of hindsight may change one's mind about plausibility. We can now realize that an s-shaped response, $a(\rho) < 1$, is not at all convoluted, nor unnatural, nor special to passive fiscal regimes.

The fiscal theory model literature, covered in Section 24.2, estimates much-elaborated versions of (22.3)–(22.6), with time-varying regimes or Markov-switching between regimes. These models typically also specify an AR(1) for the disturbance $\{u_{s,t}\}$. Together with $\alpha = \gamma$, then, an s-shaped surplus process indicates a passive fiscal regime, and active fiscal policy is tied to the counterfactual predictions of Section 4.2. The AR(1) or other restriction on the disturbance is crucial for this identification, as a more general process can produce the s-shaped response all on its own. (Sections 24.1 and 24.2 contain reviews of this literature.)

As we saw, a surplus process with $\gamma = 0$, $s_{t+1} = u_{s,t+1}$, and a positively correlated disturbance with $a(\rho) > 1$ is deeply counterfactual. How, then, do the models find *any* periods of active fiscal policy? Well, by also restricting $\theta = \phi$ and the monetary policy disturbance $\{u_{i,t}\}$, the models impose a different set of counterfactual predictions for active monetary policy. My guess, then, is that such models find active-fiscal passive-monetary policy in times such as the 1970s, as they typically do, when there is a lot of inflation volatility so the $a(\rho) > 1$ counterfactual predictions don't look so bad, but interest rates do not move much with inflation, so $\phi = \theta > 1$ is a particularly bad fit. They then find active-money passive-fiscal regimes in times such as the 1980-2008 period when the $a(\rho) > 1$ surplus predictions are really hurtful to model fit, and $\phi = \theta > 1$ better fits Fed behavior. The chosen regime is a compromise of which identifying assumption makes the model fit least badly.

The result is a misapprehension of what fiscal theory is and does. The active fiscal or "fiscal dominant" regime is usually seen as the bad regime, when inflation is volatile, when monetary policy is forced to cave in to inflationary fiscal pressure. The active money or "money dominant" regime is seen as the good one, when fiscal policy follows monetary commands to lower inflation. People use "fiscal dominance" as a synonym for a fiscal shock generating a large unexpected inflation.

If we loosen the identifying restrictions, we can fit the data better in all time periods—allowing an s-shaped surplus process with active fiscal policy, along with interest rates that react less than one for one to inflation when needed. And fiscal theory changes character. Fiscal theory can apply at all times, describe the whole sample, and it can describe policies and institutions that stabilize and quiet inflation.

22.4 Plausibility and Other Evidence

Observational equivalence only applies to equilibrium time series. We can still look at the plausibility of different regimes, and we can look at information in institutions, rules, mandates, legal limitations, and statements that fiscal and monetary authorities make to communicate off-equilibrium behavior. We can analyze choices governments make in difficult times. I quickly summarize the previous arguments that the active money equilibrium selection mechanism is implausible and inconsistent with this kind of evidence, while the active fiscal regime is plausible and consistent.

Observational equivalence only applies to equilibrium time series. We can and should use additional information. We can and should examine the *plausibility* of different regimes, off-equilibrium behaviors, and identifying assumptions. What do fiscal and monetary authorities say they would do in various circumstances? What kinds of behavior are encoded in the legal and institutional structures and restrictions of monetary and fiscal policy? Why, and in response to what historical experiences, were those structures chosen? How do people in the economy expect those authorities to behave? When we see governments making hard choices, say between unpopular and distortionary taxation or spending cuts versus inflation or devaluation, what do those choices tell us about the economic constraints governments perceive?

We examine plausibility of identifying assumptions and off-equilibrium behavior to overcome problems everywhere in economics and finance. Behavioral and rational asset pricers admit observational equivalence given time series of prices, dividends, and economic variables—though often grudgingly—but then question the plausibility of the alternative interpretations, or their consistency with other sources of information. As they should.

How plausible *are* the off-equilibrium stories, in the light of all this other evidence? We spent a lot of time on this issue, for just this reason that it is central, once observational equivalence knocks out formal tests.

Looking at the new-Keynesian equilibrium condition in the form $E_t(\pi_{t+1} - \pi^*_{t+1}) = \phi(\pi_t - \pi^*_t)$, I object that no central bank responds to inflation with more inflation to select equilibria, and people do not expect them to do so. Looking at operating procedures, our central banks do not limit money supplies. Money demand is interest elastic and has lost any meaning in a plethora of liquid assets and electronic transactions.

Fiscal theory critics offer similar objections to the plausibility of "active" fiscal policy, and whether people could expect such a thing. For this reason, I have argued that the fiscal commitment to refuse to adapt surpluses to variation in debt caused by unexpected inflation, while often repaying debt accumulated from past deficits, is a reasonable description of current institutions, expectations, and sound government policy.

Monetary and fiscal policies are full of *institutions,* rules, and traditions that help the government to commit to and communicate off-equilibrium behavior and equilibrium selection policies that cannot be directly observed from macroeconomic time series. The gold standard, foreign exchange pegs, backing promises, currency boards, balanced budget rules, inflation targets, Taylor rules, legal restrictions against inflationary finance and central bank actions, and the institutional separation of monetary and fiscal policy, are all examples. Central banks' repeated statements about how they would react to events in speeches, testimony, and formal strategy pronouncements, and their eternal silence about equilibrium selection via a hyperinflationary threat tell us a lot about their off-equilibrium behavior.

Events often suggest one or another of *possible* interpretations is more *plausible.* A country—Venezuela, say—has large persistent deficits, inflation, and prints a lot of money. Now it's *possible* that the central bank went nuts, printed up a lot of money, caused inflation, and the fiscal authorities though fully able to raise taxes or cut spending went along "passively," because the central bank is supposed to be in charge of inflation. But that's a pretty implausible story, though it satisfies the letter of observational equivalence.

In moments of stress we see decisions that reflect the choices that governments see in front of them. A government in a crisis chooses between distorting taxes and the distortions of inflation. Its choices, and the mechanisms it puts into place to avoid another crisis, tell us a lot. Does it put into place a rule demanding any inflation be met with higher inflation ($\phi > 1$)? Or does it put into place institutions that react to inflation with fiscal tightening?

Plausibility arguments can go on. The 50-year-old behavioral versus rational finance debate is exhibit A of that observation, with Keynesians versus monetarists, and then versus general equilibrium going on for even longer. But that is how we learn, once we rightly abandon hope that a formal test will settle things once and for all.

22.5 Laugh Tests

Apparently easy armchair laugh tests likewise fail. The present value relation is part of all theories, so does not distinguish them. Deficits are higher in recessions, and lower in booms, yet inflation goes the other way, lower in recessions and higher in booms. What about Japan, and other countries with high debts and no inflation? The fiscal theory does not predict a tight relationship between deficits or debt and inflation. For both cyclical and cross-country comparisons, variation in the discount rate may matter more than variation in expected surpluses to understand the price level.

Many commenters dismiss fiscal theory by apparently easy armchair rejections, or laugh tests. Recessions feature deficits and less inflation. Expansions feature surpluses and more inflation. The sign is wrong! Countries with large debts or deficits seem no more likely to experience currency devaluation or inflation. What about Japan, with debt more than 200% of GDP and no inflation? What about the United

States, at least before 2021, with large debts and deficits, annual warnings from the CBO of yawning fiscal gaps to come? Contrariwise, many currency crashes, and inflations, such as the late 1990s East Asian currency collapses, were not preceded by large deficits or government debts. Doesn't all this invalidate the fiscal theory?

No, as observational equivalence and existence of discount factor theorems should indicate. First, equivalence is equivalence. The present value equation is part of new-Keynesian and monetarist theories. If somehow the present value equation fails, it rejects those theories equally. Moreover, there *is* an expectation of future surpluses and a discount factor that makes sense of these observations. And they are not entirely unreasonable.

Fiscal theory does not predict a tight relationship, or even a positive correlation, between deficits and inflation. The fiscal theory ties the price level to the present value of future surpluses, not to current surpluses. On average, debts that raised revenue to finance deficits must be followed by surpluses, and do not forecast inflation, or investors won't lend in the first place. Big inflations and currency crashes, and ends of inflations and stabilizations, happen when important news about future surpluses and deficits emerges, not a slow predictable pressure of current debt. As with stock market prices or bank runs, just what the piece of information is that changes investors' minds is not necessarily easy to see. There is no asset for which economists can forecast payments and make even vaguely correct guesses about the price.

CBO projections are clearly warnings about what will happen if law does not change, not conditional means. When the CBO tells you that its own projection is "unsustainable," that means it won't happen. Bond investors may still believe that the U.S. government will undertake straightforward reforms before driving the country to a catastrophic debt crisis.

Discount rates vary. Discount rates are lower in recessions and higher in booms, driving a time-series correlation of inflation with business cycles. The steady downward trend of real interest rates from 1990 to 2020 suggestively correlates with high and rising values of debt in advanced economies, together with low and declining inflation. Just *why* real interest rates are so low is a good economic question. But it is an economic question for all theories, not a question that distinguishes fiscal theory from other theories of price level determination and equilibrium selection.

Japan has low real rates. Simplistic $r < g$ calculations say that its present value puzzle is the absence of much greater deflation! (More on $r < g$ in Section 6.4.) In addition, though Japan's gross debt-to-GDP ratio is indeed high, 264% as I write in 2021, its net debt-to-GDP ratio is 154%. The Japanese government has a lot of assets. Japan accumulated foreign assets during a long period of trade surpluses. Japan's debt is largely long-term, held by Japanese people and domestic financial institutions. Japan has an inheritance tax. And, perhaps, just wait. Most of all, again, the government debt valuation equation is equally a part of new-Keynesian and monetarist theories. Equivalence is equivalence. If it fails, all these theories fail.

None of this is proof, nor offered as such. The point is that armchair tests based on these and related facts are not tests of fiscal versus new-Keynesian or monetarist price level determination regimes. The theorems tell us that there *is* a fiscal-theory story. I only claim here that there are not totally unreasonable stories.

22.6 Chicken and Regimes

> The proverbial game of chicken between treasury and central bank is a conceptually useful fable, but an unrealistic description of policy formation. We do not *have* to model policy formation by such a game. The game of chicken does not apply to interest rate targets, in which the central bank must actively hyperinflate to select equilibria rather than refuse to monetize debts. In the end, the government must produce a coordinated policy. How it does so leaves no tracks in the time series.

The question of active versus passive regime is often told as a game of chicken, the game in which two drivers face head on and the one who swerves is the chicken. Sargent and Wallace (1981) famously used this metaphor for a situation in which money supply and fiscal policy are in conflict. It has been almost too influential, leaving the false impression that we *have* to describe policy in these terms, with one of monetary or fiscal policy completely passive and the other one completely in charge.

In the end, the government must provide a coordinated fiscal and monetary policy: a setting of interest rates or money supplies, and surpluses and debts, that generates a unique equilibrium price level. When two tools conflict, the government needs to figure out settings that do not conflict. From a historical, political science, or just common sense view, the game of chicken embodies a stylized and unrealistic story of how a government forms a coordinated policy. Government is composed of many interested players with conflicting objectives, who hash out the intricate negotiation that occupies daily media coverage of public affairs. Even treasury versus central bank conflicts are negotiated.

The central point: We do not *have to* describe government policy as the outcome of a two-agent ultimatum game. We don't have to describe any objectives at all, as I mostly have not done in this book. Most of the time we do not bother modeling the inner workings of government or other economic agents. If government maximizes something, it is a unitary objective. We likewise don't describe consumers with a little angel on one shoulder and a devil on the other playing an ultimatum game.

Yes, studying how governments come up with decisions is a separate and fruitful investigation. A "passive" fiscal policy must run into a Laffer limit at some point. Strong and independent central banks that dislike inflation can pressure treasuries to difficult but necessary probity. Modeling how household preferences result from internal bargaining games is interesting and fruitful as well. But this consideration goes into the bucket of political economy, of dynamic public finance, of figuring out what overall government preferences for distorting taxes versus inflation are; not of figuring out how inflation and other aggregates react once a coordinated policy is in place. The observational equivalence theorem drives home this point: Given the policy, observable variables are the same no matter which policy is active or passive.

Sargent and Wallace (1981) consider $MV = Py$ as the monetary alternative, in which control of the money supply completely controls the price level. Active money is a refusal to act, a refusal to print money to finance deficits, or in response to inflation. Active fiscal policy is a refusal to adapt surpluses and deficits to inflation-induced changes in the value of debt.

The game of chicken story makes less sense in comparing fiscal theory of monetary policy to the new-Keynesian model, in which monetary policy controls expected inflation in either regime. The active/passive question is now whether a central bank controls unexpected inflation by equilibrium selection threat $\phi(\pi_t - \pi_t^*)$, or whether fiscal policy directly determines unexpected inflation. Active fiscal policy is still a refusal to act, but active monetary policy requires the central bank to actively exercise its threats. It was already unclear just how a threat to hyperinflate makes the private sector jump to the central bank's desired equilibrium, but how does that threat force a reluctant treasury to tighten fiscal policy? "We're not printing money to finance your deficits, good luck in the bond market," is a sensible threat. "If you don't tighten up, we're going to hyperinflate with explosive interest rate targets" seems a lot less realistic. The new-Keynesian tradition does not in fact analyze the $\phi(\pi_t - \pi_t^*)$ threat as a way of enforcing passive policy on a reluctant treasury, but merely as a coordinating device for private expectations, with a vision that the treasury meekly follows inflation and deflation with whatever surpluses are required.

Pure active/passive regimes and chicken games are good stories to tell in order to understand theoretical possibilities and to understand some episodes. But they are not a necessary part of a policy specification, or, in the end, that useful.

22.7 Inconsistent or Undetermined Regimes

> I fill in the two other possibilities: undetermined and inconsistent regimes. Neither makes sense, emphasizing that it is unwise to test for these stylized regimes.

The conventional taxonomy following Leeper (1991) lists four parameter regions, not two. If $\phi > 1$ and $\gamma = 0$, both policies are active. Now, with the rule against nominal explosions, inflation is overdetermined. Similarly, if monetary policy fixes M and fiscal policy fixes surpluses at an inconsistent value, no equilibrium can form. If, on the other hand, $\phi < 1$ or M is passive and fiscal policy is also passive, $\gamma > 0$, then we are back to indeterminacy.

"Equilibrium cannot form" makes no sense here, however, any more than it did in our efforts to trim multiple equilibria. Suppose that the central bank cuts the money supply in half and leaves it there. That should cut the price level in half. But that would double the value of government debt. What if fiscal authorities refuse to raise taxes, or they are at the top of the Laffer curve and cannot raise tax revenues? Well, bondholders see government bonds as overvalued at the low price level so they try to sell before the inevitable default. That raises aggregate demand and pushes the price level up. Money will become scarce, with troubles in markets and financial institutions. People will start using scrip or foreign currency. In the short run, whether government finances or the money demand curve is the more flexible economic relationship will determine the outcome. In the long run, either the treasury or the central bank will have to give in. "Equilibrium can't form" just means you've written down an incomplete theory. A treasury running intractable deficits and a central bank exercising its threat to hyperinflate at the same time is an even more unrealistic picture.

A double passive policy would also fall apart. Inflation is always something, the price level is a real number. Thus, any model that stops at "indeterminate" is just missing an ingredient, even if the ingredient is sunspots. Inflation or deflation comes from some source—a frost raises the price of orange juice, say. Money adapts. Fiscal policy responds with spending or austerity. Inflation slowly becomes unhinged. Authorities soon figure out they need a better policy.

These configurations are useful for telling stories and exploring how theory works, but they are not realistic policy configurations to consider in applications, to try to measure or test. We will not observe a government in an overdetermined or underdetermined regimes. If one wants to follow these directions, better game theory is the answer.

22.8 Regimes and Practice

Observational equivalence suggests that we modify procedures so that the choice of regime is, where possible, less important. We can estimate models written in terms of observable quantities, without identifying restrictions to tie equilibrium selection parameters to observables. We still need to think to ask interesting policy questions: If the central bank raises interest rates, should we include a contemporaneous fiscal shock? One should at least calculate and examine implicit fiscal predictions of active money views. Regimes affect central doctrines. Since the alternative monetary and interest rate models are not consistent with what we know about how governments behave and financial markets are structured, we should get on with the business of seeing *how* the only theory we have does work.

What should we *do*, in light of observational equivalence? From the fact that tests depend on identification assumptions, one might be led to a search for better identification assumptions and get back to testing regimes. But 30 years of search haven't gotten that far, similar efforts elsewhere in economics and finance have not borne fruit, and I have argued that the active money regimes don't make much sense. This does not seem a productive path.

One view is that anything unobservable shouldn't matter that much. So put aside these controversies and adopt modeling and empirical procedures in which the choice of regime is of minor importance. Chapter 5 showed how we can estimate models by just studying equilibrium conditions, where starred values equal their unstarred counterparts.

That's all we need for fitting data or even for simulating models. It does not matter whether unexpected inflation causes following surpluses or vice versa for that purpose, or why the interest rate and surplus shocks display whatever correlation one finds in the data. The only issue is why unexpected inflation is one particular value and not another. Add a footnote stating that one could support the equilibrium by specifying either an active interest rate-based equilibrium selection policy, or by specifying active fiscal policy. (Werning (2012) innovated this clever strategy, including the footnote.)

This attitude does not free us from thinking about foundations when analyzing policy. If one wants to ask, “what happens if the Fed raises interest rates?” we have to know if it is *interesting* to specify a contemporaneous fiscal contraction to that monetary contraction. It also does not excuse the typical new-Keynesian habit of ignoring fiscal implications. One should at least calculate and examine the implicit fiscal predictions of active-money models’ simulations.

Regimes also still affect central doctrines: Can the central bank set an interest rate peg without causing volatile indeterminate inflation? Must a central bank raise observable interest rates more than one for one with inflation?

The he new-Keynesian equilibrium selection story such as $i_t = i_t^* + \phi(\pi_t - \pi_t^*)$, and its monetarist counterparts, do not correspond to how we know the world works. Some version of the active fiscal regime is the only coherent complete model of the price level we have, that is vaguely consistent with current institutions. In this context, observational equivalence is a feature not a bug. It means that time series tests can’t prove fiscal theory wrong, and it offers a recipe for translating existing models to fiscal foundations. In addition to observational equivalence and nonidentification, there is no point in trying to test for regimes that don’t make any sense. What do we do? Let’s get on with understanding the world using the only sensible regime we have. Write coordinated monetary–fiscal policies in which the ultimate foundation of price level determinacy is fiscal and see *how* to specify the models and policies so that the models are useful. This is not easy work.

Part V

Past, Present, and Future

Chapter Twenty Three

Past and Present

I HAVE KEPT extensive pointers to and reviews of literature out of the main text of this book to focus on the issues and to keep it readable. Here, I point to some of the crucial work in the development of fiscal theory, focusing on work that I have not described already, and I outline some recent and current work. I see a fundamental way that much recent work can be improved to fit better, to fit all data with fiscal theory, and to reorient its basic message away from a theory of failed institutions that create unwanted inflation to a theory of successful institutions that control inflation.

The fiscal theory is an active research field. I have barely touched on many current efforts, and this review will necessarily be incomplete as well. I close with some speculation about future steps for fiscal theory.

23.1 The Rise of Fiscal Theory

Leeper (1991) "Equilibria Under Active and Passive Monetary Policies" is the fiscal theory watershed. Leeper considers interest rate rules, rather than money growth rules, to characterize monetary policy, and thus connects with contemporary macroeconomics. Leeper shows that active fiscal policy can uniquely determine inflation even with passive $\phi < 1$ monetary policy. Even an interest rate peg can have a stable, determinate inflation. Boiling it down to a simple model, Leeper analyzes a model of the form

$$i_t = E_t \pi_{t+1} \tag{23.1}$$

$$i_t = \phi \pi_t + u_{i,t} \tag{23.2}$$

$$\tilde{s}_{t+1} = \gamma v_t + u_{s,t+1} \tag{23.3}$$

$$\rho v_{t+1} = v_t + i_t - \pi_{t+1} - \tilde{s}_{t+1}. \tag{23.4}$$

As we have seen many times, this model needs a forward-looking eigenvalue to tie down unexpected inflation $\Delta E_{t+1} \pi_{t+1}$. We can achieve that result with active monetary and passive fiscal policy, $\phi > 1$, $\gamma > 0$, or with active fiscal and passive monetary policy $\phi < 1$, $\gamma = 0$. Leeper's singular contribution is to point out the latter possibility. The fiscal theory is born. Leeper includes a sticky price model, along the lines of the simple models investigated here. That is an immediately important generalization, though the algebra hides the determinacy and equilibrium selection issues.

(As a reminder, we can substitute for $\tilde{s}_{t+1}$ and i_t to write

$$\rho v_{t+1} = (1-\gamma)v_t - \Delta E_{t+1}\pi_{t+1} - u_{s,t+1}.$$

If $\gamma > 0$, the value of debt grows more slowly than the interest rate, so any unexpected inflation $\Delta E_{t+1}\pi_{t+1}$ is consistent with the transversality condition. If $\gamma = 0$, then only one value of unexpected inflation keeps debt from exploding.)

Sims (1994) writes a full nonlinear model, emphasizing the possibility that controlling money might not determine the price level, and emphasizing the stability and determinacy of an interest rate peg under fiscal theory. Woodford (1995) shows that fiscal theory can give a determinate price level with a passive money supply policy, thereby titling his paper "price level Determinacy Without Control of a Monetary Aggregate." Woodford also first uses the term "fiscal theory of the price level" that I have found. Woodford (2001) shows determinacy under an interest rate peg.

I got involved in the late 1990s with Cochrane (1998a). This paper shows the observational equivalence theorem. I express the issue in terms of on-equilibrium versus off-equilibrium responses. I tangle with the data. I show how the AR(1) surplus cannot work, because it predicts a tight connection between inflation, debt, and deficits. I present a two-component model that generates an s-shaped response, I show that we need such a process to fit the data, and that fiscal theory is compatible with such a process, though not the full range of counterfactuals distilled in this book. I show that the s-shape or surpluses that respond to debt are not signs of passive fiscal policy. I discuss the need for discount rate variation to make sense of the data, and I offer simple version of the linearizations, VAR, and inflation variance decomposition reported here. I also explore long-term debt.

I mistitled the paper "A Frictionless View." I should have titled it "A Fiscal View." I was enthralled with the idea that fiscal theory allows one to think about the price level in models with no monetary or pricing frictions, and that such frictionless models might go a long way to understanding the data; an abstraction on which one could add price stickiness later to get better-fitting dynamics. I didn't even include the simple sticky price models that Leeper (1991) had already taught us how to use. Though a nice point, however, the frictionless possibility and benchmark are not the central message of fiscal theory. The paper gained comparatively little attention. Observational equivalence, the s-shaped surplus, and more, did not impact subsequent work. Title your papers well, and edit them better.

Cochrane (2001) explores long-term debt more deeply, giving rise to most of the treatment of long-term debt you see here. I also show that the two-component s-shaped surplus process is not recoverable from VARs that exclude the value of debt. That argument was complex, involving spectral densities. The version in Online Appendix Section A2.1 is a lot clearer.

Just after these first steps, Woodford also produced his magisterial Woodford (2003) *Interest and Prices*, putting in one place the emerging new-Keynesian model. Yet, despite Woodford's leading role in bringing fiscal theory to life, he abandoned it in this book, relying exclusively on explosive inflation threats to select equilibria.

23.2 Precursors

With hindsight, one can see many precursors. We all stand on the shoulders of giants.

The 1970s and 1980s saw an outpouring of work on monetary and fiscal policy in the intertemporal rational-expectations general-equilibrium revolution, from which the fiscal theory sprang. However, though we can now see these roots, reevaluation of central monetarist doctrines, and many important fiscal theory propositions, much of that literature retained the central concern with money versus bonds, seigniorage, targeting monetary aggregates rather than interest rates, rate of return distortions, separate central bank balance sheets, and so forth. Much of it took place within the overlapping generations framework, which adds dynamic inefficiency questions and is difficult to relate to actual money, since money turns over more than once per generation. Only after Leeper did fiscal theory move the baseline to a completely cashless economy, nominal debt, and interest rate targets, with monetary frictions tacked on as needed but not central to price level determination.

Sargent and Wallace (1981) "Unpleasant Monetarist Arithmetic," and Sargent (1982b) "Ends of Hyperinflations," surveyed above in Sections 14.3 and 19.6, were a huge impetus. They combine clear simple theory, an evident match to experience, and relevance to contemporary policy.

Sargent (1982a), surveying the intertemporal revolution, suggests we

> begin with the initial working hypothesis that the government is like a firm and that its debt is priced according to the same sorts of equilibrium asset-pricing theories developed for pricing bonds and equities... the return stream backing the government's debt is the prospective excess of its explicit tax collections over its expenditures. (p. 383.)

But Sargent immediately retrenches with "this approach is valuable, if only for the qualifications that it immediately invites." Most of those qualifications center around non-interest bearing cash and financial distortions induced by regulation, which we now understand are important extensions but not central to price level determination.

Sargent offers the first use I have seen of the term "Ricardian regime." He imagines

> two polar monetary–fiscal regimes. In the first or Ricardian regime, the issuing of additional interest-bearing government securities is always accompanied by a planned increase in future explicit tax collections just sufficient to repay the debt... In the second polar regime, increased government interest-bearing securities will be paid off ... by eventually collecting seigniorage through issuing base money.

The s-shaped surplus begins. By implication, a regime can lie between the "polar extremes."

Aiyagari and Gertler (1985) notice that monetarist propositions rely on fiscal backing, what Leeper later calls passive fiscal policy. They consider a model with money, induced by overlapping generations, and government debt, but no pricing frictions. Following Sargent, they analyze a "non-Ricardian regime" in which "the central bank fully accommodates a fiscal deficit by financing the new debt with current and future money creation." In their non-Ricardian regime, the price level becomes proportional to the total supply of government debt, as in the fiscal theory with an AR(1) surplus, and the price level becomes independent of the composition of government debt, the split between M and B.

Wallace (1981) proves that open market operations can be irrelevant, in a "Modigliani-Miller" theorem:

> Monetary policy determines the composition of the government's portfolio. Fiscal policy . . . determines the path of net government indebtedness . . . alternative paths of the government's portfolio consistent with a single path of fiscal policy *can be* irrelevant. . .

This model is also based on overlapping generations. Indeed, the paper includes an apologia to economists who find that framework a strained parable for money (me). As this irrelevance theorem is not true when there is a standard monetary distortion, one might be forgiven for having seen it as just confirming the liquidity trap or as an overlapping generations curiosity.

Sargent and Wallace (1982) show that a real bills doctrine with passive money supply can lead to a determinate price level, and is indeed optimal, but again in an overlapping-generations context. Contrariwise, Sargent and Wallace (1985) find that paying market interest on reserves gives a "continuum of equilibria" in an overlapping generations model of money, a modern version of a liquidity trap. But if full interest on reserves is financed by taxes, it is consistent with a determinate price level. This is an early simple version of the fiscal theory proposition that full interest on reserves can leave the price level determinate.

Monetary economists long recognized the importance of monetary–fiscal interactions, if for nothing else that fiscal stress leads governments to finance deficits by printing money. Friedman (1948), though quite different from his later thoughts, is a program for monetary *and fiscal* stability. Patinkin (1965) emphasizes a wealth effect of government bonds, which we can see in fiscal theory. Wesley Clair Mitchell (1903) and Irving Fisher (e.g. Fisher (1912)) debated the quantity theory versus fiscal backing of greenback inflation. The intuition of the fiscal theory is already reflected by Adam Smith, quoted in the epigraph.

The "chartalist" school includes some elements of fiscal theory. Knapp (1924) wrote in 1905 The "State Theory of Money." He views "money" as defined by legal status in paying debts, including and especially debts to the state. He writes (p. 95) that the key "test" of money is whether it is "accepted in payments made to the State's offices." Metallic content is not relevant to Knapp, and we should think of even metallic money as a "token," "ticket" or "Charta." (Knapp coined the word "Chartal," p. 32.) But Knapp's work is mostly devoted to the philosophical question "what is money?" and classifying money into a schema of properties such as "morphic," "authylistic," "lytric," "hylogenic," "autogenic," "amphithropic," "monotropic," and so forth. He is not concerned with the *value* of money, the price level, or inflation, other than the price of gold, silver, and foreign exchange rates. One can regard the book as a precursor to the legal restrictions school, which regards demand for fiat money as generated by legal restrictions on the forms of payment, rather than fiscal theory.

In “Functional Finance,” Lerner (1943) recognizes that taxes can soak up extra currency and hence stop inflation. His view is basically a static L-shaped aggregate supply curve. More demand induced by printing more money or by borrowing first raises output and employment, and then inflation, unless soaked up by taxes. More recently, “modern monetary theorists” such as Kelton (2020) have taken on the mantle following Lerner and the chartalist school. However, they mix one good idea—money can be soaked up by taxes to prevent inflation—with a great number of wrong ideas to produce sharply different analysis. (See, among other reviews, Cochrane (2020).)

That money is valued if it is backed by some real claim is an idea stretching back millennia, along with the realization that money useful in transactions and limited in supply can gain a higher value than its backing. Pure fiat money is the newcomer on the intellectual block. That paper money devalues when governments print it to finance spending was seen and understood time and again. The conventional view sees in our relatively stable inflation the capstone to the slow development of institutions that limit money printing under $MV = Py$. But perhaps we should see it instead as an equally slowly-won but perhaps temporary victory of institutions by which sovereigns commit to repay nominal debts rather than default or inflate them away.

23.3 Disputes

The fiscal theory entered a period of theoretical controversies. Is the fiscal theory even right? How can an agent “threaten to violate an intertemporal budget constraint?” Among others, Buiter (1999), Buiter (2002), Buiter (2017) calls the fiscal theory “fatally flawed” and a “fallacy” for mistreating a “budget constraint.” Kocherlakota and Phelan (1999), Bohn (1998b), and Ljungqvist and Sargent (2018) more charitably write that fiscal theory assumes that the government has a special ability to violate a budget constraint at off-equilibrium prices, but thereby validate the idea that the government debt valuation equation is a budget constraint. Marimon (2001), while recognizing fiscal theory as analogous to a “financial theory of the firm,” still characterizes the fiscal theory as “a theory that does not respect Walras’ law.” Even Woodford (2003) (p. 691 ff.) endorses the view that the valuation equation is a “budget constraint” but the government is special.

I wrote “Money as Stock” Cochrane (2005b) to address this critique. As you’ve seen many times in this book, the fiscal theory is based on a valuation equation, an equilibrium condition, not a “budget constraint.”

“Money as Stock” also discusses whether it is *plausible* that a government refuses to adapt surpluses to changes in the valuation of debt brought on by inflation and deflation, once one admits that it is possible in a way that violating budget constraints is impossible. The long and better discussion in this book started there. This issue owes a lot to persistent discussions with Marty Eichenbaum and Larry Christiano, for which I am grateful. Christiano and Fitzgerald (2000) put some of this thought in writing. The bottom line as expressed here is simple, but subtle.

Just because we see governments often “respond” with surpluses to higher debt generated by past deficits in equilibrium, does not mean that they would respond to higher debt generated by off-equilibrium deflation with the same extra surpluses. Repaying one’s debts is different than validating a deflation-induced windfall to bondholders, with 1933 a prime example.

Controversy on this point is understandable. The valuation equation is a lot closer to an “intertemporal budget constraint” in a model with real debt and no default. Economists had spent decades studying such models. That it works differently with nominal debt, and without a gold or foreign exchange peg, is not obvious. The distinction between budget constraint and valuation equation is subtle. I used the word “intertemporal budget constraint” as well in Cochrane (1998a), before the distinction dawned on me.

Niepelt (2004) offers a different critique, calling the theory a “Fiscal Myth.” To Niepelt, the fiscal theory is wrong because it cannot start from a period with no outstanding nominal debt. The government, selling initial nominal debt in return for goods, must promise additional future surpluses. To Niepelt, this fact means that fiscal policy must be “Ricardian,” and “the notion of fiscal price level determination therefore collapses.” We have seen in this book so many debt sales with higher future surpluses that I hope it’s abundantly clear such operations do not contravene fiscal theory.

Starting up a fiscal-theory economy is straightforward. The government can simply give money to people at the beginning of the first period. Or the government can exchange new money or government debt in exchange for old money, as in the introduction of the Euro. Daniel (2007) rebuts this critique, and I discuss it in Section 2.2.

I have not spent much time in this book on theoretical controversies because I think they are settled, so not worth carrying along. The point of the book is to make fiscal theory useful, and deeply reviewing or rehashing these arguments seems unproductive. A lot of right theories do not organize events and are ignored. If the fiscal theory is not useful, nobody will long care about theoretical underpinnings.

I emphasize an approach to fiscal theory via simple Walrasian equilibrium. Bassetto (2002) spells out game theoretic foundations for dynamic equilibria involving government policies. This work parallels similar game-theoretic equilibrium-selection foundations for new-Keynesian models in Atkeson, Chari, and Kehoe (2010), Christiano and Takahashi (2018), and the extensive literature on game-theoretic foundations of general equilibrium theory. Bassetto and Sargent (2020) is a beautiful use of this framework, thinking of government policies as “strategies” and mapping the joint monetary–fiscal analysis to events in U.S. history.

This approach is surely right in a deep sense. My verbal discussion of how governments react to nonequilibrium prices, my v versus v^* and π versus π^* distinctions and my long discussions of institutions to guide expectations of off-equilibrium behavior, qualify as Bassetto’s “more complex than the simple budgetary rules usually associated with the fiscal theory,” such as simple $s_{t+1} = \gamma v_t$ or $i_t = \phi \pi_t$ feedback that I also criticize.

Why then does this book not adopt and survey game theoretic foundations in its hundreds of pages? I hope in this book to make fiscal theory *useful*. Hopefully, we don’t *have to* spell out game theory foundations in order to use fiscal theory productively, just as standard Walrasian general equilibrium theory is useful though game theoretic foundations can be more satisfying, and as most applied new-Keynesian work ignores its parallel game theoretic foundations. If we always have to spell out such foundations the theory will be much less useful. Bassetto and Sargent (2020) challenge this view. They show the practical usefulness of game-theoretic foundations by mapping government actions in those episodes to such concepts. But the counterexample proves the larger theorem: Sophisticated approaches will catch on, as they should, to the extent and in applications where they are useful. One must also accept comparative advantage, and mine does not lie in clarifying game theoretic foundations of equilibrium. So my silence on these questions does not signal that they are not important or potentially productive.

McCallum (2001), McCallum (2009a), McCallum and Nelson (2005), and Christiano (2018) add "learnability" to the definition of equilibrium, and view the active-money passive-fiscal equilibria as learnable, while the passive-money active-fiscal equilibrium is not learnable. I argue the opposite case for new-Keynesian models in Cochrane (2009), and survey this issue in Section 16.10 above. Since we do not observe $\pi_t \neq \pi_t^*$, there is no way to learn ϕ in $i_t = i_t^* + \phi(\pi_t - \pi_t^*)$ of the new-Keynesian model.

Learnability is an addition to the standard Walrasian paradigm, as is the restriction to locally bounded equilibria. We don't *need* game theory, learnability, or a restriction to locally bounded equilibria to say that supply and demand determines the price of tomatoes. Having thrown out one Walrasian equilibrium condition, authors need to add something else. But then one must extend the definition of Walrasian equilibrium in order to write *any* model, no matter how simple, that determines the price level. If so, maybe one needs a better model! Fiscal theory alone still offers the Occam's razor simple *possibility* that the price level can be determined by Walrasian equilibrium with no frictions, additional rules, equilibrium selection philosophies, and so forth.

Chapter Twenty Four

Tests, Models, and Applications

24.1 Tests

For contemporary macroeconomists, the first instinct with a new theory is to run econometric tests, including grand tests for one class of theory versus another. I cover some of these tests in Section 22.3, with a focus on observational equivalence.

The main test in Canzoneri, Cumby, and Diba (2001) is based on the finding that surplus innovations lower the value of debt. They acknowledge both interpretations of this result and argue against the plausibility of the s-shaped surplus, as discussed in Sections 4.2 and 22.3.

They start, however, by considering and disclaiming the obvious test: Run a regression of (23.3), $s_{t+1} = \gamma v_t + \varepsilon_{t+1}$, and see if $\gamma > 0$, if surpluses respond to debt. Such a test would parallel Clarida, Galí, and Gertler (2000), who ran (boiled down) $i_t = \phi \pi_t + \varepsilon_t$ to test $\phi > 1$ for active monetary policy. As Cumby, Canzoneri, and Diba point out, we see $\gamma > 0$ in the data, as surpluses were higher in the early post-WWII era than in the 1970s, and shown in regressions by Bohn (1998a). But Cumby, Canzoneri and Diba recognize that we can see $\gamma > 0$ in both active and passive fiscal regimes. Recall the v versus v^* example in Section 5.4, or that a surplus moving average with $a(\rho) < 1$ generates a regression coefficient $\gamma > 0$.

Their careful analysis of this point did not stop $\gamma > 0$ from being a persistent informal argument against fiscal theory, as in Christiano and Fitzgerald (2000) for example, and a core identifying restriction for the whole fiscal theory modeling literature covered in the next section.

24.2 Fiscal Theory Models

Most application of fiscal theory in the last two decades has taken the form of model-building rather than purely econometric tests. These models describe the economy completely, including fiscal and monetary policy rules. Leeper and Leith (2016) is an excellent review survey including its own advances in the state of the art.

These models are specified in much more detail than any model in this book. The models include ingredients such as detailed fiscal policy rules, often separating taxes and spending, distorting taxes, valuable government spending, labor supply, sticky wages, more complex preferences, production with capital and investment, financial frictions, explicit microfoundations, nonlinear solution methods, optimal policy, commitment versus discretion, and other elaborations. They typically describe full microfoundations, rather than jump to linearized aggregate equilibrium conditions as I have. Most are estimated or calibrated to realistic parameters.

Following DSGE macro tradition, authors simulate the effects of policies and other shocks. By computing impulse-response functions, these models give concrete advice, and provide an account of history. They thus move beyond testing a theory in the abstract and on to using the theory to answer practical questions.

So far, however, this style of model building and evaluation has remained a subdiscipline. It has not infused fiscal roots into the larger DSGE model construction and evaluation enterprise. We should consider why not, and how to foster that jump.

I offer a general overview, and then a review of specific papers.

The models in this literature take a different approach than I have in this book, on the central question of how to specify regimes and how to integrate monetary and fiscal policy, what to *do* with fiscal theory. I pursue the goal of using fiscal theory to describe the whole sample, guided by observational equivalence. To that end, I generalize the description of policy to the form

$$i_t = \theta\pi_t + \phi(\pi_t - \pi_t^*) + u_{i,t} \tag{24.1}$$
$$\tilde{s}_{t+1} = \alpha v_t^* + \gamma(v_t - v_t^*) + u_{s,t+1}, \tag{24.2}$$

and I thereby allow any form of the surplus process, including $a(\rho) \ll 1$ if needed. I thereby fit the data at least as well as a new-Keynesian model, but I lose the ability to identify and test active money versus active fiscal regimes, to measure and test ϕ and γ, from equilibrium time series. This is a feature, not a bug, as above.

The current fiscal theory models in this literature are written in generalized forms of (23.1)-(23.4),

$$i_t = \phi\pi_t + u_{i,t}$$
$$\tilde{s}_{t+1} = \gamma v_t + u_{s,t+1},$$

with restrictions (typically an AR(1)) on the disturbances. These models look for time periods in which $\phi > 1$ and $\gamma > 0$, active monetary and passive fiscal policy, versus time periods in which $\phi < 1$ and $\gamma = 0$, passive monetary and active fiscal policy. Such measurements with standard errors are also regime tests.

You see the familiar identifying assumptions: Monetary and fiscal policy are tied to equilibrium selection policies $\theta = \phi$ and $\alpha = \gamma$, which are not realistic. Most of all, an active fiscal policy cannot generate an s-shaped surplus response.

As a result, the active fiscal regime must fit data quite badly. A single equation regression estimate may lose $\gamma > 0$ in standard errors, but a full model estimate faces the counterfactual correlations and puzzles of Section 4.2, in particular that deficits lower the value of debt. The model can estimate an active fiscal regime only in a time such as the 1970s, when high and volatile inflation hides those counterfactual predictions, and when the $\phi > 1$ parameterization of identified active monetary policy does even more violence to the data. In a time such as the 1980s, with less volatile inflation, the counterfactual predictions of the restricted surplus process, and in which $\phi > 1$ fits better, the models find active money.

The result is profound. Fiscal theory is seen as a rare and unfortunate outcome, when monetary authorities lose a game of chicken and large volatile inflation breaks out. Active money is seen as the good state of affairs with low inflation. Indeed "fiscal dominance" is often used as a synonym for large or uncontrolled fiscal inflation, not for a set of institutions that can commit to and produce a steady price level.

This is a broad picture, which may not characterize every paper. But we *know* that any paper that produces estimates and tests of regimes, which does not report a flat likelihood function, has imposed some identifying restriction, and that restriction limits each regime's ability to describe data.

It is natural that authors proceeded this way. This is the most natural thing to do with models written in the standard form, and before one really digests observational equivalence and identification issues and works to write the models in a form that expresses the identifying assumptions. Estimating ϕ by regressions that identify monetary policy with equilibrium selection policy is the standard thing to do in new-Keynesian literature as well. And there is a path dependence in most investigations. Once one starts building on a structure such as (23.1)-(23.4), it is natural to focus on elaborations and not rewriting and reorienting the basic idea.

But now that we have the clarity of the observational equivalence theorems, now that we can express monetary and fiscal policy in terms of on- versus off-equilibrium reactions, now that we can separate monetary or fiscal policy from their equilibrium selection policies, now that we can write (24.1)-(24.2), we know that measuring regimes must rely on strong and unrealistic identifying restrictions, which artificially limit each regime's ability to describe the data. Moreover, these restrictions ($\theta = \phi$, $\alpha = \gamma$) artificially limit the models' fit in either estimated regime, and thus its overall fit. Any period of active fiscal policy contains all the counterfactual predictions of Section 4.2. Allowing an s-shaped surplus process in periods when $\phi = \theta < 1$ must improve model fit, likely a lot.

How did we not notice? Curiously, the DSGE literature does not emphasize goodness of fit measures or forecasting ability, cornerstones of earlier model building, and focuses on policy evaluation. In a sense, all models fit perfectly, because they add enough shocks to every equation to fit the data. But the size of the shocks is large, and becomes the predominant part of the model's explanatory power. For example, if one fits the data with the simple three-equation model presented here, inflation volatility comes almost entirely from inflation shocks, shocks to the Phillips curve, innovations $\varepsilon_{\pi,t}$ to

$$\begin{aligned}\pi_t &= \beta E_t \pi_{t+1} + \kappa x_t + u_{\pi,t}\\ u_{\pi,t} &= \eta_\pi u_{\pi,t-1} + \varepsilon_{\pi,t}.\end{aligned}$$

Expected inflation and output don't explain much variation in current inflation; the Phillips curve has low R^2; inflation versus output gap plots are a big cloud. Where one might hope for a model to say that lower inflation volatility since 1980 derives from fewer monetary policy shocks, or a change in the monetary policy rule that reduces the influence of other shocks on inflation, in fact, a variance accounting throws up its hands and says inflation became less volatile because the gods sent us fewer inflation shocks. (Sims and Zha (2006) have a sophisticated calculation of this point.) But such variance accounting is no longer a common part of model evaluation. It is common to compare selected impulse-response functions to estimates. They may fit well, but if the corresponding shocks do not account for much variance, the model may still fit the data badly.

How can detailed and carefully estimated fiscal theory models miss the glaringly counterfactual puzzles induced by large $a(\rho)$? Because by and large they do not look, or the stylized facts are lost in dry model evaluation statistics. And, to be fair, if the goal is to match an estimated response function for a monetary policy shock, to think about the marginal effect of monetary policy shocks, even if such shocks contribute a small fraction of output and inflation forecast-error variance, the bad overall fit is not a salient fact.

How did regime identification and estimation go on despite warnings of observational equivalence from Cochrane (1998a) and Canzoneri, Cumby, and Diba (2001)? The answer may be that a clear and simple alternative was not readily at hand, so those papers' inconvenient but abstract points didn't make it into mainstream consciousness. While the King (2000) representation $i_t = i_t^* + \phi(\pi_t - \pi_t^*)$ was available in 2000, its implication for monetary policy identification didn't show up until Cochrane (2011a). The point remains contentious, with many critics of that work feeling there are reasonable identification restrictions one can make to measure ϕ. King's representation is still not a part of the regular toolkit and textbook expression of new-Keynesian models. The parallel way to write fiscal policy that distinguishes in-equilibrium responses, responses to past deficits and real interest rates, from responses to multiple equilibrium inflation—for example $\tilde{s}_{t+1} = \alpha v_t^* + \gamma(v_t - v_t^*)$—is, as far as I know, original in summer 2020 manuscripts of Cochrane (2021b) and this book. That's hardly common technology. To write a model, you need the technology to do it, not just general theorems and whining from commenters. Moreover, it was obviously not clear how much the identifying assumptions hurt the model's ability to match facts. All estimates include identifying assumptions. Put all this together, along with the enduring wish, perpetual referee demand, and a hard-to-break habit of a literature to measure and test regimes, and the literature's progression is understandable.

But now we have these tools, which open an orchard of low-hanging fruit. We have a range of interesting work, and detailed, well-worked out models needing only slight modification. Indeed, much of what these papers do is suggestive. The papers measure the correlations between on-equilibrium policy variables. They measure θ in $i_t^* = \theta\pi_t^*$ or α in $s_t^* = \alpha v_t^*$. This measurement remains interesting and important. Observable parameters vary over time and such shifts are important, even if they do not document a switch between regimes, a switch of unobservable parameters.

Specific examples follow.

Davig and Leeper (2006) is a foundational paper in this line. Davig and Leeper estimate interest rate and surplus policy rules that depend on inflation, output, and in the latter case lagged debt, with uncorrelated disturbances. In a central and widely followed innovation, the policy rule coefficients vary between active fiscal and active money according to a Markov process, switching between $\gamma = 0$, $\phi < 1$ and $\gamma > 0$, $\phi > 1$. They embed these estimates in a detailed DSGE model with nominal rigidities and calculate the responses to policy shocks. The perpetual possibility of changing to a different regime plays an important role in these responses. Davig and Leeper (2006) warn against leaving out regime switches: "Many estimates of policy rules…condition on sub-samples in which a particular regime prevailed…embedding the estimated rules in fixed-regime DSGE models can lead to seriously misleading ... inferences..."

Markov switching captures a larger and important theoretical point, also in Davig and Leeper (2007): If an economy is currently in what looks like a passive fiscal regime, but people expect a switch to what looks like an active fiscal regime, then that future active fiscal policy selects equilibria. We may have $\phi > 1$ and $\gamma > 0$, but if inflation really gets out of control, the government will switch to active fiscal policy. Well, then we are in the active fiscal regime all along.

This fact means that measuring regimes is doubly hard—and thus, in my view, doubly impossible and doubly pointless. It is not enough to surmount on- versus off-equilibrium identification issues in estimating *current* responses of surpluses and interest rates to inflation and debt. We have to estimate the structure of Markov switching and the cumulative probability of ending up in one versus another regime. We have to find which variable actually explodes as time goes forward and regimes switch back and forth. At a minimum, we need different language. A time with $\phi < 1$ should be something like "temporarily passive" monetary policy, not "passive" without qualification. (We have thought about nonlinear functions $i = \Phi(\Pi)$, and "locally" active or passive policies that reflect derivatives near the steady state, $\Phi'(\Pi^*)$. Nonlinearity could change a policy's "global" properties. Here we think about variation in the policy rule over time, rather than over a wider range of a state variable.)

Regime-switching authors are right that policy rule parameters likely vary over time and in response to economic outcomes. They are right that we should look at the economy as a single meta-rule, or meta-regime, in which policy parameters vary over time, people expect such variation, and such variation should be incorporated in expectations and response calculations. For example, a big part of the story for persistently high ex post real interest rates in the 1980s may well be that people put some weight on a return to 1970s policy. Responses to monetary policy and other shocks should include changing assessments of the chance of such changes.

But it is not obvious that such parameter variation is best modeled by Markov switching rather than conventional time series models for parameters. As a modeling approximation, there is some sense to Markov switching. In history, policy parameters have arguably changed somewhat discontinuously. Pre- and post-1980, the zero bound era, and pre-war, 1940-1945, and postwar era are suggestively discretely different regimes, stable within but shifting discontinuously across. But that is a modeling choice, and it is not entirely obvious. There is also lots of policy drift within regimes. Moreover, the Markov assumption, with exactly two (or even N) states assigns zero probability that people consider other possibilities or other regimes.

So why not adopt simpler, more flexible models of policy rule parameter evolution? Here I think that linking policy rule changes to equilibrium selection regime changes ($\theta = \phi$, $\gamma = \alpha$) is a core trouble. Shifting from active-fiscal passive-money to active-money passive-fiscal requires a discrete shift in parameter values to move eigenvalues from stable to unstable. But if we are simply viewing shifting correlations between equilibrium variables or monetary and fiscal policy rules, a shift of θ in $i_t^* = \theta\pi_t^*$ and α in $\tilde{s}_{t+1} = \alpha v_t$, it has no such momentous or discontinuous consequences.

Leeper, Davig, and Chung (2007) show that apparently active-money passive-fiscal policy is not enough to insulate the economy from inflationary fiscal shocks. Following the usual restriction, only their (temporarily) passive-fiscal active-money policy can have an s-shaped response, so that regime is needed for debt repayment and for fiscal shocks not to result in immediate inflation. But that policy may not last long enough to repay debt. The expected switch to active fiscal means that fiscal shocks affect inflation immediately, even in the temporarily active-money passive-fiscal repayment regime. Again, though, the possibility of active fiscal policy with an s-shaped surplus, $\alpha \neq \gamma$ would remove that result. And a switch from active fiscal with an s-shaped surplus to active fiscal that inflates away debt would reinforce the result. We do not need to tie policy parameter changes to equilibrium selection regime changes to see the central point.

Leeper, Traum, and Walker (2017) present a detailed sticky price model allowing a fiscal theory solution, aimed at evaluating the output effects of fiscal stimulus. They specify fiscal policy as an AR(1) (p. 2416) along with one-period debt. They include an indirect mechanism that buffers the AR(1) surplus conundrum and allows a bit of $a(\rho) < 1$. Surpluses respond to output. So, a deficit leads to inflation, which raises output, which raises tax revenues, and leads to higher later surpluses. But that mechanism is not necessarily large enough to generate substantial repayment of large debts, $a(\rho) \ll 1$. That the regime is identified means there is some restriction.

Bianchi and Melosi (2013) offer an interesting application of these regime-switching ideas. They call the active-money passive-fiscal regime "virtuous," because as in these other papers, they assume that only a passive fiscal regime can repay debts with $a(\rho) < 1$, so in the opposite (sinful?) passive-money active-fiscal regime any surplus shock results in inflation. But Markov switching allows a similar change of story. A temporary lapse in virtue—a temporarily active fiscal regime with $a(\rho) > 1$–can nonetheless see little inflation, if people expect a reversion to "virtue" and its s-shaped surplus process. But a fiscal expansion with inadequate expectation of reversion to virtuous policy can give rise to large immediate inflation. In this way, they account for episodes in which persistent deficits and accommodative monetary policy do not give rise to inflation, or only give rise to slow inflation, and others in which deficits lead quickly to inflation. Again the same ideas could easily be present in an entirely active-fiscal equilibrium selection regime, with parameters that vary over time. The $a(\rho)$ can vary over time, which one could model in my parametric v, v^* form by varying the correlation of unexpected inflation with the surplus shock β_s over time.

Bianchi and Melosi also describe a "dormant" shock, expectations of future fiscal policy that causes inflation today, leaving conventional analysis puzzled about the source of the inflation, "if an external observer were monitoring the economy focusing exclusively on output and inflation, he would detect a run-up in inflation and an increase in volatility without any apparent explanation." We have seen many parallel analyses.

Bianchi and Melosi (2017) show how fiscal theory accounts for the absence of deflation in response to a preference shock, the zero bound puzzle of new-Keynesian models studied here in Section 20.2, and how expectations of a switch between regimes affects responses to shocks. Bianchi and Melosi specify that taxes follow an AR(1) that also responds to output. Their model switches between a temporarily passive fiscal regime in which surpluses respond to debt and a temporarily active fiscal regime that does not do so (their equation (6) p. 1041). Government spending also follows an AR(1) that responds to output (p. 1040).

Bianchi and Ilut (2017) address an important issue and come to an appealing conclusion: The inflation of the 1970s came from loose fiscal policy, and the disinflation of the 1980s followed a fiscal reform. This paper begins to fill the great gaping hole of applied fiscal theory analysis: In a fiscal theory narrative, just what went wrong in the 1970s, and what fixed it in the 1980s? They augment a new-Keynesian model with a fiscal block and a geometric term structure for government debt, an important and often-overlooked generalization. They also posit monetary and fiscal rules that feed back from inflation and output. They specify Markov switching between temporarily active fiscal and temporarily active money regimes, finding temporarily passive fiscal policy in the 1980s and temporarily active fiscal policy in the 1970s. This paper is a good concrete example of that general finding, which I referred to earlier. Chen, Leeper, and Leith (2021) follow with a more comprehensive view of fiscal policy underpinnings of the 1980 shift.

Bhattarai, Lee, and Park (2016) likewise add fiscal policy to a DSGE model. They split the sample pre- and post-Volcker. They find both monetary and fiscal policy passive pre-Volcker, and thus "equilibrium indeterminacy in the pre-Volcker era," modeled as sunspot shocks. They include standard fiscal and monetary policy rules that implicitly identify equilibrium selection regimes from the policy rules, and do not allow an s-shaped response in the active fiscal regime.

Bianchi and Melosi (2019) study a situation of temporarily uncoordinated policy, thinking about how a large stock of debt such as the United States has in 2021 will play out. Will the government choose high taxes or inflation? Both fiscal and monetary policy are temporarily active, both $\phi > 1$ and $\gamma = 0$ for a while. Again, active fiscal policy disallows debt repayment. Eventually one policy loses the game of chicken, and agents expect that fact ahead of time. If fiscal policy wins, which in their restricted specification means that fiscal policy refuses to repay debt, then "hawkish monetary policy backfires" and creates additional inflation. As I digest the result, $\phi > 1$ policy is "hawkish" in that it tries to push the economy to a low-inflation equilibrium, including the fiscal authorities, by threatening higher interest rates and higher inflation. If that threat does not work, then we see the higher interest rates, and higher inflation. The result is similar to that of Section 17.4.2, in which monetary policy cannot work, even in an active money regime, if the "passive" fiscal austerity does not follow.

Beck-Friis and Willems (2017) construct a clean new-Keynesian model with fiscal theory, to address the government spending multiplier. They study the standard model similar to (20.5) and (20.6), except that their government spending provides utility, so g enters alongside consumption, which equals output, x in the IS curve (20.5) as well as in the Phillips curve as in (20.6). They contrast the effect of government spending shocks with active money, $\phi > 1$ in $i_t = \phi\pi_t$ and passive fiscal policy, $\gamma = 0$ in

$$\begin{aligned}\tau_t &= \gamma b_{t-1} + \varepsilon_{\tau,t}\\ g_t &= \eta g_{t-1} + \varepsilon_{g,t},\end{aligned}$$

where τ = taxes, with the same experiment under active fiscal policy $\gamma > 0$ and $\phi < 1$. They find important differences in the multipliers across the active money versus active fiscal regime. With the benefit of hindsight, we see that their surpluses are i.i.d. in the active fiscal regime, so government cannot repay any debts and finances all spending shocks by inflating away debt. What happens if one allows the fiscal regime also to repay debts, with $a(\rho) < 1$? Or how much of the result comes from asking different questions of the government spending shock, holding monetary policy constant in a different way? Analysis of fiscal multipliers along this line is more low-hanging fruit.

In the frictionless model, the interest rate target sets expected inflation, and fiscal policy sets unexpected inflation. Caramp and Silva (2021) offer a generalized decomposition that applies to sticky price models. Monetary policy governs intertemporal substitution while fiscal policy operates through a wealth effect. Their decomposition includes changes in real interest rates and discount rates induced by monetary policy. They also include long-term debt, finding that higher interest rates without a change in surpluses only lower inflation in the presence of long-term debt. In general, they highlight "the necessity of a strong, contractionary fiscal backing to overturn the presence of this [neo-Fisherian] force," and produce a negative inflation response. More generally,

> In the New Keynesian model, the magnitude of the wealth effect depends on the fiscal response to monetary policy rather than on the change in the path of the nominal interest rate per se.

and

> the inverse relation between the nominal interest rate and inflation under the Taylor equilibrium is driven entirely by a negative wealth effect. In the absence of such wealth effects, not only does the monetary authority lose control of initial inflation, but the effect on future inflation has the opposite sign than in the standard result.

These are now familiar conclusions to a reader of this book, but expressed in a different and interesting way, and analyzed in a more detailed set of models, including heterogeneous agents models and capital.

By focusing on the possibilities for refinement and for future work, I do not mean to diminish the substantial accomplishment. We have here a body of detailed and careful fiscal theory modeling, and an indication of the range of historical experience and policy analysis it can apply to. These papers take on the challenge of *using* fiscal theory, by the DSGE rules of the game of modern macroeconomics, to analyze data and policies.

However, in my view, this line of work got stuck, tying monetary and fiscal policy to equilibrium selection policy, $\theta = \phi$ and $\alpha = \gamma$, and forcing active fiscal policy to always inflate and not repay debts, with $a(\rho) > 1$, restricting the model's ability to fit the data in the quest to measure equilibrium selection regimes.

So an opportunity beckons. We can build on all the hard work in this literature, by slightly generalizing the fiscal specification so that the models can fit all the data, better, entirely with a fiscal regime, and reinterpreting regime-switching models as parameter-switching models within that regime. This opportunity parallels the opportunity to adapt new-Keynesian models via observational equivalence and then include their fiscal implications. As a recipe for writing papers, better fitting data, and for addressing important issues without having to build a whole set of new models from scratch, this is great news.

24.3 Exchange Rates

If we are to replace $MV = Py$ or interest rate targets at the foundation of price determination, exchange rates are a natural place to apply ideas. As a measure of the value of the dollar, exchange rates are less sticky and better measured than price indices. And exchange rates have been a perpetual puzzle. Traditional theory either starts with $MV = Py$ and tries to relate exchange rates to relative money stocks, or starts with Keynesian models and relates exchange rates to interest differentials. The disconnect between exchange rates and "fundamentals" has been one of many constant puzzles in this literature.

The world is not all darkness. Exchange rates do line up with interest rate differentials. Some fiscal connections are evident. Exchange rates appreciate on good news of countries' growth rates. Well, more growth means better government finances. Exchange rate collapses are often connected to bad fiscal news. Exchange rates often fall suddenly without much "fundamental" news, though on fears about the future, which our present value formulation encourages.

Dupor (2000) brings fiscal theory to exchange rates. In classic passive-fiscal theory, if countries peg interest rates rather than money supplies, or if people can use either country's money, then the exchange rate is indeterminate, mirroring the indeterminacy of the price level under interest rate pegs and passive money. For example, Kareken and Wallace (1981) showed indeterminacy in the then-popular overlapping generations setup, driven by the assumption that people can use either country's money. Dupor introduces fiscal theory, but he emphasizes the case that one country runs persistent deficits and the other persistent surpluses. Two currencies vie for a common pool of surpluses, so the exchange rate is indeterminate. When two countries with separate currencies pay off their own debts, exchange rates are determinate under the fiscal theory, determined by the present value of each country's surpluses. Daniel (2001b) responds directly, making this point, and giving an explicit model why governments would choose to run separate surplus streams, giving a determinate exchange rate.

Daniel (2001a) has an early and innovative analysis of currency crises. Crises happen when the present value of primary surpluses can no longer support a pegged exchange rate. Daniel brings to international economics the stabilizing potential of long-term debt:

> In the absence of long-term government bonds, the exchange rate collapse must be instantaneous. With long-term government bonds, the collapse can be delayed at the discretion of the monetary authority... Fiscal policy is responsible for the inevitability of a crisis, while monetary policy determines ... the timing of the crisis and the magnitude of exchange rate depreciation.

Daniel (2010) has a dynamic fiscal theory model of currency crises. An exchange rate peg implies a passive fiscal policy, but there is an upper bound on debt and surpluses. When that limit is reached, policy must switch, including depreciation.

Daniel applies the model to the 2001 Argentine crisis.

Burnside, Eichenbaum, and Rebelo (2001) was, to me, a watershed, though they do not pitch it as fiscal theory. This paper shows that the East Asian currency crises of the late 1990s were precipitated by bad news about *prospective* deficits. The countries did not have large debts, and were not experiencing bad *current* deficits, nor did they exhibit current monetary loosening. But these countries were suddenly likely to have intractable *future* and contingent deficits. The governments were poised to bail out banks, and banks had taken on a lot of short-term foreign-currency debt. A run on banks then becomes a run on the government. The lesson that contingent liabilities can undermine fiscal and monetary affairs is one we might pay attention to more broadly, given the large size of the United States' implicit and explicit bailout and income support guarantees. Burnside, Eichenbaum, and Rebelo (2001) also point to fiscal benefits of inflation and devaluation. For example, inflation lowers the real value of sticky government employee salaries.

Jiang (2021) and Jiang (2022) bring fiscal theory directly to exchange rates. Jiang shows that exchange rates fall when forecasts of future deficits rise. This is a good case in which a positively correlated surplus process seems to work. The s-shape is not always and everywhere, especially in bad news for emerging markets.

24.4 Applications

Reading history, policies, and institutions through the lens of the fiscal theory, finding simple parables that help pave the way to more fundamental understanding, is what a lot of this book is about. Here I list a few efforts not already mentioned.

Leeper and Walker (2013) and Cochrane (2011e), Cochrane (2011d) are attempts in real time to confront how fiscal theory accounts for the 2008 recession and to look through the fog to see what lay ahead. The combination of large debts, large prospective deficits, and low growth sounds some sort of alarm bell, but just what is it? Most macroeconomics imagines monetary policy alone able to control inflation, but the new situation calls that faith into question. Historically, some debts have been managed successfully, others lead to creeping inflation, others lead to crisis. What will ours do?

Thinking about these issues led me to ponder many mechanisms echoed here. I interpreted "flight to quality" as a lower discount rate for government debt, an increase in demand for government debt, which on its own is deflationary. I analyzed many policies from stimulus to QE as efforts to raise the supply of government debt. I considered stimulus from a fiscal theory perspective, as I have analyzed here, noting that the "stimulative" effect depends on expected future deficits. Hence promises to repay later are not useful for stimulus in this framework. In retrospect, however, one sees the tension between trying to engineer a default through inflation now while preserving a reputation for repaying debt to allow future borrowing. I offered the analysis of quantitative easing offered here: neutral to first-order but potentially stimulative as an inflation rearrangement with long-term debt. I worried then as now about fiscal inflation. I noted what we see here in greater detail; that fiscal inflation can come slowly, not just a price level jump. I worried about real and contingent liabilities. I introduced the present value Laffer curve analysis echoed here. I emphasized the run-like unpredictable nature of a fiscal inflation, and how the central bank may be powerless to stop it, and how central and dangerous short-term financing is to that scenario.

Leeper and Walker (2013) start by reminding us that a fiscal inflation can break out without seigniorage, by devaluing nominal bonds directly, the point of Section 19.6. Leeper and Walker also stress that the prospective deficits of Social Security and Medicare in the United States pose a central fiscal challenge, analyzed in detail in Davig, Leeper, and Walker (2010). Leeper and Walker also include long-term debt, which alters dynamics substantially as we have seen.

These papers were written in the immediate aftermath of the 2008 recession and its then-shocking increase in debt, before large primary deficits continued during the economic expansion of the late 2010s, before arguments for additional large deliberate fiscal expansion, and before the COVID-19 era debt expansion. But they were also written before the era of persistent negative real rates, lowering discount rates and interest costs. Just how large debts will play out, and the role inflation will play, remain good questions. Even the best theory in the world is hard to deploy in real time for soothsaying, as Sargent and Wallace discovered 40 years ago.

Sims (2013) used his AEA presidential address to "bring FTPL down to earth." This is a lovely summary and exposition of many fiscal theory issues. Sims starts with a mechanism we have seen in the sticky price analysis: A rise in interest rates not accompanied by fiscal contraction will be inflationary by raising discount rates and interest costs, as well as by potentially raising expected inflation. Loyo (1999) cites examples in Brazil in which higher interest rates raise interest costs on debt, do not provoke a fiscal contraction to pay those costs, and so seem to bring on higher inflation. The mechanism may apply to the United States and Europe, in the shadow of our large debts and deficits. Sims explains as I have that $MV(i) = Py$ does not determine the price level, and that fixes to restore determinacy essentially involve adding fiscal theory, backing money at some point with taxes.

Sims points to the fiscal foundations of the euro, and interactions between central banks and treasuries when there is an institutional separation between their balance sheets, at least for a while. He sees ultimate fiscal backing of an independent central bank in recapitalization, as I have, but points to some doubts that such recapitalization might happen (p. 567). Sims explains clearly the distinction between real and nominal debt, and that nominal debt is a "cushion" like equity.

The fiscal foundation of the euro is an obvious case of fiscal–monetary interaction. If a central bank is committed to printing money as needed, to do "whatever it takes" to keep each country from defaulting, and countries can borrow freely, there is an obvious problem. Sims (1997) and Sims (1999) presciently think about the foundations of the euro in explicitly fiscal theory terms. While not directly fiscal theory, the parallels between fiscal affairs in the early United States and those of European fiscal integration underlying the euro in Sargent (2012) are deeply insightful.

Sims (2001), mentioned above, opined that Mexico would do well not to dollarize, so as to maintain an equity-like cushion. One can, as I did above, question the judgment, valuing the repayment precommitments of dollarization, while agreeing entirely with the analysis of the options and appreciating the use of fiscal theory to think about an important issue.

Chapter Twenty Five

The Future

It's tough to make predictions, especially about the future. Nonetheless, I close with some thoughts about where the fiscal theory may go, or at least avenues on my ever-growing list of possibilities to explore.

25.1 Episodes

As many papers by Tom Sargent with coauthors have shown us, the analysis of historical episodes through the lens of monetary theory with monetary–fiscal interactions can be deeply revealing. Sargent and Velde's (2003) "History of Small Change," Sargent and Velde's (1995) "Macroeconomics of the French Revolution," Velde's (2009) 'Chronicle of Deflation," Hall and Sargent's (2014) tale of which debts the United States paid and which it did not or inflated away, and Sargent's (2012) contrast between ninetheenth-century United States and today's euro are some of my particular favorites.

The emergence of inflation in the United States and worldwide in the 1970s and its decline in the 1980s still needs a more comprehensive and well-documented fiscal theory narrative. We have the beginnings: for example, work like Bianchi and Ilut (2017) and Sims (2011). But the purely monetary conventional narrative—an insufficiently aggressive $\phi < 1$ Taylor rule giving instability in the 1970s, followed by tough-love $\phi > 1$ in the 1980s —developed on thousands of papers and their digestion. The new-Keynesian narrative—multiple equilibrium indeterminacy $\phi < 1$ in the 1970s followed by $\phi > 1$ determinacy—likewise stands on a large body of work. Developing a durable fiscal theory narrative that has a chance of unseating such solidified conventional wisdoms will be a challenge, even if it is right.

Summarizing and extending previous comments, in particular in Sections 6.1 and 8.4, there are many tantalizing fiscal clues. Inflation emerged in the late 1960s along with the fiscal pressure of the Great Society and Vietnam War. The United States did have a major crisis ending with its abandoning the remaining gold standard and devaluing the dollar in 1971. But one must address just why the deficits of this episode provoked inflation and our much larger deficits did not, at least until 2021. The restrictions of the Bretton Woods system and closed international financial markets surely play a role.

The 1970s saw a productivity and growth slowdown. An apparently lower trend of GDP growth is terrible news for the present value of surpluses. They saw a break in the traditional cyclical behavior of primary surpluses. The year 1975 saw the worst deficit by far since WWII, with no bright future in sight.

The 1980s saw a 20-year resumption in growth and, as it turned out, tax receipts, despite lower tax rates. In retrospect, 1980 looks a lot like a classic inflation stabilization combined with fiscal and pro-growth reform, such as inflation targeting countries introduced. The fiscal and pro-growth reform came after monetary policy changes, and may have been partly induced by the interest expense provoked by higher interest rates. The interest expense channel can provoke fiscal reform rather than spark a doom loop. Or, the fiscal reform may have been the clean-up effort that made the monetary tightening stick. Many attempted monetary tightenings have failed when promised fiscal reforms did not materialize.

In 1933, I argued, the United States refused to accommodate a surprise deflation by fiscal austerity to pay a windfall to bondholders. Starting in 1980, the United States did exactly the opposite. Investors who bought bonds at the high nominal interest rates of the late 1970s, expecting a low real return and continuing inflation, instead got a windfall, repaid in sharply more valuable dollars, courtesy of the U.S. taxpayer.

Fiscal and monetary policies are intertwined. The Sims (2011) vision of interest rate increases that temporarily reduce inflation, but without fiscal support eventually make it worse, has a 1970s flair to it needing quantitative exploration, or deeper investigation with more detailed models of the temporary negative inflation effect of interest rate increases. Likewise, the model of 17.4.2 in which higher interest rates without fiscal backing do not lower inflation may apply to the 1970s/1980 divergence, as well as sound a cautionary note for future stabilization efforts in the shadow of debt.

But this is storytelling, not economic history. The fiscal roots of this inflation and its conquest need a closer, quantitative, model-based look. I opined several times that the slow inflation various models produce in response to a fiscal shock is reminiscent of the 1970s. Reminiscent isn't good enough. (Bordo and Levy (2020) have a good summary of fiscal–monetary affairs through the inflation and disinflation.)

Cross-country comparisons are revealing. What *about* Japan? And Europe? In some extreme events we can see a direct correlation between contemporaneous deficits, debts, and inflation. Høien (2016) includes an example from Russia 2012-2015, in which primary deficit and inflation march hand in hand.

Latin American monetary and fiscal history has not so far been widely studied by U.S. and European economists. Yet it includes a menagerie of monetary–fiscal experiences and institutions. The comprehensive Kehoe and Nicolini (2021) "Monetary and Fiscal History of Latin America" together with its impressive data collection and dissemination effort[1], should jump-start our understanding of inflation, and in just about every case its fiscal roots. The history is subtle, with successful and unsuccessful stabilizations, a variety of institutions to control inflation and attempt fiscal commitments, great lessons of plans that worked and plans that fell apart. There is not a different economics for Latin America or emerging markets, and what happened there can happen here.

[1] `https://mafhola.uchicago.edu`

25.2 Theory and Models

Obviously, we need more comprehensive theory. And it is easy to describe the list of ingredients that one should add to the soup. But one must be careful. Good economic theory does not consist of merely stirring tasty ingredients into the pot.

Inflation is always a choice: The government can inflate, default (haircut, reschedule), raise distorting taxes, or cut spending. The fiscal theory is a part of dynamic public finance—the discipline which asks which distorting taxes are better than others—and political economy. Contrariwise, by understanding the decisions governments take, we gain some understanding of what the tradeoffs are; we learn about economics not visible in time series from a settled regime in equilibrium. Leeper, Plante, and Traum (2010) is an example of the dynamic DSGE tradition exploring these issues without a nominal side, with a good literature review. It is waiting for integration with real/nominal issues via fiscal theory.

Fiscal theory is a part of the larger question of sovereign debt management and sustainability. The full range of time consistency, reputation building, and other concerns, which already consider inflation as a form of default, can productively be merged with a fiscal theory that recognizes means other than seigniorage by which inflation comes about, and the dynamics seen in models here.

I have emphasized the importance of institutions, including fiscal precommitments, the separation between central bank and treasury, the legal structures preventing inflationary finance, and so forth. Institutions are if nothing else good ways to communicate off-equilibrium commitments. That whole question needs deeper study, both in the historical and institutional vein, and in the more modern game theory tradition.

I have preached enough about how to integrate fiscal theory with the DSGE tradition, so I'll just repeat again how technically easy but fertile that enterprise ought to be. Likewise, slightly modifying the existing fiscal theory models to remove the restrictions they impose in the vain attempt to measure equilibrium selection regimes is an easy path to take.

The end of this long book is really just a beginning.

Bibliography

Aiyagari, S. Rao and Mark Gertler. 1985. "The Backing of Government Bonds and Monetarism." *Journal of Monetary Economics* 16 (1):19–44.

———. 1991. "Asset Returns with Transactions Costs and Uninsured Individual Risk." *Journal of Monetary Economics* 27 (3):311–331.

Akerlof, George A. and Ross D. Milbourne. 1980. "Irving Fisher on His Head II: The Consequences of the Timing of Payments for the Demand for Money." *The Quarterly Journal of Economics* 95 (1):145–157.

Alesina, Alberto, Carlo Favero, and Francesco Giavazzi. 2019. *Austerity: When It Works and When It Doesn't*. Princeton: Princeton University Press.

Allais, Maurice. 1947. *Économie et intérêt*. Paris: Librairie des publications officielles.

Alstadheim, Ragna and Dale W. Henderson. 2006. "Price-Level Determinacy, Lower Bounds on the Nominal Interest Rate, and Liquidity Traps." *The B.E. Journal of Macroeconomics (Contributions)* 6 (1). Article 12.

Alves, Felipe, Greg Kaplan, Benjamin Moll, and Gianluca L. Violante. 2020. "A Further Look at the Propagation of Monetary Policy Shocks in HANK." *Journal of Money, Credit and Banking* 52 (S2):521–559.

Andersen, Leonall C. and Jerry L. Jordan. 1968. "Monetary and Fiscal Actions: A Test of Their Relative Importance in Economic Stabilization." *St. Louis Fed Economic Review* 50:11–24.

Angeletos, George-Marios. 2002. "Fiscal Policy with Noncontingent Debt and the Optimal Maturity Structure." *The Quarterly Journal of Economics* 117 (3):1105–1131.

Atkeson, Andrew, Vardarajan V. Chari, and Patrick J. Kehoe. 2010. "Sophisticated Monetary Policies." *Quarterly Journal of Economics* 125 (1):47–89.

Auclert, Adrien, Matthew Rognlie, and Ludwig Straub. 2020. "Micro Jumps, Macro Humps: Monetary Policy and Business Cycles in an Estimated HANK Model." Manuscript.

Bansal, Ravi and Wilbur John Coleman. 1996. "A Monetary Explanation of the Equity Premium, Term Premium, and Risk-Free Rate Puzzles." *Journal of Political Economy* 104:1135–1171.

Bansal, Ravi, Dana Kiku, and Amir Yaron. 2012. "An Empirical Evaluation of the Long-Run Risks Model for Asset Prices." *Critical Finance Review* 1 (1):183–221.

Bansal, Ravi and Amir Yaron. 2004. "Risks for the Long Run: A Potential Resolution of Asset Pricing Puzzles." *The Journal of Finance* 59 (4):1481–1509.

Barro, Robert J. 1974. "Are Government Bonds Net Wealth?" *Journal of Political Economy* 82:1095–1117.

———. 1979. "On the Determination of the Public Debt." *Journal of Political Economy* 87 (5, Part 1):940–971.

Barro, Robert J. and Herschel I. Grossman. 1971. "A General Disequilibrium Model of Income and Employment." *The American Economic Review* 61 (1):82–93.

Bassetto, Marco. 2002. "A Game-Theoretic View of the Fiscal Theory of the Price Level." *Econometrica* 70:2167– 2195.

———. 2004. "Negative Nominal Interest Rates." *American Economic Review* 94:104–108.

Bassetto, Marco and Gherardo Caracciolo. 2021. "Monetary/Fiscal Interactions with Forty Budget Constraints." Manuscript.

Bassetto, Marco and Wei Cui. 2018. "The Fiscal Theory of the Price Level in a World of Low Interest Rates." *Journal of Economic Dynamics and Control* 89:5–22.

Bassetto, Marco and Todd Messer. 2013. "Fiscal Consequences of Paying Interest on Reserves." *Fiscal Studies* 34 (4):413–436.

Bassetto, Marco and Thomas J. Sargent. 2020. "Shotgun Wedding: Fiscal and Monetary Policy." Federal Reserve of Minneapolis Staff Report 599.

Baumol, William J. 1952. "The Transactions Demand for Cash: An Inventory Theoretic Approach." *The Quarterly Journal of Economics* 66 (4):545–556.

Beck-Friis, Peder and Tim Willems. 2017. "Dissecting Fiscal Multipliers under the Fiscal Theory of the Price Level." *European Economic Review* 95:62–83.

Beeler, Jason and John Y. Campbell. 2012. "The Long-Run Risks Model and Aggregate Asset Prices: An Empirical Assessment." *Critical Finance Review* 1 (1):141–182.

Benhabib, Jess, Stephanie Schmitt-Grohé, and Martín Uribe. 2001a. "Monetary Policy and Multiple Equilibria." *American Economic Review* 91 (1):167–186.

———. 2001b. "The Perils of Taylor Rules." *Journal of Economic Theory* 96 (1):40–69.

———. 2002. "Avoiding Liquidity Traps." *Journal of Political Economy* 110 (3):535–563.

Berentsen, Aleksander and Christopher Waller. 2018. "Liquidity Premiums on Government Debt and the Fiscal Theory of the Price Level." *Journal of Economic*

Dynamics and Control 89:173–182.

Berg, Claes and Lars Jonung. 1999. "Pioneering Price Level Targeting: The Swedish Experience 1931-1937." *Journal of Monetary Economics* 43:525–551.

Bernanke, Ben S. 1983. "Nonmonetary Effects of the Financial Crisis in the Propagation of the Great Depression." *The American Economic Review* 73 (3):257–276.

Bhattarai, Saroj, Jae Won Lee, and Woong Yong Park. 2016. "Policy Regimes, Policy Shifts, and US Business Cycles." *The Review of Economics and Statistics* 98:968–983.

Bianchi, Francesco and Cosmin Ilut. 2017. "Monetary/Fiscal Policy Mix and Agents' Beliefs." *Review of Economic Dynamics* 26:113–139.

Bianchi, Francesco and Leonardo Melosi. 2013. "Dormant Shocks and Fiscal Virtue." *NBER Macroeconomics Annual* 28:1–46.

———. 2017. "Escaping the Great Recession." *American Economic Review* 107 (4):1030–58.

———. 2019. "The Dire Effects of the Lack of Monetary and Fiscal Coordination." *Journal of Monetary Economics* 104:1–22.

Blanchard, Olivier. 2019. "Public Debt and Low Interest Rates." *American Economic Review* 109:1197–1229.

Blanchard, Olivier Jean and Charles M. Kahn. 1980. "The Solution of Linear Difference Models under Rational Expectations." *Econometrica* 48 (5):1305–1311.

Bloom, Nicholas, Chad Jones, John Van Reenen, and Michael Webb. 2020. "Are Ideas Getting Harder to Find?" *American Economic Review* 110:1104–1144.

Bohn, Henning. 1995. "The Sustainability of Budget Deficits in a Stochastic Economy." *Journal of Money, Credit and Banking* 27 (1):257–271.

———. 1998a. "The Behavior of U. S. Public Debt and Deficits." *The Quarterly Journal of Economics* 113 (3):949–963.

———. 1998b. "Comment." *NBER Macroeconomics Annual* 13:384–389.

Bordo, Michael D. 2018. "The Imbalances of the Bretton Woods System 1965 to 1973: U.S. Inflation, The Elephant in the Room." Working Paper 25409, National Bureau of Economic Research.

Bordo, Michael D. and Mickey D. Levy. 2020. "Do Enlarged Fiscal Deficits Cause Inflation: The Historical Record." Working Paper 28195, National Bureau of Economic Research.

Brash, Donald T. 2002. "Inflation Targeting 14 Years On." URL http://www.bis.org/review/r020121b.pdf. Speech given at 2002 American Economics Association Conference.

Brock, William A. 1974. "Money and Growth: The Case of Long run Perfect

Foresight." *International Economic Review* 15:750–777.

———. 1975. "A Simple Perfect Foresight Monetary Model." *Journal of Monetary Economics* 1:133–150.

Brunnermeier, Markus K., Sebastian Merkel, and Yuliy Sannikov. 2020. "The Fiscal Theory of the Price Level with a Bubble." Manuscript.

Buera, Francisco and Juan Pablo Nicolini. 2021. "The Case of Argentina." In *A Monetary and Fiscal History of Latin America, 1960–2017*, edited by Timothy J. Kehoe and Juan Pablo Nicolini. Minneapolis: University of Minnesota Press.

Buiter, Willem H. 1999. "The Fallacy of the Fiscal Theory of the Price Level." Working Paper 7302, National Bureau of Economic Research.

———. 2002. "The Fiscal Theory of the Price Level: A Critique." *Economic Journal* 112:459–480.

———. 2017. "The Fallacy of the Fiscal Theory of the Price Level - Once More." *CEPR Discussion Paper* No. DP11941.

Buiter, Willem H. and Anne C. Siebert. 2007. "Deflationary Bubbles." *Macroeconomic Dynamics* 11:431–454.

Burdekin, Richard C. K. and Marc D. Weidenmier. 2001. "Inflation is Always and Everywhere a Monetary Phenomenon; Richmond vs. Houston in 1864." *American Economic Review* 91:1621–1630.

Burnside, Craig, Martin Eichenbaum, and Sergio Rebelo. 2001. "Prospective Deficits and the Asian Currency Crisis." *Journal of Political Economy* 109:1155–1197.

Cachanosky, Nicolás and Federico Julián Ferrelli Mazza. 2021. "Why Did Inflation Targeting Fail in Argentina?" *Quarterly Review of Economics and Finance* 80:102–116.

Cagan, Phillip. 1956. "The Monetary Dynamics of Hyperinflation." In *Studies in the Quantity Theory of Money*, edited by Milton Friedman. University of Chicago Press, 25–117.

Campbell, John Y. and John Ammer. 1993. "What Moves the Stock and Bond Markets? A Variance Decomposition for Long-Term Asset Returns." *The Journal of Finance* 48 (1):3–37.

Campbell, John Y. and N. Gregory Mankiw. 1987. "Are Output Fluctuations Transitory?" *The Quarterly Journal of Economics* 102 (4):857–880.

Campbell, John Y., Carolin E. Pflueger, and Luis M. Viceira. 2020. "Macroeconomic Drivers of Bond and Equity Risks." *Journal of Political Economy* 128:3148–3185.

Campbell, John Y. and Robert J. Shiller. 1988. "The Dividend-Price Ratio and Expectations of Future Dividends and Discount Factors." *The Review of Financial Studies* 1 (3):195–228.

Canzoneri, Matthew B., Robert E. Cumby, and Behzad T. Diba. 2001. "Is the Price Level Determined by the Needs of Fiscal Solvency?" *American Economic Review* 91:1221–1238.

Caramp, Nicolas and Djanir H. Silva. 2021. "Fiscal Policy and the Monetary Transmission Mechanism." Manuscript.

Carlozzi, Nicholas and John B. Taylor. 1985. "International Capital Mobility and the Coordination of Monetary Rules." In *Exchange Rate Management Under Uncertainty*, edited by J. Bhandhari. Cambridge, MA: MIT Press, 186–211.

Chamley, Christophe. 1986. "Optimal Taxation of Capital Income in General Equilibrium with Infinite Lives." *Econometrica* 54 (3):607–622.

Chen, Xiaoshan, Eric M. Leeper, and Campbell Leith. 2021. "Strategic Interactions in U.S. Monetary and Fiscal Policies." *Quantitative Economics* Forthcoming.

Chiang, Alpha C. 1992. *Elements of Dynamic Optimization.* New York: McGraw-Hill.

Christiano, Lawrence J. 1987. "Cagan's Model of Hyperinflation under Rational Expectations." *International Economic Review* 28 (1):33–49.

———. 2018. "Comment." *NBER Macroeconomics Annual* 32:227–245.

Christiano, Lawrence J., Martin Eichenbaum, and Charles Evans. 1999. "Monetary Policy Shocks: What Have We Learned and To What End?" In *Handbook of Macroeconomics*, edited by Michael Woodford and John B. Taylor. Amsterdam: North-Holland, 65–148.

———. 2005. "Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy." *Journal of Political Economy* 113:1–45.

Christiano, Lawrence J., Martin Eichenbaum, and Sergio Rebelo. 2011. "When Is the Government Spending Multiplier Large?" *Journal of Political Economy* 119:78–121.

Christiano, Lawrence J. and Terry J. Fitzgerald. 2000. "Understanding the Fiscal Theory of the Price Level." *Federal Reserve Bank of Cleveland Economic Review* 36:1–37.

Christiano, Lawrence J. and Yuta Takahashi. 2018. "Discouraging Deviant Behavior in Monetary Economics." Manuscript, Northwestern University.

Clarida, Richard, Jordi Galí, and Mark Gertler. 2000. "Monetary Policy Rules and Macroeconomic Stability: Evidence and Some Theory." *Quarterly Journal of Economics* 115:147–180.

Cochrane, John H. 1988. "How Big Is the Random Walk in GNP?" *Journal of Political Economy* 96 (5):893–920.

———. 1989. "The Sensitivity of Tests of the Intertemporal Allocation of Consumption to Near-Rational Alternatives." *American Economic Review* 79:319–337.

———. 1991a. "A Critique of the Application of Unit Root Tests." *Journal of*

Economic Dynamics and Control 15 (2):275–284.

———. 1991b. “Volatility Tests and Efficient Markets: A Review Essay.” *Journal of Monetary Economics* 27:463–485.

———. 1992. “Explaining the Variance of Price-Dividend Ratios.” *Review of Financial Studies* 5:243–280.

———. 1994a. “Comment on “What Ends Recessions?” by David and Christina Romer.” *NBER Macroeconomics Annual* 9:58–74.

———. 1994b. “Permanent and Transitory Components of GNP and Stock Prices.” *The Quarterly Journal of Economics* 109 (1):241–265.

———. 1994c. “Shocks.” *Carnegie-Rochester Conference Series on Public Policy* 41:295–364.

———. 1998a. “A Frictionless View of U.S. Inflation.” *NBER Macroeconomics Annual* 13:323–384.

———. 1998b. “What do the VARs Mean? Measuring the Output Effects of Monetary Policy.” *Journal of Monetary Economics* 41 (2):277–300.

———. 2001. “Long Term Debt and Optimal Policy in the Fiscal Theory of the Price Level.” *Econometrica* 69:69–116.

———. 2003. “Stocks as Money: Convenience Yield and the Tech-Stock Bubble.” In *Asset Price Bubbles*, edited by George G. Kaufman William C. Hunter and Michael Pomerleano. Cambridge: MIT Press, 175–204.

———. 2005a. *Asset Pricing, Revised Edition.* Princeton: Princeton University Press.

———. 2005b. “Money as Stock.” *Journal of Monetary Economics* 52:501–528.

———. 2008. “The Dog That Did Not Bark: A Defense of Return Predictability.” *The Review of Financial Studies* 21 (4):1533–1575.

———. 2009. “Can Learnability Save new-Keynesian Models?” *Journal of Monetary Economics* 56 (8):1109–1113.

———. 2011a. “Determinacy and Identification with Taylor Rules.” *Journal of Political Economy* 119:565–615.

———. 2011b. “Determinacy and Identification with Taylor Rules, Manuscript with Technical Appendix.” Manuscript.

———. 2011c. “Discount Rates: American Finance Association Presidential Address.” *Journal of Finance* 66:1047–1108.

———. 2011d. “Inflation and Debt.” *National Affairs* 9:56–78.

———. 2011e. “Understanding Policy in the Great Recession: Some Unpleasant Fiscal Arithmetic.” *European Economic Review* 55 (1):2–30.

———. 2012. “Continuous-Time Linear Models.” *Foundations and Trends in Fi-*

nance 6 (3):165–219.

———. 2014a. "Comments on Eggertsson and Mehrotra, 'A Model of Secular Stagnation'." Manuscript.

———. 2014b. "Monetary Policy with Interest on Reserves." *Journal of Economic Dynamics and Control* 49:74–108.

———. 2014c. "Toward a Run-Free Financial System." In *Across the Great Divide: New Perspectives on the Financial Crisis*, edited by Martin Neil Baily and John B. Taylor. Stanford: Hoover Institution Press, 197–249.

———. 2015a. "Continuous Time." Manuscript.

———. 2015b. "A New Structure For U. S. Federal Debt." In *The $13 Trillion Question: Managing the U.S. Government's Debt*, edited by David Wessel. Brookings Institution Press, 91–146.

———. 2015c. "A Response to Sims (2013)." Manuscript.

———. 2016. "Comments on 'A Behavioral New-Keynesian Model' by Xavier Gabaix." Manuscript.

———. 2017a. "Macro-Finance." *Review of Finance* 21:945–985.

———. 2017b. "Michelson-Morley, Fisher and Occam: The Radical Implications of Stable Inflation at Near-Zero Interest Rates, Manuscript with Appendix." Manuscript.

———. 2017c. "The New-Keynesian Liquidity Trap." *Journal of Monetary Economics* 92:47–63.

———. 2017d. "Online Appendix to 'The New-Keynesian Liquidity Trap'." URL https://doi.org/10.1016/j.jmoneco.2017.09.003.

———. 2017e. "Stepping on a Rake: The Fiscal Theory of Monetary Policy." *European Economic Review* 101:354–375.

———. 2018. "Michelson-Morley, Fisher, and Occam: The Radical Implications of Stable Quiet Inflation at the Zero Bound." *NBER Macroeconomics Annual* 32:113–226.

———. 2019. "The Value of Government Debt." Manuscript.

———. 2020. "'The Deficit Myth' Review: Years of Magical Thinking." *Wall Street Journal* June 5.

———. 2021a. "The Fiscal Roots of Inflation." *Review of Economic Dynamics* Forthcoming.

———. 2021b. "A Fiscal Theory of Monetary Policy with Partially Repaid Long-Term Debt." *Review of Economic Dynamics* Forthcoming.

———. 2021c. "$r < g$?" Manuscript.

Cochrane, John H., John B. Taylor, and Volcker Wieland. 2020. "Evaluating Rules in the Fed's Report and Measuring Discretion." In *Strategies for Monetary Policy*, edited by John H. Cochrane and John B. Taylor. Stanford: Hoover Institution Press, 217–258.

Cogley, Timothy and Argia M. Sbordone. 2008. "Trend Inflation, Indexation, and Inflation Persistence in the New Keynesian Phillips Curve." *American Economic Review* 98 (5):2101–2126.

Cole, Harold L. and Lee E. Ohanian. 2004. "New Deal Policies and the Persistence of the Great Depression: A General Equilibrium Analysis." *Journal of Political Economy* 112 (4):779–816.

Congressional Budget Office. 2020. "The 2020 Long-Term Budget Outlook."

Copeland, Adam, Darrell Duffie, and David Yilin Yang. 2021. "Reserves Were Not So Ample After All." NBER Working Paper 29090.

Daniel, Betty C. 2001a. "A Fiscal Theory of Currency Crises." *International Economic Review* 42 (4):969–988.

———. 2001b. "The Fiscal Theory of the Price Level in an Open Economy." *Journal of Monetary Economics* 48 (2):293–308.

———. 2007. "The Fiscal Theory of the Price Level and Initial Government Debt." *Review of Economic Dynamics* 10 (2):193–206.

———. 2010. "Exchange Rate Crises and Fiscal Solvency." *Journal of Money, Credit and Banking* 42 (6):1109–1135.

Davig, Troy and Eric M. Leeper. 2006. "Fluctuating Macro Policies and the Fiscal Theory." In *NBER Macroeconomics Annual 2006*. Cambridge: MIT Press, 247–298.

———. 2007. "Generalizing the Taylor Principle." *American Economic Review* 97 (3):607–635.

Davig, Troy, Eric M. Leeper, and Todd B. Walker. 2010. "'Unfunded Liabilities' and Uncertain Fiscal Financing." *Journal of Monetary Economics* 57 (5):600–619.

Del Negro, Marco and Christopher A. Sims. 2015. "When Does a Central Bank's Balance Sheet Require Fiscal Support?" *Journal of Monetary Economics* 73:1–19.

Di Tella, Sebastian and Robert Hall. 2021. "Risk Premium Shocks Can Create Inefficient Recessions." Manuscript.

Diamond, Douglas W. and Phillip H. Dybvig. 1983. "Bank Runs, Deposit Insurance, and Liquidity." *Journal of Political Economy* 91:401–419.

Diamond, Douglas W. and Raghuram G. Rajan. 2012. "Illiquid Banks, Financial Stability, and Interest Rate Policy." *Journal of Political Economy* 120 (3):552–591.

Dowd, Kevin. 1994. "A Proposal to End Inflation." *The Economic Journal* 104 (425):828–840.

Duffie, Darrell and Elizabeth Economy, editors. 2022. *Digital Currencies: The US, China, And The World At A Crossroads*. Stanford: Hoover Institution Press.

Dupor, William. 2000. "Exchange Rates and the Fiscal Theory of the Price Level." *Journal of Monetary Economics* 45:613–630.

Edwards, Sebastian. 2002. "The Great Exchange Rate Debate After Argentina." *The North American Journal of Economics and Finance* 13 (3):237–252.

———. 2018. *American Default: The Untold Story of FDR, the Supreme Court, and the Battle over Gold*. Princeton: Princeton University Press.

Eggertsson, Gauti B. 2008. "Great Expectations and the End of the Depression." *American Economic Review* 98 (4):1476–1516.

———. 2010. "The paradox of toil." *Federal Reserve Bank of New York Staff Report N. 433* .

———. 2011. "What Fiscal Policy is Effective at Zero Interest Rates?" *NBER Macroeconomics Annual* 25:59–112.

———. 2012. "Was the New Deal Contractionary?" *The American Economic Review* 102:524–555.

Eggertsson, Gauti B., Andrea Ferrero, and Andrea Raffo. 2014. "Can Structural Reforms Help Europe?" *Journal of Monetary Economics* 61:2–22.

Eggertsson, Gauti B., Neil R. Mehrotra, and Jacob A. Robbins. 2019. "A Model of Secular Stagnation: Theory and Quantitative Evaluation." *American Economic Journal: Macroeconomics* 11 (1):1–48.

Ekeland, Ivar and José A. Scheinkman. 1986. "Transversality Conditions for Some Infinite Horizon Discrete Time Optimization Problems." *Mathematics of Operations Research* 11:216–229.

Evans, George W. and Seppo Honkapohja. 2001. *Learning and Expectations in Macroeconomics*. Princeton: Princeton University Press.

Fama, Eugene F. 1970. "Efficient Capital Markets: A Review of Theory and Empirical Work." *The Journal of Finance* 25 (2):383–417.

Fama, Eugene F. and Kenneth R. French. 1988a. "Dividend Yields and Expected Stock Returns." *Journal of Financial Economics* 22 (1):3–25.

———. 1988b. "Permanent and Temporary Components of Stock Prices." *Journal of Political Economy* 96 (2):246–273.

Federal Open Market Committee. 2020. "Statement on Longer-Run Goals and Monetary Policy Strategy."

Fernández-Villaverde, Jesús, Juan F. Rubio-Ramírez, Thomas J. Sargent, and Mark W. Watson. 2007. "ABCs (and Ds) of Understanding VARs." *American Economic Review* 97 (3):1021–1026.

Fisher, Irving. 1912. "'The Equation of Exchange' for 1911, and Forecast." *The*

American Economic Review 2 (2):302–319.

Fleckenstein, Mattias, Francis A. Longstaff, and Hanno Lustig. 2014. "The TIPS-Treasury Bond Puzzle." *The Journal of Finance* 69 (5):2151–2197.

Friedman, Milton. 1948. "A Monetary and Fiscal Framework for Economic Stability." *The American Economic Review* 38 (3):245–264.

———. 1968. "The Role of Monetary Policy." *The American Economic Review* 58:1–17.

———. 1969. "The Optimum Quantity of Money." In *The Optimum Quantity of Money and Other Essays*. Chicago: Aldine, 1–50.

———. 1992. *Money Mischief: Episodes in Monetary History*. New York: Harcourt.

Friedman, Milton and Anna Jacobson Schwartz. 1963. *A Monetary History of the United States, 1867-1960*. Princeton: Princeton University Press.

Gabaix, Xavier. 2014. "A Sparsity-Based Model of Bounded Rationality." *Quarterly Journal of Economics* 129:1661–1710.

———. 2020. "A Behavioral New Keynesian Model." *American Economic Review* 110:2271–2327.

Gagnon, Joseph E. and Brian Sack. 2019. "Recent Market Turmoil Shows That the Fed Needs a More Resilient Monetary Policy Framework." *Real Time Economics Issues Watch* September 26.

Galí, Jordi. 2015. *Monetary Policy, Inflation and the Business Cycle: An Introduction to the New Keynesian Framework*. Princeton: Princeton University Press, 2nd ed.

García-Schmidt, Mariana and Michael Woodford. 2019. "Are Low Interest Rates Deflationary? A Paradox of Perfect-Foresight Analysis." *American Economic Review* 109:86–120.

Garín, Julio, Robert Lester, and Eric Sims. 2018. "Raise Rates to Raise Inflation? Neo-Fisherianism in the New Keynesian Model." *Journal of Money, Credit and Banking* 50 (1):243–259.

Giannoni, Marc P. and Michael Woodford. 2005. "Optimal Inflation Targeting Rules." In *Inflation Targeting*, edited by Benjamin S. Bernanke and Michael Woodford. Chicago: University of Chicago Press.

Gordon, Robert J. 2016. *The Rise and Fall of American Growth: The U.S. Standard of Living since the Civil War*. Princeton: Princeton University Press.

Gorton, Gary and Andrew Metrick. 2012. "Securitized Banking and the Run on Repo." *Journal of Financial Economics* 104 (3):425–451.

Granger, C. W. J. 1969. "Investigating Causal Relations by Econometric Models and Cross-Spectral Methods." *Econometrica* 37 (3):424–438.

Granger, Clive W. J. 2004. "Time Series Analysis, Cointegration, and Applications." *American Economic Review* 94 (3):421–425.

Greenlaw, David, James D. Hamilton, Ethan S. Harris, and Kenneth D. West. 2018. "A Skeptical View of the Impact of the Fed's Balance Sheet." Manuscript.

Greenwood, Robin, Samuel G. Hanson, Joshua S. Rudolph, and Lawrence H. Summers. 2015. "Debt Management Conflicts between the U.S. Treasury and the Federal Reserve." In *The $13 Trillion Question: Managing the U.S. Government's Debt*, edited by David Wessel. Washington, D.C.: Brookings Institution Press, 43–90.

Hall, George J., Jonathan Payne, and Thomas J. Sargent. 2018. "US Federal Debt 1776–1960: Quantities and Prices." Manuscript.

Hall, George J. and Thomas J. Sargent. 2011. "Interest Rate Risk and Other Determinants of Post WWII U.S. Government Debt/GDP Dynamics." *American Economic Journal: Macroeconomics* 3:192–214.

———. 2014. "Fiscal Discriminations in Three Wars." *Journal of Monetary Economics* 61:148–166.

———. 2018. "A History of U.S. Debt Limits." *Proceedings of the National Academy of Sciences* 115:2942–2945.

———. 2019. "Complications for the United States from International Credits: 1913-1940." In *Debt and Entanglements Between the Wars*, edited by Era Dabla-Norris. IMF.

———. 2021. "Three World Wars: Fiscal-Monetary Consequences." *Manuscript, Brandeis and NYU* .

Hall, Robert E. 1984. "A Free Market Policy To Stabilize the Purchasing Power of the Dollar." In *Money in Crisis: The Federal Reserve, the Economy, and Monetary Reform*, edited by Barry N. Siegel. Cambridge, MA: Ballinger Publishing, 303–321.

Hall, Robert E. and Marianna Kudlyak. 2021. "Why Has the US Economy Recovered So Consistently from Every Recession in the Past 70 Years?" Manuscript.

Hamilton, James D. 1996. "The Daily Market for Federal Funds." *Journal of Political Economy* 104 (1):26–56.

Hansen, Alvin H. 1939. "Economic Progress and Declining Population Growth." *The American Economic Review* 29 (1):1–15.

Hansen, Lars Peter and Scott F. Richard. 1987. "The Role of Conditioning Information in Deducing Testable Restrictions Implied by Dynamic Asset Pricing Models." *Econometrica* 55 (3):587–613.

Hansen, Lars Peter, William Roberds, and Thomas J. Sargent. 1992. "Implications of Present Value Budget Balance and of Martingale Models of Consumption and Taxes." In *Rational Expectations Econometrics*, edited by Lars Peter Hansen and Thomas J. Sargent. London: Westview Press, 121–161.

Hansen, Lars Peter and Thomas J. Sargent. 1981. "A Note on Wiener-Kolmogorov

Prediction Formulas for Rational Expectations Models." Staff report, Federal Reserve Bank of Minneapolis.

Harrison, J. Michael and David Kreps. 1979. "Martingales and Arbitrage in Multiperiod Securities Markets." *Journal of Economic Theory* 20:381–409.

Hetzel, Robert. 1991. "A Better Way to Fight Inflation." *Wall Street Journal* April 25, A14.

Høien, Torgeir. 2016. "Prices and Policies: A Primer on the Fiscal Theory of the Price Level." *SKAGEN Funds, Stavanger, Norway* .

Holden, Thomas. 2020. "A Robust Monetary Rule." Manuscript.

Holston, Kathryn, Thomas Laubach, and John C. Williams. 2017. "Measuring the Natural Rate of Interest: International Trends and Determinants." *Journal of International Economics* 108:S39–S75. Supplement 1.

Jacobson, Margaret M., Eric M. Leeper, and Bruce Preston. 2019. "Recovery of 1933." Working Paper 25629, National Bureau of Economic Research.

Jiang, Zhengyang. 2021. "US Fiscal cycle and the dollar." *Journal of Monetary Economics* 124:91–106.

———. 2022. "Fiscal Cyclicality and Currency Risk Premia." *The Review of Financial Studies* 35 (3):1527–1552.

Jiang, Zhengyang, Hanno N. Lustig, Stijn Van Nieuwerburgh, and Mindy Z. Xiaolan. 2019. "The U.S. Public Debt Valuation Puzzle." Manuscript.

Jones, Chad. 2020a. "The End of Economic Growth? Unintended Consequences of a Declining Population." Manuscript.

———. 2020b. "Taxing Top Incomes in a World of Ideas." Manuscript.

Judd, Kenneth L. 1999. "Optimal Taxation and Spending in General Competitive Growth Models." *Journal of Public Economics* 71 (1):1–26.

Kamihigashi, Takashi. 2000. "A Simple Proof of Ekeland and Scheinkman's Result on the Necessity of a Transversality Condition." *Economic Theory* 15:463–468.

Kaplan, Greg, Benjamin Moll, and Giovanni L. Violante. 2018. "Monetary Policy According to HANK." *American Economic Review* 108 (3):697–743.

Kareken, John and Neil Wallace. 1981. "On the Indeterminacy of Equilibrium Exchange Rates." *The Quarterly Journal of Economics* 96 (2):207–222.

Kehoe, Timothy J. and Juan Pablo Nicolini, editors. 2021. *A Monetary and Fiscal History of Latin America, 1960–2017.* Minneapolis: University of Minnesota Press.

Kehoe, Timothy J. and Edward C. Prescott. 2007. *Great Depressions of the Twentieth Century.* Minneapolis, MN: Federal Reserve Bank of Minneapolis.

Kelton, Stephanie. 2020. *The Deficit Myth: Modern Monetary Theory and the Birth of the People's Economy.* Public Affairs.

Kiley, Michael T. 2016. "Policy Paradoxes in the New Keynesian Model." *Review of Economic Dynamics* 21:1–15.

Kimball, Miles. 2020. "How and Why to Eliminate the Zero Lower Bound: A Reader's Guide." *Confessions of a Supply-Side Liberal Blog* URL https://blog.supplysideliberal.com/emoney.

King, Robert G. 2000. "The New IS-LM Model: Language, Logic, and Limits." *Federal Reserve Bank of Richmond Economic Quarterly* 86 (3):45–104.

King, Robert G., Charles I. Plosser, and Sergio T. Rebelo. 1988. "Production, Growth and Business Cycles: I. The Basic Neoclassical Model." *Journal of Monetary Economics* 21 (2):195–232.

Knapp, Georg Friedrich. 1924. *The State Theory of Money*. London: Macmillian.

Kocherlakota, Narayana. 2010. "Inside the FOMC." URL https://www.minneapolisfed.org/news-and-events/presidents-speeches/inside-the-fomc. Speech.

Kocherlakota, Narayana and Christopher Phelan. 1999. "Explaining the Fiscal Theory of the Price Level." *Federal Reserve Bank of Minneapolis Quarterly Review* 23:14–23.

Kotlikoff, Laurence J. and Adam N. Michel. 2015. "Closing America's Enormous Fiscal Gap: Who Will Pay?" Mercatus Working Paper, Mercatus Center at George Mason University, Arlington, VA.

Krishnamurthy, Arvind and Annette Vissing-Jorgensen. 2012. "The Aggregate Demand for Treasury Debt." *Journal of Political Economy* 120:233–267.

Kroszner, Randall. 2003. "Is it Better to Forgive Than to Receive?: Repudiation of the Gold Indexation Clause in Long-Term Debt during the Great Depression." Manuscript.

Kuhn, Thomas. 1962. *The Structure of Scientific Revolutions*. Chicago: University of Chicago Press.

Kydland, Finn E. and Edward C. Prescott. 1982. "Time to Build and Aggregate Fluctuations." *Econometrica* 50 (6):1345–1370.

Lamont, Owen A. and Richard H. Thaler. 2003. "Can the Market Add and Subtract? Mispricing in Tech Stock Carve-outs." *Journal of Political Economy* 111:227–266.

Leeper, Eric M. 1991. "Equilibria Under 'Active' and 'Passive' Monetary and Fiscal Policies." *Journal of Monetary Economics* 27:129–147.

Leeper, Eric M., Troy Davig, and Hess Chung. 2007. "Monetary and Fiscal Policy Switching." *Journal of Money, Credit and Banking* 39:809– 842.

Leeper, Eric M. and Campbell B. Leith. 2016. "Understanding Inflation as a Joint Monetary-Fiscal Phenomenon." In *Handbook of Macroeconomics Volume 2*, edited by John B. Taylor and Harald Uhlig. Amsterdam: Elsevier, 2305–2415.

Leeper, Eric M. and Bing Li. 2017. “Surplus-Debt Regressions.” *Economics Letters* 151:10–15.

Leeper, Eric M., Michael Plante, and Nora Traum. 2010. “Dynamics of Fiscal Financing in the United States.” *Journal of Econometrics* 156 (2):304–321.

Leeper, Eric M., Nora Traum, and Todd B. Walker. 2017. “Clearing Up the Fiscal Multiplier Morass.” *American Economic Review* 107 (8):2409–2454.

Leeper, Eric M. and Todd B. Walker. 2013. “Perceptions and Misperceptions of Fiscal Inflation.” In *Fiscal Policy After the Financial Crisis*, edited by Alberto Alesina and Francesco Giavazzi. Chicago: University of Chicago Press, 255–299.

Leeper, Eric M., Todd B. Walker, and Shu-Chun Susan Yang. 2013. “Fiscal Foresight and Information Flows.” *Econometrica* 81 (3):1115–1145.

Lerner, Abba P. 1943. “Functional Finance and the Federal Debt.” *Social Research* 10 (1):38–51.

Lerner, Eugene M. 1956. “Inflation in the Confederacy, 1861-65.” In *Studies in the Quantity Theory of Money*, edited by Milton Friedman. Chicago: University of Chicago Press, 163–75.

LeRoy, Stephen F. and Richard D. Porter. 1981. “The Present-Value Relation: Tests Based on Implied Variance Bounds.” *Econometrica* 49 (3):555–574.

Ljungqvist, Lars and Thomas J. Sargent. 2018. *Recursive Macroeconomic Theory, 4th edition*. Cambridge, MA: MIT Press.

Long, John B. and Charles I. Plosser. 1983. “Real Business Cycles.” *Journal of Political Economy* 91 (1):39–69.

Loyo, Eduardo. 1999. “Tight Money Paradox on the Loose: A Fiscalist Hyperinflation.” Manuscript.

Lucas, Robert E. 1972. “Expectations and the Neutrality of Money.” *Journal of Economic Theory* 4:103–124.

———. 1973. “Some International Evidence on Output-Inflation Tradeoffs.” *The American Economic Review* 63 (3):326–334.

———. 1976. “Econometric Policy Evaluation: A Critique.” *Carnegie-Rochester Conference Series on Public Policy* 1:19–46.

———. 1980. “Equilibrium in a Pure Currency Economy.” *Economic Inquiry* 18 (2):203–220.

———. 1984. “Money in a Theory of Finance.” *Carnegie-Rochester Conference Series on Public Policy* 21:9–45.

———. 1988. “Money Demand in the United States: A Quantitative Review.” *Carnegie-Rochester Conference Series on Public Policy* 29:137–167.

Lucas, Robert E. and Thomas J. Sargent. 1981. “After Keynesian Macroeconomics.” *Rational Expectations and Econometric Practice* 1:295–319.

Lucas, Robert E. and Nancy L. Stokey. 1983. "Optimal Fiscal and Monetary policy in an Economy Without Capital." *Journal of Monetary Economics* 12 (1):55–93.

———. 1987. "Money and Interest in a Cash-in-Advance Economy." *Econometrica* 55 (3):491–513.

Lustig, Hanno, Christopher Sleet, and Şevin Yeltekin. 2008. "Fiscal Hedging With Nominal Assets." *Journal of Monetary Economics* 55 (4):710–727.

Mankiw, N. Gregory. 2018. "An Effective Marginal Tax Rate." *Greg Mankiw's Blog* URL http://gregmankiw.blogspot.com/2018/06/an-effective-marginal-tax-rate.html.

Mankiw, N. Gregory and Jeffrey A. Miron. 1991. "Should the fed smooth interest rates? the case of seasonal monetary policy." *Carnegie-Rochester Conference Series on Public Policy* 34:41–69.

Mankiw, N. Gregory and Ricardo Reis. 2002. "Sticky Information versus Sticky Prices: A Proposal to Replace the New Keynesian Phillips Curve." *Quarterly Journal of Economics* 117:1295–1328.

Marimon, Ramon. 2001. "The Fiscal Theory of Money as an Unorthodox Financial Theory of the Firm." In *Monetary Theory as a Basis for Monetary Policy*, edited by Axel Leijonhufvd. New York: Palgrave.

Matsuyama, Kiminori. 1990. "Sunspot Equilibria (Rational Bubbles) in a Model of Money-in-the-Utility-Function." *Journal of Monetary Economics* 25:137–144.

———. 1991. "Endogenous Price Fluctuations in an Optimizing Model of a Monetary Economy." *Econometrica* 59:1617–1631.

McCallum, Bennett T. 1981. "Price Level Determinacy with an Interest Rate Policy Rule and Rational Expectations." *Journal of Monetary Economics* 8:319–329.

———. 2001. "Indeterminacy, Bubbles, and the Fiscal Theory of Price Level Determination." *Journal of Monetary Economics* 47:19–30.

———. 2003. "Multiple-Solution Indeterminacies in Monetary Policy Analysis." *Journal of Monetary Economics* 50:1153–75.

———. 2009a. "Inflation Determination with Taylor Rules: Is New-Keynesian Analysis Critically Flawed?" *Journal of Monetary Economics* 56 (8):1101–1108.

———. 2009b. "Rejoinder to Cochrane." *Journal of Monetary Economics* 56 (8):1114–1115.

McCallum, Bennett T. and Edward Nelson. 2005. "Monetary and Fiscal Theories of the Price Level: The Irreconcilable Differences." *Oxford Review of Economic Policy* 21 (4):565–583.

McCandless, George T. 1996. "Money, Expectations, and the U.S. Civil War." *The American Economic Review* 86 (3):661–671.

McDermott, John and Rebecca Williams. 2018. "Inflation Targeting in New Zealand: An Experience in Evolution." URL https://www.rbnz.

govt.nz/-/media/ReserveBank/Files/Publications/Speeches/2018/Speech-Inflation-Targeting-in-New-Zealand.pdf. A speech delivered to the Reserve Bank of Australia conference on central bank frameworks, in Sydney.

Mehrotra, Neil and Dmitriy Sergeyev. 2021. "Debt Sustainability in a Low Interest Rate World." *Journal of Monetary Economics* Forthcoming.

Miller, David S. 2021. "A Monetary-Fiscal Theory of Sudden Inflations and Currency Crises." Manuscript.

Miller, Merton H. and Daniel Orr. 1966. "A Model of the Demand for Money by Firms." *The Quarterly Journal of Economics* 80 (3):413–435.

Minford, Patrick and Naveen Srinivasan. 2011. "Determinacy in New Keynesian Models: a Role for Money After All?" *International Finance* 14:211–229.

Mitchell, Wesley Clair. 1903. *A History of the Greenbacks.* Chicago: University of Chicago Press.

Mulligan, Casey. 2012. *The Redistribution Recession: How Labor Market Distortions Contracted the Economy.* New York: Oxford University Press.

Murray, John. 2018. "Bank of Canada's Experience with Inflation Targeting: Partnering with the Government." URL https://www.brookings.edu/wp-content/uploads/2017/12/murray-slides.pdf.

Mussa, Michael. 1986. "Nominal Exchange Rate Regimes and the Behavior of Real Exchange Rates: Evidence and Implications." *Carnegie-Rochester Conference Series on Public Policy* 25:117–214.

Nelson, Edward. 2020. *Milton Friedman and Economic Debate in the United States, 1932–1972, Volume 1.* Chicago: University of Chicago Press.

Niepelt, Dirk. 2004. "The Fiscal Myth of the Price Level." *The Quarterly Journal of Economics* 119 (1):277–300.

Obstfeld, Maurice. 1984. "Multiple Stable Equilibria in an Optimizing Perfect-Foresight Model." *Econometrica* 52:223–228.

Obstfeld, Maurice and Kenneth Rogoff. 1983. "Speculative Hyperinflations in Maximizing Models: Can we Rule them Out?" *Journal of Political Economy* 91:675–687.

———. 1986. "Ruling out Divergent Speculative Bubbles." *Journal of Monetary Economics* 17:349–362.

———. 2021. "Revisiting Speculative Hyperinflations in Monetary Models." *Review of Economic Dynamics* 40:1–11.

Patinkin, Donald. 1965. *Money, Interest, and Prices.* New York: Harper and Row.

Peltzman, Sam. 1975. "The Effects of Automobile Safety Regulation." *Journal of Political Economy* 83 (4):677–725.

Piazzesi, Monika. 2005. "Bond Yields and the Federal Reserve." *Journal of Political*

Economy 113:311–344.

Poole, William. 1970. "Optimal Choice of Monetary Policy Instruments in a Simple Stochastic Macro Model*." *The Quarterly Journal of Economics* 84 (2):197–216.

Poterba, James M. and Lawrence H. Summers. 1988. "Mean Reversion in Stock Prices: Evidence and Implications." *Journal of Financial Economics* 22 (1):27–59.

Rabushka, Alvin. 2008. *Taxation in Colonial America.* Princeton, N.J.: Princeton University Press.

Ramey, Valerie. 2016. "Macroeconomic Shocks and Their Propagation." In *Handbook of Macroeconomics Vol. 2*, edited by John B. Taylor and Harald Uhlig. Amsterdam: Elsevier, 71–162.

Reis, Ricardo. 2021. "The Constraint on Public Debt When $r < g$ but $g < m$." Manuscript.

Restuccia, Diego. 2021. "The Case of Venezuela." In *A Monetary and Fiscal History of Latin America, 1960–2017*, edited by Timothy J. Kehoe and Juan Pablo Nicolini. Minneapolis: University of Minnesota Press.

Rogoff, Kenneth. 2017. *The Curse of Cash: How Large-Denomination Bills Aid Crime and Tax Evasion and Constrain Monetary Policy.* Princeton: Princeton University Press.

Romer, Christina D. and David H. Romer. 1989. "Does Monetary Policy Matter? A New Test in the Spirit of Friedman and Schwartz." *NBER Macroeconomics Annual* 4:121–170.

Rueff, Jacques. 1972. *The Monetary Sin of the West.* Macmillan. Translated by Roger Glémet.

Rusnak, Marek, Tomas Havranek, and Roman Horvath. 2013. "How to Solve the Price Puzzle? A Meta-Analysis." *Journal of Money, Credit and Banking* 45 (1):37–70.

Sargent, Thomas J. 1982a. "Beyond Demand and Supply Curves in Macroeconomics." *The American Economic Review* 72 (2):382–389.

———. 1982b. "The Ends of Four Big Inflations." In *Inflation: Causes and Effects*, edited by Robert E. Hall. University of Chicago Press, for the NBER, 41–97.

———. 1987. *Macroeconomic Theory.* Bingley: Emerald Group, 2nd ed.

———. 2012. "Nobel Lecture: United States Then, Europe Now." *Journal of Political Economy* 120 (1):1–40.

———. 2013. *Rational Expectations and Inflation (3d Edition).* Princeton: Princeton University Press.

Sargent, Thomas J. and François R. Velde. 1995. "Macroeconomic Features of the French Revolution." *Journal of Political Economy* 103 (3):474–518.

———. 2003. *The Big Problem of Small Change*. Princeton: Princeton University Press.

Sargent, Thomas J. and Neil Wallace. 1973. "Rational Expectations and the Dynamics of Hyperinflation." *International Economic Review* 14 (2):328–350.

———. 1975. "'Rational' Expectations, the Optimal Monetary Instrument, and the Optimal Money Supply Rule." *Journal of Political Economy* 83 (2):241–254.

———. 1981. "Some Unpleasant Monetarist Arithmetic." *Federal Reserve Bank of Minneapolis Quarterly Review* 5:1–17.

———. 1982. "The Real-Bills Doctrine versus the Quantity Theory: A Reconsideration." *Journal of Political Economy* 90 (6):1212–1236.

———. 1985. "Interest on Reserves." *Journal of Monetary Economics* 15 (3):279–290.

Schmitt-Grohé, Stephanie and Martín Uribe. 2000. "Price Level Determinacy and Monetary Policy Under a Balanced Budget Requirement." *Journal of Monetary Economics* 45:211–246.

———. 2007. "Optimal Inflation Stabilization in a Medium-Scale Macroeconomic Model." In *Monetary Policy Under Inflation Targeting*, edited by Klaus Schmidt-Hebbel and Rick Mishkin. Santiago, Chile: Central Bank of Chile, 125–186.

———. 2014. "Liquidity Traps: An Interest-Rate-Based Exit Strategy." *The Manchester School* 81 (S1):1–14.

Schultz, George P., Arthur F. Burns, Paul McCracken, Milton Friedman, William E. Simon, Alan Greenspan, Charles S. Walker, Michel T. Halbouty, Murray L. Weidenbaum, Jack Kemp, Caspar W. Weinberger, James T. Lynn, and Walter B. Wriston. 1980. "Economic Strategy for the Reagan Administration." URL https://www.hoover.org/sites/default/files/economic-strategy-memo-nov-1980.pdf. A Report to President-Elect Ronald Reagan from His Coordinating Committee on Economic Policy.

Shiller, Robert J. 1981. "Do Stock Prices Move Too Much to be Justified by Subsequent Changes in Dividends?" *American Economic Review* 71 (3):421–436.

Shlaes, Amity. 2019. *Great Society: A New History*. New York: Harper Perennial.

Sims, Christopher A. 1980. "Macroeconomics and Reality." *Econometrica* 48 (1):1–48.

———. 1994. "A Simple Model for Study of the Determination of the Price Level and the Interaction of Monetary and Fiscal Policy." *Economic Theory* 4 (3):381–399.

———. 1997. "Fiscal Foundations of Price Stability in Open Economies." Manuscript.

———. 1999. "The Precarious Fiscal Foundations of EMU." *De Economist* 147:415–436.

———. 2001. "Fiscal Consequences for Mexico of Adopting the Dollar." *Journal of Money, Credit and Banking* 33:597–616.

———. 2004. "Fiscal Aspects of Central Bank Independence." In *European Monetary Integration, CESifo Seminar Series*, edited by Hans-Werner Sinn, Mika Widgrén, and Marko K othenbürger. Cambridge: MIT Press, 103–116.

———. 2005. "Limits to Inflation Targeting." In *The Inflation-Targeting Debate, Vol. 32 of National Bureau of Economic Research Studies in Business Cycles*, edited by Ben S. Bernanke. and Michael Woodford, chap. 7. Chicago, IL: University of Chicago Press, 283–299.

———. 2011. "Stepping on a Rake: The Role of Fiscal Policy in the Inflation of the 1970s." *European Economic Review* 55:48–56.

———. 2013. "Paper Money." *American Economic Review* 103 (2):563–84.

———. 2016. "Active Fiscal, Passive Money Equilibrium in a Purely Backward-Looking Model." Manuscript.

Sims, Christopher A. and Tao Zha. 2006. "Were There Regime Switches in U.S. Monetary Policy?" *American Economic Review* 96 (1):54–81.

Smets, Frank and Raf Wouters. 2007. "Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach." *American Economic Review* 97:586–606.

Straub, Ludwig and Iván Werning. 2020. "Positive Long-Run Capital Taxation: Chamley-Judd Revisited." *American Economic Review* 110 (1):86–119.

Sturzenegger, Federico. 2019. "Macri's Macro: The Elusive Road to Stability and Growth." In *Brookings Papers on Economic Activity*, vol. Fall, edited by Janice Eberly and James H. Stock. Washington, DC: Brookings Institution Press, 339–411.

Summers, Lawrence. 2014. "U.S. Economic Prospects: Secular Stagnation, Hysteresis, and the Zero Lower Bound." *Business Economics* 49:65–73.

Taylor, John B. 1993. "Discretion Versus Policy Rules in Practice." *Carnegie-Rochester Conference Series on Public Policy* 39:195–214.

———. 1999. "The Robustness and Efficiency of Monetary Policy Rules as Guidelines for Interest Rate Setting by the European Central Bank." *Journal of Monetary Economics* 43:655–679.

Tobin, James. 1956. "The Interest-Elasticity of Transactions Demand For Cash." *The Review of Economics and Statistics* 38 (3):241–247.

———. 1958. "Liquidity Preference as Behavior Towards Risk." *The Review of Economic Studies* 25 (2):65–86.

———. 1970. "Money and Income: Post Hoc Ergo Propter Hoc?" *The Quarterly Journal of Economics* 84 (2):301–317.

———. 1980. *Asset Accumulation and Economic Activity: Reflections on Contemporary Macroeconomic Theory.* Chicago: University of Chicago Press.

Uribe, Martìn. 2018. "The Neo-Fisher Effect: Econometric Evidence from Empirical and Optimizing Models." NBER Working Paper 25089.

Velde, François R. 2009. "Chronicle of a Deflation Unforetold." *Journal of Political Economy* 117 (4):591–634.

Wallace, Neil. 1981. "A Modigliani-Miller Theorem for Open-Market Operations." *The American Economic Review* 71 (3):267–274.

Weidenmier, Marc. 2002. "Money and Finance in the Confederate States of America." URL http://eh.net/encyclopedia/money-and-finance-in-the-confederate-states-of-america/.

Werning, Iván. 2012. "Managing a Liquidity Trap: Monetary and Fiscal Policy." Manuscript.

Wicksell, Knut. 1898. *Geldzins und Güterpreise.* Jena, Germany: Gustav Fischer.

———. 1965. *Interest and Prices.* New York: Augustus M. Kelley.

Wieland, Johannes. 2019. "Are Negative Supply Shocks Expansionary at the Zero Lower Bound?" *Journal of Political Economy* 127:973–1007.

Williamson, Stephen D. 2018. "Can the Fiscal Authority Constrain the Central Bank?" *Journal of Economic Dynamics and Control* 89:154–172.

Woodford, Michael. 1994. "Monetary Policy and Price Level Determinacy in a Cash-in-Advance Economy." *Economic Theory* 4:345–380.

———. 1995. "Price-Level Determinacy Without Control of a Monetary Aggregate." *Carnegie-Rochester Conference Series on Public Policy* 43:1–46.

———. 2001. "Fiscal Requirements for Price Stability." *Journal of Money, Credit and Banking* 33:669–728.

———. 2003. *Interest and Prices.* Princeton: Princeton University Press.

———. 2011. "Simple Analytics of the Government Expenditure Multiplier." *American Economic Journal: Macroeconomics* 3 (1):1–35.

———. 2012. "Methods of Policy Accommodation at the Interest-Rate Lower Bound." In *The Changing Policy Landscape.* Federal Reserve Bank of Kansas City, 185–288.

Zweig, Martin E. 1973. "An Investor Expectations Stock Price Predictive Model Using Closed-End Fund Premiums." *Journal of Finance* 28:67–78.

Part VI

Online Appendix to *The Fiscal Theory of the Price Level*

Chapter One

Algebra and extensions

This chapter collects algebra for several results in the main text along with some extensions of the analysis.

A1.1 The transversality condition

> The transversality condition $\lim_{T\to\infty} E_t\left(\beta^T B_{T-1}/P_T\right) = 0$ results from a "no-Ponzi" condition, that people cannot borrow, consume, and roll their debt over debt forever, $\lim_{T\to\infty} E_t\left(\beta^T B_{T-1}/P_T\right) \geq 0$, plus an optimality condition, that people should consume rather than let wealth grow forever $\lim_{T\to\infty} E_t\left(\beta^T B_{T-1}/P_T\right) \leq 0$.

The transversality condition, introduced in Section 2.1, remains an object of much confusion. My general advice is to make sure a model makes sense with a finite horizon and then take limits.

The transversality condition takes us from flow budget constraints and dynamic trading in a sequence of markets to the present value budget constraint and a description of the economy in terms of equivalent time-zero contingent claims markets. For example, with a constant real interest rate, perfect foresight, and no money, the consumer's flow budget constraint (2.2) is

$$B_{t-1} + P_t y = P_t c_t + P_t s_t + \frac{1}{R}\frac{P_t}{P_{t+1}} B_t$$

$$\frac{B_{t-1}}{P_t} + y = c_t + s_t + \frac{1}{R}\frac{B_t}{P_{t+1}}. \tag{A1.1}$$

Iterate forward to

$$\frac{B_{t-1}}{P_t} = \sum_{j=0}^{\infty} \frac{1}{R^j}\left(c_t + s_t - y\right) + \lim_{T\to\infty} \frac{1}{R^T}\frac{B_{t+T}}{P_{t+T+1}}. \tag{A1.2}$$

The first term of (A1.2) is the present value budget constraint, which is what you would write down for a consumer buying contingent claims in time 0 markets. It is equivalent to the sequence of period budget constraints (A1.1) plus the transversality condition, that the second term of (A1.2) goes to zero. Trading in a sequence of markets offers an opportunity to borrow and roll over debt forever, which we have to rule out to give the same result as the time 0 budget constraint.

The lower limit $\lim_{T\to\infty} E_t\left(\beta^T B_{T-1}/P_T\right) \geq 0$ (later $\lim_{T\to\infty} E_t(\Lambda_T B_{T-1}/P_T) > 0$ with Λ_T a stochastic discount factor) is a genuine budget constraint. The consumer can't borrow, eat, and roll over the debt forever. It stems from a "no-Ponzi" condition imposed in various ways, such as a borrowing limit and $B_t \geq 0$ here. The upper limit $\lim_{T\to\infty} E_t\left(\beta^T B_{T-1}/P_T\right) \leq 0$ is a condition of consumer optimization. No budget constraint stops you from accumulating infinite amounts of debt. But if you were to do so, you could improve utility by consuming a bit more at each date.

As a simple example, consider an equilibrium with constant income y, constant surplus $s_t = s$, and no uncertainty. The initial price level should satisfy

$$\frac{B_{t-1}}{P_t} = \sum_{j=0}^{\infty} \beta^j s = \frac{s}{1-\beta}.$$

Suppose that the initial price level is too low, but the other conditions for equilibrium hold – intertemporal optimization and no money demand. Now debt evolves as

$$\frac{B_{t-1}}{P_t} = s + \beta \frac{B_t}{P_{t+1}}$$

$$\frac{B_{t-1}}{P_t} - \frac{s}{1-\beta} = \beta\left(\frac{B_t}{P_{t+1}} - \frac{s}{1-\beta}\right)$$

$$\frac{B_{t+T}}{P_{t+T+1}} = \beta^{-(T+1)}\left(\frac{B_{t-1}}{P_t} - \frac{s}{1-\beta}\right) + \frac{s}{1-\beta}$$

The terminal or transversality term does not go to zero. The valuation equation reads

$$\frac{B_{t-1}}{P_t} = E_t \sum_{j=0}^{T} \beta^j s_{t+j} + \beta^{T+1} \frac{B_{t+T}}{P_{t+T+1}}.$$

$$\frac{B_{t-1}}{P_t} = E_t \sum_{j=0}^{T} \beta^j s + \left(\frac{B_{t-1}}{P_t} - \frac{s}{1-\beta}\right) + \beta^{T+1} \frac{s}{1-\beta}.$$

and taking the limit,

$$\frac{B_{t-1}}{P_t} = E_t \sum_{j=0}^{\infty} \beta^j s + \left(\frac{B_{t-1}}{P_t} - \frac{s}{1-\beta}\right).$$

What's wrong? The consumer would prefer to reallocate some of this terminal wealth to consumption. Specifically, facing prices $\{P_t\}$, an initial debt $\{B_{t-1}\}$, surpluses and endowment s and y, and facing real interest rates β^{-1}, the consumer's T-period budget constraint is

$$\frac{B_{t-1}}{P_t} = E_t \sum_{j=0}^{T} \beta^j \left[s + c_{t+j} - y\right] + \beta^{T+1} \frac{B_{t+T}}{P_{t+T+1}}.$$

The limit of the last term must be non-negative as part of the budget constraint. But suppose it is positive.

The first-order conditions for intertemporal allocation still hold, so optimal consumption will be flat at all dates. Suppose the consumer raises consumption by Δc at each date, $c_t = y + \Delta c$, proposing to consume just a little more and buy a bit less government debt. Debt now accumulates as

$$\frac{B_{t-1}}{P_t} = s + \Delta c + \beta \frac{B_t}{P_{t+1}}$$

and hence

$$\frac{B_{t+T}}{P_{t+T+1}} - \frac{s + \Delta c}{1-\beta} = \beta^{-(T+1)} \left(\frac{B_{t-1}}{P_t} - \frac{s+\Delta c}{1-\beta} \right).$$

So long as the initial price level is still too low,

$$\frac{B_{t-1}}{P_t} > \frac{s+\Delta c}{1-\beta} \tag{A1.3}$$

terminal debt still explodes at the interest rate. The new allocation satisfies optimal intertemporal allocation, budget constraints, and improves utility. Indeed, the increase Δc by which (A1.3) holds with equality is the optimal choice. But with $\Delta c > 0$, i.e. $c > y$, this choice is not an equilibrium, as the goods market does not clear. Prices rise until it does.

The transversality condition is, in general, weighted by contingent claims prices, which are equal marginal utility by the consumer's first-order condition,

$$\lim_{T\to\infty} E_t \left(\Lambda_T B_{T-1}/P_T \right) = 0$$
$$\beta^T u'(c_T) = \Lambda_T.$$

When confused, return to the question: Facing the prices of a hypothesized equilibrium, keeping intertemporal and asset pricing first-order conditions intact, can the consumer raise utility by reducing the object that does not go to zero on the right-hand side of a present value?

A1.2 Derivation of the linearized identities

I derive the linearized flow identity (3.17)

$$\rho v_{t+1} = v_t + r_{t+1} - g_{t+1} - \tilde{s}_{t+1}. \tag{A1.4}$$

Along the way, I express the useful nonlinear flow and present value identities.

First, I derive the linearized flow identity (3.17), repeated as (A1.4) just above. Denote by

$$V_t = M_t + \sum_{j=0}^{\infty} Q_t^{(t+1+j)} B_t^{(t+1+j)}$$

the nominal end-of-period market value of debt, where M_t is non-interest-bearing money, $B_t^{(t+j)}$ is zero-coupon nominal debt outstanding at the end of period t and due at the beginning of period $t+j$, and $Q_t^{(t+j)}$ is the time t price of that bond, with $Q_t^{(t)} = 1$. It turns out to be a bit prettier to consider this end-of-period value rather than the beginning-of-period convention we have used so far. Taking logs, denote by

$$v_t \equiv \log\left(\frac{V_t}{y_t P_t}\right)$$

the log market value of the debt divided by GDP, where P_t is the price level and y_t is real GDP or another stationarity-inducing divisor such as consumption, potential GDP, population, etc. Denote by

$$R_{t+1}^n \equiv \frac{M_t + \sum_{j=1}^{\infty} Q_{t+1}^{(t+j)} B_t^{(t+j)}}{M_t + \sum_{j=1}^{\infty} Q_t^{(t+j)} B_t^{(t+j)}} \tag{A1.5}$$

the nominal return on the portfolio of government debt, i.e. how the change in prices overnight from the end of t to the beginning of $t+1$ affects the value of debt held overnight, and

$$r_{t+1}^n \equiv \log(R_{t+1}^n)$$

is the log nominal return on that portfolio. As usual,

$$\pi_{t+1} \equiv \log \Pi_{t+1} = \log\left(\frac{P_{t+1}}{P_t}\right),\ g_{t+1} \equiv \log G_{t+1} = \log\left(\frac{y_{t+1}}{y_t}\right)$$

are log inflation and GDP growth rate.

Now, I establish the nonlinear flow and present value identities. In period $t+1$, we have the flow identity

$$M_t + \sum_{j=1}^{\infty} Q_{t+1}^{(t+j)} B_t^{(t+j)} = P_{t+1} s p_{t+1} + M_{t+1} + \sum_{j=1}^{\infty} Q_{t+1}^{(t+1+j)} B_{t+1}^{(t+1+j)}. \tag{A1.6}$$

Money M_{t+1} at the end of period $t+1$ is equal to money brought in from the previous period M_t plus the effects of bond sales or purchases at price $Q_{t+1}^{(t+j)}$, less money soaked up by real primary surpluses sp_{t+1}.

Using the definition of return, (A1.6) becomes

$$\left(M_t + \sum_{j=1}^{\infty} Q_t^{(t+j)} B_t^{(t+j)}\right) R_{t+1}^n = P_{t+1} s p_{t+1} + \left(M_{t+1} + \sum_{j=1}^{\infty} Q_{t+1}^{(t+1+j)} B_{t+1}^{(t+1+j)}\right),$$

or,

$$V_t R_{t+1}^n = P_{t+1} s p_{t+1} + V_{t+1}. \tag{A1.7}$$

The nominal value of government debt is increased by the nominal rate of return, and decreased by primary surpluses. This seems easy. The algebra all comes from properly defining the return on the portfolio of government debt.

Expressing the result as ratios to GDP, we have a flow identity

$$\frac{V_t}{P_t y_t} \times \frac{R^n_{t+1}}{G_{t+1}} \frac{P_t}{P_{t+1}} = \frac{sp_{t+1}}{y_{t+1}} + \frac{V_{t+1}}{P_{t+1} y_{t+1}}. \tag{A1.8}$$

We can iterate this flow identity (A1.8) forward to express the nonlinear government debt valuation identity as

$$\frac{V_t}{P_t y_t} = \sum_{j=1}^{\infty} \prod_{k=1}^{j} \frac{1}{R^n_{t+k}/(\Pi_{t+k} G_{t+k})} \frac{sp_{t+j}}{y_{t+j}}. \tag{A1.9}$$

I assume here that the right-hand side converges. If you want to examine this issue, keep the limiting debt term or iterate a finite number of periods.

I linearize the flow equation (A1.8) to get its linearized counterpart (3.17) and then I iterate that forward to obtain (3.18), the linearized version of (A1.9). Taking logs of (A1.8),

$$v_t + r^n_{t+1} - \pi_{t+1} - g_{t+1} = \log\left(\frac{sp_{t+1}}{y_{t+1}} + \frac{V_{t+1}}{P_{t+1} y_{t+1}}\right). \tag{A1.10}$$

I linearize in the *level* of the surplus, not its log as one conventionally does in asset pricing, since the surplus is often negative. Taylor expand the last term of (A1.10),

$$v_t + r^n_{t+1} - \pi_{t+1} - g_{t+1} = \log\left(sy_{t+1} + e^{v_{t+1}}\right)$$

$$v_t + r^n_{t+1} - \pi_{t+1} - g_{t+1} = \log(e^v + sy) + \frac{e^v}{e^v + sy}(v_{t+1} - v) + \frac{1}{e^v + sy}\left(sy_{t+1} - sy\right)$$

where

$$sy_{t+1} \equiv \frac{sp_{t+1}}{y_{t+1}} \tag{A1.11}$$

denotes the surplus to GDP ratio, and variables without subscripts denote a steady state of (A1.10). With $r \equiv r^n - \pi$, steady states obey

$$r - g = \log\left(\frac{e^v + sy}{e^v}\right).$$

Then,

$$\begin{aligned}
&v_t + r^n_{t+1} - \pi_{t+1} - g_{t+1} = \\
&= \left[\log(e^v + sy) - \frac{e^v}{e^v + sy}\left(v + \frac{sy}{e^v}\right)\right] + \frac{e^v}{e^v + sy} v_{t+1} + \frac{e^v}{e^v + sy}\frac{sy_{t+1}}{e^v} \\
&= \left[v + r - g - \frac{e^v}{e^v + sy}\left(v + \frac{e^v + sy}{e^v} - 1\right)\right] + \rho v_{t+1} + \rho \frac{sy_{t+1}}{e^v},
\end{aligned}$$

or,

$$v_t + r^n_{t+1} - \pi_{t+1} - g_{t+1} = \left[r - g + (1-\rho)(v-1)\right] + \rho v_{t+1} + \rho \frac{sy_{t+1}}{e^v} \tag{A1.12}$$

where

$$\rho \equiv e^{-(r-g)}. \tag{A1.13}$$

Suppressing the small constant, and thus interpreting variables as deviations from means, the linearized flow identity is

$$v_t + r^n_{t+1} - \pi_{t+1} - g_{t+1} = \rho \frac{sy_{t+1}}{e^v} + \rho v_{t+1}. \tag{A1.14}$$

I use the symbol $\tilde{s}_t$ in the linearized formulas to refer to the surplus/GDP ratio scaled by the steady-state value to GDP ratio,

$$\tilde{s}_{t+1} \equiv \rho \frac{sy_{t+1}}{e^v}.$$

We have the linearized flow identity (3.17),

$$\rho v_{t+1} = v_t + r_{t+1} - g_{t+1} - \tilde{s}_{t+1}.$$

Iterating forward and if the terms converge, we have the linearized present value formula (valueid)

$$v_t = \sum_{j=1}^{T} \rho^{j-1} \tilde{s}_{t+j} + \sum_{j=1}^{T} \rho^{j-1} g_{t+j} - \sum_{j=1}^{T} \rho^{j-1} r_{t+j} + \rho^T v_{t+T}.$$

There is nothing wrong with expanding about $r = g$, $\rho = 1$, in which case the constant in the identity is zero. The point of linearization need not be the sample mean. For most time-series applications v_t is stationary, so $\lim_{T\to\infty} E_t v_{t+T} = 0$ even without discounting by ρ^T. We usually apply linearizations to variables that have been demeaned, or to understand second moments of the data, so the constant drops in that case as well.

Cochrane (2021a) evaluates the accuracy of approximation, by comparing the surplus calculated from the exact nonlinear flow identity to the surplus calculated from the linearized identity. I find it reasonably close outside of the extreme deficits of early WWII.

A1.3 Geometric maturity formulas

A1.3.1 Nonlinear geometric maturity structure formulas

A geometric maturity structure $B_{t-1}^{(t+j)} = \omega^j B_{t-1}$ in discrete time and $B_t^{(t+j)} = \varpi e^{-\varpi j} B_t$ in continuous time is analytically convenient. I present formulas for the examples in Figure 3.1 and Figure 7.1.

To maintain the geometric structure, the government must roll over debt, and gradually sell more debt of each coupon as its date approaches.

A geometric maturity structure $B_{t-1}^{(t+j)} = \omega^j B_{t-1}$ is analytically convenient. A perpetuity is $\omega = 1$, and one-period debt is $\omega = 0$. Here I work out exact formulas for one-time shocks. This analysis is a counterpart to the linearized formulas in Section 3.5.3. I use these formulas in Figure 3.1.

Suppose the interest rate $i_{t+j} = i$ is expected to last forever, and suppose surpluses are constant s. The bond price is then $Q_t^{(t+j)} = 1/(1+i)^j$. The valuation equation at time 0 becomes

$$\frac{\sum_{j=0}^{\infty} Q_0^{(j)} \omega^j B_{-1}}{P_0} = \sum_{j=0}^{\infty} \frac{\omega^j}{(1+i)^j} \frac{B_{-1}}{P_0} = \frac{1+i}{1+i-\omega} \frac{B_{-1}}{P_0} = \frac{1+r}{r} s. \tag{A1.15}$$

Start at a steady state $B_{-1} = B$, $P_{-1} = P$, $i_{-1} = r$. In this steady state we have

$$\frac{1+r}{1+r-\omega} \frac{B}{P} = \frac{1+r}{r} s. \tag{A1.16}$$

Now suppose at time 0 the interest rate rises unexpectedly and permanently from r to i. We can express (A1.15) as

$$\frac{P_0}{P} = \frac{(1+i)}{(1+r)} \frac{(1+r-\omega)}{(1+i-\omega)}. \tag{A1.17}$$

These formulas are prettier in continuous time. The valuation equation is

$$\frac{\int_{j=0}^{\infty} Q_t^{(t+j)} B_t^{(t+j)} dj}{P_t} = E_t \int_{j=0}^{\infty} e^{-rj} s_{t+j} dj.$$

With maturity structure $B_t^{(t+j)} = \varpi e^{-\varpi j} B_t$, and a constant interest rate $i_t = i$,

$$\varpi \int_{j=0}^{\infty} e^{-ij} e^{-\varpi j} dj \frac{B_t}{P_t} = \frac{\varpi}{i+\varpi} \frac{B_t}{P_t} = \frac{s}{r}. \tag{A1.18}$$

Here $\varpi = 0$ is the perpetuity and $\varpi = \infty$ is instantaneous debt. They are related by $\omega = e^{-\varpi}$. (I use the overbar here to contrast the continuous time parameter from the discrete- time parameter. Elsewhere, I use the same symbol ω without overbar in both cases.) B_t is predetermined. P_t can jump.

Starting from the $i_t = r$, $t < 0$ steady state, if i_0 jumps to a new permanently higher value i, we now have

$$\frac{P_0}{P} = \frac{r+\varpi}{i+\varpi} \tag{A1.19}$$

in place of (A1.17).

In the case of one-period debt, $\omega = 0$ or $\varpi = \infty$, $P_0 = P$ and there is no downward jump. In the case of a perpetuity, $\omega = 1$ or $\varpi = 0$, (A1.17) becomes

$$P_0 = \frac{1+i}{1+r} \frac{r}{i} P. \tag{A1.20}$$

and (A1.19) becomes

$$P_0 = \frac{r}{i} P. \tag{A1.21}$$

The price level P_0 jumps down as the interest rate rises, and proportionally to the interest rate rise.

This is potentially a large effect; a rise in interest rates from $r = 3\%$ to $i = 4\%$ occasions a 25% price level drop. However, our governments maintain much shorter maturity structures, monetary policy changes in interest rates are not permanent, and they are often preannounced, each factor reducing the size of the effect. With $\omega = 0.8$, the permanent interest rate rise graphed in Figure 3.1 leads to a 3.5% price level drop. The forward guidance of Figure 7.1 leads to a 1.6% price level drop. A mean-reverting interest rate rise has a smaller effect still. Price stickiness also makes the effect smaller, because higher real interest rates also devalue the right-hand side of the valuation equation, a countervailing inflationary effect.

When the government announces at time 0 that interest rates will rise from r to i starting at time T, equation (7.3) reads

$$\left[\sum_{j=0}^{T} \frac{\omega^j}{(1+r)^j} + \sum_{j=T+1}^{\infty} \frac{\omega^T}{(1+r)^T} \frac{\omega^{(j-T)}}{(1+i)^{(j-T)}}\right] \frac{B_{-1}}{P_0} = \frac{s}{1-\beta}$$

and with a bit of algebra

$$\frac{P_0}{P} - 1 = \left(\frac{\omega}{1+r}\right)^T \left[\frac{(1+i)}{(1+r)} \frac{(1+r-\omega)}{(1+i-\omega)} - 1\right],$$

generalizing (A1.17). In continuous time, we have

$$\left[\varpi \int_0^T e^{-rj} e^{-\varpi j} dj + \varpi \int_T^{\infty} e^{-rT-i(j-T)} e^{-\varpi j} dj\right] \frac{B_0}{P_0} = \frac{s}{r},$$

leading to

$$\frac{P_0}{P} - 1 = e^{-(r+\varpi)T} \left(\frac{r+\varpi}{i+\varpi} - 1\right),$$

generalizing (A1.19).

The price level P_0 still jumps—forward guidance works. Longer T or shorter maturity structures—lower ω or larger ϖ—give a smaller price level jump for a given interest rate rise. As $T \to \infty$, the downward price level jump goes to zero.

A geometric maturity structure needs tending, except in a knife-edge case that surpluses are also nonstochastic and geometric. To see the needed bond sales, write bond sales as

$$B_t^{(t+j)} - B_{t-1}^{(t+j)} = \omega^{j-1} B_t - \omega^j B_{t-1}.$$

Thus, to maintain a steady state,

$$B_t^{(t+j)} - B_{t-1}^{(t+j)} = \omega^{j-1} (1-\omega) B = \frac{1-\omega}{\omega} B_{t-1}^{(t+j)}.$$

In order to pay off maturing debt B_{t-1}, in addition to the current surplus s_t, the government must issue new debt. It issues debt across the maturity spectrum, in the same geometric pattern as debt outstanding. Equivalently, the government issues more and more of each bond as it approaches maturity, again with a geometric pattern. This is roughly what our governments do, since they issue short-term bonds while older long-term bonds have the same maturity.

A1.3.2 Geometric maturity structure linearizations

I derive linearized identities for geometric maturity structures. The return and price obey

$$r^n_{t+1} \approx \omega q_{t+1} - q_t.$$

The bond price is negative the weighted sum of future returns,

$$q_t = -\sum_{j=1}^{\infty} \omega^j r^n_{t+j}.$$

Taking innovations, we obtain (3.21),

$$\Delta E_{t+1} r^n_{t+1} = -\sum_{j=1}^{\infty} \omega^j \Delta E_{t+1} \left(r^n_{t+1+j} \right)$$

$$= -\sum_{j=1}^{\infty} \omega^j \Delta E_{t+1} \left[\left(r^n_{t+1+j} - \pi_{t+1+j} \right) + \pi_{t+1+j} \right].$$

Under the expectations hypothesis we also have

$$i_t = E_t r^n_{t+1}$$
$$i_t = \omega E_t q_{t+1} - q_t.$$

Denote the maturity structure by

$$\omega_{j,t} \equiv \frac{B_t^{(t+j)}}{B_t^{(t+1)}}$$

and denote $B_t \equiv B_t^{(t+1)}$. Then the end of period t nominal market value of debt is

$$\sum_{j=1}^{\infty} B_t^{(t+j)} Q_t^{(t+j)} = B_t \sum_{j=1}^{\infty} \omega_{j,t} Q_t^{(t+j)}.$$

(I ignore money to keep the formulas simple.) Define the price of the government debt portfolio

$$Q_t = \sum_{j=1}^{\infty} \omega_{j,t} Q_t^{(t+j)}.$$

The return on the government debt portfolio is then

$$R_{t+1}^n = \frac{\sum_{j=1}^{\infty} B_t^{(t+j)} Q_{t+1}^{(t+j)}}{\sum_{j=1}^{\infty} B_t^{(t+j)} Q_t^{(t+j)}} = \frac{\sum_{j=1}^{\infty} \omega_{j,t} Q_{t+1}^{(t+j)}}{\sum_{j=1}^{\infty} \omega_{j,t} Q_t^{(t+j)}} = \frac{1 + \sum_{j=1}^{\infty} \omega_{j+1,t} Q_{t+1}^{(t+1+j)}}{Q_t}. \tag{A1.22}$$

I loglinearize around a geometric maturity structure, $B_t^{(t+j)} = B_t \omega^{j-1}$, or equivalently $\omega_{j,t} = \omega^{j-1}$. I use variables with no subscripts to denote the linearization points.

When we linearize, we move bond prices holding the maturity structure at its steady-state, geometric value, and then we move the maturity structure while holding bond prices at their steady-state value. As a result, changes in maturity structure have no first-order effect on the linearized bond return. At the steady state $Q^{(j)} = 1/(1+i)^j$,

$$R_{t+1}^n = \frac{\sum_{j=1}^{\infty} \omega_{j,t}/(1+i)^{j-1}}{\sum_{j=1}^{\infty} \omega_{j,t}/(1+i)^j} = (1+i)$$

independently of $\{w_{j,t}\}$. Intuitively, at the steady-state bond prices, all bonds give the same return, so all portfolios of bonds give the same return. Moreover, maturity structure is a time-t variable in the definition of return R_{t+1}^n. The return from t to $t+1$ is not affected by the time $t+1$ maturity structure. (Changes in maturity structure might affect returns if there is price pressure in bond markets. These are formulas for measurement, however, and such effects would show up as changes in measured prices coincident with changes in quantities.)

Maturity structure has a second-order time-$t+1$ *interaction* effect on the bond portfolio return. For example, suppose yields decline throughout the maturity structure. Now, a longer maturity structure at t results in a larger bond portfolio return at $t+1$. A longer maturity structure at t likewise raises the expected return if the yield curve at t is also temporarily upward sloping. But a first-order decomposition does not include interaction effects.

In empirical work I measure the bond portfolio return r_{t+1}^n directly, and exactly, and such a measure includes all variation in maturity structure. The linearization only affects the decomposition of the bond portfolio return to future inflation and future expected returns or other calculations one makes with the linearized formula.

The term of the linearization with steady-state bond prices and changing maturity thus adds nothing. The linearization only includes a linearization with steady-state, geometric maturity structure and changing bond prices. Linearizing (A1.22), we have

$$R_{t+1}^n = \frac{1 + \sum_{j=1}^{\infty} \omega^{j+1} Q_{t+1}^{(t+1+j)}}{Q_t} = \frac{1 + \omega Q_{t+1}}{Q_t}$$

$$r_{t+1}^n = \log\left(1 + \omega e^{q_{t+1}}\right) - q_t \approx \log\left(\frac{1+\omega Q}{Q}\right) + \frac{\omega Q}{1+\omega Q}\tilde{q}_{t+1} - \tilde{q}_t \tag{A1.23}$$

where as usual variables without subscripts are steady state values and tildes are deviations from steady state. In a steady state,

$$Q = \sum_{j=1}^{\infty} \omega^{j-1} \frac{1}{(1+i)^j} = \left(\frac{1}{1+i}\right)\left(\frac{1}{1-\frac{\omega}{1+i}}\right) = \frac{1}{1+i-\omega}. \tag{A1.24}$$

The limits are $\omega = 0$ for one-period bonds, which gives $Q = 1/(1+i)$, and $\omega = 1$ for perpetuities, which gives $Q = 1/i$. The terms of the approximation (A1.23) are then

$$\frac{1+\omega Q}{Q} = 1+i$$
$$\frac{\omega Q}{1+\omega Q} = \frac{\omega}{1+i}$$

so we can write (A1.23) as

$$r_{t+1}^n \approx i + \frac{\omega}{1+i}\tilde{q}_{t+1} - \tilde{q}_t.$$

since $i < 0.05$ and $\omega \approx 0.7$, I further approximate to

$$r_{t+1}^n \approx i + \omega\tilde{q}_{t+1} - \tilde{q}_t. \tag{A1.25}$$

In empirical work, I find the value of ω that best fits the return identity, rather than measure the maturity structure directly, so the difference between ω and $\omega/(1+i)$ makes no practical difference.

To derive the bond return identity (3.21), iterate (A1.25) forward to express the bond price in terms of future returns,

$$\tilde{q}_t = -\sum_{j=1}^{\infty} \omega^j \tilde{r}_{t+j}^n.$$

Yes, this equation holds ex post, not just in expectation. Take innovations, move the first term to the left-hand side, and divide by ω,

$$\Delta E_{t+1}\tilde{r}_{t+1}^n = -\sum_{j=1}^{\infty} \omega^j \Delta E_{t+1}\tilde{r}_{t+1+j}^n. \tag{A1.26}$$

Then add and subtract inflation to get (3.21),

$$\Delta E_{t+1}\tilde{r}_{t+1}^n = -\sum_{j=1}^{\infty} \omega^j \Delta E_{t+1}\left[(\tilde{r}_{t+1+j}^n - \tilde{\pi}_{t+1+j}) + \tilde{\pi}_{t+1+j}\right].$$

The expectations hypothesis states that expected returns on bonds of all maturities are the same,

$$E_t r_{t+1}^n = i_t$$
$$i + \omega E_t \tilde{q}_{t+1} - \tilde{q}_t = i_t$$
$$\omega E_t \tilde{q}_{t+1} - \tilde{q}_t = \tilde{i}_t.$$

All variables are deviations from steady state, so I drop the tilde notation.

A1.4 Flow and present value in continuous time

The next two sections connect flow and present value statements in continuous time. The following section derives the present value statements with money in continuous time. There is nothing exciting here, but the algebra isn't totally obvious. Here it is.

A1.4.1 Continuous time with short-term debt

I connect the flow and present value relations in continuous time with short-term debt, (3.29)

$$B_t i_t dt = P_t s_t dt + dB_t \tag{A1.27}$$

and (3.31), (3.32),

$$\frac{B_t}{P_t} = E_t \int_{\tau=t}^{\infty} \frac{\Lambda_\tau}{\Lambda_t} s_\tau d\tau. \tag{A1.28}$$

$$\frac{B_t}{P_t} = \int_{\tau=t}^{\infty} \frac{W_t}{W_\tau} s_\tau d\tau. \tag{A1.29}$$

To connect the flow (3.29) and present value relations (3.31) (3.32) of Section 3.6.1, reproduced as (A1.27) and (A1.28) (A1.29), note

$$r_t dt = -E_t \left(\frac{d\Lambda_t}{\Lambda_t} \right)$$

$$i_t dt = -E_t \left[d\left(\frac{\Lambda_t}{P_t} \right) / \left(\frac{\Lambda_t}{P_t} \right) \right] \tag{A1.30}$$

$$i_t dt = -\frac{d\left[1/\left(P_t W_t\right)\right]}{1/\left(P_t W_t\right)}.$$

Equation (A1.30) takes a few lines of algebra starting from (3.33),

$$\frac{dW_t}{W_t} = dR_t = i_t dt + \frac{d\left(1/P_t\right)}{1/P_t}. \tag{A1.31}$$

Then, work either up or down,

$$\frac{B_t}{P_t} = E_t \int_{\tau=t}^{\infty} \frac{\Lambda_\tau}{\Lambda_t} s_\tau d\tau$$

$$\Lambda_t \frac{B_t}{P_t} = E_t \int_{\tau=t}^{\infty} \Lambda_\tau s_\tau d\tau$$

$$
\begin{aligned}
d\left(\Lambda_t \frac{B_t}{P_t}\right) &= -s_t \Lambda_t dt \\
\Lambda_t \frac{dB_t}{P_t} + B_t E_t \left[d\left(\frac{\Lambda_t}{P_t}\right)\right] &= -\Lambda_t s_t dt \\
dB_t + B_t E_t \left[\frac{d\left(\frac{\Lambda_t}{P_t}\right)}{\frac{\Lambda_t}{P_t}}\right] &= -P_t s_t dt \\
B_t i_t dt &= P_t s_t dt + dB_t.
\end{aligned}
$$

Similarly, for the rate of return as discount factor, work either up or down,

$$
\begin{aligned}
\frac{B_t}{P_t} &= \int_{\tau=t}^{\infty} \frac{W_t}{W_\tau} s_\tau d\tau \\
\frac{1}{W_t}\frac{B_t}{P_t} &= \int_{\tau=t}^{\infty} \frac{s_\tau}{W_\tau} d\tau \\
d\left(\frac{1}{W_t}\frac{B_t}{P_t}\right) &= -\frac{s_t}{W_t} dt \\
\frac{1}{W_t}\frac{dB_t}{P_t} + B_t d\left(\frac{1}{P_t W_t}\right) &= -\frac{s_t}{W_t} dt \\
\frac{1}{W_t}\frac{dB_t}{P_t} - \frac{B_t}{P_t W_t} i_t dt &= -\frac{s_t}{W_t} dt \\
-dB_t + B_t i_t dt &= P_t s_t dt.
\end{aligned}
$$

A1.4.2 Continuous time with long-term debt

I connect the flow relation with long-term debt (3.34),

$$
B_t^{(t)} dt = P_t s_t dt + \int_{j=0}^{\infty} Q_t^{(t+j)} dB_t^{(t+j)} dj, \tag{A1.32}
$$

to the present value relations with a stochastic discount factor (3.35)

$$
V_t = \frac{\int_{j=0}^{\infty} Q_t^{(t+j)} B_t^{(t+j)} dj}{P_t} = E_t \int_{\tau=t}^{\infty} \frac{\Lambda_\tau}{\Lambda_t} s_\tau d\tau \tag{A1.33}
$$

and an ex post return (3.36)

$$
V_t = \frac{\int_{j=0}^{\infty} Q_t^{(t+j)} B_t^{(t+j)} dj}{P_t} = \int_{\tau=t}^{\infty} \frac{W_t}{W_\tau} s_\tau d\tau. \tag{A1.34}
$$

This section connects the long-term debt flow relation (3.34) to the present value relations with a stochastic discount factor (3.35) and an ex post return (3.36), presented in Section 3.6.2, and reproduced above as (A1.32)-(A1.34).

Start from the definition of return (3.39)

$$dV_t = -s_t dt + V_t dR_t. \tag{A1.35}$$

Write (3.39) as

$$\frac{dV_t}{V_t} = -\frac{s_t}{V_t}dt + \frac{dW_t}{W_t}. \tag{A1.36}$$

To connect flow and present value relations using the discount factor, note that the definition of a discount factor Λ_t implies the basic pricing relation

$$E_t\left[d\left(\Lambda_t W_t\right)\right] = 0$$

hence

$$E_t\left(\frac{d\Lambda_t}{\Lambda_t} + \frac{dW_t}{W_t} + \frac{d\Lambda_t}{\Lambda_t}\frac{dW_t}{W_t}\right) = 0.$$

From (A1.36), which in turn came from the flow relation, we have

$$\frac{dW_t}{W_t} = \frac{dV_t}{V_t} + \frac{s_t}{V_t}dt.$$

So,

$$E_t\left(\frac{d\Lambda_t}{\Lambda_t} + \frac{dV_t}{V_t} + \frac{d\Lambda_t}{\Lambda_t}\frac{dV_t}{V_t}\right) = -\frac{s_t}{V_t}dt$$

$$E_t\left[d\left(\Lambda_t V_t\right)\right] = -\Lambda_t s_t dt$$

$$V_t\Lambda_t = \int_{\tau=t}^{\infty} \Lambda_\tau s_\tau d\tau,$$

and vice versa.

To connect flow and present value relations discounting with the ex post return, note that at non-jump points, (A1.36) implies

$$\frac{dV_t^2}{V_t^2} = \frac{dW_t dV_t}{W_t V_t} = \frac{dW_t^2}{W_t^2}.$$

Thus,

$$d\left(\frac{V_t}{W_t}\right) = \frac{V_t}{W_t}\left(\frac{dV_t}{V_t} - \frac{dW_t}{W_t} - \frac{dW_t}{W_t}\frac{dV_t}{V_t} + \frac{dW_t^2}{W_t^2}\right)$$

$$d\left(\frac{V_t}{W_t}\right) = \frac{V_t}{W_t}\left(\frac{dV_t}{V_t} - \frac{dW_t}{W_t}\right)$$

$$d\left(\frac{V_t}{W_t}\right) = -\frac{V_t}{W_t}\frac{s_t}{V_t}dt = -\frac{s_t}{W_t}dt.$$

Integrating,

$$\frac{V_t}{W_t} - \lim_{T\to\infty}\frac{V_T}{W_T} = -\int_{\tau=0}^{\infty} d\left(\frac{V_\tau}{W_\tau}\right) = \int_{\tau=0}^{\infty}\frac{s_t}{V_t}dt$$

$$\frac{V_t}{W_t} = \int_{\tau=t}^{\infty} \frac{s_\tau}{W_\tau} d\tau. \tag{A1.37}$$

From (A1.36), V grows more slowly than W, so the limit is zero.

At jump points (A1.36) implies that the jumps obey

$$\frac{dW}{W} = \frac{dP}{P}.$$

At the jump points $d(V_t/W_t) = 0$ so they do not affect the integral (A1.37).

To go backwards, take the differential of the final integral. (The same steps allow us to express a stock's price as the present value of its dividend stream, discounted by the ex post return, in continuous time. Start with

$$\frac{dW}{W} = dR = \frac{d}{P}dt + \frac{dP}{P}. \tag{A1.38}$$

Follow the same steps to conclude

$$\frac{P_t}{W_t} = \int_{\tau=t}^{\infty} \frac{d_\tau}{W_\tau} d\tau \tag{A1.39}$$

and vice versa.)

A1.4.3 Money in continuous time

I derive expressions for the government debt valuation equation with money in continuous time, from Section 3.6.4. The flow equation is

$$dM_t = i_t B_t dt + i_t^m M_t dt - P_t s_t dt - dB_t.$$

With no interest on money, we can write seignorage in terms of money creation,

$$\frac{B_t}{P_t} = E_t \int_{\tau=t}^{\infty} \frac{\Lambda_\tau}{\Lambda_t} \left(s_\tau d\tau + \frac{dM_\tau}{P_\tau} \right).$$

We can also write seignorage as a saving of interest costs,

$$\frac{M_t + B_t}{P_t} = E_t \int_{\tau=t}^{\infty} \frac{\Lambda_\tau}{\Lambda_t} \left[s_\tau + (i_\tau - i_\tau^m) \frac{M_\tau}{P_\tau} \right] d\tau.$$

Discounting with the ex post return gives the same formulas with the cumulative value of a portfolio that invests in all government debt including money W_t in place of the discount factor Λ_t.

This section presents the algebra behind the expressions of the government debt valuation equation with money in continuous time, Section 3.6.4. I repeat some of that discussion for completeness, so you don't have to flip back and forth.

The nominal and real flow conditions in continuous time are

$$dM_t = i_t B_t dt + i_t^m M_t dt - P_t s_t dt - dB_t. \tag{A1.40}$$

$$\frac{B_t}{P_t} i_t dt + \frac{M_t}{P_t} i_t^m dt = s_t dt + \frac{dB_t}{P_t} + \frac{dM_t}{P_t}.$$

To express seigniorage as money creation, specialize to $i_t^m = 0$, rearrange (3.52), and substitute the definition of the nominal interest rate,

$$\begin{aligned} \frac{dB_t}{P_t} + E_t\left[d\left(\frac{\Lambda_t}{P_t}\right)/\left(\frac{\Lambda_t}{P_t}\right)\right]\frac{B_t}{P_t} &= -s_t dt - \frac{dM_t}{P_t} \\ \frac{\Lambda_t}{P_t} dB_t + E_t\left[d\left(\frac{\Lambda_t}{P_t}\right)B_t\right] &= -\Lambda_t\left(s_t dt + \frac{dM_t}{P_t}\right) \\ E_t\left[d\left(\Lambda_t \frac{B_t}{P_t}\right)\right] &= -\Lambda_t\left(s_t dt + \frac{dM_t}{P_t}\right) \end{aligned} \tag{A1.41}$$

Now we can integrate, and impose the transversality condition to obtain

$$\frac{B_t}{P_t} = E_t \int_{\tau=t}^{\infty} \frac{\Lambda_\tau}{\Lambda_t}\left(s_\tau d\tau + \frac{dM_\tau}{P_\tau}\right). \tag{A1.42}$$

To express seigniorage in terms of interest cost, including the case that money pays interest $0 < i_t^m < i_t$, start again from (A1.40), and write

$$\frac{d(M_t + B_t)}{P_t} - i_t \frac{(B_t + M_t)}{P_t} dt = -s_t dt - (i_t - i_t^m)\frac{M_t}{P_t} dt$$

$$\frac{d(M_t + B_t)}{P_t} + E_t\left[d\left(\frac{\Lambda_t}{P_t}\right)/\left(\frac{\Lambda_t}{P_t}\right)\right]\frac{(B_t + M_t)}{P_t} = -s_t dt - (i_t - i_t^m)\frac{M_t}{P_t} dt$$

$$\Lambda_t \frac{d(M_t + B_t)}{P_t} + E_t\left[d\left(\frac{\Lambda_t}{P_t}\right)\right](B_t + M_t) = -\Lambda_t\left(s_t + (i_t - i_t^m)\frac{M_t}{P_t}\right) dt$$

$$E_t\left[d\left(\Lambda_t \frac{M_t + B_t}{P_t}\right)\right] = -\Lambda_t\left(s_t + (i_t - i_t^m)\frac{M_t}{P_t}\right) dt.$$

Integrating again,

$$\frac{M_t + B_t}{P_t} = E_t \int_{\tau=t}^{\infty} \frac{\Lambda_\tau}{\Lambda_t}\left[s_\tau + (i_\tau - i_\tau^m)\frac{M_\tau}{P_\tau}\right] d\tau. \tag{A1.43}$$

To discount with the ex post return, define W_t^n and W_t as the cumulative nominal and real values of investment in short-term debt, so dW_t/W_t is the ex post real return. Then,

$$\begin{aligned} \frac{dW_t^n}{W_t^n} &= i_t dt \\ P_t W_t &= W_t^n \\ d\left(\frac{1}{P_t W_t}\right) &= -\frac{1}{W_t^n}\frac{dW_t^n}{W_t^n} = -\frac{1}{W_t^n} i_t dt = -\frac{1}{P_t W_t} i_t dt \end{aligned}$$

$$i_t dt = -d\left(\frac{1}{P_t W_t}\right) / \left(\frac{1}{P_t W_t}\right). \tag{A1.44}$$

(P_t and W_t may jump here, but $P_t W_t$ is differentiable.) Start again with the nominal flow condition (A1.40), rearrange and divide by W_t to give.

$$\frac{dB_t}{P_t W_t} - i_t \frac{B_t}{P_t W_t} dt = -\frac{1}{W_t}\left(s_t dt + \frac{dM_t}{P_t}\right). \tag{A1.45}$$

Substituting (A1.44) for i_t,

$$\frac{dB_t}{P_t W_t} + d\left(\frac{1}{P_t W_t}\right) B_t = -\frac{1}{W_t}\left(s_t dt + \frac{dM_t}{P_t}\right)$$

$$d\left(\frac{1}{W_t}\frac{B_t}{P_t}\right) = -\frac{1}{W_t}\left(s_t dt + \frac{dM_t}{P_t}\right)$$

Integrating,

$$\frac{B_t}{P_t} = \int_{\tau=t}^{\infty} \frac{W_t}{W_\tau}\left(s_\tau d\tau + \frac{dM_\tau}{P_\tau}\right).$$

To discount at the ex post rate of return, expressing seigniorage as an interest saving, and allowing money to pay interest, start at (A1.45), and write

$$\begin{aligned}
\frac{d\left(B_t + M_t\right)}{P_t W_t} - i_t \frac{\left(B_t + M_t\right)}{P_t W_t} dt &= -\frac{1}{W_t}\left[s_t + \left(i_t - i_t^m\right)\frac{M_t}{P_t}\right] dt \\
\frac{d\left(B_t + M_t\right)}{P_t W_t} + d\left(\frac{1}{P_t W_t}\right)\left(B_t + M_t\right) &= -\frac{1}{W_t}\left[s_t + \left(i_t - i_t^m\right)\frac{M_t}{P_t}\right] dt \\
d\left(\frac{B_t + M_t}{P_t W_t}\right) &= -\frac{1}{W_t}\left[s_t + \left(i_t - i_t^m\right)\frac{M_t}{P_t}\right] dt \\
\frac{B_t + M_t}{P_t} &= \int_{\tau=t}^{\infty} \frac{W_t}{W_\tau}\left[s_\tau + \left(i_t - i_t^m\right)\frac{M_\tau}{P_\tau}\right] d\tau.
\end{aligned}$$

To write the discount factor as a rate of return that mixes the bond rate of return and the lower (zero) money rate of return, define W^{nm} and W^m as the cumulative nominal and real value of an investment in the overall government bond portfolio, now including money.

$$\begin{aligned}
\frac{dW_t^{nm}}{W_t^{nm}} &= \frac{B_t}{B_t + M_t} i_t dt + \frac{M_t}{B_t + M_t} i_t^m dt \\
P_t W_t^m &= W_t^{nm} \\
d\left(\frac{1}{P_t W_t^m}\right) &= -\frac{1}{W_t^n}\frac{dW_t^n}{W_t^n} = -\frac{1}{P_t W_t^m}\left(\frac{B_t}{B_t + M_t} i_t dt + \frac{M_t}{B_t + M_t} i_t^m dt\right) \\
d\left(\frac{1}{P_t W_t^m}\right) &= -\frac{1}{W_t^m}\frac{1}{B_t + M_t}\left(\frac{B_t}{P_t} i_t dt + \frac{M_t}{P_t} i_t^m dt\right) \\
\left(B_t + M_t\right) W_t^m d\left(\frac{1}{P_t W_t^m}\right) &= -\left(\frac{B_t}{P_t} i_t dt + \frac{M_t}{P_t} i_t^m dt\right).
\end{aligned}$$

Again start at (A1.45), and substitute,

$$\frac{d\left(M_t+B_t\right)}{P_t}-i_t\frac{B_t}{P_t}dt-i_t^m\frac{M_t}{P_t}dt=-s_tdt$$

$$\frac{d\left(M_t+B_t\right)}{P_tW_t^m}+\left(B_t+M_t\right)d\left(\frac{1}{P_tW_t^m}\right)=-\frac{1}{W_t^m}s_tdt$$

$$d\left(\frac{B_t+M_t}{P_tW_t^m}\right)=-\frac{1}{W_t^m}s_tdt$$

$$\frac{B_t+M_t}{P_t}=\int_{\tau=t}^{\infty}\frac{W_t^m}{W_\tau^m}s_\tau d\tau. \tag{A1.46}$$

A1.5 Discrete time sticky price models

In this section I document the algebra for the discrete time sticky price models, both fiscal theory versions, i.e. with passive monetary and active fiscal policy, and the traditional new-Keynesian models with active monetary and passive fiscal policy. The treatment isn't unified, as each section here documents the algebra of a particular section in the main text.

A1.5.1 sticky price model analytical solution

I derive the explicit solutions (5.5)-(5.14), for inflation and output given the equilibrium path of interest rates,

$$\pi_{t+1}=\frac{\sigma\kappa}{\lambda_1-\lambda_2}\left[i_t+\sum_{j=1}^{\infty}\lambda_1^{-j}i_{t-j}+\sum_{j=1}^{\infty}\lambda_2^{j}E_{t+1}i_{t+j}\right]+\sum_{j=0}^{\infty}\lambda_1^{-j}\delta_{t+1-j}. \tag{A1.47}$$

$$\kappa x_{t+1}=\frac{\sigma\kappa}{\lambda_1-\lambda_2}\left[\left(1-\beta\lambda_1^{-1}\right)\sum_{j=0}^{\infty}\lambda_1^{-j}i_{t-j}+\left(1-\beta\lambda_2^{-1}\right)\sum_{j=1}^{\infty}\lambda_2^{j}E_{t+1}i_{t+j}\right]+$$
$$+\left(1-\beta\lambda_1^{-1}\right)\sum_{j=0}^{\infty}\lambda_1^{-j}\delta_{t+1-j}. \tag{A1.48}$$

Here I derive the explicit solutions (5.5)-(5.14), reproduced as (A1.47)-(A1.48) in the above box, for inflation and output given the equilibrium path of interest rates. This section comes from Cochrane (2017b), which includes a generalization to a model with money that pays interest $i^m \leq i$.

The simple model (5.1)-(5.2) is

$$x_t=E_tx_{t+1}-\sigma(i_t-E_t\pi_{t+1})$$

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t.$$

Expressing the model in lag operator notation,

$$E_t(1 - L^{-1})x_t = \sigma E_t L^{-1}\pi_t - \sigma i_t$$

$$E_t(1 - \beta L^{-1})\pi_t = \kappa x_t$$

Forward-differencing the second equation,

$$E_t(1 - L^{-1})(1 - \beta L^{-1})\pi_t = E_t(1 - L^{-1})\kappa x_t$$

Then substituting,

$$E_t(1 - L^{-1})\left(1 - \beta L^{-1}\right)\pi_t = \sigma\kappa E_t L^{-1}\pi_t - \sigma\kappa i_t$$

$$E_t\left[(1 - L^{-1})\left(1 - \beta L^{-1}\right) - \sigma\kappa L^{-1}\right]\pi_t = -\sigma\kappa i_t$$

$$E_t\left[1 - (1 + \beta + \sigma\kappa)L^{-1} + \beta L^{-2}\right]\pi_t = -\sigma\kappa i_t.$$

Factor the lag polynomial

$$E_t(1 - \lambda_1 L^{-1})(1 - \lambda_2 L^{-1})\pi_t = -\sigma\kappa i_t$$

where

$$\lambda_i = \frac{(1 + \beta + \sigma\kappa) \pm \sqrt{(1 + \beta + \sigma\kappa)^2 - 4\beta}}{2}.$$

Since $\lambda_1 > 1$ and $\lambda_2 < 1$, reexpress the result as

$$E_t\left[(1 - \lambda_1^{-1}L)(1 - \lambda_2 L^{-1})\lambda_1 L^{-1}\pi_t\right] = \sigma\kappa i_t$$

$$E_t\left[(1 - \lambda_1^{-1}L)(1 - \lambda_2 L^{-1})\pi_{t+1}\right] = \sigma\kappa\lambda_1^{-1} i_t.$$

The bounded solutions are

$$\pi_{t+1} = E_{t+1}\frac{\lambda_1^{-1}}{(1 - \lambda_1^{-1}L)(1 - \lambda_2 L^{-1})}\sigma\kappa i_t + \frac{1}{(1 - \lambda_1^{-1}L)}\delta_{t+1}$$

where δ_{t+1} is a sequence of unpredictable random variables, $E_t\delta_{t+1} = 0$. I follow the usual practice and I rule out solutions that explode in the forward direction.

Using a partial fractions decomposition to break up the right-hand side,

$$\frac{\lambda_1^{-1}}{\left(1 - \lambda_1^{-1}L\right)(1 - \lambda_2 L^{-1})} = \frac{1}{\lambda_1 - \lambda_2}\left(1 + \frac{\lambda_1^{-1}L}{1 - \lambda_1^{-1}L} + \frac{\lambda_2 L^{-1}}{1 - \lambda_2 L^{-1}}\right).$$

So,

$$\pi_{t+1} = \frac{1}{\lambda_1 - \lambda_2}E_{t+1}\left(1 + \frac{\lambda_1^{-1}L}{1 - \lambda_1^{-1}L} + \frac{\lambda_2 L^{-1}}{1 - \lambda_2 L^{-1}}\right)\sigma\kappa i_t + \frac{1}{(1 - \lambda_1^{-1}L)}\delta_{t+1}$$

or in sum notation,

$$\pi_{t+1} = \sigma\kappa \frac{1}{\lambda_1 - \lambda_2}\left(i_t + \sum_{j=1}^{\infty} \lambda_1^{-j} i_{t-j} + \sum_{j=1}^{\infty} \lambda_2^{j} E_{t+1} i_{t+j}\right) + \sum_{j=0}^{\infty} \lambda_1^{-j}\delta_{t+1-j}. \quad \text{(A1.49)}$$

The long-run impulse-response function is 1:

$$\frac{1}{(1-\lambda_1^{-1})(1-\lambda_2)}\frac{\sigma\kappa}{\lambda_1} = -\frac{\sigma\kappa}{(1-\lambda_1)(1-\lambda_2)}$$
$$= -\frac{\sigma\kappa}{(1-(\lambda_1+\lambda_2)+\lambda_1\lambda_2)} = -\frac{\sigma\kappa}{(1-(1+\beta+\sigma\kappa)+\beta)} = 1.$$

Having found the path of π_t, we can find output by

$$\kappa x_t = \pi_t - \beta E_t \pi_{t+1}.$$

In lag operator notation, and shifting forward one period,

$$\kappa x_{t+1} = E_{t+1}\left[(1-\beta L^{-1})\pi_{t+1}\right]$$

$$\kappa x_{t+1} = \frac{\sigma\kappa}{\lambda_1-\lambda_2} E_{t+1}\left[(1-\beta L^{-1})\left(1 + \frac{\lambda_1^{-1}L}{1-\lambda_1^{-1}L} + \frac{\lambda_2 L^{-1}}{1-\lambda_2 L^{-1}}\right) i_t\right]$$
$$+ E_{t+1}\frac{(1-\beta L^{-1})}{(1-\lambda_1^{-1}L)}\delta_{t+1}.$$

Now,

$$(1-\beta L^{-1})\left(1 + \frac{\lambda_1^{-1}L}{1-\lambda_1^{-1}L} + \frac{\lambda_2 L^{-1}}{1-\lambda_2 L^{-1}}\right) = \frac{1-\beta\lambda_1^{-1}}{1-\lambda_1^{-1}L} + \frac{(1-\beta\lambda_2^{-1})\left(\lambda_2 L^{-1}\right)}{1-\lambda_2 L^{-1}}.$$

(To get to the last expression, merge everything together and rederive the partial fractions decomposition.) We can then rewrite the polynomials to give

$$\kappa x_{t+1} = \frac{\sigma\kappa}{\lambda_1-\lambda_2} E_{t+1}\left[\frac{1-\beta\lambda_1^{-1}}{1-\lambda_1^{-1}L} + \frac{(1-\beta\lambda_2^{-1})\left(\lambda_2 L^{-1}\right)}{1-\lambda_2 L^{-1}}\right] i_t + E_{t+1}\left[\frac{1-\beta\lambda_1^{-1}}{1-\lambda_1^{-1}L}\right]\delta_{t+1}.$$

(In the second term, I use $E_t\left[\beta L^{-1}\delta_{t+1}\right] = 0$.) In sum notation,

$$\kappa x_{t+1} = \frac{\sigma\kappa}{\lambda_1-\lambda_2}\left[\left(1-\beta\lambda_1^{-1}\right)\sum_{j=0}^{\infty}\lambda_1^{-j} i_{t-j} + \left(1-\beta\lambda_2^{-1}\right)\sum_{j=1}^{\infty}\lambda_2^{j} E_{t+1} i_{t+j}\right] +$$
$$+ \left(1-\beta\lambda_1^{-1}\right)\sum_{j=0}^{\infty}\lambda_1^{-j}\delta_{t+1-j}$$

A1.5.2 Matrix solution method

I present the standard matrix solution method for discrete time linear models. Write the model in standard form

$$z_{t+1} = Az_t + B\varepsilon_{t+1} + C\delta_{t+1}.$$

Then, eigenvalue-decompose the matrix A, solve unstable eigenvalues forward and stable eigenvalues backward. With as many forward-looking eigenvalues as there are expectational errors δ, we obtain a unique solution.

Here I present the standard solution method for the discrete time linear models, introduced by Blanchard and Kahn (1980). I present the method in the context of the sticky price fiscal-theory model with a simple monetary policy rule and AR(1) surplus process.

First express the system in standard form

$$Az_{t+1} = Bz_t + C\varepsilon_{t+1} + D\delta_{t+1} + Fw_t. \tag{A1.50}$$

The economic variables x_t, π_t, etc. go in the vector z_t. Structural shocks to the behavioral equations and policy shocks go into ε_{t+1}. For example, $\varepsilon_{i,t+1}$ is a "structural" shock to the monetary policy rule $i_t = \theta\pi_t + u_{i,t}$, $u_{i,t+1} = \eta_u u_{i,t} + \varepsilon_{i,t+1}$. I use the notation δ_{t+1} to denote expectational errors in equations that only determine expectations. For example, I write the Phillips curve

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t$$

as

$$\beta\pi_{t+1} = \pi_t - \kappa x_t + \beta\delta_{\pi,t+1}.$$

The structural shocks ε are known and exogenous shocks to the model. All the model says is that $E_t\delta_{t+1} = 0$. Solving the model means also finding δ_{t+1} in terms of other variables.

As an example, I add to the simple model (5.15)-(5.16), a simple monetary policy rule, so we can see how to include such rules. The model is

$$x_t = E_t x_{t+1} - \sigma\left(i_t - E_t\pi_{t+1}\right) \tag{A1.51}$$
$$\pi_t = \beta E_t\pi_{t+1} + \kappa x_t \tag{A1.52}$$
$$\rho v_{t+1} = v_t + i_t - \pi_{t+1} - \tilde{s}_{t+1} \tag{A1.53}$$
$$i_t = \theta_{i\pi}\pi_t + u_{i,t} + w_t \tag{A1.54}$$
$$s_t = u_{s,t} \tag{A1.55}$$
$$u_{i,t+1} = \eta_i u_{i,t} + \varepsilon_{i,t+1} \tag{A1.56}$$
$$u_{s,t+1} = \eta_s u_{s,t} + \varepsilon_{s,t+1}. \tag{A1.57}$$

To calculate the permanent unexpected interest rate rise of Figure 5.1 I use $\theta_{i\pi} = 0$, $\eta_i = 1$, $w_t = 0$. To calculate the expected interest rate rise of Figure 5.2, I use $\theta_{i\pi} = 0$, $\varepsilon_{i,t} = 0$ and w_t that rises from 0 to 1 at $t = 1$.

The w_t are variables known ahead of time. I use such a w to compute the effect of an expected interest rate rise. For example, to calculate the effects of an interest rate rise at time 5, I use $w_5 = 1$, but this w_5 is known at time 1. In this VAR(1) context, the alternative is to introduce variables that are known k periods ahead of time, and then carry around an extra k variables in the state vector.

Since (A1.54) and (A1.55) just define one variable in terms of others at the same time, I use them to eliminate i_t and s_t. Then, I write

$$\begin{aligned} E_t x_{t+1} + \sigma E_t \pi_{t+1} &= x_t + \sigma\left(\theta_{i\pi}\pi_t + u_{i,t} + w_t\right) \\ \beta E_t \pi_{t+1} &= \pi_t - \kappa x_t \\ \rho v_{t+1} + \pi_{t+1} + u_{s,t+1} &= v_t + \theta_{i\pi}\pi_t + u_{i,t} + w_t \end{aligned}$$

and in matrix form,

$$\begin{bmatrix} 1 & \sigma & 0 & 0 & 0 \\ 0 & \beta & 0 & 0 & 0 \\ 0 & 1 & \rho & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_{t+1} \\ \pi_{t+1} \\ v_{t+1} \\ u_{i,t+1} \\ u_{s,t+1} \end{bmatrix} = \begin{bmatrix} 1 & \sigma\theta_{i\pi} & 0 & \sigma & 0 \\ -\kappa & 1 & 0 & 0 & 0 \\ 0 & \theta_{i\pi} & 1 & 1 & 0 \\ 0 & 0 & 0 & \eta_i & 0 \\ 0 & 0 & 0 & 0 & \eta_s \end{bmatrix} \begin{bmatrix} x_t \\ \pi_t \\ v_t \\ u_{i,t} \\ u_{s,t} \end{bmatrix}$$

$$+ \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \varepsilon_{i,t+1} \\ \varepsilon_{s,t+1} \end{bmatrix} + \begin{bmatrix} 1 & \sigma \\ 0 & \beta \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \delta_{x,t+1} \\ \delta_{\pi,t+1} \end{bmatrix} + \begin{bmatrix} \sigma \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} w_t$$

This case is simple enough to invert the leading matrix analytically and still get a pretty answer,

$$\begin{bmatrix} x_{t+1} \\ \pi_{t+1} \\ v_{t+1} \\ u_{i,t+1} \\ u_{s,t+1} \end{bmatrix} = \begin{bmatrix} 1 + \frac{\kappa\sigma}{\beta} & \sigma\left(\theta_{i\pi} - \frac{1}{\beta}\right) & 0 & \sigma & 0 \\ -\frac{\kappa}{\beta} & \frac{1}{\beta} & 0 & 0 & 0 \\ \frac{\kappa}{\rho\beta} & \frac{1}{\rho}\left(\theta_{i\pi} - \frac{1}{\beta}\right) & \frac{1}{\rho} & \frac{1}{\rho} & -\frac{1}{\rho}\eta_s \\ 0 & 0 & 0 & \eta_i & 0 \\ 0 & 0 & 0 & 0 & \eta_s \end{bmatrix} \begin{bmatrix} x_t \\ \pi_t \\ v_t \\ u_{i,t} \\ u_{s,t} \end{bmatrix}$$

$$+ \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & -\frac{1}{\rho} \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \varepsilon_{i,t+1} \\ \varepsilon_{s,t+1} \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & -\frac{1}{\rho} \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \delta_{x,t+1} \\ \delta_{\pi,t+1} \end{bmatrix} + \begin{bmatrix} \sigma \\ 0 \\ \frac{1}{\rho} \\ 0 \\ 0 \end{bmatrix} w_t,$$

The eigenvalues of the transition matrix are

$$\rho^{-1}, \eta_i, \eta_s, \lambda_+, \lambda_-$$

with

$$\lambda_{+,-} = \frac{1 + \beta + \kappa\sigma \pm \sqrt{(1 + \beta + \kappa\sigma^2) - 4\beta\left(1 + \kappa\sigma\theta_{i\pi}\right)}}{2\beta}.$$

With two linearly independent expectational errors, we need two eigenvalues greater or equal to one. Conventional new-Keynesian models wipe out the v equation with passive fiscal policy, often just deleting it from the analysis with a footnote assuming lump-sum taxes move to satisfy the valuation equation, and assume $\theta_{i\pi} > 1$ so both λ are larger than one. For a fiscal-theory solution, use $\theta_{i\pi} < 1$, as ρ^{-1} provides the extra explosive eigenvalue.

Next, write (A1.50) as

$$z_{t+1} = A^{-1}Bz_t + A^{-1}C\varepsilon_{t+1} + A^{-1}D\delta_{t+1} + A^{-1}Fw_t.$$

Eigenvalue decompose the transition matrix $A^{-1}B$, and transform the dynamics.

$$z_{t+1} = Q\Lambda Q^{-1}z_t + A^{-1}C\varepsilon_{t+1} + A^{-1}D\delta_{t+1} + A^{-1}Fw_t$$

$$Q^{-1}z_{t+1} = \Lambda Q^{-1}z_t + Q^{-1}A^{-1}C\varepsilon_{t+1} + Q^{-1}A^{-1}D\delta_{t+1} + Q^{-1}A^{-1}Fw_t$$

where Λ is a diagonal matrix of the eigenvalues λ_i of $A^{-1}B$, and Q is the corresponding matrix of eigenvectors. Using hats to denote transformed variables $\hat{z} = Q^{-1}z$, $\hat{\varepsilon} = Q^{-1}A^{-1}C\varepsilon$, etc., and k to denote elements of vectors, the system decouples into a set of scalar difference equations,

$$\hat{z}_{k,t+1} = \lambda_k \hat{z}_{k,t} + \hat{\varepsilon}_{k,t+1} + \hat{\delta}_{k,t+1} + \hat{w}_{k,t} \tag{A1.58}$$

We solve the stable eigenvalues backwards. Rather than write out the solution, we can just calculate response functions from (A1.58).

We solve the unstable eigenvalues $\lambda_k \geq 1$ forward. We are looking for bounded, stable solutions, in which $E_t\hat{z}_{k,t+j}$ does not explode. Taking E_t of (A1.58), and solving forward with $E_t\varepsilon_{t+j} = E_t\delta_{t+j} = 0$, and expressing the result at time $t+1$,

$$\hat{z}_{k,t+1} = -\sum_{j=1}^{\infty} \lambda_k^{-j}\hat{w}_{k,t+j}.$$

Without the w, which are deterministic and thus known ahead of time, the right-hand side would be zero. Taking innovations $E_{t+1} - E_t$,

$$\hat{\delta}_{k,t+1} = -\hat{\varepsilon}_{k,t+1}. \tag{A1.59}$$

We have determined the expectational errors in terms of structural shocks. In order to have a unique locally bounded solution, we need exactly as many unstable eigenvalues $\lambda_k > 1$ as there are linearly independent expectational shocks δ. This result is not magic, and usually has strong economic intuition. Stock prices jump when there is a change to expected dividends, consumption jumps when there is a change to expected income, and the price level jumps in this model when there is a change to expected surpluses.

Explicitly, denote $Q^{-1}_{\lambda<1}$ a matrix composed of the rows of Q^{-1} corresponding to stable eigenvalues, and likewise $Q^{-1}_{\lambda>1}$ a matrix composed of the rows of Q^{-1} corresponding to unstable eigenvalues. Equation (A1.59) then implies

$$Q^{-1}_{\lambda>1}A^{-1}D\delta_{t+1} = -Q^{-1}_{\lambda>1}A^{-1}C\varepsilon_{t+1}.$$

When there are as many explosive eigenvalues as expectational shocks δ we can invert,

$$\delta_{t+1} = -\left[Q_{\lambda>1}^{-1} A^{-1} D\right]^{-1} Q_{\lambda>1}^{-1} A^{-1} C \varepsilon_{t+1},$$

and then write

$$\hat{\delta}_{t+1} = -Q^{-1} A^{-1} D \left[Q_{\lambda>1}^{-1} A^{-1} D\right]^{-1} Q_{\lambda>1}^{-1} A^{-1} C \varepsilon_{t+1}. \tag{A1.60}$$

We can now write the system dynamics as

$$\begin{aligned} \lambda_k < 1 &: \hat{z}_{k,t+1} = \lambda_k \hat{z}_{k,t} + \hat{\varepsilon}_{k,t+1} + \hat{\delta}_{k,t+1} + \hat{w}_{k,t} \\ \lambda_k \geq 1 &: \hat{z}_{k,t+1} = -\sum_{j=1}^{\infty} \lambda_k^{-j} \hat{w}_{k,t+j}. \end{aligned}$$

Then we find the original variables by

$$z_t = Q \hat{z}_t.$$

A1.5.3 State variable solution method

> We can also solve these sorts of models by a guess and check approach. Find the state variables of the economy. Guess that the endogenous variables are functions of the state variables. Use the model equilibrium conditions to find the coefficients of the functions relating endogenous variables to state variables.

An alternative "minimum state variable," "method of undetermined coefficients" approach, similar to dynamic programming, is often even easier. However, it obscures the logic by which one rules out non-stationary solutions.

For example, write the simplest model

$$i_t = E_t \pi_{t+1} \tag{A1.61}$$

$$\rho v_{t+1} = v_t + i_t - \pi_{t+1} - s_{t+1} \tag{A1.62}$$

$$i_t = u_{i,t}$$

$$s_t = u_{s,t}$$

$$u_{i,t+1} = \eta_i u_{i,t} + \varepsilon_{i,t+1} \tag{A1.63}$$

$$u_{s,t+1} = \eta_s u_{s,t} + \varepsilon_{s,t+1}. \tag{A1.64}$$

We know the answer already,

$$\pi_{t+1} = i_t + \Delta E_{t+1} \sum_{j=0}^{\infty} \rho^j s_{t+1+j} = u_{i,t} + \frac{1}{1-\rho\eta_s} \varepsilon_{s,t+1}$$

$$v_t = E_t \sum_{j=0}^{\infty} \rho^j s_{t+1+j} = \frac{\eta_s}{1-\rho\eta_s} s_t = \frac{\eta_s}{1-\rho\eta_s} u_{s,t}.$$

Now, we explore another way to get there.

We guess state variables $u_{i,t}$, $u_{s,t}$, v_t, i.e. we guess that in equilibrium the endogenous variables are functions of state variables and their innovations. We guess

$$\begin{bmatrix} \pi_{t+1} \\ v_t \end{bmatrix} = A \begin{bmatrix} u_{i,t} \\ u_{s,t} \end{bmatrix} + B \begin{bmatrix} \varepsilon_{i,t+1} \\ \varepsilon_{s,t+1} \end{bmatrix}.$$

We could also guess that the endogenous variables are functions of a larger state vector $\begin{bmatrix} u_{i,t} & u_{i,t+1} & u_{s,t} & u_{s,t+1} \end{bmatrix}'$. We normally would include i_t and s_t as endogenous variables, but they are trivial in this case. We plug this guess into the equilibrium conditions to see if we can find matrices A and B to make it work.

From $i_t = E_t\pi_{t+1}$ we know $u_{i,t} = E_t\pi_{t+1}$ and hence

$$A = \begin{bmatrix} A_{\pi i} & A_{\pi s} \\ A_{vi} & A_{vs} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ A_{vi} & A_{vs} \end{bmatrix}.$$

Write (A1.62) as

$$\pi_{t+1} - v_t = -\rho v_{t+1} + u_{i,t} - \eta_s u_{s,t} - \varepsilon_{s,t+1}$$

$$\begin{bmatrix} 1 & -1 \end{bmatrix} \left(A \begin{bmatrix} u_{i,t} \\ u_{s,t} \end{bmatrix} + B \begin{bmatrix} \varepsilon_{i,t+1} \\ \varepsilon_{s,t+1} \end{bmatrix} \right) = \begin{bmatrix} 0 & -\rho \end{bmatrix} \left(A \begin{bmatrix} u_{i,t+1} \\ u_{s,t+1} \end{bmatrix} + B \begin{bmatrix} \varepsilon_{i,t+2} \\ \varepsilon_{s,t+2} \end{bmatrix} \right)$$
$$+ \begin{bmatrix} 1 & -\eta_s \end{bmatrix} \begin{bmatrix} u_{i,t} \\ u_{s,t} \end{bmatrix} + \begin{bmatrix} 0 & -1 \end{bmatrix} \begin{bmatrix} \varepsilon_{i,t+1} \\ \varepsilon_{s,t+1} \end{bmatrix}.$$

If the guess is going to work, then

$$\begin{bmatrix} 0 & -\rho \end{bmatrix} \begin{bmatrix} B_{\pi i} & B_{\pi s} \\ B_{vi} & B_{vs} \end{bmatrix} \begin{bmatrix} \varepsilon_{i,t+2} \\ \varepsilon_{s,t+2} \end{bmatrix} = 0,$$
$$\begin{bmatrix} B_{\pi i} & B_{\pi s} \\ B_{vi} & B_{vs} \end{bmatrix} = \begin{bmatrix} B_{\pi i} & B_{\pi s} \\ 0 & 0 \end{bmatrix}.$$

In words, v_t can't respond to ε_{t+1}. Using (A1.63)-(A1.64), then,

$$\begin{bmatrix} 1 & -1 \end{bmatrix} \left(A \begin{bmatrix} u_{i,t} \\ u_{s,t} \end{bmatrix} + B \begin{bmatrix} \varepsilon_{i,t+1} \\ \varepsilon_{s,t+1} \end{bmatrix} \right) = \begin{bmatrix} 0 & -\rho \end{bmatrix} \left(A \begin{bmatrix} \eta_i & 0 \\ 0 & \eta_s \end{bmatrix} \begin{bmatrix} u_{i,t} \\ u_{s,t} \end{bmatrix} + \begin{bmatrix} \varepsilon_{i,t+1} \\ \varepsilon_{s,t+1} \end{bmatrix} \right)$$
$$+ \begin{bmatrix} 1 & -\eta_s \end{bmatrix} \begin{bmatrix} u_{i,t} \\ u_{s,t} \end{bmatrix} + \begin{bmatrix} 0 & -1 \end{bmatrix} \begin{bmatrix} \varepsilon_{i,t+1} \\ \varepsilon_{s,t+1} \end{bmatrix}.$$

The coefficients on the u and ε must be separately equal, and hold for any values of the state variables. Therefore,

$$\begin{bmatrix} 1 & -1 \end{bmatrix} A = \begin{bmatrix} 0 & -\rho \end{bmatrix} A \begin{bmatrix} \eta_i & 0 \\ 0 & \eta_s \end{bmatrix} + \begin{bmatrix} 1 & -\eta_s \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 \end{bmatrix} B = \begin{bmatrix} 0 & -\rho \end{bmatrix} A + \begin{bmatrix} 0 & -1 \end{bmatrix}.$$

Solving, we find the right answer

$$A = \begin{bmatrix} 1 & 0 \\ 0 & \frac{\eta_s}{1-\rho\eta_s} \end{bmatrix}, \; B = \begin{bmatrix} 0 & \frac{1}{1-\rho\eta_s} \\ 0 & 0 \end{bmatrix}.$$

Although the model is linear, and the endogenous variables are linear functions of state, the coefficients A and B are not necessarily linear functions of the model coefficients. Analytic solutions are not always easy. Determinacy comes here from the assumption that v_t is a time-invariant function of the state variables.

A1.5.4 Sticky price fiscal model with long-term debt

This section presents algebra for Section 5.2. The model from (5.17)-(5.21) is

$$\begin{aligned} x_t &= E_t x_{t+1} - \sigma\left(i_t - E_t \pi_{t+1}\right) \\ \pi_t &= \beta E_t \pi_{t+1} + \kappa x_t \\ \rho v_{t+1} &= v_t + r^n_{t+1} - \pi_{t+1} - \tilde{s}_{t+1} \\ E_t r^n_{t+1} &= i_t \\ r^n_{t+1} &= \omega q_{t+1} - q_t. \end{aligned}$$

Adding an interest rate rule and surplus process

$$\begin{aligned} i_t &= \theta_{i,\pi}\pi_t + u_{i,t} + w_t \\ s_t &= u_{s,t} \\ u_{i,t+1} &= \eta_i u_{i,t} + \varepsilon_{i,t+1} \\ u_{s,t+1} &= \eta_s u_{s,t} + \varepsilon_{s,t+1} \end{aligned}$$

where w_t is deterministic, we have

$$\begin{aligned} x_{t+1} &= \left(1 + \frac{\sigma\kappa}{\beta}\right) x_t + \sigma\left(\theta_{\iota,\pi} - \frac{1}{\beta}\right)\pi_t + \sigma u_{i,t} + \sigma w_t + \delta_{x,t+1} \\ \pi_{t+1} &= -\frac{\kappa}{\beta} x_t + \frac{1}{\beta}\pi_t + \delta_{\pi,t+1} \\ v_{t+1} &= \frac{1}{\rho}\frac{\kappa}{\beta} x_t + \frac{1}{\rho}\left(\theta_{i\pi} - \frac{1}{\beta}\right)\pi_t + \frac{1}{\rho} v_t - \frac{\eta_s}{\rho} s_t + \frac{1}{\rho} u_{i,t} + \frac{1}{\rho} w_t \\ &\quad - \frac{1}{\rho}\varepsilon_{s,t+1} - \frac{1}{\rho}\delta_{\pi,t+1} + \frac{\omega}{\rho}\delta_{q,t+1} \\ q_{t+1} &= \frac{\theta_{\iota,\pi}}{\omega}\pi_t + \frac{1}{\omega} q_t + \frac{1}{\omega} u_{i,t} + \frac{1}{\omega} w_t + \delta_{q,t+1}. \end{aligned}$$

Putting it all together,

$$\begin{bmatrix} x_{t+1} \\ \pi_{t+1} \\ v_{t+1} \\ q_{t+1} \\ u_{i,t+1} \\ u_{s,t+1} \end{bmatrix} = \begin{bmatrix} 1 + \frac{\kappa\sigma}{\beta} & \sigma\left(\theta_{i\pi} - \frac{1}{\beta}\right) & 0 & 0 & \sigma & 0 \\ -\frac{\kappa}{\beta} & \frac{1}{\beta} & 0 & 0 & 0 & 0 \\ \frac{\kappa}{\rho\beta} & \frac{1}{\rho}\left(\theta_{i\pi} - \frac{1}{\beta}\right) & \frac{1}{\rho} & 0 & \frac{1}{\rho} & -\frac{1}{\rho}\eta_s \\ 0 & \frac{\theta_{\iota,\pi}}{\omega} & 0 & \frac{1}{\omega} & \frac{1}{\omega} & 0 \\ 0 & 0 & 0 & 0 & \eta_i & 0 \\ 0 & 0 & 0 & 0 & 0 & \eta_s \end{bmatrix} \begin{bmatrix} x_t \\ \pi_t \\ v_t \\ q_t \\ u_{i,t} \\ u_{s,t} \end{bmatrix}$$

$$+\left[\begin{array}{cc} 0 & 0 \\ 0 & 0 \\ 0 & -\frac{1}{\rho} \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{array}\right]\left[\begin{array}{c} \varepsilon_{i,t+1} \\ \varepsilon_{s,t+1} \end{array}\right]+\left[\begin{array}{ccc} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -\frac{1}{\rho} & \frac{\omega}{\rho} \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{array}\right]\left[\begin{array}{c} \delta_{x,t+1} \\ \delta_{\pi,t+1} \\ \delta_{q,t+1} \end{array}\right]+\left[\begin{array}{c} \sigma \\ 0 \\ \frac{1}{\rho} \\ \frac{1}{\omega} \\ 0 \\ 0 \end{array}\right] w_t.$$

This is the standard form

$$z_{t+1} = Az_t + B\varepsilon_{t+1} + C\delta_{t+1}.$$

I then use the matrix solution formulas from Section A1.5.2.

A1.5.5 sticky price fiscal theory model with policy rules

This section presents algebra for Section 5.5. I solve the model in the standard way. I reduce it to $Az_{t+1} = Bz_t + C\varepsilon_{t+1} + D\delta_{t+1}$. I solve unstable eigenvalues forward and stable eigenvalues backward using the formulas from Section A1.5.2.

We reduced the equilibrium conditions (5.51)-(5.62) to (5.64)-(5.73),

$$x_t = E_t x_{t+1} - \sigma(i_t - E_t\pi_{t+1}) \tag{A1.65}$$
$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t \tag{A1.66}$$
$$i_t = \theta_{i\pi}\pi_t + \theta_{ix}x_t + u_{i,t} \tag{A1.67}$$
$$u_{i,t+1} = \eta_i u_{i,t} + \varepsilon_{i,t+1} \tag{A1.68}$$
$$\Delta E_{t+1}\pi_{t+1} = -\beta_s \varepsilon_{s,t+1} - \beta_i \varepsilon_{i,t+1} \tag{A1.69}$$
$$\tilde{s}_{t+1} = \theta_{s\pi}\pi_{t+1} + \theta_{sx}x_{t+1} + \alpha v_t + u_{s,t+1} \tag{A1.70}$$
$$u_{s,t+1} = \eta_s u_{s,t} + \varepsilon_{s,t+1} \tag{A1.71}$$
$$\rho v_{t+1} = v_t + r^n_{t+1} - \pi_{t+1} - \tilde{s}_{t+1} \tag{A1.72}$$
$$E_t r^n_{t+1} = i_t \tag{A1.73}$$
$$r^n_{t+1} = \omega q_{t+1} - q_t. \tag{A1.74}$$

Substituting (A1.66) into (A1.65), using (A1.67) to eliminate i_t and (A1.70) to eliminate s_t, and introducing expectational errors δ, we have

$$\begin{aligned}
x_{t+1} &= \left(1 + \frac{\sigma\kappa}{\beta} + \sigma\theta_{ix}\right) x_t + \left(\sigma\theta_{i\pi} - \frac{\sigma}{\beta}\right)\pi_t + \sigma u_{i,t} + \delta_{x,t+1} \\
\pi_{t+1} &= -\frac{\kappa}{\beta}x_t + \frac{1}{\beta}\pi_t - (\beta_s\varepsilon_{s,t+1} + \beta_i\varepsilon_{i,t+1}) \\
q_{t+1} &= \frac{\theta_{ix}}{\omega}x_t + \frac{\theta_{i\pi}}{\omega}\pi_t + \frac{1}{\omega}q_t + \frac{1}{\omega}u_{i,t} + \delta_{q,t+1} \\
u_{i,t+1} &= \eta_i u_{i,t} + \varepsilon_{i,t+1} \\
u_{s,t+1} &= \eta_s u_{s,t} + \varepsilon_{s,t+1}.
\end{aligned}$$

Other variables follow from these. The model is then in standard matrix form,

$$
\begin{bmatrix} x_{t+1} \\ \pi_{t+1} \\ q_{t+1} \\ u_{i,t+1} \\ u_{s,t+1} \end{bmatrix} = \begin{bmatrix} 1+\frac{\sigma\kappa}{\beta}+\sigma\theta_{ix} & \sigma\theta_{i\pi}-\frac{\sigma}{\beta} & 0 & \sigma & 0 \\ -\frac{\kappa}{\beta} & \frac{1}{\beta} & 0 & 0 & 0 \\ \frac{\theta_{ix}}{\omega} & \frac{\theta_{i\pi}}{\omega} & \frac{1}{\omega} & \frac{1}{\omega} & 0 \\ 0 & 0 & 0 & \eta_i & 0 \\ 0 & 0 & 0 & 0 & \eta_s \end{bmatrix} \begin{bmatrix} x_t \\ \pi_t \\ q_t \\ u_{i,t} \\ u_{s,t} \end{bmatrix}
$$
$$
+ \begin{bmatrix} 0 & 0 \\ -\beta_i & -\beta_s \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \varepsilon_{i,t+1} \\ \varepsilon_{s,t+1} \end{bmatrix}
+ \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \delta_{x,t+1} \\ \delta_{q,t+1} \end{bmatrix}.
$$

A1.6 The standard new-Keynesian model

> This section discusses how I solve the classic new-Keynesian model. I illustrate the matrix method, undetermined coefficients, and lag operator methods.

The standard new-Keynesian model is (17.15)-(17.17), with policy rule expressed as in (17.15)-(17.21) and AR(1) shocks,

$$x_t = E_t x_{t+1} - \sigma r_t + u_{x,t} \tag{A1.75}$$
$$i_t = r_t + E_t \pi_{t+1} \tag{A1.76}$$
$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t + u_{\pi,t} \tag{A1.77}$$
$$i_t = \phi \pi_t + u_{i,t} \tag{A1.78}$$
$$u_{i,t} = \eta u_{i,t-1} + \varepsilon_{i,t}. \tag{A1.79}$$

There is nothing special about the solution method, which is the same as the fiscal theory sticky price models of the previous section. We use different parameters, so this section details the algebra.

There are (at least) four ways to approach a model of this form. First, express it in a standard matrix AR(1) form; eigenvalue decompose the transition matrix; and solve stable roots backwards and unstable roots forwards as outlined in Section A1.5.2. This method is the easiest to apply to large models as all the work is done by computers, but it often hides intuition. Second, substitute until you have a lag-operator expression for the variable of interest, π_t here. Factor the lag polynomial, solve unstable roots forward and stable roots backward, to express π_t as a two-sided moving average of the forcing variables, as in (5.5). This form shows analytically how the variable of interest responds to the shock of interest, so it is useful for intuition. Third, guess that the final answer is a function of state variables, substitute that guess in (A1.75)-(A1.79) and use the method of undetermined coefficients. This is often the quickest way to get an analytic solution, but it hides the economics and especially how the model gets rid of multiple equilibria. Fourth, one can simplify algebra considerably by rewriting the rule in the form $i_t = i_t^* + \phi(\pi_t - \pi_t^*)$. I use each method according to which makes the particular point clearest.

A1.6.1 Matrix method

I set the standard new-Keynesian model up for the matrix solution method.

Section 17.4.3 presents calculations of this model's responses to AR(1) monetary policy shocks. I use the matrix method in this case.

Eliminate i_t and r_t and rearrange, leaving

$$\begin{bmatrix} E_t x_{t+1} \\ E_t \pi_{t+1} \end{bmatrix} = \frac{1}{\beta} \begin{bmatrix} \beta + \sigma\kappa & -\sigma(1-\beta\phi) \\ -\kappa & 1 \end{bmatrix} \begin{bmatrix} x_t \\ \pi_t \end{bmatrix} + \frac{1}{\beta} \begin{bmatrix} -1 & \sigma & \sigma\beta \\ 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} u_{x,t} \\ u_{\pi,t} \\ u_{i,t} \end{bmatrix}. \tag{A1.80}$$

This equation is the generalization of the equilibrium condition

$$E_t \pi_{t+1} = \phi \pi_t + u_{i,t} \tag{A1.81}$$

of the frictionless model.

The eigenvalues of the transition matrix in (A1.80) are

$$\lambda = 1 + \frac{1}{2\beta}\left[(1-\beta+\sigma\kappa) \pm \sqrt{(1-\beta+\sigma\kappa)^2 - 4\beta\sigma\kappa(\phi-1)}\right]. \tag{A1.82}$$

The + eigenvalue is greater than one. But if $\phi < 1$ the − eigenvalue is less than one, i.e. stable. Thus, with $\phi < 1$, we solve one part of the system backward. Since the left-hand side of (A1.80) determines only the expectations of future variables, we need two forward-looking roots and a rule against explosions to get rid of multiple equilibria, so with $\phi < 1$ we have multiple equilibria. If $\phi > 1$ then both eigenvalues are greater than one, and unstable. We solve the system forward and determine uniquely the expectational shocks in both x_t and π_t, in order to have a locally bounded solution. This is the generalization of the idea that led to $\phi > 1$ and then solving the frictionless model (A1.81) forward. From (A1.80) we can apply the matrix machinery of Section A1.5.2 directly. The logic is the same as the frictionless case and the simplified case, though the algebra is considerably worse. Models of this complexity and more are typically solved on a computer, as the formulas for eigenvalues get worse quickly. Cochrane (2011b) contains the most general analytic formulas I know of.

In this case as well, $\lambda < -1$ or λ complex with modulus greater than one also lead to local determinacy. The oscillating hyperinflation threat is as good, or indeed better, if we wish to "coordinate equilibria" by ruling out unreasonable expectations. Here

$$\phi < -\left(1 + 2\frac{1+\beta}{\sigma\kappa}\right) \tag{A1.83}$$

serves just as well to rule out multiple equilibria. In models with more complex policy rules including responses to output and expected future inflation, complicated possibilities emerge. Cochrane (2011b) contains plots of the determinacy regions for a variety of such models. The lesson here is even clearer: $\phi_\pi > 1$ is neither necessary nor sufficient to generate explosive eigenvalues, so this model really does not really embody the standard intuition about the Taylor rule.

Usually, one finds eigenvalues and eigenvectors of the transition matrix and solves the model numerically. You can follow the above approach analytically. Cochrane (2011b) finds eigenvalues and eigenvectors and writes the most general analytic solutions to this class of model that I know of. It's just a question of how much algebra you can stand.

A1.6.2 Undetermined coefficients

I solve the new-Keynesian model by undetermined coefficients, quickly giving an analytic solution.

$$\begin{aligned}
\pi_t &= -\frac{1}{\phi - \eta + \frac{(1-\beta\eta)(1-\eta)}{\sigma\kappa}} u_{i,t} \\
x_t &= \frac{1-\beta\eta}{\kappa}\pi_t \\
i_t &= \left[\eta - \frac{(1-\beta\eta)(1-\eta)}{\sigma\kappa}\right]\pi_t.
\end{aligned}$$

You get to the same answer more quickly with the method of undetermined coefficients. Specializing to the monetary policy shock only, guess an answer of the form

$$\begin{aligned}
\pi_t &= \alpha_\pi u_{i,t} \\
x_t &= \alpha_x u_{i,t}.
\end{aligned}$$

Substitute this guess into (17.15)-(17.21), giving

$$\begin{aligned}
\alpha_x u_{i,t} &= \eta\alpha_x u_{i,t} - \sigma\left(\phi\alpha_\pi u_{i,t} + u_{i,t} - \eta\alpha_\pi u_{i,t}\right) \\
\alpha_\pi u_{i,t} &= \beta\eta\alpha_\pi u_{i,t} + \kappa\alpha_x u_{i,t}.
\end{aligned}$$

Since these equations must hold for any $u_{i,t}$, conclude

$$\begin{aligned}
\alpha_x &= \eta\alpha_x - \sigma\left[1 + (\phi-\eta)\,\alpha_\pi\right] \\
\alpha_\pi &= \beta\eta\alpha_\pi + \kappa\alpha_x,
\end{aligned}$$

$$(1-\eta)\alpha_x = -\sigma\left[1 + (\phi-\eta)\,\alpha_\pi\right]$$

$$(1-\beta\eta)\alpha_\pi = \kappa\alpha_x. \tag{A1.84}$$

Eliminating α_x and solving,

$$(1-\beta\eta)(1-\eta)\alpha_\pi = -\sigma\kappa\left[1+(\phi-\eta)\,\alpha_\pi\right]$$

$$\left[(1-\beta\eta)(1-\eta)+\sigma\kappa\,(\phi-\eta)\right]\alpha_\pi = -\sigma\kappa$$

and finally, therefore

$$\pi_t = -\frac{1}{\phi-\eta+\frac{(1-\beta\eta)(1-\eta)}{\sigma\kappa}}u_{i,t} \tag{A1.85}$$

$$x_t = \frac{1-\beta\eta}{\kappa}\pi_t \tag{A1.86}$$

$$i_t = \left[\eta-\frac{(1-\beta\eta)(1-\eta)}{\sigma\kappa}\right]\pi_t. \tag{A1.87}$$

I used (17.20) and (A1.84) in the latter two equations. Yes, undetermined coefficients gets you to the answer quickly!

You can see the inflation response (A1.87) is a natural generalization of the simple sticky price model (17.12),

$$\pi_t = -\frac{1}{\phi-\eta+\frac{1-\eta}{\sigma\kappa}}u_{i,t},$$

and of the frictionless model (16.7),

$$\pi_t = -\frac{1}{\phi-\eta}u_{i,t}.$$

A1.6.3 Lag operator solution

I use lag operator methods to exhibit the model solution in a way that displays dynamics directly.

$$\pi_t = \frac{\sigma\kappa}{1+\sigma\kappa\phi}\frac{1}{\lambda_1-\lambda_2}E_t\left(-\lambda_1\sum_{j=0}^{\infty}\lambda_1^j u_{i,t+j}+\lambda_2\sum_{j=0}^{\infty}\lambda_2^j u_{i,t+j}\right).$$

Here, I use lag operator techniques to write the solution for inflation analytically. I calculate the three-equation model of Figure 17.8 by this method. It allows us to see dynamics directly.

From (A1.75)-(A1.79), substitute out i_t and r_t again, and drop the $u_{\pi,t}$ shock for simplicity, leaving

$$x_t = E_t x_{t+1} - \sigma(\phi\pi_t + u_{i,t} - E_t\pi_{t+1})$$
$$\pi_t = \beta E_t\pi_{t+1} + \kappa x_t$$

Expressing the model in lag operator notation,

$$E_t(1-L^{-1})x_t = \sigma E_t\left(L^{-1}-\phi\right)\pi_t - \sigma u_{i,t}$$

$$E_t(1-\beta L^{-1})\pi_t = \kappa x_t$$

Forward-differencing the second equation,

$$E_t(1-L^{-1})(1-\beta L^{-1})\pi_t = E_t(1-L^{-1})\kappa x_t$$

Then substituting into the first equation,

$$E_t(1-L^{-1})\left(1-\beta L^{-1}\right)\pi_t = \sigma\kappa E_t\left(L^{-1}-\phi\right)\pi_t - \sigma\kappa u_{i,t}$$

$$E_t\left[1-\frac{1+\beta+\sigma\kappa}{1+\sigma\kappa\phi}L^{-1}+\frac{\beta}{1+\sigma\kappa\phi}L^{-2}\right]\pi_t = -\frac{\sigma\kappa}{1+\sigma\kappa\phi}u_{i,t}.$$

Factor the lag polynomial

$$E_t(1-\lambda_1 L^{-1})(1-\lambda_2 L^{-1})\pi_t = -\frac{\sigma\kappa}{1+\sigma\kappa\phi}u_{i,t}$$

where

$$\lambda = \frac{1+\beta+\sigma\kappa \pm \sqrt{(1+\beta+\kappa\sigma)^2 - 4\beta\left(1+\phi\sigma\kappa\right)}}{2\left(1+\sigma\kappa\phi\right)}$$

These lag operator roots are the inverse of the eigenvalues of the usual transition matrix. The system is stable and solved backward for $\lambda > 1$; it is unstable and solved forward for $\lambda < 1$.

The standard new-Keynesian model uses $\phi > 1$ so both roots are unstable, $\lambda_1 < 1$ and $\lambda_2 < 1$. Then, we can write

$$E_t(1-\lambda_1 L^{-1})(1-\lambda_2 L^{-1})\pi_t = -\frac{\sigma\kappa}{1+\sigma\kappa\phi}u_{i,t}$$

$$\pi_t = -E_t\frac{1}{(1-\lambda_1 L^{-1})(1-\lambda_2 L^{-1})}\frac{\sigma\kappa}{1+\sigma\kappa\phi}u_{i,t}$$

$$\pi_t = E_t\frac{1}{\lambda_1-\lambda_2}\left(\frac{-\lambda_1}{1-\lambda_1 L^{-1}}+\frac{\lambda_2}{1-\lambda_2 L^{-1}}\right)\frac{\sigma\kappa}{1+\sigma\kappa\phi}u_{i,t}$$

$$\pi_t = \frac{\sigma\kappa}{1+\sigma\kappa\phi}\frac{1}{\lambda_1-\lambda_2}E_t\left(-\lambda_1\sum_{j=0}^{\infty}\lambda_1^j u_{i,t+j}+\lambda_2\sum_{j=0}^{\infty}\lambda_2^j u_{i,t+j}\right)$$

Using the AR(1) form of the disturbance v^i,

$$\pi_t = \frac{\sigma\kappa}{1+\sigma\kappa\phi}\frac{1}{\lambda_1-\lambda_2}\left(-\lambda_1\sum_{j=0}^{\infty}\lambda_1^j\eta^j+\lambda_2\sum_{j=0}^{\infty}\lambda_2^j\eta^j\right)u_{i,t}$$

$$\pi_t = \frac{\sigma\kappa}{1+\sigma\kappa\phi}\frac{1}{\lambda_1-\lambda_2}\left(-\frac{\lambda_1}{1-\lambda_1\eta}+\frac{\lambda_2}{1-\lambda_2\eta}\right)u_{i,t}$$

$$\pi_t = \frac{\sigma\kappa}{1+\sigma\kappa\phi}\frac{1}{\lambda_1-\lambda_2}\left(\frac{\lambda_2(1-\lambda_1\eta)-\lambda_1(1-\lambda_2\eta)}{(1-\lambda_1\eta)(1-\lambda_2\eta)}\right)v_t^i$$
$$\pi_t = -\frac{\sigma\kappa}{1+\sigma\kappa\phi}\left(\frac{1}{(1-\lambda_1\eta)(1-\lambda_2\eta)}\right)u_{i,t}$$

Thus, to produce Figure 17.8, I simply simulate the AR(1) impulse-response, for $\{u_{i,t}\}$, calculate π_t by the last equation, and calculate $i_t = \phi\pi_t + u_{i,t}$.

A1.7 Continuous time sticky price models

A1.7.1 Algebra of the continuous time sticky price analytical solution

I derive the analytic solution (5.94)

$$\pi_t = C_0 e^{-\lambda_2 t} + \left(\frac{1}{\lambda_2}+\frac{1}{\lambda_1}\right)^{-1}\left[\int_{\tau=0}^{t} e^{-\lambda_2\tau} i_{t-\tau}d\tau + \int_{\tau=0}^{\infty} e^{-\lambda_1\tau} i_{t+\tau}d\tau\right]$$

to the continuous time sticky price model.

This section presents the algebra of the analytic solution (5.94), reproduced above, to the continuous time sticky price model (5.91)-(5.93),

$$\frac{dx_t}{dt} = \sigma(i_t - \pi_t) \tag{A1.88}$$
$$\frac{d\pi_t}{dt} = \rho\pi_t - \kappa x_t \tag{A1.89}$$
$$\frac{dv_t}{dt} = i_t - \pi_t + rv_t - \tilde{s}_t. \tag{A1.90}$$

Differentiating (A1.89) and using (A1.88) to eliminate x_t,

$$\frac{d^2\pi_t}{dt^2} - \rho\frac{d\pi_t}{dt} - \kappa\sigma\pi_t = -\kappa\sigma i_t.$$

To solve this differential equation, express it as

$$(D-\lambda_1)(D+\lambda_2)\pi_t = -\kappa\sigma i_t;\ D \equiv d/dt.$$

with

$$\lambda_1 = \frac{\rho+\sqrt{\rho^2+4\kappa\sigma}}{2}; \lambda_2 = \frac{-\rho+\sqrt{\rho^2+4\kappa\sigma}}{2}.$$

(The equalities $\lambda_1\lambda_2 = \kappa\sigma$, and $\lambda_1 - \lambda_2 = \rho$ come in handy.) Now solve it as

$$\pi_t = -\frac{1}{(D-\lambda_1)(D+\lambda_2)}\kappa\sigma i_t \tag{A1.91}$$
$$= -\frac{1}{\lambda_1+\lambda_2}\left[\frac{1}{(D-\lambda_1)} - \frac{1}{(D+\lambda_2)}\right]\kappa\sigma i_t$$

$$= -\left(\frac{1}{\lambda_2}+\frac{1}{\lambda_1}\right)^{-1}\left[\frac{1}{(D-\lambda_1)}-\frac{1}{(D+\lambda_2)}\right]i_t.$$

To express the right-hand side in terms of integrals, note that if

$$(D-a)y_t = z_t,$$

i.e.

$$\frac{dy_t}{dt} = ay_t + z_t,$$

then we solve forward, and the stationary solution is

$$y_t = -\int_{\tau=0}^{\infty} e^{-a\tau} z_{t+\tau}d\tau.$$

If, on the other hand

$$(D+b)y_t = z_t,$$

then we solve backward, and the stationary solution is

$$y_t = Ce^{-bt} + \int_{\tau=0}^{t} e^{-b\tau} z_{t-\tau}d\tau.$$

The solution to (A1.91), and thus to the pair (A1.88)-(A1.89), is the sum of the last two integral expressions.

$$\pi_t = C_0e^{-\lambda_2 t} + \left(\frac{1}{\lambda_2}+\frac{1}{\lambda_1}\right)^{-1}\left[\int_{\tau=0}^{t} e^{-\lambda_2\tau}i_{t-\tau}d\tau + \int_{\tau=0}^{\infty} e^{-\lambda_1\tau}i_{t+\tau}d\tau\right]. \quad \text{(A1.92)}$$

Cochrane (2012) describes the analogy between the D operator here and the L operator of discrete time in more detail.

A1.7.2 Continuous time matrix metod

The continuous time linear models are in the form

$$dz_t = Az_tdt + Bd\varepsilon_t + Cd\delta_t$$

where $d\varepsilon_t$ are structural shocks and $d\delta_t$ are expectational errors.

Eigenvalue decompose the transition matrix A,

$$A = Q\Lambda Q^{-1}.$$

Defining $\tilde{z}_t \equiv Q^{-1}z_t$,

$$d\tilde{z}_t = \Lambda\tilde{z}_tdt + Q^{-1}Bd\varepsilon_t + Q^{-1}Cd\delta_t. \quad \text{(A1.93)}$$

I offer two notations for the answer. First, defining by a + and − subscript rows corresponding to explosive eigenvalues and stable eigenvalues, we have

$$\tilde{z}_{+t} = 0,$$

an autoregressive representation

$$d\tilde{z}_{-t} = \Lambda_{-t}\tilde{z}_{-t}dt + Q_{-}^{-1}\left[I - C\left[Q_{+}^{-1}C\right]^{-1}Q_{+}^{-1}\right]Bd\varepsilon_t,$$

and a moving average representation

$$\tilde{z}_{-t} = e^{\Lambda_{-}t}\tilde{z}_{-0} + \int_{s=0}^{t} e^{\Lambda_{-}s}Q_{-}^{-1}\left[I - C\left[Q_{+}^{-1}C\right]^{-1}Q_{+}^{-1}\right]Bd\varepsilon_{t-s}.$$

Reassembling $\tilde{z}_t$ and with $z_t = Q\tilde{z}_t$ we have the solution.

Second, defining matrices P and M that select rows of Q^{-1} corresponding to explosive and nonexplosive eigenvalues, we can express the whole operation as an autoregressive representation

$$d\tilde{z}_t = \Lambda^*\tilde{z}_t dt + M'MQ^{-1}\left[I_{N_v} - C\left[PQ^{-1}C\right]^{-1}PQ^{-1}\right]Bd\varepsilon_t.$$

and moving average representation,

$$\tilde{z}_t = e^{\Lambda^* t}\tilde{z}_0 + \int_{s=0}^{t} e^{\Lambda^* t}M'MQ^{-1}\left[I_{N_v} - C\left[PQ^{-1}C\right]^{-1}PQ^{-1}\right]Bd\varepsilon_{t-s}$$

where

$$\Lambda^* \equiv M'M\Lambda M'M.$$

In this section I adapt the standard matrix solution method from Section A1.5.2 to continuous time models.

The linear models we study can all be written in the form

$$dz_t = Az_t dt + Bd\varepsilon_t + Cd\delta_t$$

where $d\varepsilon_t$ are structural shocks and $d\delta_t$ are expectational errors, as in discrete time. We find the expectational errors in terms of the structural shocks, and then find an autoregressive and then a moving average representation for the equilibrium x_t.

I use this method to solve the fiscal theory model from Section 5.7.3. Equating starred and unstarred variables, the model is

$$\begin{aligned}
dx_t &= \sigma(i_t - \pi_t)dt + d\delta_{x,t} \\
d\pi_t &= (\rho\pi_t - \kappa x_t)\,dt \ + d\delta_{\pi,t} \\
dp_t &= \pi_t dt \\
dq_t &= \left[(r+\omega)\,q_t + i_t\right]dt + d\delta_{q,t} \\
dv_t &= (rv_t + i_t - \pi_t - \tilde{s}_t)\,dt + d\delta_{q,t} \\
di_t &= -\zeta_i\left[i_t - (\theta_{i\pi}\pi_t + \theta_{ix}x_t + u_{i,t})\right]dt + \theta_{i\varepsilon}d\varepsilon_{i,t}
\end{aligned}$$

$$\begin{aligned}
\tilde{s}_t &= \theta_{s\pi}\pi_t + \theta_{sx}x_t + \alpha v_t + u_{s,t} \\
d\pi_t - E_t d\pi_t &= -\beta_s d\varepsilon_{s,t} - \beta_i d\varepsilon_{i,t} \\
du_{i,t} &= -\eta_i u_{i,t} + d\varepsilon_{i,t} \\
du_{s,t} &= -\eta_s u_{s,t} + d\varepsilon_{s,t}.
\end{aligned}$$

We write the model in standard form matrix representation as

$$d\begin{bmatrix} x_t \\ \pi_t \\ q_t \\ v_t \\ i_t \\ u_{i,t} \\ u_{s,t} \end{bmatrix} = \begin{bmatrix} 0 & -\sigma & 0 & 0 & \sigma & 0 & 0 \\ -\kappa & \rho & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & r+\omega & 0 & 1 & 0 & 0 \\ -\theta_{sx} & -1-\theta_{s\pi} & 0 & r-\alpha & 1 & 0 & -1 \\ \zeta_i\theta_{ix} & \zeta_i\theta_{i\pi} & 0 & 0 & -\zeta_i & \zeta_i & 0 \\ 0 & 0 & 0 & 0 & 0 & -\eta_i & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -\eta_s \end{bmatrix} \begin{bmatrix} x_t \\ \pi_t \\ q_t \\ v_t \\ i_t \\ u_{i,t} \\ u_{s,t} \end{bmatrix} dt +$$

$$+ \begin{bmatrix} 0 & 0 \\ -\beta_i & -\beta_s \\ 0 & 0 \\ 0 & 0 \\ \theta_{i\varepsilon} & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} d\varepsilon_{i,t} \\ d\varepsilon_{s,t} \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} d\delta_{x,t} \\ d\delta_{q,t} \end{bmatrix}.$$

The surplus process determines $d\pi_t - E_t d\pi_t$, and thus determines $d\delta_{\pi,t}$. In the process of setting starred equal to unstarred variables we already used one forward-looking root, which determines $d\delta_{\pi,t}$.

In the case with $\alpha = 0$ instead we must find $d\delta_{\pi,t}$ to match the initial value of debt. I highlight the changes in boxes.

$$\begin{aligned}
dx_t &= \sigma(i_t - \pi_t)dt + d\delta_{x,t} \\
d\pi_t &= (\rho\pi_t - \kappa x_t)\, dt \; \boxed{+d\delta_{\pi,t}} \\
dq_t &= [(r+\omega)\, q_t + i_t]dt + d\delta_{q,t} \\
dv_t &= \Big[(i_t - \pi_t) + \Big(r - \boxed{0}\Big)\, v_t - (\theta_{s\pi}\pi_t + \theta_{sx}x_t + u_{s,t})\Big]\, dt + d\delta_{q,t} \\
di_t &= -\zeta_i\left[i_t - (\theta_{i\pi}\pi_t + \theta_{ix}x_t + u_{i,t})\right] dt \\
du_{i,t} &= -\eta_i u_{i,t} + d\varepsilon_{i,t} \\
du_{s,t} &= -\eta_s u_{s,t} + d\varepsilon_{s,t}.
\end{aligned}$$

(The $d\delta_{\pi,t}$ isn't by itself a change; the change is that there is no later equation you can use to eliminate $d\delta_{\pi,t}$.

Then the matrix representation is

$$d\begin{bmatrix} x_t \\ \pi_t \\ q_t \\ v_t \\ i_t \\ u_{i,t} \\ u_{s,t} \end{bmatrix} = \begin{bmatrix} 0 & -\sigma & 0 & 0 & \sigma & 0 & 0 \\ -\kappa & \rho & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & r+\omega & 0 & 1 & 0 & 0 \\ -\theta_{sx} & -1-\theta_{s\pi} & 0 & r-\boxed{0} & 1 & 0 & -1 \\ \zeta_i\theta_{ix} & \zeta_i\theta_{i\pi} & 0 & 0 & -\zeta_i & \zeta_i & 0 \\ 0 & 0 & 0 & 0 & 0 & -\eta_i & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -\eta_s \end{bmatrix} \begin{bmatrix} x_t \\ \pi_t \\ q_t \\ v_t \\ i_t \\ u_{i,t} \\ u_{s,t} \end{bmatrix} dt +$$

$$+\begin{bmatrix} 0 & 0 \\ \boxed{0} & \boxed{0} \\ 0 & 0 \\ 0 & 0 \\ \theta_{i\varepsilon} & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} d\varepsilon_{i,t} \\ d\varepsilon_{s,t} \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & \boxed{1} & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} d\delta_{x,t} \\ \boxed{d\delta_{\pi,t}} \\ d\delta_{q,t} \end{bmatrix}.$$

Eigenvalue decomposing the transition matrix A,

$$A = Q\Lambda Q^{-1}$$

where Λ is a diagonal matrix of eigenvalues, we can premultiply by Q^{-1} and defining $\tilde{z}_t \equiv Q^{-1}z_t$ we have

$$d\tilde{z}_t = \Lambda\tilde{z}_t dt + Q^{-1}Bd\varepsilon_t + Q^{-1}Cd\delta_t. \tag{A1.94}$$

The goal of this section is an autoregressive and then a moving average representation for $\tilde{z}_t$ and consequently $z_t = Q\tilde{z}_t$.

We partition the system (A1.94) into the rows with explosive (real part greater than zero) eigenvalues and the rows with stable (real part less than or equal to zero) eigenvalues. Let Q_+^{-1}, $\tilde{z}_{+t}$ denote the rows of these matrices corresponding to explosive eigenvalues, and Λ_+ the diagonal matrix with positive eigenvalues. Then, the explosive eigenvalues obey

$$d\tilde{z}_{+t} = \Lambda_+\tilde{z}_{+t}dt + Q_+^{-1}Bd\varepsilon_t + Q_+^{-1}Cd\delta_t.$$

To have $E_t\tilde{z}_{t+j}$ not explode, we must have

$$\tilde{z}_{+t} = 0$$

and hence

$$Q_+^{-1}Cd\delta_t = -Q_+^{-1}Bd\varepsilon_t$$

$$d\delta_t = -\left[Q_+^{-1}C\right]^{-1}Q_+^{-1}Bd\varepsilon_t.$$

The explosive eigenvalues tell us the expectational errors as functions of the structural shocks – so long as there as are many explosive eigenvalues as there are expectational errors, i.e. $\left[Q_+^{-1}C\right]$ is invertible.

The rows with stable eigenvalues then give us

$$d\tilde{z}_{-t} = \Lambda_-\tilde{z}_{-t}dt + Q_-^{-1}Bd\varepsilon_t + Q_-^{-1}Cd\delta_t$$

$$d\tilde{z}_{-t} = \Lambda_- \tilde{z}_{-t} dt + Q_-^{-1} B d\varepsilon_t - Q_-^{-1} C \left[Q_+^{-1} C\right]^{-1} Q_+^{-1} B d\varepsilon_t$$

$$d\tilde{z}_{-t} = \Lambda_- \tilde{z}_{-t} dt + Q_-^{-1} \left[I - C \left[Q_+^{-1} C\right]^{-1} Q_+^{-1}\right] B d\varepsilon_t.$$

This gives us an autoregressive representation for the $\tilde{z}_{it}$ with stable eigenvalues. Integrating, we have a moving average representation

$$\tilde{z}_{-t} = e^{\Lambda_- t} \tilde{z}_{-0} + \int_{s=0}^{t} e^{\Lambda_- s} Q_-^{-1} \left[I - C \left[Q_+^{-1} C\right]^{-1} Q_+^{-1}\right] B d\varepsilon_{t-s}.$$

Here by $e^{\Lambda t}$ I mean

$$e^{\Lambda t} \equiv \left[\begin{array}{cccc} e^{\lambda_1 t} & 0 & 0 & \\ 0 & e^{\lambda_2 t} & 0 & \cdots \\ 0 & 0 & e^{\lambda_3 t} & \\ & \vdots & & \ddots \end{array}\right],$$

element by element exponentiation and not including the off diagonal elements. We reassemble $\tilde{z}_t$ from $\tilde{z}_{-t}$ and $\tilde{z}_{+t} = 0$. Then, the original values are

$$z_t = Q\tilde{z}_t.$$

The matrix carpentry of this solution may seem inelegant. At the cost of a bit of notation we can do the same thing with matrices and obtain somewhat more elegant formulas. To do this, let N_v denote the number of variables, so A is $N_v \times N_v$, let N_ε be the number structural shocks so B is $N_v \times N_\varepsilon$, and let N_δ be the number of expectational errors, so C is $N_v \times N_\delta$. There are N_δ explosive eigenvalues with positive real parts. Then let P be a $N_\delta \times N_v$ matrix that selects rows of Q^{-1} corresponding to eigenvalues with positive real parts, and let M be an $(N_v - N_\delta) \times N_v$ matrix that selects rows corresponding to eigenvalues with non-positive real parts. For example, if

$$\Lambda = \left[\begin{array}{ccc} 0.1 & 0 & 0 \\ 0 & -0.1 & 0 \\ 0 & 0 & 0.2 \end{array}\right]$$

then

$$P = \left[\begin{array}{ccc} 1 & 0 & 0 \\ 0 & 0 & 1 \end{array}\right]$$
$$M = \left[\begin{array}{ccc} 0 & 1 & 0 \end{array}\right].$$

The matrix P selects the first and third row, and M selects the second row. In terms of the notation of the last section, $Q_+^{-1} = PQ^{-1}$, $\tilde{z}_{+t} = P\tilde{z}_t$, etc. The matrices P' and M' then put things back in the original rows, so $P'P + M'M = I_{N_v}$. We start again from (A1.93),

$$d\tilde{z}_t = \Lambda \tilde{z}_t dt + Q^{-1} B d\varepsilon_t + Q^{-1} C d\delta_t$$

$$P d\tilde{z}_t = P\Lambda \tilde{z}_t dt + PQ^{-1} B d\varepsilon_t + PQ^{-1} C d\delta_t$$

to have $E_t \tilde{z}_{t+j}$ not explode, we must have

$$P\tilde{z}_t = 0$$

and hence

$$PQ^{-1}Cd\delta_t = -PQ^{-1}Bd\varepsilon_t$$

$$d\delta_t = -\left[PQ^{-1}C\right]^{-1} PQ^{-1}Bd\varepsilon_t.$$

Again, the explosive eigenvalues tell us the expectational errors as functions of the structural shocks – so long as there as are many explosive eigenvalues as there are expectational errors, i.e. $PQ^{-1}C$ is invertible.

The rows with stable eigenvalues then give us from (A1.93),

$$Md\tilde{z}_t = M\Lambda\tilde{z}_t dt + MQ^{-1}Bd\varepsilon_t + MQ^{-1}Cd\delta_t$$

$$Md\tilde{z}_t = M\Lambda\tilde{z}_t dt + MQ^{-1}Bd\varepsilon_t - MQ^{-1}C\left[PQ^{-1}C\right]^{-1} PQ^{-1}Bd\varepsilon_t$$

$$dM\tilde{z}_t = M\Lambda\left(P'P + M'M\right)\tilde{z}_t dt + MQ^{-1}\left[I_{N_v} - C\left[PQ^{-1}C\right]^{-1} PQ^{-1}\right]Bd\varepsilon_t.$$

With $P\tilde{z}_t = 0$,

$$d\left(M\tilde{z}_t\right) = M\Lambda M'\left(M\tilde{z}_t\right)dt + MQ^{-1}\left[I_{N_v} - C\left[PQ^{-1}C\right]^{-1} PQ^{-1}\right]Bd\varepsilon_t$$

We can reassemble the whole $\tilde{z}$ vector with

$$d\tilde{z} = \left(P'P + M'M\right)d\tilde{z}$$

$$d\tilde{z} = M'Md\tilde{z}$$

$$d\tilde{z}_t = \Lambda^*\tilde{z}_t dt + M'MQ^{-1}\left[I_{N_v} - C\left[PQ^{-1}C\right]^{-1} PQ^{-1}\right]Bd\varepsilon_t$$

where

$$\Lambda^* \equiv M'M\Lambda M'M$$

is the $N_v \times N_v$ diagonal matrix of eigenvalues, with zeros in place of the explosive eigenvalues.

This is the autoregressive representation of $\tilde{z}$. The moving average representation is

$$\tilde{z}_t = e^{\Lambda^* t}\tilde{z}_0 + \int_{s=0}^{t} e^{\Lambda^* t}M'MQ^{-1}\left[I_{N_v} - C\left[PQ^{-1}C\right]^{-1} PQ^{-1}\right]Bd\varepsilon_{t-s}$$

and the impulse-response function, i.e. to a single $d\varepsilon_0$ starting at $z_0 = 0$ is

$$\tilde{z}_t = e^{\Lambda^* t}M'MQ^{-1}\left[I_{N_v} - C\left[PQ^{-1}C\right]^{-1} PQ^{-1}\right]Bd\varepsilon_0 = e^{\Lambda^* t}Kd\varepsilon_0$$

Then, the original values are

$$z_t = Q\tilde{z}_t = Qe^{\Lambda^* t}Kd\varepsilon_0 \tag{A1.95}$$

To compute terms of the linearized identities as in Table 5.2, rather than sum up terms of the response functions, we can find the terms of the decomposition directly from the solution (A1.95). Let $a_x' z_t$ select variable x from the state vector z_t. Then the terms of the weighted-inflation identity (3.51) are

$$\int_{\tau=0}^{\infty} e^{-(r+\omega)\tau} a_\pi' z_\tau d\tau = -\int_{\tau=0}^{\infty} e^{-r\tau} a_s' z_\tau d\tau + \int_{\tau=0}^{\infty} e^{-rt}\left(1-e^{-\omega\tau}\right)\left(a_i' - a_\pi'\right) z_t d\tau$$

$$\begin{aligned} a_\pi' Q \left\{\int_{\tau=0}^{\infty} e^{[\Lambda^* - (r+\omega)I]\tau} d\tau\right\} K d\varepsilon_0 &= -a_s' Q \left\{\int_{\tau=0}^{\infty} e^{(\Lambda^* - rI)\tau} d\tau\right\} K d\varepsilon_0 \\ &+ \left(a_i' - a_\pi'\right) Q \left\{\int_{\tau=0}^{\infty} \left(e^{(\Lambda^* - rI)\tau} - e^{(\Lambda^* - (r+\omega)I)\tau}\right) d\tau\right\} K_0 d\tau \end{aligned}$$

Each of the terms in curly brackets is a diagonal matrix, e.g.

$$\left\{\int_{\tau=0}^{\infty} e^{(\Lambda^* - rI)\tau} d\tau\right\} = \begin{bmatrix} \frac{1}{r-\lambda_1} & & \\ & \frac{1}{r-\lambda_2} & \\ & & \ddots \end{bmatrix}.$$

The surplus is not directly an element of the state vector, so use

$$a_s = \theta_{s\pi} a_\pi + \theta_{sx} a_x + \alpha a_v + a_{ui}.$$

A1.8 Future sales

Section 7.3.3 described verbally the effect of expected future debt sales, with no change in surpluses, on the path of the price level. I present here an explicit version of that analysis.

With no long-term debt outstanding at time 0, expected future bond sales do not affect P_0. An expected future bond sale lowers P_1 and raises P_2, raising bond price $Q_0^{(1)}$ and lowering $Q_0^{(2)}$.

With long-term debt outstanding, expected future bond sales can affect P_0 as well. The sign depends on how much time 1 versus time 2 debt is sold at time 0, relative to the amount outstanding. If the government sells proportionally more time 1 debt than time 2 debt,

$$\frac{B_0^{(1)} - B_{-1}^{(1)}}{B_0^{(1)}} > \frac{B_0^{(2)} - B_{-1}^{(2)}}{B_0^{(2)}}$$

then expected future debt sales $B_1^{(2)} - B_0^{(2)} > 0$ lower the price level P_0, and vice versa.

The effects of QE-like bond purchases depend on expected future purchases and sales.

Section 7.3 calculated the effects on the price level of a variety of purchases and sales of long-term debt, while holding surpluses constant. Section 7.3.3 discussed verbally how expected *future* bond sales affect current prices, future prices, and hence long-term interest rates. I present the algebraic treatment here. The algebra quickly gets more tedious than enlightening, so I pursue a three-period example. Figure A1.1 illustrates.

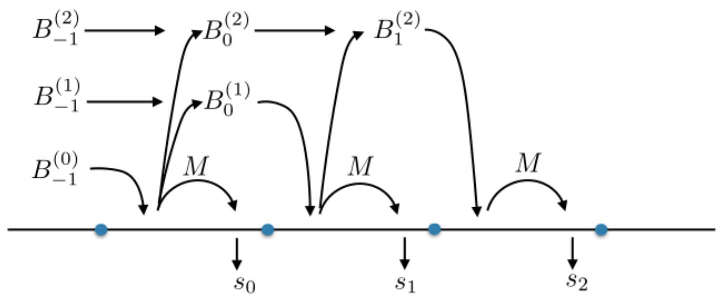

Figure A1.1: Long term debt example, illustrating the effects of future purchases and sales.

The novelty in this case is the additional sale $B_1^{(2)} - B_0^{(2)}$ during period 1. The formulas get complex because now we have outstanding period 0, 1, and 2 debt, time 0 sales of time 1 and time 2 debt and time 1 sales of time 2 debt to consider.

To solve this example, start at the final period 2. Debt $B_1^{(2)}$ is outstanding, so the price level is determined by

$$\frac{B_1^{(2)}}{P_2} = s_2. \tag{A1.96}$$

Total two-period debt $B_1^{(2)} = B_0^{(2)} + (B_1^{(2)} - B_0^{(2)})$ affects P_2.

- *Expected future bond sales and purchases $B_1^{(2)} - B_0^{(2)}$ enter symmetrically with time zero sales $B_0^{(2)} - B_{-1}^{(2)}$ in determining the expected price P_2*

The flow condition for period 1 gives us P_1,

$$\frac{B_0^{(1)}}{P_1} = s_1 + \beta \frac{\left(B_1^{(2)} - B_0^{(2)}\right)}{B_1^{(2)}} E_1(s_2). \tag{A1.97}$$

Expected future debt sales $B_1^{(2)} - B_0^{(2)}$ can affect the expected P_1

- *If the government leaves outstanding debt at time 0, $[B_0^{(2)} > 0]$, then expected sales $[B_1^{(2)} - B_0^{(2)}]$ of additional long-term debt can lower the expected price level P_1, and therefore raise the bond price $Q_0^{(1)}$.*

To find P_0, start with the period 0 flow condition

$$\frac{B_{-1}^{(0)}}{P_0} = s_0 + \beta E_0 \left(\frac{1}{P_1}\right)\left(B_0^{(1)} - B_{-1}^{(1)}\right) + \beta^2 E_0 \left(\frac{1}{P_2}\right)\left(B_0^{(2)} - B_{-1}^{(2)}\right).$$

Substituting in the prices P_1 and P_2 from (A1.96) and (A1.97), we have an expression for P_0,

$$\frac{B_{-1}^{(0)}}{P_0} = s_0 + \frac{\left(B_0^{(1)} - B_{-1}^{(1)}\right)}{B_0^{(1)}} E_0 \left[\beta s_1 + \frac{\left(B_1^{(2)} - B_0^{(2)}\right)}{B_1^{(2)}} \beta^2 s_2\right] + E_0 \left[\frac{\left(B_0^{(2)} - B_{-1}^{(2)}\right)}{B_1^{(2)}} \beta^2 s_2\right]. \tag{A1.98}$$

To make sense of this expression, I consider a few special cases of this special case.

A1.8.1 No outstanding long-term debt

Suppose there is no long-term debt outstanding, $B_{-1}^{(1)} = 0$ and $B_{-1}^{(2)} = 0$, and suppose the government sells some debt $B_0^{(1)}$ and $B_0^{(2)}$ at time 0. Equation (A1.98) reduces once again to

$$\frac{B_{-1}^{(0)}}{P_0} = s_0 + E_0 \left(\beta s_1 + \beta^2 s_2\right),$$

so the expected bond sale $B_1^{(2)} - B_0^{(2)}$ has no effect on P_0. Again, there must be debt outstanding to dilute in order for bond sales to affect the price level at time 0.

With P_0 fixed, future price levels translate to bond prices.

- *Expected future bond sales and purchases $B_1^{(2)} - B_0^{(2)}$ enter symmetrically with time zero sales $B_0^{(2)} - B_{-1}^{(2)}$ in determining the bond price $Q_0^{(2)}$*

This fact has an important implication for quantitative easing:

- *The effects of a bond sale or purchase $B_0^{(2)}$ on the long-term bond price $Q_0^{(2)}$ can be undone by expected future bond sales or purchases.*

When the government sells some debt at the end of time 0, expected future debt sales $B_1^{(2)} - B_0^{(2)}$ can affect the expected P_1 and hence, with P_0 fixed, the bond price $Q_0^{(1)}$.

- *If the government sells some long-term debt at time 0, [$B_0^{(2)} > 0$], then expected sales [$B_1^{(2)} - B_0^{(2)}$] of additional long-term debt raise the bond price $Q_0^{(1)}$.*

In sum, with no debt outstanding

- *The timing of expected future bond sales and purchases affects intermediate price levels and bond prices, even though it has no effect on the terminal price level* *and its time 0 price.*

A1.8.2 Outstanding long-term debt

When long-term debt *is* outstanding at time 0, $B_{-1}^{(j)} > 0$, expected future sales $B_1^{(2)} - B_0^{(2)}$ can affect the price level P_0.

$$\frac{B_{-1}^{(0)}}{P_0} = s_0 + \frac{\left(B_0^{(1)} - B_{-1}^{(1)}\right)}{B_0^{(1)}} E_0 \left[\beta s_1 + \frac{\left(B_1^{(2)} - B_0^{(2)}\right)}{B_1^{(2)}} \beta^2 s_2\right] + E_0 \left[\frac{\left(B_0^{(2)} - B_{-1}^{(2)}\right)}{B_1^{(2)}} \beta^2 s_2\right]. \tag{A1.99}$$

Note all the time 1 debt sales $B_1^{(2)} - B_0^{(2)}$ (implicit in $B_1^{(2)}$ terms) multiply time 0 debt sales, $B_0^{(1)} - B_{-1}^{(1)}$ or $B_0^{(2)} - B_{-1}^{(2)}$. If there are no time 0 sales, then there is no effect of time 1 sales on P_0. Put another way, time 1 sales only change the effects of time 0 sales.

- *Expected future sales only have an interaction effect on the initial price level P_0, modifying the dilution effects of time 0 sales in the presence of outstanding debt.*

With that in mind, it is easier to see how expected time 1 sales modify each of the time 0 sales in turn.

If there is no time 0 sale of time 1 debt, $B_0^{(1)} - B_{-1}^{(1)} = 0$, then the price level at time 0 simplifies to

$$\frac{B_{-1}^{(0)}}{P_0} = s_0 + E_0 \left[\beta^2 \frac{\left(B_0^{(2)} - B_{-1}^{(2)}\right)}{B_{-1}^{(2)} + \left(B_0^{(2)} - B_{-1}^{(2)}\right) + \left(B_1^{(2)} - B_0^{(2)}\right)} s_2\right].$$

The time 0 sale $B_0^{(2)} - B_{-1}^{(2)}$ dilutes expected future debt $B_1^{(2)} - B_0^{(2)}$ equally as it dilutes outstanding debt $B_{-1}^{(2)}$ as a claim to the time-2 surplus. Conversely, fixing the time 0 sale, the time 1 sale $B_1^{(2)} - B_0^{(2)}$ adds to the total, reducing the dilution from the time 0 sale. and raising P_0. (Don't also consider $B_{-1}^{(1)} = 0$, as then the fraction in the second term of (A1.98) is 0/0).

If there is no time 0 sale of time 2 debt, $B_0^{(2)} - B_{-1}^{(2)} = 0$ then the price level at time 0 simplifies to

$$\frac{B_{-1}^{(0)}}{P_0} = s_0 + \frac{\left(B_0^{(1)} - B_{-1}^{(1)}\right)}{B_0^{(1)}} \left[\beta E_0 \left(s_1\right) + E_0 \left(\left[\frac{B_1^{(2)} - B_0^{(2)}}{B_0^{(2)} + \left(B_1^{(2)} - B_0^{(2)}\right)}\right] \beta^2 s_2\right)\right]. \tag{A1.100}$$

The first term is the straightforward devalution effect. The second term shows the interaction. An expected debt sale at time 1 transfers resources from time 2 to time 1, and thereby enhances the effects of a time 0 sale of time 1 debt.

The general interaction mechanism is easiest to see in the last term if we write it as

$$\frac{B_{-1}^{(0)}}{P_0} = ... + E_0 \left[... + \beta^2 \frac{\left(B_0^{(2)} - B_{-1}^{(2)}\right)}{B_{-1}^{(2)} + \left(B_0^{(2)} - B_{-1}^{(2)}\right) + \left(B_1^{(2)} - B_0^{(2)}\right)} s_2 \right]. \tag{A1.101}$$

Selling additional debt at time 0 when there is debt outstanding $B_0^{(2)} - B_{-1}^{(2)}$ can raise revenue and affect the price P_0. The twist is that the denominator includes expected future debt sales as well as outstanding debt. Dilution occurs relative to all expected claims, even future ones. Conversely, greater debt sales $B_1^{(2)} - B_0^{(2)}$ dilute the time 0 sales $B_0^{(2)} - B_{-1}^{(2)}$, raising revenue for period 1 at the expense of period 0, and thus raising P_0.

The second-to last term of (A1.100) is more subtle. The first part $(B_0^{(1)} - B_{-1}^{(1)})/B_0^{(1)}$ expresses revenue raised at 0 by the dilution of outstanding time 1 debt. But time 1 debt is a claim to the revenues gained by diluting time 2 debt, as well as to s_1. That claim forms the interaction term.

The last two terms of (A1.100) partially offset. Expected future sales $B_1^{(2)} - B_0^{(2)}$ raise the value of the time 1 claim, and lower the value of the time-2 claim. The weights of the two terms are the fractions of each maturity's debt outstanding at the end of time 0 that was sold at time 0. When those two fractions are equal, when

$$\frac{B_0^{(1)} - B_{-1}^{(1)}}{B_0^{(1)}} = \frac{B_0^{(2)} - B_{-1}^{(2)}}{B_0^{(2)}}$$

the last two terms offset, and expected future debt sales have no effect.

- *The effect of expected future debt sales* $\left(B_1^{(2)} - B_0^{(2)}\right)$ *on* P_0 *depends on how much time 1 and time 2 debt is sold at time 0, relative to the amount outstanding. If the government sells proportionally more time 1 debt than time 2 debt,*

$$\frac{\left(B_0^{(1)} - B_{-1}^{(1)}\right)}{B_0^{(1)}} > \frac{\left(B_0^{(2)} - B_{-1}^{(2)}\right)}{B_0^{(2)}}$$

then expected future debt sales $B_1^{(2)} - B_0^{(2)} > 0$ *lower the price level* P_0*, and vice versa.*

Chapter Two

How not to test fiscal theory

> Many apparent tests and puzzles of the fiscal theory forget hard-won wisdom from time-series econometrics and empirical asset pricing. Leaving the value of debt out of a forecasting VAR is a mistake. Agents have more information than we do, so one cannot use such VAR forecasts to test the present value relation. With the value of debt in the VAR, and discounting by returns, the present value relation is an identity. The state of the art in asset pricing and macroeconomics examines which terms of the present value identity matter, as we have done, and tests discount factor models, but does not try to test the present value relation per se.
>
> Even with completely exogenous surpluses, we expect a positive regression coefficient of surpluses on debt, and debt to Granger-cause surpluses.

Just how best to evaluate and use fiscal theory remains an open question. But we can avoid the many false starts of our predecessors. Asset pricing spent decades figuring out how to empirically analyze present value relations. Macroeconomics spent decades wrestling with forward-looking relations such as the permanent income hypothesis of consumption. Both disciplines followed a number of attractive but in the end fruitless paths. This section adds to the comments on testing for regimes in Chapter 22 by a reminder of the lessons of this large literature, and linking those lessons to fiscal theory.

It is naturally appealing to forecast surpluses and model discount rates, and then come up with a prediction of what the value of debt should be, and given the nominal value of debt to predict the price level. It is natural to want to see if inflation lines up with changes in such values. It is natural to view such calculations as the fiscal theory's "prediction" for the value of debt, and a failure of that "prediction" as a "rejection" of fiscal theory. It is naturally appealing to look for Granger causality or other tests of the causal logic of active versus passive fiscal policy. More broadly, it is naturally appealing to search for a definitive time-series test of fiscal theory, on its own or versus some other theory. It is natural to criticize fiscal theory for lacking such an all-encompassing "testable predictions."

But all these natural impulses failed in asset pricing and macroeconomics during the 1980s and 1990s, and time-series econometrics analyzes that failure. We do not now "test" asset pricing by using independent (i.e. ignoring prices) forecasts of dividends, discounting them, and testing whether our independent calculation lines up with observed stock prices. We do not now "test" the theory of consumption by forming an independent forecast of permanent income. Yet those theories remain useful! So, let us remember and not rediscover the hard-won lessons of the past.

We should at least start by using the present value fiscal theory as we use other present value relations, and at a minimum not repeat fallacies that took a long time to understand in those other contexts. With the advantage of hindsight, it is difficult to remember just how much derision greeted forward-looking present value relations in economics and finance, for violating ancient intuition. The glass today may not be completely full, but it is hardly as empty as first thought.

For decades financial economists struggled to test present value relations. Lining up prices with forecasts of dividends from statistical models or surveys never seemed to work. Many armchair refutations of present value thinking seemed at hand, just as armchair refutations of fiscal theory seem easy in the correlations of inflation, debt, and deficits.

Volatility tests (Shiller (1981), LeRoy and Porter (1981)) seemed to formalize a rejection of the present value relation, that prices are wildly far from the present value of dividends, even though monthly returns are poorly predictable. But the extra assumptions of that work turned out to be consequential. It took a decade of dissection culminating in Campbell and Shiller (1988) to understand that present value tests and long-run return forecasts from dividend/price ratios are the same thing, that all controversy is only about the source of expected return variation, that it is pointless to test the present value relation per se. In Campbell and Shiller's work, as in fiscal theory discounted by the ex post return or other by-construction discount factor, the present value relation is an untestable identity. Campbell and Shiller forecast dividend growth and returns using observed price/dividend ratios, and measure terms of the present value identity which spits out exactly the same price/dividend ratio as goes in. They do try to forecast returns and dividend growth without using prices, and use the present value identity to make an independent measure of what the price/dividend ratio should be. But it remains interesting to measure which terms (cash flow versus discount rate) move to account for price changes. Puzzles come in reconciling cash flows and discount rates with economic or, someday, psychological models, not in "testing" the present value per se. That's what asset pricing does now, and it is a fruitful precedent for fiscal theory. (Cochrane (1991b), Cochrane (1992) are my contributions to this literature; reviews in Cochrane (2005a) and Cochrane (2011c).)

This approach looks easy in retrospect, but it was hard-won knowledge. In the 1960s it seemed that one could test market efficiency by looking at returns alone, looking for random walk stock prices for example. The discount factor existence theorems removed that hope. Volatility tests seemed to offer a way to test and reject the present value identity. But the reconciliation of volatility tests with long-term return studies removed the same hope.

Fiscal-theory empirical work followed much of the same path with a two to three decade lag. Most analysis links inflation only to changes in surpluses, not to changes in discount rates. People still try to use independent forecasts of surpluses, from statistical models, surveys, or government agencies; discount them, and compare present value calculations to the actual value of debt. They attempt to measure fiscal versus monetary regimes, to test fiscal theory, and they proclaim failure or puzzle when it doesn't work. (Jiang et al. (2019) is a recent and notable example.) Such failures are chalked up as rejections of the underlying fiscal theory or the present value relation, not as a puzzle of discount rates, or the reflection of restrictions on the surplus forecasts. These repeat the misconceptions of 1980s asset pricing and macroeconomics.

In particular, if we forecast surpluses and discount rates including the value of debt in the forecasts, and if we discount with ex post returns, the present value relationship works exactly. But it is a non-testable identity. To test the present value relation, to obtain a predicted value of debt that is not mechanically equal to the observed value, we must restrict the discount factor process or leave the value of debt out of the forecast. But, as I will show below in some detail, leaving the value of debt out of the forecast is a mistake. It leads to false rejections even when the fiscal theory is true. There always *is* a discount factor that produces the observed price, so we are back to arguing, correctly, about the economic foundations of discount factors, not the present value relation per se. That does not mean there is nothing to do. We can follow Campbell and Shiller, and include the value of debt in the VAR to forecast surpluses and returns, resulting in an interesting decomposition but recognizing the present value relation per se is an untestable identity.

The fading lessons of 1980s time-series econometrics bear as well. Measuring the long string of higher-order correlations that add up to $a(\rho) = \sum_{j=0}^{\infty} \rho^j a_j$ of a moving average representation $s_t = a(L)\varepsilon_t$ is hard. Standard time-series methods, focused on short-run forecasts, and using univariate or restricted VARs, can fail miserably.

More generally, I think we have all learned that it is a bad idea to try to test whole classes of theories. All theories rely on auxiliary modeling assumptions. We can and should construct models, surplus forecasts, discount rate models, and then construct present values and compare them with data. But when they fail, that tells us only that we need a better model.

In this chapter I summarize and apply some of this classic time-series and present value history and apply it to government debt.

A2.1 Time-series lessons

Measuring the long string of higher order correlations that drive $a(\rho)$ is hard. The lessons of time-series econometrics emphasize that one should include the value of debt when forming long-run surplus or discount rate forecasts, just as one should include the price/dividend ratio when forming long-run return or dividend growth forecasts, and one should include consumption when forming long-run income forecasts. Omitting these variables can not only lead to a dramatic loss of power, it can lead to flat-out mistakes. There are several different perspectives behind this advice.

A2.1.1 Beware the ARMA(1,1)

The most likely, s-shaped, form of the surplus process has features that make its long-run forecasts particularly difficult to measure if one excludes the value of debt from the forecast. This fact is easiest to see in the example from Section 5.5. There we studied a simple process (5.22)-(5.23), which I simplify further to

$$s_{t+1} = \alpha v_t + \varepsilon_{s,t+1} \tag{A2.1}$$

$$\rho v_{t+1} = v_t + \beta_s \varepsilon_{s,t+1} - s_{t+1} \tag{A2.2}$$

that is equivalent to an s-shaped moving average representation (5.26),

$$s_{t+1} = \frac{\left(1 - \frac{1}{\rho}L\right) + \beta_s \frac{\alpha}{\rho} L}{1 - \frac{1-\alpha}{\rho} L} \varepsilon_{s,t+1}.$$

I keep $\alpha > 1 - \rho$ so that debt remains stationary, and the denominator coefficient $(1-\alpha)/\rho < 1$. As we have seen, this sort of process captures the central facts in the data.

We can write the surplus moving average

$$s_{t+1} = a(L)\varepsilon_{s,t+1} = \left(\frac{1 - \frac{1-\alpha\beta_s}{\rho} L}{1 - \frac{1-\alpha}{\rho} L} \right) \varepsilon_{s,t+1} = \left(1 - \frac{\frac{\alpha(1-\beta_s)}{\rho}}{1 - \frac{1-\alpha}{\rho} L} L \right) \varepsilon_{s,t+1}. \tag{A2.3}$$

The empirically and theoretically relevant case is that $\beta = a(\rho)$ is zero or a small positive number.

The right-hand expression writes the response function as 1 in one direction followed by a small and geometrically decaying set of responses in the other direction.

The second equality writes the response function in more conventional ARMA format. In the case $\beta_s = 1$, we recover the i.i.d. shock $s_{t+1} = \varepsilon_{t+1}$. For smaller β_s, the numerator coefficient on the lag operator is slightly larger than the denominator coefficient and we have an ARMA(1,1) with nearly canceling roots – a classic econometric trap. The long tail of small responses all in the same direction dramatically affects the long-run properties of the series $a(\rho)$, and especially of its cumulation – debt cumulates surpluses, levels cumulate growth rates.

We have already seen in Figure 4.3 how close the true process is to an approximating AR(1), yet how different the weighted *sum* $a(\rho)$ of response functions can be. Conventional time series estimation techniques minimize one-step ahead prediction errors $\min \operatorname{var}(s_{t+1} - E_t s_{t+1})$ that do not much weight these long-run features. Adding uncertainty over the true process – add $a_u(L)\varepsilon_{s,t+1}$ – emphasizes even more the wisdom of experience: If you want to learn the long-run behavior of a time series, involving discounted sums of moving average coefficients, finding the long-run implications of short-run ARMA models is a dangerous procedure. These statements are a summary of the lessons of Cochrane (1988), Campbell and Mankiw (1987). The long-run risks literature following Bansal and Yaron (2004), Bansal, Kiku, and Yaron (2012) faces similar difficulties. See for example Beeler and Campbell (2012).

One could try techniques that put more weight on fitting long-run forecasts, as this literature explores. But when we have a forward-looking variable such as the value of debt, price-dividend ratio, or consumption-income ratio, that reveals long-run forecasts, including that variable in the forecasting VAR easily outperforms long-run-oriented time-series estimates that ignore those variables. The estimates based on a simple VAR with a forward-looking variable in Cochrane (1994b) are far preferable to the univariate variance-ratio estimate in Cochrane (1988). Similarly, regressions of returns on dividend yields in Fama and French (1988a) uncover long-run forecastability better than long-run autocorrelations in Fama and French

(1988b) and Poterba and Summers (1988).

A2.1.2 Include cointegrating vectors and slow-moving forecasters

The bottom line of the unit root/long-run estimation literature is this: *Include a cointegrating vector in the VAR.* (I learned this lesson in Cochrane (1994b)). A cointegrating vector captures how far away variables are from their long-run values, and acts as a state variable for the long sum of future forecasts in $a(\rho)$.

In forecasting long-run consumption and income, include the consumption/income ratio as a forecasting variable. If that ratio is far from its mean, it indicates long steady forecastable growth in one of consumption or income. In forecasting long-run stock returns and dividend growth, include the price/dividend ratio as a forecasting variable. If prices are much higher than dividends, we can forecast that the level of prices will decline, or the level of dividends will rise, i.e. a period of low long-run returns or high long-run dividend growth.

The econometric lesson does not require cointegration. In this context, (A2.1)-(A2.2) are almost identical to a vector autoregression of return or dividend growth (in the place of s_t) and dividend yield (v_t). The value of debt is a slow-moving stationary variable that forecasts surpluses, accumulates surpluses, and thus captures long-run surplus forecasts that are hard, or as we will shortly see impossible, to measure from the history of surpluses themselves.

A2.1.3 Beware the non-invertible representation

For $\beta_s = a(\rho) = 0$, the example A2.3 simplifies to

$$s_{t+1} = \left(\frac{1 - \frac{1}{\rho} L}{1 - \frac{1-\alpha}{\rho} L} \right) \varepsilon_{s,t+1}. \tag{A2.4}$$

The numerator coefficient is greater than one. This ARMA(1,1) is not invertible, and hence it *cannot* be recovered by any autoregression, or any other time series technique using the history of surpluses, or excluding the value of debt in the VAR. Leeper, Walker, and Yang (2013) have several excellent examples of the perils of non-fundamental representations. Fernández-Villaverde et al. (2007) give a nice treatment, emphasizing the role of state variables that agents see but we do not include in the VAR.

If you generate data by (A2.4), run an autoregression, and find the implied moving average, you recover

$$s_{t+1} = \left(\frac{1 - \rho L}{1 - \frac{1-\alpha}{\rho} L} \right) w_{s,t+1}. \tag{A2.5}$$

The shocks are one-step ahead prediction errors from the autoregression, $w_{s,t+1} = s_{t+1} - E(s_{t+1}|s_t, s_{t-1}, ...)$. This fitted process has

$$a(\rho) = \frac{1 - \rho^2}{\alpha}$$

not the correct answer $a(\rho)=0$! An autoregression recovers the Wold moving average representation, which is invertible. The spectral density $S(\omega)=a(e^{i\omega})a(e^{-i\omega})$ of (A2.5) and (A2.4) is the same, so (A2.5) is the Wold moving average representation of the true process (A2.5).

This general observation holds beyond the specific univariate ARMA(1,1) example (A2.4). A government that pays back its debts runs a surplus process with $a(\rho)=0$. $\rho\le 1$. The condition for invertibility is that all zeros of the moving average representation are outside the unit circle. So the moving average representation of the surplus $s_t=a(L)\varepsilon_{s,t}$ *must* be non-invertible. *The project of estimating a surplus process without including the value of debt to test whether governments pay back their debts is doomed.* As Hansen, Roberds, and Sargent (1992) (p. 122) put it concisely "any vector autoregressive representation for $[\{s_t\}]$ must correspond to a moving average representation that violates this restriction" $[a(\rho)=0]$ – even if the data are generated by a government that obeys the restriction."

The true ε shocks and the true non-invertible moving average can still be recovered if we include debt in the VAR. We can write (A2.1)-(A2.2)

$$s_{t+1}=\alpha v_t+\varepsilon_{s,t+1} \tag{A2.6}$$

$$v_{t+1}=\frac{1-\alpha}{\rho}v_t-\frac{1-\beta_s}{\rho}\varepsilon_{s,t+1}. \tag{A2.7}$$

$$\left[\begin{array}{c} s_{t+1}\\ v_{t+1}\end{array}\right]=\left[\begin{array}{cc} 0 & \alpha\\ 0 & \frac{1-\alpha}{\rho}\end{array}\right]\left[\begin{array}{c} s_t\\ v_t\end{array}\right]+\left[\begin{array}{c} 1\\ -\frac{1-\beta_s}{\rho}\end{array}\right]\varepsilon_{s,t+1}. \tag{A2.8}$$

The eigenvalues of the transition matrix are $(1-\alpha)/\rho$ and 0, both less than one, so this is the joint Wold moving average representation. The true *surplus* shock can be recovered from the history of *debt*, because debt reflects and reveals to us the expectations of future surpluses that we need to identify the true surplus process.

The non-invertibility problem occurs for any $\beta_s<(1-\rho)/\alpha$. But the difficulty of estimating a process with nearly canceling and nearly non-invertible roots, and how easy it is to estimate that process when one includes the value of debt, extends for larger values of β_s.

A2.1.4 An MA(1) example

The MA(1) gives a simple and clear though unrealistic example. Suppose the surplus follows

$$s_t=a(L)\varepsilon_t=\varepsilon_t+\theta\varepsilon_{t-1}.$$

Directly, the value of debt is

$$\frac{B_{t-1}}{P_t}=E_t\sum_{j=0}^{\infty}\beta^j s_{t+j}=(1+\beta\theta)\varepsilon_t+\theta\varepsilon_{t-1}. \tag{A2.9}$$

If a government wishes no unexpected inflation, the surplus process must follow

$$a(\beta)=1+\beta\theta=0.$$

Therefore, the process must follow $\theta = -R = 1/\beta$,

$$s_t = \varepsilon_t - R\varepsilon_{t-1} \tag{A2.10}$$

$$\frac{B_{t-1}}{P_t} = \varepsilon_t - R\varepsilon_{t-1} + \beta R\varepsilon_t = -R\varepsilon_{t-1}. \tag{A2.11}$$

In words, this government issues debt $B_{t-1}/P_t = \varepsilon_{t-1}$ at time $t-1$ to fund the surplus shock ε_{t-1}, and then pays it back one period later, with interest R.

But the moving average (A2.10) is not invertible, so it cannot be estimated by regressions of s_t on the history of s, or any other technique using that data. If we try to invert the true process (A2.10),

$$\frac{s_t}{1-RL} = \varepsilon_t$$

$$s_t + Rs_{t-1} + R^2 s_{t-2} + ... = \varepsilon_t \tag{A2.12}$$

we see exploding coefficients on the left-hand side.

What would happen if you ran autoregressions of surpluses from data generated by (A2.10)? You would recover the wrong coefficient, the wrong error, and you would predict inflation volatility where there is none. Run the autoregression

$$b(L)s_t = w_t.$$

From this regression you recover a stationary and invertible $b(L)$. When you invert it, you find an MA(1),

$$s_t = w_t - \beta w_{t-1}. \tag{A2.13}$$

Comparing (A2.10) and (A2.13), you recover a moving average coefficient $\theta = -\beta = -1/R$ not the correct $\theta = -R$, and you recover $w_t \neq \varepsilon_t$, the wrong shock. (To show (A2.13), match autocovariances.)

Most of all, using (A2.9), (A2.13) implies

$$(E_t - E_{t-1})\sum_{j=0}^{\infty} \beta^j s_{t+j} = (1-\beta^2)w_t,$$

not zero. You predict volatile inflation, and you are puzzled to see constant inflation. The mistake, really, is using the same symbol E_t to mean expectation conditional on agent's information, which includes the ε_t, and expectation conditional on our information, just the set of current and past s_t.

Now, consider the *joint* process of surplus and debt. From (A2.10) and (A2.11), the fundamental (using structural shocks ε) joint moving average is

$$s_t = \varepsilon_t - R\varepsilon_{t-1} \tag{A2.14}$$

$$\frac{B_t}{P} = -R\varepsilon_t. \tag{A2.15}$$

(Here, since $P_t = P$ constant, I locate B_t/P_{t+1} in the time t information set, which clarifies the example.) Inverting this moving average, we find an autoregressive representation

$$\begin{aligned} s_t &= \varepsilon_t + B_{t-1}/P \\ B_t/P &= -R\varepsilon_t. \end{aligned}$$

Or, to be super explicit,

$$\begin{bmatrix} s_t \\ B_t/P \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} s_{t-1} \\ B_{t-1}/P \end{bmatrix} + \begin{bmatrix} \varepsilon_t \\ -R\varepsilon_t \end{bmatrix}. \tag{A2.16}$$

The terms here do converge unlike (A2.12). The right-hand variables are uncorrelated with the error term. So OLS regressions uncover exactly (A2.16). You can work backwards: If you run this *vector* autoregression, including debt on the right-hand side,

$$\begin{bmatrix} s_t \\ B_t/P \end{bmatrix} = A \begin{bmatrix} s_{t-1} \\ B_{t-1}/P \end{bmatrix} + \begin{bmatrix} w_t^s \\ w_t^b \end{bmatrix},$$

this is a consistent estimate of the structural VAR (A2.16). Inverting that VAR you can estimate the structural impulse-response function (A2.14) or (A2.16). Equation (A2.15) provides the key – the value of debt, which we can observe, reveals to us agents' information about the structural shock, just as the value of equity reveals to us a slice of agents' information about future dividends.

A2.1.5 Invertibility with a general moving average

The same points hold in general, though less transparently than in the MA(1) or VAR(1) examples. The surplus follows a general moving average based on structural shocks

$$s_t = a(L)\varepsilon_t$$

with $a(\rho) = 0$.

One cannot estimate $a(L)$ and test $a(\rho) = 0$ from *any* autoregression. The condition for a moving average representation to correspond to an autoregression is that all the zeros of $a(z)$ lie outside the unit circle. The condition $a(\rho) = 0$ means that a zero lies inside the unit circle, so this structural representation is not invertible.

Factor $a(L)$,

$$a(L) = \frac{(1-\lambda_1 L)(1-\lambda_2 L)...(1-\rho^{-1}L)}{(1-\delta_1 L)(1-\delta_2 L)...}. \tag{A2.17}$$

If we have $a(\rho) = 0$, one of the numerator factors must be $(1-\rho^{-1}L)$ as written. But since $\rho^{-1} > 1$, you can't invert to write this surplus process in autoregressive form

$$\frac{(1-\delta_1 L)(1-\delta_2 L)...}{(1-\lambda_1 L)(1-\lambda_2 L)} \frac{1}{(1-\rho^{-1}L)} s_t = \varepsilon_t$$

since the $(1-\rho^{-1}L)$ root blows up going backwards. If you do run an autoregression you recover a representation that is invertible, by the Wold decomposition theorem. If all the other δ and λ in the structural representation are appropriately less than one, an autoregression yields

$$\frac{(1-\delta_1 L)(1-\delta_2 L)...}{(1-\lambda_1 L)(1-\lambda_2 L)}\frac{1}{(1-\rho L)}s_t = w_t.$$

You have both the wrong root, ρ not ρ^{-1}, and the regression error is not the structural shock $w_t \neq \varepsilon_t$. After you invert, the resulting estimate of $a(L)$ does not have $a(\rho) = 0$ even in infinite data. Thus, it implies innovations to inflation that do not exist. Though the surplus follows an exogenous univariate process, the whole procedure of estimating a surplus process and discounting it is wrong. (You know this is the autoregressive representation by matching spectral densities $a(e^{-i\omega})a(e^{i\omega})$.)

Given a surplus process with $a(\rho) = 0$, debt follows the structural moving average representation

$$v_t = \sum_{j=0}^{\infty} \rho^j s_{t+1+j} = \frac{a(L)\varepsilon_{t+1}}{1-\rho L^{-1}} = \frac{s_{t+1}}{1-\rho L^{-1}}. \tag{A2.18}$$

Yes, debt equals the ex post as well as expected present value of surpluses.

We can get to (A2.18) by

$$\rho v_{t+1} = v_t - \Delta E_t \pi_{t+1} - s_{t+1}$$

with $a(\rho) = 0$, we have $\Delta E_{t+1}\pi_{t+1} = 0$ always, so

$$\rho v_{t+1} = v_t - s_{t+1}$$

and thus (A2.18). The fact that people know the government will adjust surpluses $\{s_{t+j+1}\}$ to offset shocks to s_{t+1} to give a constant price level is the key in this example. More elegantly, for general $a(L)$ the value of the debt follows a variant of the Hansen and Sargent (1981) prediction formula for geometric sums

$$E_t \sum_{j=0}^{\infty} \rho^j s_{t+1+j} = \frac{[a(L) - a(\rho)]\, L^{-1}}{1-\rho L^{-1}}\varepsilon_t. \tag{A2.19}$$

(To derive this equation, note the $a(L)$ term gives the present value of ex post surpluses, and then see that the $a(\rho)$ term subtracts off all the future shocks ε_{t+j}. See Sargent (1987) p. 381-385.) Thus, if $a(\rho) = 0$ we have (A2.18).

Now, express (A2.18) using the factor representation of $a(L)$, (A2.17)

$$\begin{aligned} v_t &= \frac{a(L)L^{-1}}{1-\rho L^{-1}}\varepsilon_t = -\frac{a(L)\rho^{-1}}{1-\rho^{-1}L}\varepsilon_t \\ &= -\frac{\rho^{-1}}{(1-\rho^{-1}L)}\frac{(1-\lambda_1 L)(1-\lambda_2 L)...(1-\rho^{-1}L)}{(1-\delta_1 L)(1-\delta_2 L)...}\varepsilon_t \end{aligned}$$

and thus,

$$= -\rho^{-1}\frac{(1-\lambda_1 L)(1-\lambda_2 L)...}{(1-\delta_1 L)(1-\delta_2 L)...}\varepsilon_t. \tag{A2.20}$$

Debt, though it is a strange-looking present value of future surpluses, is in fact a proper function of current and past shocks ε_t, because the surplus process wipes out any shocks to that present value. The non-invertible root cancels – the debt is an invertible moving average of the structural shock. Thus, one can recover the structural shock to *surpluses* from an autoregression of *debt* on past *debt*.

Equation (A2.20) is therefore also the moving average representation of that autoregression of debt on past debt, up to a normalization of the size of the shock ε_t. It, together with (4.1) and (A2.17),

$$s_t = a(L)\varepsilon_t = \frac{(1-\lambda_1 L)(1-\lambda_2 L)...(1-\rho^{-1}L)}{(1-\delta_1 L)(1-\delta_2 L)...}\varepsilon_t \tag{A2.21}$$

are now the moving average representation of the debt and surplus VAR, which is invertible.

This analysis also puts to rest a related temptation, to test the Granger-causality between surpluses and debt to test whether fiscal policy is active or passive. We see here that in the vector autoregressive representation for surpluses and debt together, regression shocks to debt help to forecast surpluses. Fiscal theory is active, yet debt Granger-causes surpluses.

One might get excited by these VAR and Granger-causality examples. Yes, estimating a surplus process that excludes debt and discounting the forecasted surplus will not work. But it seems one can run an autoregression that includes *debt*, recover the structural shocks ε_t, run a regression of surpluses on current and past debt shocks as in (A2.21), and test whether $a(\rho) = 0$. Hansen, Roberds, and Sargent (1992) propose this test, and generalize to the case that some of the other zeros of $a(L)$ are inside the unit circle.

Alas, this test does not extend to a time-varying discount rate, and time-varying discount rates are central to making sense of the data. When we do extend to time-varying discount rates, the restriction $a_s(\rho) - a_r(\rho) = 0$ holds as a non-testable identity.

A2.1.6 People have more information than we do

So far, we see that it is *wise* to include the value of debt in a surplus forecasting regression, but it is not yet *wrong* to omit the value of debt unless we face a non-invertible moving average. The fact that agents have more information than we do makes it generically wrong to leave out the value of debt. And only by leaving out the value of debt can we try to test the present value relation, i.e. make a prediction about the value of debt that is not tautologically true. So, here we have in a nutshell why testing the present value relation per se is a hopeless cause.

The general argument is simple. Let Ω denote the information set of people in the economy. Then the valuation equation

$$v_t = \left(E \sum_{j=0}^{\infty} \beta^j s_{t+1+j} \middle| \Omega_t \right) \tag{A2.22}$$

implies a present value relation using our forecasts on the right-hand side

$$v_t = \left(E \sum_{j=0}^{\infty} \beta^j s_{t+1+j} \middle| I_t \subset \Omega_t \right) \tag{A2.23}$$

where I_t is the VAR information set, *only if* we include the value of debt in the VAR, $v_t \in I_t$, or if agents use no more information than we have in the VAR, $I_t = \Omega_t$ – a very restrictive assumption. Otherwise, we have $E(v_t|I_t)$ on the left-hand side. Leaving v_t out of the VAR, if $v_t \notin I_t$, the present value relation (A2.22) does not imply the relation (A2.23) that one tests with the VAR.

In (A2.22) we see that the value of debt reveals *agent's* expectations of the present value of surpluses, including the larger information set that we do not observe. That makes it such a useful forecasting variable – as consumption and stock prices are useful forecasting variables.

How to adapt econometric procedure to the fact that agents have more information than we have took a long time. Faced with a present value relation – $B_{t-1}/P_t = E_t \sum_{j=0}^{\infty} \beta^j s_{t+j}$, for example – one's first and natural instinct is to fit a time series process to s_t by forecasting regressions, or examine analyst or survey forecasts, compute the right-hand side, and compare it to the left-hand side. When the two calculations don't match up, one declares a puzzle.

This situation is exactly what faced macro and financial economists in the late 1970s, studying present value relations in finance and the permanent income hypothesis in macroeconomics. Starting with

$$p_t = E_t \sum_{j=1}^{\infty} \beta^j d_{t+j},$$

what could be more natural than to model dividends, say as an AR(1)

$$d_{t+1} = \eta_d d_t + \varepsilon_{t+1},$$

to calculate a present value,

$$E_t \sum_{j=1}^{\infty} \beta^j d_{t+j} = \frac{\beta \eta_d}{1 - \beta \eta_d} d_t,$$

and to compare the result to p_t? The result is a disaster – prices do not move one for one with dividends, or with VAR forecasts of dividends that exclude the price (really price/dividend ratio), or analyst or survey forecasts.

Similarly, start with the permanent income model of consumption c and income y,

$$c_t = c_{t-1} + r\beta \sum_{j=0}^{\infty} \beta^j \left(E_t - E_{t-1}\right) y_{t+j}.$$

What could be more natural than to model income as

$$y_t = \eta_y y_{t-1} + \varepsilon_t,$$

compute the present value, and compare it to consumption? The resulting model predicts a tight relation between consumption and income,

$$c_t - c_{t-1} = \frac{r\beta}{1-\beta\eta_y}(y_t - \rho y_{t-1}).$$

This result is not quite as awful, but it is easy to reject statistically. The 100% R^2 prediction fails – there is no error term in the latter relation – and other variables help to predict consumption growth.

As illustrative exercises and as models, there is nothing wrong with these calculations. These calculations are really simple general equilibrium models. Such models are useful for generating patterns reminiscent of those in the data and understanding mechanisms. But they are easily falsifiable as literal, testable representations of reality. As in these examples, the models typically contain 100% R^2 relations between variables, "stochastic singularities," unavoidable when a model has more variables than shocks.

Thus, as tests of the present value relation, these procedures make several crucial mistakes. Vital here, these tests presume that agents, forming prices and setting consumption, have no more information than we do in specifying the dividend or income time-series models. This assumption is patently wrong. One should ask of any test in macroeconomics or finance, does this test (usually implicitly) assume agents have no more information than we use? Too many tests still fail that question. (These tests also presume constant expected returns, and they mistreat unit roots in dividends, prices, and income. We end up fixing all three issues.)

When we model surplus as an AR(1),

$$s_{t+1} = \eta_s s_t + \varepsilon_{t+1},$$

compute present values such as

$$v_t = E_t \sum_{j=0}^{\infty} \rho^j s_{t+1+j} = \frac{\eta_s}{1-\rho\eta_s} s_t$$

$$\Delta E_{t+`}\pi_{t+1} = -a(\rho)\varepsilon_{t+1} = -\frac{1}{1-\beta\eta_s}(s_{t+1} - \eta_s s_t),$$

and if we interpret the evident and large empirical failures of these calculations as rejections of the present value relation or rejections of the fiscal theory, we repeat exactly this mistake. The latter interpretation is doubly wrong, since the present value relation holds under both active and passive fiscal policy.

This failure is more general than an AR(1). If we add extra variables to a VAR that forecasts s_t, omitting the value of debt v_t itself, and follow the same procedure, we still assume that agents only see the variables of our VAR and no more.

What can we do? *Include the value of debt in the VAR.* If the resulting VAR shows an s-shaped surplus process and no more puzzle, well, too bad, the puzzle (such as proclaimed by Jiang et al. (2019)) hinges on the assumption that agents have no more information than we have, so it isn't a puzzle.

A2.2 So how do we test present value relations?

OK, you can't omit the value of debt from the VAR that forecasts surpluses and discount rates. But suppose you put the value of debt in the VAR. Now, how do you test the present value relation?

Repeating previews of the last section a bit, the short answer is, you can't. If we allow time-varying expected returns, the present value relation is an identity. Apparent tests are tests of auxiliary hypotheses, such as agents don't have more information than the history of surpluses, or expected returns are constant over time, that are surely false, or tests of surplus or discount rate models that may be true. Specifying and testing models of expected return variation is interesting and important. It's how we make theories useful. But such tests are not tests of the present value relation per se.

The culmination of this sort of exercise in finance, the literature following Campbell and Shiller (1988), no longer pretends to test the present value relation per se. Instead, it investigates the terms of the present value identity. Do prices rise on news of higher future dividends or lower future discount rates? When do those events occur? It has to be one of the two. "Neither" is not a coherent answer. (Cochrane (2008) is a whole paper devoted to this point.) The worst such a calculation can do is to point to large or puzzling discount rate variation, that one may find implausible or hard to model, but it cannot reject the present value identity. We should learn rather than rediscover this lesson.

To see the point explicitly, suppose that data including surplus and value of debt follow a VAR,

$$z_{t+1} = Az_t + \varepsilon_{t+1}.$$

The flow identity (3.17)

$$\rho v_{t+1} = v_t + r^n_{t+1} - \pi_{t+1} - g_{t+1} - s_{t+1} \tag{A2.24}$$

implies that the VAR coefficients must satisfy

$$(I - \rho A)a'_v = \left(-a'_{r^n} + a'_\pi + a'_g + a'_s\right) A. \tag{A2.25}$$

These are not restrictions we need to impose. Since the data, if properly constructed, must obey (A2.24), the estimated parameters will automatically obey (A2.25).

Now, let us try to test the present value relation, (3.18),

$$v_t = E_t \left[\sum_{j=1}^{\infty} \rho^{j-1} s_{t+j} + \sum_{j=1}^{\infty} \rho^{j-1} g_{t+j} - \sum_{j=1}^{\infty} \rho^{j-1} \left(r^n_{t+j} - \pi_{t+j}\right) \right]. \tag{A2.26}$$

We compute the terms on the right-hand side from the VAR as

$$\left(a'_s + a'_g - a'_{r^n} + a'_\pi\right)\left(I - \rho A\right)^{-1} Az_t.$$

so the present value holds if

$$a'_v \stackrel{?}{=} \left(a'_s + a'_g - a'_{r^n} + a'_\pi\right)\left(I - \rho A\right)^{-1} A.$$

So long as the variables are stationary, the eigenvalues of A are less than one, this restriction is identical to the restriction coming from the flow identity (A2.25). With v_t in the VAR, and without restrictions on expected returns $E_t r^n_{t+1}$ (or the other variables, but that one is most common) the constructed present value of surpluses comes out to be each day's value of debt, exactly, and by construction. Equation (A2.26) reduces to $v_t = v_t$.

In the rear-view mirror, this statement is obvious. After all, we derived (A2.26) by iterating forward (A2.24), so it is unsurprising that the result is a present value *identity*. The $(I - \rho A)^{-1}$ operation just does the forward iteration that we did by hand to derive (A2.26). We're looking at a tautology, not a test. (Throughout there is extra content in convergence of the present value, i.e. the absence of explosive terminal values. This alternative is not the one usually in mind, so I leave out the discussion of such "rational bubbles" here.)

A2.2.1 Point nulls are pointless.

The unit root literature spent a lot of time testing unit roots against the alternative of a root less than one, indicating a stationary process. The asymptotic distribution theory is sharply different for a root of exactly one. But common sense should warn us that a root of 1.000 versus a root of 0.999 cannot possibly make a difference in a finite sample. Unit-root asymptotics can be a better guide to small sample distributions even if one knows the true root is 0.999. Pre-testing for unit roots and then imposing that form in subsequent analysis is a classic econometric trap. The test does not measure costs and benefits of the imposition. (My view is in Cochrane (1991a).)

The same situation occurs in fiscal theory tests. If we think $\gamma = 0$ versus $\gamma > 0$ in a regression $s_t = \gamma v_{t-1} + b(L)s_{t-1} + \varepsilon_t$ is the distinguishing characteristic of active versus passive fiscal policy, including $\alpha = 0.001$, then clearly we are asking a question that cannot make a difference for our sample. To the observation that we cannot reject $\gamma = 0$ in a regression $s_{t+1} = ... + \gamma v_t ... + ... \varepsilon_{t+1}$, we should answer that we also cannot reject positive numbers. To fail to reject is not to accept. There is no reason that zero is a default null hypothesis. Of course, we should add that $\gamma = 0$ versus $\gamma > 0$ is not a test of active versus passive policy in the first place, and that γ is not identified, as the counterexample of Section 5.4 emphasizes.

A2.3 Summary: What can and cannot be tested

The quantity $a(\rho)$ and the restriction $a(\rho) = 0$ can be estimated and tested. Run a VAR including at least the value of debt, estimate and test.

We can also estimate $a(\rho)$ and the present value restriction $a(\rho) = 0$ in a model with a constant discount rate. This is the insight of Hansen, Roberds, and Sargent (1992). Section 4.2 suggests that the most potent pieces of evidence are likely to be cross-equation restrictions, in particular whether the value of debt rises or declines when there is a shock to surpluses. Here one alternative to $a(\rho) = 0$ is a constant interest rate with $a(\rho) > 0$. The alternative also includes time-varying discount rates, in which case the value of debt and inflation move with no surplus news at all. If we think of the alternative being that the present value relation does not hold, that must be because of an explosive bubble term, but such a term would render the value of debt and potentially other variables non-stationary, so the properties of a test against that alternative are challenging. The present value relation in a model with a constant discount rate is not an identity. Therefore, some of the testable content is that discount rate model.

When we enlarge our view to present value relations with a time-varying discount rate, matters change. Recall the linearized identity,

$$v_t = \sum_{j=1}^{T} \rho^{j-1} s_{t+j} - \sum_{j=1}^{T} \rho^{j-1} r_{t+j} + \rho^T v_{t+T}$$

where $r_t \equiv r_t^n - \pi_t$ denotes the ex post real return. Unlike the present value with a constant discount rate, this formula is an identity. It always holds ex post, so it always holds ex ante. When we take limits, it is possible for the terminal condition to explode, so one can view tests of the present value relation as tests of that possibility. But again, then the value of debt and potentially the surplus as well become non-stationary, the statistical properties of such a test are challenging. And, the economics of the alternative are not that interesting, at least to me.

Imposing convergence of the last term, and writing the moving average representation of surplus and returns $s_t = a_s(L)\varepsilon_t$, $r_t = a_r(L)\varepsilon_t$, both including a vector shock ε_t, and taking ΔE_{t+1} of both sides, we obtain

$$0 = a_s(\rho) - a_r(\rho).$$

This looks like a promising extension of the testable $a(\rho) = 0$. But this relation derives from an identity, so it too is an identity. If you've done things right it always holds. You can measure the relative sizes of $a_s(\rho)$ and $a_r(\rho)$ which is interesting, but there is no alternative (other than the exploding debt, rational-bubble) under which the difference is not zero.

If we add a discount factor model, then we have something testable. Indeed, the constant discount factor model is just $a_r(\rho) = 0$. As has been done in asset pricing, one could test, for example, the hypothesis that the expected return on government bonds is measured by the ex ante real rate, or some other measure, not by construction equal to the ex post return on government bonds. One can create a complex stochastic discount factor that tiptoes around using the observed ex post return on government bonds, as Jiang et al. (2019) do. Then we have a rejectable statement, but the rejection is only the discount factor model since we know it works perfectly using the ex post return on government debt. Or one can restrict the surplus process, e.g. require $a_s(\rho) > 1$, again as Jiang et al. (2019). But without restrictions or bubbles, $a_s(\rho) = a_r(\rho)$ is an identity.

In sum, the key difference between the Hansen, Roberds, and Sargent (1992) test of $a(\rho) = 0$ and a generalization to a present value formula with time-varying discount rates is that the former restricts discount rates; it has a sensible alternative in which discount rates vary. In the latter case the present value formula is already an identity, so there is no sensible alternative.

A2.4 An alternative surplus process

I explore a tractable and useful surplus process that may be more useful or compelling than the v, v^* specification and is more realistic than the MA(1). The surplus has a permanent and transitory component, $s_t = z_t + x_t$; $z_t = \eta_z z_{t-1} + \varepsilon_{z,t}$; $x_t = \eta_x x_{t-1} + \varepsilon_{x,t}$, with $\eta_z > \eta_x$. The model generates a pretty response in which temporary deficits are financed by long-lasting increases in later surpluses, shown in Figure A2.1. When we pick parameters so that all debt is repaid, $a(\rho) = 0$, the univariate surplus process is not invertible. Again, forecasting surplus using debt, one can recover the structural process, and debt Granger-causes surpluses, though by construction surpluses cause variation in the value of debt.

This section summarizes a useful surplus process that allows for an s-shaped moving average, and debts to be partially repaid. The v and v^* model introduced in Section 5.5 is more elegant, but a bit more complex and at first glance conceptually harder. The MA(1) $s_t = \varepsilon_{s,t} + \theta\varepsilon_{s,t-1}$ is conceptually simple, but unrealistic. This example generalizes and simplifies the example in Cochrane (2001).

Suppose the surplus (or surplus/GDP ratio) has a permanent component and a transitory component, each AR(1):

$$s_t = z_t + x_t \tag{A2.27}$$
$$z_t = \eta_z z_{t-1} + \varepsilon_{z,t} \tag{A2.28}$$
$$x_t = \eta_x x_{t-1} + \varepsilon_{x,t}. \tag{A2.29}$$

Think of the cyclical component x_t as resulting from temporary events like recessions, wars, or economic booms like the late 1990s. These events result from temporary spending needs or fluctuations in GDP. Think of z_t as set by tax rates or the structure of entitlement programs. These changes are more permanent both by the nature of such policies and by tax-smoothing principles. These equations describe deviations about means.

Thus, in a war or recession, the government has deficits – negative x_t. To fund the deficits, it issues debt. But in order to raise revenue from the debt sales, the government promises persistently higher taxes to pay off the debt after the war or recession is over – positive z_t. I allow $\eta_z < 1$ to avoid a pure random walk in the surplus, but $\eta_z = 1$ simplifies formulas even more and does little harm. Think of η_z as a large number, however, and η_x as a smaller number.

With this time-series model, and again using the constant discount rate short-term debt model, unexpected inflation is

$$\Delta E_{t+1}\pi_{t+1} = -\sum_{j=0}^{\infty}\rho^j \Delta E_{t+1}s_{t+1+j} = -\frac{1}{1-\rho\eta_z}\varepsilon_{z,t+1} - \frac{1}{1-\rho\eta_x}\varepsilon_{x,t+1}. \tag{A2.30}$$

We aim to understand the response to the cyclical shock $\varepsilon_{x,t}$, and how much of that deficit is financed by inflation and how much is financed by borrowing, promising higher subsequent surpluses. To that end, let the government move long-run tax policy along with the deficit, leaving out for now orthogonal movements in long-run tax policy.

$$\varepsilon_{z,t+1} = -\frac{\left[1-\left(1-\rho\eta_x\right)\beta_s\right]\left(1-\rho\eta_z\right)}{\left[1-\left(1-\rho\eta_z\right)\beta_s\right]\left(1-\rho\eta_x\right)}\varepsilon_{x,t+1}. \tag{A2.31}$$

Here, I find it useful to parameterize the response of long-run policy to short-run deficit shocks in terms of the new parameter β_s, and the other terms are reverse-engineering carpentry of the assumption to give a pretty result. When there is a transitory deficit, a negative $\varepsilon_{x,t+1}$, the government raises persistent taxes or cuts persistent spending $\varepsilon_{z,t+1}$ in order to fund that deficit.

With this specification, the surplus innovation is

$$\Delta E_{t+1}s_{t+1} \equiv \varepsilon_{s,t+1} = \varepsilon_{z,t+1} + \varepsilon_{x,t+1} = \frac{\rho\left(\eta_z-\eta_x\right)}{\left[1-\left(1-\rho\eta_z\right)\beta_s\right]\left(1-\rho\eta_x\right)}\varepsilon_{x,t+1},$$

and from (A2.30), the inflation innovation is

$$\Delta E_{t+1}\pi_{t+1} = -\beta_s\varepsilon_{s,t+1}.$$

For $\beta_s = 0$, the government fully pays back debts, and there is no inflation. For $\beta_s > 0$, the government partially repays debts and partially inflates. For $\beta_s = 1/(1-\rho\eta_x)$, we have $\varepsilon_{z,t} = 0$ and $z_t = 0$ so $s_t = x_t$. There is no long run tax response and the surplus model reduces to an AR(1).

With (A2.31), the surplus process is

$$s_t = \frac{1}{1-\eta_z L}\varepsilon_{z,t} + \frac{1}{1-\eta_x L}\varepsilon_{x,t}$$

$$s_t = \left[\frac{\left[1-\left(1-\rho\eta_z\right)\beta_s\right]\left(1-\rho\eta_x\right)}{1-\eta_x L} - \frac{\left[1-\left(1-\rho\eta_x\right)\beta_s\right]\left(1-\rho\eta_z\right)}{1-\eta_z L}\right]\frac{1}{\rho\left(\eta_z-\eta_x\right)}\varepsilon_{s,t}.$$

The difference of two AR(1) produces a pretty s-shaped and hump-shaped response function. You can quickly verify $a(\rho) = \beta_s$.

Figure A2.1 presents the response function (A2.32) for the case $\beta_s = a(\rho) = 0$. I plot the response to a unit negative $\varepsilon_t = -1$ shock, a deficit. As you can see, deficits are persistent. But deficits eventually turn to surpluses which pay back the accumulated debts.

We can also condense the surplus process into a single lag operator

$$s_t = \frac{1-\left[1-\beta_s\left(1-\rho\eta_x\right)\left(1-\rho\eta_z\right)\right]\rho^{-1}L}{\left(1-\eta_x L\right)\left(1-\eta_z L\right)}\varepsilon_{s,t}.$$

This is an ARMA(2,1) with similar AR and MA roots, already an econometric challenge. When $\beta_s = 0$, this expression reduces to

$$s_t = \frac{\left(1-\rho^{-1}L\right)}{\left(1-\eta_x L\right)\left(1-\eta_z L\right)}\varepsilon_{s,t}. \tag{A2.32}$$

You cannot recover this surplus response from running autoregressions of surpluses

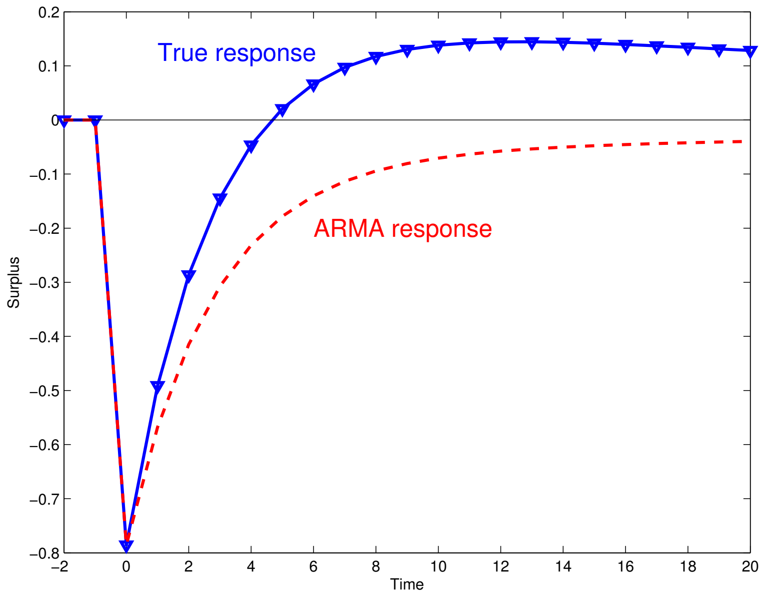

Figure A2.1: Surplus impulse-response function for the permanent-transitory model. The ARMA response is what one would infer from a regression of surpluses on past surpluses. $\eta_z = 0.975$, $\eta_x = 0.7$, $\rho = 1/1.05$.

on their past values, as (A2.32) is a non-invertible representation. If you run autoregressions or fit an ARMA model to surplus data generated by the model (A2.32), you recover an estimated model

$$s_t = \frac{(1-\rho L)}{(1-\eta_x L)(1-\eta_z L)} w_t \tag{A2.33}$$

rather than (A2.32), where the w_t are residuals from the regression of s_t on lagged s_{t-j}. You recover ρ not ρ^{-1} in the moving average term, and the regression error w_t is not the true shock ε_t.

In the not-unreasonable case $\rho = \eta_z$, you recover exactly the wrong AR(1) response function with coefficient η_x,

$$s_t = \frac{1}{1-\eta_x L} w_t,$$

as if the taxes were not there at all. You measure

$$a(\rho) = \frac{\left(1-\rho^2\right)}{(1-\eta_x\rho)(1-\eta_z\rho)},$$

not the correct answer $a(\rho) = 0$.

Figure A2.1 also presents the implied estimated response function (A2.33), the response to a single unit $w_t = -1$ shock. (The variance of the regression shocks w is also larger, so one will also misestimate the size of a one-standard-error shock. I graph the response to a unit shock to focus on the shape.) The response functions are broadly similar, but this one, fitted by a regression of surpluses on lagged surpluses, misses the rise in surpluses that pays off the debt. Hence, it predicts counterfactual surprise inflation associated with deficits.

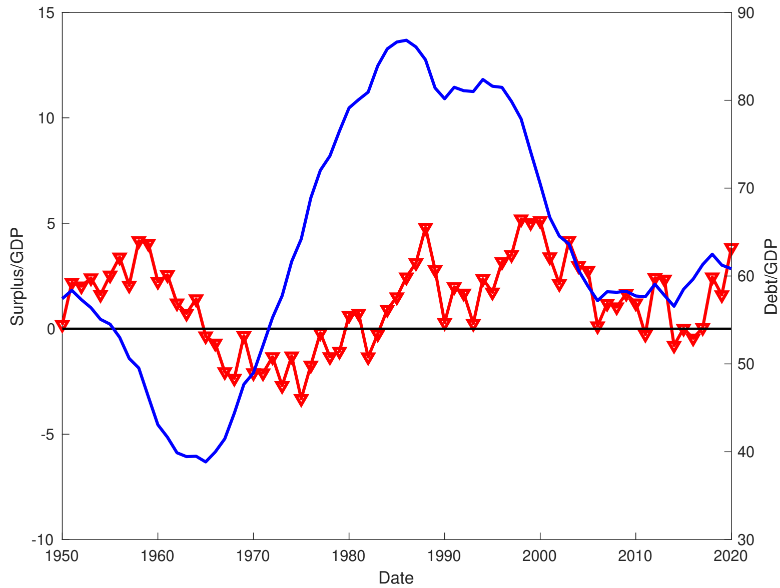

Figure A2.2: Simulation of the permanent-transitory surplus model. Parameters $z = 1.1$, $\eta_x = 0.8$, $\eta_z = 0.95$, $\sigma_\varepsilon = 2$.

Figure A2.2 presents a simulation of this permanent-transitory model. I picked parameters by eye to roughly match the dynamics of inflation and debt shown in Figure 4.2. (I add a mean $z = 1.1$.) There is no simple relation that debt, price level or inflation is proportional to surpluses. When surpluses are positive, debt falls. When surpluses are negative, debt rises. The government seems to run surpluses to pay off debts, following a passive fiscal policy, though the example is constructed under the explicitly opposite assumption.

In this case as well, you can estimate the true surplus process, if you use a VAR that includes debt. The value of debt is

$$v_t = E_t \sum_{j=1}^{\infty} \rho^{j-1} s_{t+j} = \frac{1}{1-\rho\eta_z} z_t + \frac{1}{1-\rho\eta_x} x_t.$$

Together with

$$s_t = z_t + x_t$$

we can then find the structural VAR representation.

$$\begin{bmatrix} z_t \\ x_t \end{bmatrix} = \begin{bmatrix} \eta_z & 0 \\ 0 & \eta_x \end{bmatrix} \begin{bmatrix} z_{t-1} \\ x_{t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_{z,t} \\ \varepsilon_{x,t} \end{bmatrix}$$
$$\begin{bmatrix} s_t \\ v_t \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ \frac{1}{1-\rho\eta_z} & \frac{1}{1-\rho\eta_x} \end{bmatrix} \begin{bmatrix} z_t \\ x_t \end{bmatrix}$$

At this point, the pair of surplus and debt are a non-singular transformation of a pair of AR(1). So, it should be clear that the par s_t, v_t also follow a first-order invertible VAR with stable roots. Mechanically, we have

$$\begin{bmatrix} s_t \\ v_t \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ \frac{1}{1-\rho\eta_z} & \frac{1}{1-\rho\eta_x} \end{bmatrix} \begin{bmatrix} \eta_z & 0 \\ 0 & \eta_x \end{bmatrix} \begin{bmatrix} 1 & 1 \\ \frac{1}{1-\rho\eta_z} & \frac{1}{1-\rho\eta_x} \end{bmatrix}^{-1} \begin{bmatrix} s_{t-1} \\ v_{t-1} \end{bmatrix} + \begin{bmatrix} 1 & 1 \\ \frac{1}{1-\rho\eta_z} & \frac{1}{1-\rho\eta_x} \end{bmatrix} \begin{bmatrix} \varepsilon_{z,t} \\ \varepsilon_{x,t} \end{bmatrix}$$

The construction already gives us a diagonalization of the VAR transition matrix verifying stable eigenvalues η_z, η_x. Evaluating the matrix product, we have a structural autoregressive representation,

$$\begin{bmatrix} s_t \\ v_t \end{bmatrix} = \begin{bmatrix} \eta_x + \eta_z - \rho^{-1} & \rho^{-1}(1-\rho\eta_x)(1-\rho\eta_z) \\ -\rho^{-1} & \rho^{-1} \end{bmatrix} \begin{bmatrix} s_{t-1} \\ v_{t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_{s,t} \\ \varepsilon_{v,t} \end{bmatrix}. \tag{A2.34}$$

Since this is the autoregressive representation, the structural shocks $\varepsilon_{z,t}$, $\varepsilon_{x,t}$ are recoverable from the regression residuals. This result holds for any correlation of the shocks $\varepsilon_{z,t}$ and $\varepsilon_{x,t}$ including $\beta_s = 0$ and perfect that produces a non-invertible moving average representation for s_t alone. Even in the case of perfect shock correlation (A2.31) and $\beta_s > 0$, in which s_t is in principle estimable from its own past, it is much easier to estimate a first-order VAR than it is to estimate an ARMA(2,1) with nearly-canceling roots.

This is also a pure fiscal-theory example with completely exogenous surplus process. Yet in (A2.34), the regression coefficient of surplus s_t on value v_{t-1} is positive, showing us how that coefficient does not measure passive fiscal policy. Debt helps to forecast and thus Granger-causes surpluses. And the coefficient $\rho^{-1} > 1$ of debt on lagged debt warns us to be careful about misinterpreting individual regression coefficients for eigenvalues of systems.

Chapter Three

Pruning multiple monetary equilibria

This chapter gives an overview of efforts to prune the multiple inflationary and deflationary equilibria of monetary models, from Section 19.3.2, without recourse to active fiscal policy. I demote this discussion to this appendix, as I think it belongs in the department of historical controversies that have no bearing on models of inflation we might use today, as explained in the text. Still, having made the claim that multiple equilibria are not solved by this literature, here I offer a review. The review is at least interesting for covering history of thought.

Figure 19.1 exhibits the dynamics of the money in the utility function model. I repeat the figure here as Figure A3.1 for easy reference. There is an inflationary steady state, a zero interest rate deflationary steady state, and a full range of deflationary and inflationary equilibria emanating from the inflationary steady state, either to large inflation or down to the deflationary steady state. I take up here efforts to prune all but the inflationary steady state as an equilibrium. The model is set forth in Section 19.3.2.

A3.1 Pruning deflationary equilibria

A transversality condition argument can rule out the deflationary equilibria when money growth is nonnegative $\mu \geq 0$, and for some specifications of the utility function, when there is no debt and money growth is financed by lump-sum taxation. This *is* the fiscal theory, not an alternative to fiscal theory. The result is also sensitive to assumptions. If there is nominal debt and the central bank controls money by open-market operations, it also fails. If money growth is negative $\mu \leq 0$, deflationary equilibria survive as well.

If money growth is non-negative $\mu > 0$, real money holdings in my example rise faster than the real interest rate. One might say the transversality condition is violated, ruling out these paths. We can see this outcome by manipulating (19.21) to give

$$\left(\frac{M_{t+1}}{P_{t+1}y}\right) / \left(\frac{M_t}{P_t y}\right) = (1+\delta)(1+\mu)\left[1-\theta\left(\frac{P_t y}{M_t}\right)^{\gamma}\right]$$

so, as $P_t y/M_t \searrow 0$,

$$\left(\frac{M_{t+1}}{P_{t+1}y}\right) / \left(\frac{M_t}{P_t y}\right) \nearrow (1+\delta)(1+\mu).$$

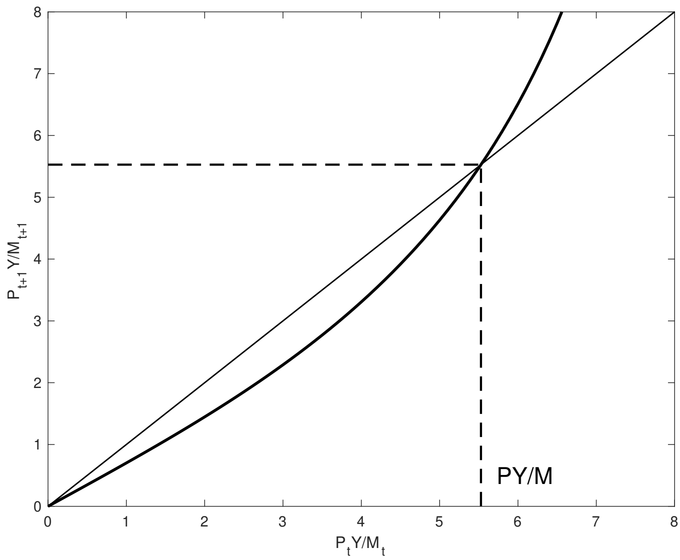

Figure A3.1: Phase diagram for the money in the utility function model with constant money growth.

Thus, if $\mu \geq 0$, real money holdings violate the condition,

$$\lim_{T\to\infty} E_t \beta^{T+1} \frac{M_{t+T}}{P_{t+T+1}y} \neq 0.$$

Consumers, seeing this rise in wealth, may try to increase consumption, and in the process drive the price level back up and away from the deflationary equilibrium.

This objection only applies to policies with positive money growth $\mu > 0$. If money growth is negative $-\delta < \mu < 0$, then even this subtlety vanishes and we unequivocally have multiple deflationary equilibria. So at best we are ruling out one set of multiple equilibria for one set of parameter values, leaving others intact.

A3.1.1 This *is* fiscal theory

When it holds, this argument *is* fiscal theory, not an alternative *to* fiscal theory. Passive fiscal policy means that fiscal policy does what it takes so that the government debt valuation holds, i.e. that the transversality condition is satisfied for any initial price level. This government refuses to raise taxes as passive policy would do, validating the too-low price level. That's active fiscal policy.

To see the argument explicitly, it helps as always to start with the simplest environment: The government issues no debt $\{B_t\} = 0$, and utility of money is bounded, $u_m(m) = 0$ for $m \geq m_{sat}$. Start having entered this region. With $u_m = 0$, the nominal interest rate must be zero $i = 0$. The difference equation for money holdings (19.18) reduces to $P_{t+1}/P_t = \beta$, deflation at the real interest rate. Real money holdings themselves are arbitrary so long as $m > m_{sat}$.

The household budget constraint (19.10) in goods-market equilibrium $c_t = y_t$ reduces to

$$\frac{M_t - M_{t-1}}{P_t} = -s_t. \tag{A3.1}$$

In this model, monetary policy *is* fiscal policy. With no debt, deficits are financed entirely by printing money. Contrariwise, any increase or decrease in money must come from seigniorage. A helicopter drop counts as a deficit.

The present value budget constraint (19.12) in goods and financial market equilibrium reads

$$\frac{M_{t-1}}{P_t} = \sum_{j=0}^{T} \beta^j s_{t+j} + \beta^{T+1} \frac{M_{t+T}}{P_{t+T+1}}.$$

If you substitute (A3.1) and $P_{t+1}/P_t = \beta$, this equation just says final money equals initial money plus added money.

$$\frac{M_{t-1}}{P_t} = \sum_{j=0}^{T} \beta^j \left(-\frac{M_{t+j} - M_{t+j-1}}{P_{t+j}} \right) + \beta^{T+1} \frac{M_{t+T}}{P_{t+T+1}}$$

$$\frac{M_{t-1}}{P_t} = \sum_{j=0}^{T} \left(-\frac{M_{t+j} - M_{t+j-1}}{P_t} \right) + \frac{M_{t+T}}{P_t}.$$

The final term grows at rate $(1+\mu)^T$. The final, transversality condition is violated. The present value of surpluses explodes to negative infinity.

This government just showers money on people and does nothing else. With positive money growth and such helicopter deficits, money piles up. But with neither a utility benefit of money nor taxes to pay, people don't want to let money pile up forever. They would prefer to lower money holdings, and increase consumption throughout time. The consumer's budget constraint is

$$\frac{M_{t-1}}{P_t} = \sum_{j=0}^{T} \beta^j \left(s_{t+j} + c_{t+j} - y \right) + \beta^{T+1} \frac{M_{t+T}}{P_{t+T+1}}.$$

Facing prices $\{P_t\}$, a real interest rate β^{-1}, endowment y, and surpluses $\{s_t\}$, the consumer would prefer to raise consumption at all dates, holding less money.

This is clearly fiscal theory logic. You can't support a positive value of debt M_{t-1}/P_t with endlessly negative surpluses. Money is debt here, as money has no utility and any explicit debt would have a zero interest rate on these price level paths.

A passive fiscal policy in this circumstance would adjust surpluses so that the present value equation holds, for any given initial price level. Thus, a passive policy must have at some point positive surpluses. And with monetary policy indistinguishable from fiscal policy, that means we must in this model have a negative money growth rate $M_{t+1} - M_t < 0$, violating the setup of the example.

More generally, there is no equilibrium initial price level, with perpetually positive money growth, perpetually negative surpluses, and no liquidity value $u_m = 0$, whether fiscal policy is active or passive.

Indeed, a government that finances itself exclusively with non-interest paying money, or equivalently perpetually zero nominal interest rates on debt, is a good fiscal theory exercise. Such a government can have a determinate price level, even with no money demand at all. The nominal interest rate stays at zero, prices decline at the real rate to generate a positive return. But the present value of surpluses must be positive, meaning enough periods of negative money growth and positive surplus to soak up money issued in periods of positive money growth and negative surplus, so the nominal money supply is eventually constant or declining.

A3.1.2 No transversality violation with open-market operations

Adding debt explicitly adds a different possibility. Maintaining enough money so $u_m = 0$, money and debt are perfect substitutes, so the above analysis holds with M_t simply denoting the sum $M_t + B_t$. But now we have another possibility for interpreting the instruction to let money keep growing: Rather than grow money by printing it and handing it out, money may be increased by open-market operations that exchange money for debt. This is the conventional interpretation of monetary policy after all. In this case the transversality condition is not violated in the first place.

Now with goods $c_t = y_t$ and asset market equilibrium $Q_t = 1$, the household's period budget constraint (19.10) leads to

$$\frac{B_{t-1} + M_{t-1}}{P_t} = \frac{B_t + M_t}{P_t} + s_t$$

and the present value condition (19.12) is

$$\frac{M_{t-1} + B_{t-1}}{P_t} = \sum_{j=0}^{T} \beta^j s_{t+j} + \beta^{T+1} \frac{M_{t+T} + B_{t+T}}{P_{t+T+1}} = \sum_{j=0}^{T} \beta^j s_{t+j} + \frac{M_{t+T} + B_{t+T}}{P_t}.$$

For simplicity, let the primary surplus be constant,

$$s_t = s = (1 - \beta) \frac{B_{t-1} + M_{t-1}}{P_t}$$

which yields a constant real total debt,

$$\frac{B_t + M_t}{P_{t+1}} = \frac{B_{t-1} + M_{t-1}}{P_t}$$

and steadily declining nominal debt

$$B_t + M_t = \beta \left(B_{t-1} + M_{t-1} \right).$$

Interpret the policy of perpetual money growth $M_{t+1}/M_t = 1 + \mu$ as one commanding the central bank to undertake open-market operations, issuing money and buying debt. Now, despite ever-increasing money, the total value of government debt is the same, and the transversality condition holds. The transversality condition applies to the *sum* of the two forms of (here perfect substitutes) of government debt. Yes, the *individual* quantities of money and debt explode in opposite directions. In particular, nominal debt becomes

$$B_t + M_t = \beta^t (B_0 + M_0)$$

$$B_t = \beta^t (B_0 + M_0) - (1+\mu)^t M_0.$$

The second term grows without bound, so government debt becomes negative. The government issues more and more money, using the result first to buy its own debt back, and then to invest in a larger and larger sovereign wealth fund.

A3.1.3 Optimality with valued money

In the case that the marginal utility of money does not decline to zero $u_m > 0$, the analysis is more subtle. The ever-growing money supply may not actually violate the transversality condition.

To see the issue, consider the last period of a finite-time version of the model. Since the world ends at time $T+1$, we normally conclude that the consumer leaves no money or debt outstanding $M_T + B_T = 0$. The last period budget constraint reads

$$\frac{B_{T-1} + M_{T-1}}{P_T} = \frac{Q_T B_T + M_T}{P_T} + (s_T + c_T - y_T). \tag{A3.2}$$

It seems that $M_T > 0$ is suboptimal, as the consumer could raise consumption c_T instead.

But this is a mistake. It is worth keeping some money M_T, as it enters utility at time T, even though money is useless the next morning $T+1$. The first-order conditions in this last period are, from $\max u(c_T, M_T/P_T)$ s.t. (A3.2),

$$u_c(c_T, M_T/P_T) = u_m(c_T, M_T/P_T).$$

Positive and finite marginal utility of consumption means positive and finite marginal utility of money, and hence a positive and finite money demand, even though money is left over, apparently useless after the end of the world. This finite-horizon case gives the intuition behind cases in which the proper transversality condition holds, even though real money balances increase perpetually.

A3.1.4 Deflationary equilibria literature

> I offer a quick survey of the vast literature on deflationary equilibria with alternative specifications of the money in utility function.

The above examples leave one hungry for more. What if utility is non-separable? What exactly is the correct transversality condition? This question gives rise to a large literature. The general consensus is as I have described – for money growth between negative of the real rate and zero, -$\delta \leq \mu < 0$, there are multiple deflationary equilibria. For non-negative money growth, this fiscal (transversality condition violation) argument sometimes rules them out, but sometimes does not.

This result remains contentious. General transversality conditions for this classic model are, surprisingly, not well established. Since limiting properties of the utility function also matter – the value of money – analysis involves a good deal of mathematical horsepower, for example Ekeland and Scheinkman (1986), Kamihigashi (2000). As often, general proofs make assumptions contrary to practice, such as bounded utility. Utility non-separable between consumption and money is plausible, as it disentangles risk aversion from money demand elasticity, but complicates the analysis. In one survey, Buiter and Siebert (2007) write:

> A striking feature of the current and past macroeconomic literature on deflationary bubbles is the divergence of opinion over the correct specification of both the transversality condition in models where money is the only financial asset and the correct specification of the transversality and long-run solvency, or "no-Ponzi-game," conditions in models where there are both money and bonds.

They offer a lucid and concise review of widely-varying, and differing published opinions, including Brock (1974), Brock (1975), Obstfeld and Rogoff (1983), Obstfeld and Rogoff (1986), Ljungqvist and Sargent (2018), Woodford (2003) and many others one might turn to for guidance here including Matsuyama (1990), Matsuyama (1991), Woodford (1994). Opinion differs on whether there are two conditions, one for money and one for bonds, or one condition, for aggregate terminal wealth.

The conclusion of Buiter and Siebert (2007) mirrors the claim I started with,

> We demonstrate that deflationary bubbles cannot occur when money growth is strictly positive ($\mu > 1$). We show, however, that when the money supply is contracting, but at a lower rate than the discount factor ($\beta < \mu < 1$) deflationary bubbles can occur; indeed, any separable utility function satisfying the usual regularity conditions can produce a deflationary bubble.

However, even they do not get the last word. In particular, they write

> "deflationary bubbles accompanied by strictly positive money growth in Woodford (2003) and Benhabib et. al. (2002a) cannot exist."

This statement appears to invalidate my previous analysis, such as Figure 16.2. But that analysis didn't have any money at all, and it followed an interest rate target in which, if there is money, it can grow at a slow rate. Moreover the point of Benhabib, Schmitt-Grohé, and Uribe (2002) was exactly that by *adding* unbacked money or debt growth, an essentially fiscal policy, they could escape the deflation. Woodford (1994) explores these issues in detail in a cash and credit good cash in advance model, with interest-elastic money demand. His central point is that an interest rate target allows the zero-bound equilibrium, in a way that a money-growth target does not do. The debate will continue.

Even the classic source on optimization Chiang (1992) writes "their [transversality conditions] validity is sometimes called into question... it is only fair to warn the reader... that there exists a controversy surrounding this aspect of infinite horizon problems." (p. 102) "Many writers consider the question of infinite horizon transversality conditions to be in an unsettled state." (p. 243.) He goes on to set straight several counterexamples, many from the economics literature.

Footnote 1 in Woodford (1994) offers a concise survey of multiple equilibrium issues in money-in-utility models, essentially documenting that there is no consensus general statement: "But while several authors have addressed aspects of this problem...no very general treatment exists for that class of models." Bassetto and Sargent (2020) also summarize nicely "..for many interesting preference specifications it [difference equation (19.21)] has many solutions. The lack of a nominal anchor comes from the lack of a boundary condition for equation (38) [my (19.21)]. The only candidate for such a boundary condition is the government budget balance."

I don't pursue the issue further, because the model, though a subject of a large literature – constant money growth, no debt, no surpluses, a definite money demand – is not interesting.

A3.2 Pruning inflationary equilibria

Now we turn to the equilibria with increasing inflation, to the right of the steady state in Figure A3.1. Since money holdings decline, there is no transversality condition issue. Instead, appeal is made to the same sorts of equilibrium-selection ideas as we saw for inflationary equilibria of active-money passive-fiscal interest rate models.

A3.2.1 Timing conventions in the inflationary equilibria

> The inflationary equilibria explode in finite time. They are nonetheless valid equilibria. This behavior is an unrealistic artifact of the discrete time timing conventions, which result in $u_m/u_c = i/(1+i)$ rather than $u_m/u_c = i$ which results from the continuous time model. I introduce a modification of the utility function which gives the latter first-order condition, and removes jumps to infinite price level in finite time.

Examine the inflationary equilibria of the difference equation (19.23),

$$\left(\frac{P_{t+1}y}{M_{t+1}}\right) = \left(\frac{P_t y}{M_t}\right) \frac{\left[1-\theta\left(\frac{Py}{M}\right)^{\gamma}\right]}{\left[1-\theta\left(\frac{P_t y}{M}\right)^{\gamma}\right]}. \tag{A3.3}$$

For $P_0 y/M_0 > Py/M$, the price level eventually explodes to infinity finite time. The denominator goes to zero or worse. One might hope to eliminate inflationary equilibria on this basis.

There is nothing theoretically wrong with this result. Since money is just an argument of the utility function in an endowment economy, the economy can trundle along with $c_t = y$ and no money. A path that starts with little inflation, goes to hyperinflation, and in finite time demonetizes, is a valid equilibrium of the model. first-order conditions and budget constraints hold all the way to the infinite price level and beyond.

In fact, this consideration means there is not a continuum of inflationary equilibria, but a countable number, and the denominator of (A3.3) goes exactly to zero but not below. If consumers know that the price level will be infinite at time $T+1$, then money demand at time T is

$$M_t = P_t y \left(\frac{1}{\theta}\right)^{-\frac{1}{\gamma}}.$$

so the last period price level is

$$P_T = \frac{M_T}{y}\theta^{-\frac{1}{\gamma}}.$$

People are happy to hold this much money for a day, even knowing money will be useless tomorrow. We work back from this terminal condition to find $P_0 y/M_0$. For each such T there is a different $P_0 y/M_0$.

This is the specification, with demonetization in finite time, studied in the classic models that attempt to fix these multiple equilibria, Obstfeld and Rogoff (1983), Obstfeld and Rogoff (1986), which I examine below. These authors add additional elements to the policy mix to try to trim the inflationary equilibria, which they would not need to do if the equilibria weren't valid in the first place.

Nonetheless, demonetization in finite time feels weird, and it is. It results from a pathology of the discrete time formulation of the model. This is not a good model for studying money demand in high-inflation economies, for this and many other reasons.

In this discrete time setup, the money demand function is

$$M_t = P_t y_t \left(\frac{1}{\theta}\frac{i_t}{1+i_t}\right)^{-\frac{1}{\gamma}}. \tag{A3.4}$$

In the continuous time version of the model, outlined below, we have instead

$$M_t = P_t y_t \left(\frac{i_t}{\theta}\right)^{-\frac{1}{\gamma}}. \tag{A3.5}$$

For small i_t, the difference between i_t and $i_t/(1+i_t)$ is minor. However, for large i_t, it is not minor. As inflation and interest rates approach infinity, real money demand M_t/P_t in (A3.4) approaches a constant, while real money demand in (A3.5) smoothly approaches zero. In the discrete time model, it is worth holding money for one day, even if that money will be worthless the next morning. Interest is only paid overnight, so there is no opportunity cost for holding money for one day, and the price level is constant during the day.

This behavior is not realistic. In times of very large inflation, interest is paid even during literal days, to say nothing of the month, quarter, or year periods for which we usually apply these models, and prices rise hour by hour. You cannot hold money for any discrete period of time without an opportunity cost. The continuous time first-order conditions (A3.5) reflect this fact.

The best approach is to use the continuous time model, in which pathologies due to timing conventions do not arise. We can however derive a money demand (A3.5) in this discrete time model by modifying the utility function to

$$u\left(c_t, \frac{M_t}{P_t}\right) = \frac{c_t^{1-\gamma}}{1-\gamma} + \frac{\theta}{1-\gamma}\frac{1}{1+i_t}\left(\frac{M_t}{P_t}\right)^{1-\gamma}.$$

Now, the first-order condition

$$\frac{u_m(t)}{u_c(t)} = \frac{i_t}{1+i_t}$$

becomes, in equilibrium, (A3.5).

The i_t versus $i_t/(1+i_t)$ really belongs in the budget constraint – the fact that you cannot, in reality, use money without interest cost or inflation during the day. But it's awkward at this stage to change the budget constraint we have used throughout the book, and discrete time utility doesn't mix well with a continuous time budget constraint. So, I add the $1/(1+i_t)$ to the preferences as a shortcut to get the continuous time first-order condition and limiting behavior out of the discrete time model. The preferences are an indirect utility for some unstated transactions model anyway, and if we allow the price level P_t into preferences, we can't object to an intertemporal price i_t as well.

Repeating the previous analysis with the alternative timing convention, we obtain almost exactly the same results for small interest rates i, but a smooth limit for high interest rates, and in particular a continuum of inflationary equilibria that go on forever, without demonetizing in finite time. The difference equation is, in place of (19.21),

$$\theta\left(\frac{M_t}{P_t y}\right)^{-\gamma} = \frac{1}{\beta}\frac{P_{t+1}}{P_t} - 1 = \frac{1}{\beta}\left(\frac{M_t}{P_t y}\right) / \left(\frac{M_{t+1}}{P_{t+1} y}\right)\frac{M_{t+1}}{M_t} - 1.$$

The steady state for money holdings is

$$\theta\left(\frac{M}{Py}\right)^{-\gamma} = (1+\delta)(1+\mu) - 1.$$

and in place of (19.23),

$$\left(\frac{P_{t+1}y}{M_{t+1}}\right) = \left(\frac{P_t y}{M_t}\right)\frac{\left[1+\theta\left(\frac{P_t y}{M_t}\right)^{\gamma}\right]}{\left[1+\theta\left(\frac{Py}{M}\right)^{\gamma}\right]}.$$

Having $1+\theta\left(\frac{P_t y}{M_t}\right)^{\gamma}$ in the numerator rather than $1-\theta\left(\frac{P_t y}{M_t}\right)^{\gamma}$ in the denominator makes little difference for small values of $P_t y/M_t$ but means that the price level never goes to infinity in finite time. Figure 19.1 is visually indistinguishable in the plotted range, but no longer spikes up to infinite $P_{t+1}y/M_{t+1}$ at a finite $P_t y/M_t$,

This timing seems to me like a better way to put the continuous time model in discrete time. I present the traditional model in the text more for consistency with other sources, as the point of this section is that the model does not work. But I would use this formulation or the continuous time version if I were to use the model for serious analysis.

As I criticized deflationary equilibria for angels-on-heads-of-pins study of limits with immense money holdings, so too here one should not make too much of a model's handling of money demand in 10^{15} inflation.

A3.2.2 The Obstfeld-Rogoff fix for inflationary equilibria

I review the famous Obstfeld and Rogoff (1983) fix for inflationary equilibria. Obstfeld and Rogoff specify that the government stands ready to redeem money for goods (gold) at a very high price level. Crucially, they specify that the government refuses to *sell* money for goods at the same price. Therefore, the government disallows the recovery in real money holdings that follows the ends of inflations, ruling out an equilibrium in which the equilibrium is stopped.

On deeper analysis, however, I find that the modification does not rule out the original equilibrium in which the price level jumps to infinity.

Obstfeld and Rogoff (1983), Obstfeld and Rogoff (1986) are the most famous papers cited in the effort to trim multiple inflationary equilibria without recourse to active fiscal policy.

Obstfeld and Rogoff add to the specification of monetary policy regime that at some very high price level the central bank implements a partial commodity standard. Obstfeld and Rogoff (1983) (p. 676) write:

> Speculative paths can be eliminated... provided the government fractionally backs the currency by standing ready to redeem each dollar for a small amount of capital.

Obstfeld and Rogoff's model is based on backing with capital, which is hard to imagine in practice. What are the independent real units of capital? But as they make clear, capital is a stand-in for a commodity or gold standard: Footnote 17: "We analyze capital backing rather than gold backing here in order to avoid modeling the role of gold in consumption and/or production. But our results would clearly carry over to a model in which currency is redeemable in terms of gold." A foreign exchange peg could work the same way. I study a simplified model in which the government backs the currency with the consumption good, which it obtains by lump-sum taxes.

Based on the Section 16.10 analysis of similar devices to stop multiple equilibria in models with interest rate targets, two natural questions or objections arise: First, such a commodity standard or real backing *is* the fiscal theory, it *is* an active fiscal policy, not an alternative to fiscal theory and a defense of purely monetary price level determination. To make this commitment, the government has to have the capital, commodity, gold, foreign exchange or the ability to tax to get it, either now or in the credible future. Indeed, Obstfeld and Rogoff write (p. 684):

> "Feasibility of the government's policy requires that the government have access to sufficient reserves of capital to purchase the entire money stock M at the support price ε."

This observation is not a criticism: Obstfeld and Rogoff wrote a decade before Leeper wrote, and the active/passive distinction was ever considered. They were not trying to rescue price level determination without fiscal underpinnings, and did not claim to do so. They might have been, in 1983, perfectly happy to interpret their result as a joint monetary–fiscal policy regime, with the fiscal part of the regime important for equilibrium selection. At the time, the important question was whether *any* regime could determine the price level. The distinction is important now, however, for us to understand and categorize their result. And we should not cite them for showing something they did not claim to show.

More deeply, though, their proposal runs afoul of the earlier conundrum – the difference between stopping an inflation and ruling out an equilibrium. An inflation breaks out, and gets worse and worse. At some point – maybe when the dollar is worth one cent of its original value – the commodity standard kicks in. That stops the inflation, and the economy continues on that, fiscally-determined, gold-standard enforced, price level. Great, but the inflation, its end, and the new commodity standard are all part of an equilibrium.

How did they rule out the equilibrium path? There must be a blow-up-the world threat or inconsistent policy in there somewhere. (This section simplifies Cochrane (2011a) p. 609 ff. Obstfeld and Rogoff (2021) is a response to that paper. I hope this clearer presentation settles the issue, but one never knows.)

Obstfeld and Rogoff use the model we have been studying, with separable utility, the standard discrete time timing conventions, and constant endowment. The first-order conditions, in equilibrium, lead to the same difference equation, (19.18), which they write

$$\frac{u'(y)}{P_t} - \frac{v'(M/P_t)}{P_t} = \beta \frac{u_c'(y)}{P_{t+1}}. \tag{A3.6}$$

(This is their equation (4), p. 678. In case you want to refer to the original, I use their notation, $u'(c)$ and $v'(m)$ in place of my $u_c(c)$ and $u_m(m)$.) They specify a constant money supply M, and denote the corresponding steady state by $\overline{P}$,

$$\frac{u'(y)}{\overline{P}} - \frac{v'(M/\overline{P})}{\overline{P}} = \beta \frac{u'(y)}{\overline{P}}.$$

Hyperinflationary equilibria occur in finite time. I emphasize, as discussed above, that infinite inflation in finite time is an artifact of the discrete time timing convention, which does not appear in continuous time, or the discrete time model with modified timing studied above. One might just stop here, but I soldier on.

The hyperinflationary equilibrium ends with

$$P_{T+1} = P_{T+2} = ... = \infty.$$

The price level at time T is then

$$v'\left(\frac{M}{P_T}\right) = v'\left(\frac{M}{\bar{P}}\right) = u'(y).$$

The second equality defines $\bar{P}$, the price level if people know money will be worthless the following period. (This is $\bar{P}$ with a short bar on top, where the steady state (A3.6) is $\overline{P}$ with a long bar on top. Again, I use Obstfeld and Rogoff's notation in case you want to refer to the original.)

We find earlier price levels by working back with (A3.6). Each T generates a different potential value of P_0 and a different equilibrium path.

Figure A3.2 plots this path, labeled "$\varepsilon = 0$." The figure plots $m_t = M/P_t$ with $M = 1$ for clarity, so the jump to $P_{T+1} = \infty$ is a jump of $m_{T+1} = M/P_{T+1}$ to zero.

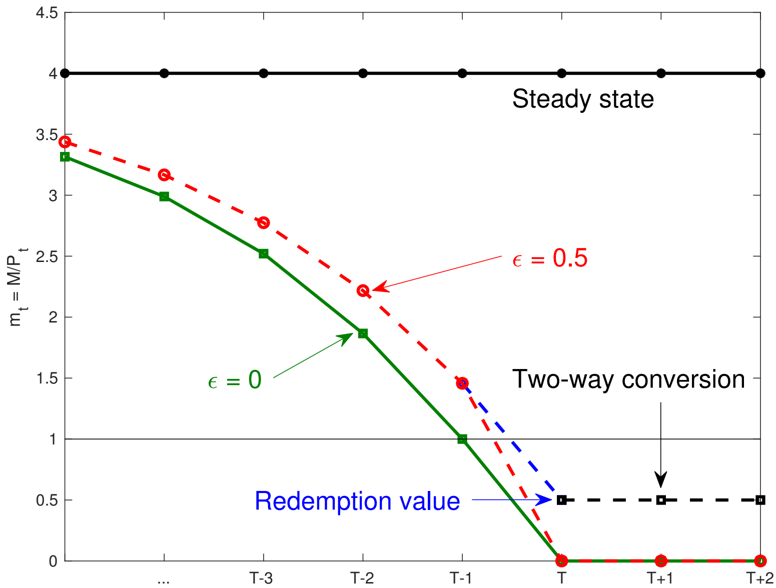

Figure A3.2: Hyperinflations in the Obstfeld-Rogoff model. "$\varepsilon = 0$" gives the hyperinflation equilibrium that we wish to rule out. "$\varepsilon = 0.5$" gives the equilibrium when the government offers to redeem money for ε consumption goods. "Redemption value" plots $M\varepsilon$, the value of money guaranteed by the government's redemption promise. "Two-way conversion" gives the equilibrium that results if the government offers to buy as well as to sell the commodity. The lower horizontal line indicates $M/\bar{P}$, money holdings at the price level where people are willing to hold money for one period though it is useless the next period. $u'(y) = 1$, $M = 1$, $\beta = 1/2$, $v(m) = m^{-1/2}$.

There is nothing wrong with these equilibria, and that is Obstfeld and Rogoff's whole point. We need something else to rule them out. Obstfeld and Rogoff make a small change (p. 684):

> "the government promises to redeem each dollar bill for ε units of capital but does not offer to sell money for capital."

I specify equivalently that the government promises to redeem each dollar for ε units of the consumption good, which it obtains by a lump-sum tax. Therefore, it seems that by arbitrage the equilibrium price level cannot be higher than

$$\overline{\overline{P}} \equiv 1/\varepsilon.$$

Here's Obstfeld and Rogoff's central claim that with this extra provision, hyperinflationary equilibrium paths are ruled out (p. 685):

> Suppose that $\{P_t\}$ is an equilibrium path with $P_0 > \overline{P}$. Let $P_T = \max\left\{P_t | P_t < \overline{\overline{P}}\right\}$. By (14) [my (A3.6)] P_T must be below $\bar{P}$, so that $u'(y) - v'(M/P_T) > 0$ while P_{T+1} must exceed P_T and therefore equal $\overline{\overline{P}}$. But there is no $M_{T+1} \leq M$ such that $u'(y) - v'(M_{T+1}/\overline{\overline{P}}) \geq 0$. Thus there is no price level P_{T+2} satisfying (14) and $\{P_t\}$ is not an equilibrium path.

The line marked "$\varepsilon = 0.5$" in Figure A3.2 plots this path. If there were a final period with $P_T = \bar{P}$, as previously hypothesized, now people would be able to turn their money in at value ε after using it. Money is more valuable. So P_T must be less than $\bar{P}$, and M/P_T higher than $\bar{P}$ as shown. But that is fine, and everything is fine up to and including period T. The issue is (again) just what happens on day $T + 1$ when the redemption promise first kicks in.

Now, you might think that after the commodity standard becomes effective, we simply move to a new equilibrium with $P_t = \overline{\overline{P}} = 1/\varepsilon$ forever, as graphed in the "two-way conversion" line of Figure A3.2 . We switch on a gold standard or foreign exchange peg, and the fiscal resources to 100% back that peg. Inflation stops. But again, stopping the inflation does not rule out the equilibrium. Quite the opposite: Stopping the inflation simply and transparently makes the equilibrium more reasonable to rationally expect in the first place.

Here the second part of the p. 684 specification is crucial: The government "does not offer to *sell* money for capital." (My emphasis.) In an inflation, with high nominal interest rates, real money demand M/P is low. When inflation ends, and interest rates perforce return to low values, real money demand increases. Governments that stop inflations can, and do, continue to print a lot of money as real money demand recovers. They *want* equilibrium to form, they want first-order and market-clearing conditions to hold, they want a successful stabilization. They do not want to set things up so that no equilibrium can form, whatever that means. Governments on the gold standard or foreign exchange peg sometimes refuse to *give you* gold or foreign exchange when you bring in money, but governments on the gold standard don't refuse give you money when *you bring them* gold! This one does.

Indeed, if the government offered a conventional, two-way commodity standard, conditional on reaching a high price level $\overline{\overline{P}}$, we would in fact see $P_t = \overline{\overline{P}} = 1/\varepsilon$ for $t = T + 1,\ T + 2, \ldots$ People would bring in as much of the commodity (capital) as needed to obtain enough money so this price level would be the new steady state, as graphed. The inflation and its end would unequivocally be an equilibrium. I emphasize this point because many readers seem to think this is what Obstfeld and Rogoff do. It is not.

Obstfeld and Rogoff's government refuses to increase the money stock, despite the huge seigniorage opportunity, and despite the crying money demand of its citizens, even if they bring gold to the window. *This* threat is the heart of the equilibrium-selection concept, and it is what rules out the $\overline{\overline{P}}$ equilibrium and its antecedents.

In this case, however, I believe even this conclusion is incorrect. The one-way redemption is not sufficient to rule out equilibrium after the inflation. Obstfeld and Rogoff left out the possibility that $P_{T+1} = \infty$, and people redeem all their money

Before we add the redemption promise, $P_{T+1} = \infty$ *is* an equilibrium, despite people's crying demand for money, despite $v'(M/P_{T+1}) \gg u'(y)$, including even an infinite $v'(M/P_{T+1}) = \infty$ or $v(M/P_{T+1}) = -\infty$. Why do people not demand more money? Are they not similarly off the first-order condition? It appears so – $u'(y) - v'(M_{T+1}/P_{T+1}) < 0$, mirroring the above p. 685 quote. But *this condition does not apply* when $P_{T+1} = \infty$. When $P_{T+1} < \infty$, an increase in nominal money M_{T+1} raises real money holdings M_{T+1}/P_{T+1}, and so the consumer can consider trading of some consumption good for some real money, by buying nominal money. But when $P_{T+1} = \infty$, buying extra nominal money does not give any increase in real money, nor any decrease in the marginal utility of consumption. At $P_{T+1} = \infty$, there is no available tradeoff between consumption goods and real money holdings. The full first-order condition requires $\left[u'(y) - v'(M_{T+1}/P_{T+1})\right]/P_{T+1} \geq 0$, and the numerator can be negative when the denominator is infinite. Obstfeld and Rogoff's $A(m)$ and $B(m)$ lines intersect again at $m = 0$. Prices are given to consumers who then choose demands. You take price limits first.

Now, let us see how the redemption promise modifies this logic. Indeed, no *finite* price level P_{T+1} is an equilibrium. But the arbitrage argument fails at $P_{T+1} = \infty$, for the same reason that the more-money-demand argument failed. At $P_{T+1} = \infty$, (and P_{T+2}, etc., $= \infty$) the optimal thing for consumers to do is to turn in all their money for the redemption value $M_T/\overline{\overline{P}} = M/\overline{\overline{P}}$. There is no point in holding on to worthless money.

One might argue the latter point – perhaps an infinite price level means one can get an infinite amount of money for a finite amount of the good, and we start a debate about limits. But if we have accepted, as Obstfeld and Rogoff have, that $P_{T+1} = \infty$ is the correct equilibrium without the redemption option, then the redemption option does not change that fact. Without the redemption option, consumers holding endowment y and money M would really like to sell some of their endowment to get some additional real money holdings. We decided that at $P_{T+1} = \infty$ they can't get any additional real money holdings. With the redemption option, consumers first redeem their money, and then show up the goods market with endowment $y + M/\overline{\overline{P}}$ to get additional real money. If they couldn't trade goods for real money before, they can't do it now.

Put another way, a government promise to exchange one good for another at a set rate only determines the relative price of those goods if the consumer holds an interior amount of the goods. Obstfeld and Rogoff's specification that the government does not sell money for goods, $M_t \leq M$, means that the price level can be lower than the peg, $P_t < \overline{\overline{P}}$, if people are at the constraint $M_t = M$. Similarly, however, the limit $M_t \geq 0$ means that the price level can exceed the peg, $P_t > \overline{\overline{P}}$, if people are at the constraint $M_t = 0$.

In sum, the jump to zero value of money, and infinite price level, in this model, is not removed by the government's promise to redeem money for a small amount of consumption good (or capital.)

Moving back, the redemption guarantee does affect the price level at time T. Previously, people held money at time T for its utility at T, even knowing it would be worthless at time $T+1$. Now, they hold money at T for its value at that time period, and also its redemption value at time $T+1$. In the presence of a redemption promise, the first-order condition at time T, in equilibrium $c_t = y$ and $M_T = M$, becomes

$$\frac{u_c(y)}{P_T} - u_m\left(\frac{M}{P_T}\right)\frac{1}{P_T} = \max\left[\frac{\beta u'(y)}{\overline{\overline{P}}}, \frac{\beta u'(y)}{P_{T+1}}\right].$$

With the latter term zero at time T, the redemption value of the former term remains. This effect gives a small decrease in the price level $P_T < \bar{P}$ and $M/P_T > M/\bar{P}$.

The dashed line in Figure A3.2 presents the equilibrium with the redemption guarantee, labeled "$\varepsilon = 0.5$." Time $T+1$ and beyond have price levels $P_t = \infty$. The time T price level is now a little lower, and time T real money M/P_T a little higher than before, because of the redemption value of money at time $T+1$. The dashed line marked "redemption value" gives the value $M/\overline{\overline{P}}$ that the consumer receives from the government at time $T+1$. This is not M_{T+1}/P_{T+1} since that is $0/\infty$. But drawing this redemption value on the graph in place of a market value of money, you can see how values propagate back in this equilibrium. Obstfeld and Rogoff study this point, with $M_{T+1} = M$ and $P_{T+1} = \overline{\overline{P}}$ as the last point of their economy. However, they claim that this point is *not* an equilibrium, and with that claim seek to rule out the path leading to it. My view here is that the point below it is the equilibrium, with $M_{T+1} = 0$ and $P_{T+1} = \infty$, and the path leading to that point remains valid.

A key to my equilibrium is that monetary policy allows de-monetization, for people to cash in their money. We could rule out this equilibrium by having monetary policy also insist that $M_{T+1} = M$. The combination of $M_T = M$ and the redemption guarantee would indeed be a policy setting for which no equilibrium can form, and we saw in Section 16.10 several proposals that amount to such "inconsistent" policy. But we dismissed inconsistent policy before, e.g., insisting simultaneously on an interest rate rule $i = \phi\pi$ together with a money growth rule that requires a lower interest rate. In my reading, Obstfeld and Rogoff do not specify an inconsistent policy. They do allow the government to undershoot the money growth target if people want to redeem their money. Alas, by specifying a consistent policy, they do not rule out multiple equilibria.

(In this treatment, I assume that people tender their money M_{t-1} to the government at the beginning of period t. Cochrane (2011a) makes the opposite timing assumption, which leads to the same answer but in a more, and unnecessarily complex, way.)

Obstfeld and Rogoff (2021) respond, calling this analysis an "error." Their central rejoinder is

> "The last Euler equation requires money to have no value on date $T+1$; that is, the price level jumps from a finite P_T to $P_{T+1} = \infty$. But the government's

> promise to redeem money remains good on date $T+1$: Any individual who deviates from the proposed equilibrium and instead carries \$1 into period $T+1$ will be able to sell it on the market to other agents at any real price less than or equal to $1/\overline{\overline{P}} = \varepsilon$ because they, in turn, can then sell the \$1 to the government for ε in output. That simple arbitrage argument implies that the market price of money on date $T+1$ simply cannot be zero. It will be at least ε and so the true price level, measured in terms of money, will be at most $\overline{\overline{P}}$; not ∞."

Here, I believe Obstfeld and Rogoff deviate from the definition of Walrasian equilibrium. An equilibrium is a set of prices (here, a price level sequence) and an allocation such that consumers maximize utility given the price level sequence and markets clear. Given a price level sequence such that $P_{T+1} = \infty$, it is optimal for consumers to sell all their money to the government at time T. This *is* an equilibrium, in the conventional definition of Walrasian equilibrium.

Somehow the price level is ∞ yet there is also an opportunity to "deviate from equilibrium" and have the price level equal $\overline{\overline{P}}$ at the same time. The rules of Walrasian equilibrium are simple – there is one $\{P_T\}$ we talk about maximization given that sequence of prices.

More charitably, Obstfeld and Rogoff are perhaps introducing a different more expanded definition of equilibrium, introducing informally some game theoretic concept in which consumers don't just maximize given prices, but they can take individual "deviations from equilibrium," in which P_{T+1} for everyone else is ∞, but I can get $1/\varepsilon$, and so on. The language "deviate from equilibrium" here, and later "individual agents would have a strong incentive to deviate from this alleged (Nash) equilibrium" adds to that interpretation.

But Obstfeld and Rogoff (1983) didn't say anything about a larger Nash equilibrium concept. And my rejoinder is only that a *Walrasian* equilibrium exists. One might find a proper game theoretic equilibrium including these concepts a potentially interesting extension of their work. But if so, the logic is that my critique is right about Obstfeld and Rogoff (1983), but the problem can be solved with an enlarged game theoretic definition of equilibrium.

A3.2.3 Interpreting Obstfeld and Rogoff

Whether or not one accepts my analysis of Obstfeld and Rogoff, it does not achieve a full price level determination with passive policy, ready to use for analyzing data or policy. The fix is fiscal, as it requires the government to have enough commodity on hand, and the government refuses the seigniorage opportunities of money demand after stopping inflation. The fix does not represent a commitment that our or any other government makes or has made, so it represents at best a policy proposal rather than a basis for description of current or historical policy. None of this is criticism, as Obstfeld and Rogoff did not claim otherwise. It is only a warning not to cite them for results they did not claim to offer.

We should not overemphasize the latter minor disagreement. 99% of the importance of Obstfeld and Rogoff's analysis for our quest does not depend on it, and can grant their view that the inflationary equilibria are ruled out.

The main point: Many authors quickly cite Obstfeld and Rogoff as showing that all multiple equilibrium problems are solved, and a small chance that governments would stop inflation by reverting to a gold standard at very high inflation restores price level determinacy by monetary policy alone. This is not what their result, even taken at face value, accomplishes.

First, it is a joint fiscal–monetary theory, not an alternative that rescues monetary price level determination with passive policy. The government must have the gold, and must not change the backstop redemption promise in response to observed prices. The government must also refuse the siren song of seigniorage that a two-way gold standard implies. If they are correct, the result challenges the generality of our earlier finding that fiscal backstops with locally passive fiscal policies do not suffice, but it is not a passive policy.

Second, the government's refusal to *take* gold in return for new money is the central ingredient for ruling out inflationary equilibrium paths. Obstfeld and Rogoff's proposal is not a simple reversion to the gold or commodity standard, which applies both ways! After the backstop price level is reached, inflation stops, and real money demand expands. If the government accommodates this demand, allowing people to bring gold in for new money, and thus allowing the steady state to re-form around the new price level $\overline{\overline{P}}$ then we successfully transition to a steady price level.

Third, this latter feature really makes the suggestion at best a proposal for threats future central banks might make, not a suggestion for how current central banks behave or are expected by anyone to behave. There is not a whiff of this commitment on the Federal Reserve's website. Many governments have stopped inflations with joint monetary–fiscal reforms. Some of those have even included gold standards or exchange rate pegs. But, as beautifully documented in Sargent (1982b), all such governments have allowed and indeed encouraged the natural recovery of nominal and hence real money holdings once inflation has stopped. No central bank has ever announced that it would refuse to *take* gold in return for new currency in a stabilization!

Fourth, ex post, a promise not to *take* gold (or foreign exchange) in return for currency, and therefore to forbid equilibrium from forming, whatever that means, is disastrous for the government and central bank's objectives. Citizens are clamoring for the central bank to satisfy a money shortage. The treasury eyes a golden opportunity for non-inflationary seigniorage. Would any central bank, ex post, inflict a non-formation-of-equilibrium on an economy?

A one-way gold standard, which rules out equilibrium formation, is not a credible specification of what people currently or historically expect of our central banks.

Finally, of course, it specifies a money growth target which our central banks do not follow. And it depends crucially on a discrete time timing convention that produces infinite inflation in finite time, with a last day of money holding, which does not appear in the continuous time model's inflation dynamics.

In these ways, if Obstfeld and Rogoff's claim to rule out inflationary paths is correct, it does not rescue $MV(i) = Py$ with passive fiscal policy as a viable framework for current or historical monetary policy analysis. That too is not a criticism of Obstfeld and Rogoff – they intended it as a piece of pure theory, and did not claim that current central banks follow their policy, or that people expect it. They write (p. 676) "Speculative paths can be eliminated..." Can, not are.

You may object that we do not see hyperinflations with constant money growth. All hyperinflations occur with immense money growth. Does that not show the inflationary equilibria are invalid? I agree that this observation shows the model that allows inflationary multiple equilibria with constant money growth is wrong, and incomplete. It needs another ingredient. In my view there is a different missing ingredient: Fiscal theory picks the price level path. Observed hyperinflations are all fiscal, that occur when the fiscal backing of the non-inflationary both disappears.

I spent a lot of time on this one paper, because so many authors casually cite Obstfeld and Rogoff as having solved all these problems and rescued $MV(i) = Py$ as a purely monetary price level determination, even with $V(i)$, and in a realistic way that we can use in analysis of actual economies. Even their claimed result does not achieve this goal – nor was it intended to do so. I spend more time because with an unconventional reading of a cited paper I think it's important to be extra careful – how am I right and the other 461 Google scholar citers wrong? But that criticism just adds to the same bottom line for our purposes here.

A3.2.4 Nonseparable utility and more indeterminacy

Utility nonseparable between money and consumption is possible, and plausible. In this case, it is possible that our constant money growth model leads to multiple stable equilibria around the steady state. The phase diagram of Figure 19.1 can cut from above at the steady state.

A CES money in the utility function separates the interest elasticity of money demand from risk aversion. It also parameterizes how money growth distorts the relationship between consumption and interest rates.

When utility is non-separable between consumption and money, with $u_{mc}(c_t, m_t) \neq 0$, our first-order conditions (19.15)-(19.16),

$$Q_t = \frac{1}{1+i_t} = E_t\left(\beta \frac{u_c(c_{t+1}, m_{t+1})}{u_c(c_t, m_t)} \frac{P_t}{P_{t+1}}\right) \tag{A3.7}$$

$$\frac{u_m(c_t, m_t)}{u_c(c_t, m_t)} = \frac{i_t}{1+i_t} = 1 - Q_t \tag{A3.8}$$

no longer separate so cleanly.

The marginal rate of substitution or discount factor for asset pricing in (A3.7) now contains money holdings as well as consumption. The usual approximation, precise in continuous time[1], gives

$$\beta \frac{u_c(c_{t+1}, m_{t+1})}{u_c(c_t, m_t)} \approx 1 - \delta - \gamma \Delta c_{t+1} - \eta \Delta m_{t+1}.$$

$$r^f \approx \delta + \gamma E_t(\Delta c_{t+1}) + \eta E_t(\Delta m_{t+1})$$

$$\gamma = -\frac{c u_{cc}}{uc}; \eta = -\frac{u_{cm}}{u_c}.$$

The relation between interest rates and consumption growth is distorted by money. Covariances with money growth will drive risk premia. Expected money growth will drive a wedge between risk free rates and expected consumption growth.

A nonseparable utility, in which variation in some additional variable moves the discount factor along with consumption growth, is widespread in finance. One can categorize almost all of the innovations in macro-finance as nonseparable utilities in which some variable other than consumption growth affects the discount factor (Cochrane (2017a).) Habits, housing, durable goods, recursive non-state-separable utility, and many others are of this form.

A nonseparable utility is also considered a realistic specification for money in the utility function. Money should be essential, in some sense, to procuring consumption. The point of money is not to enjoy Scrooge McDuck swims in it, but because money makes purchasing consumption and selling endowments easier. That thought leads to a nonseparable utility. Cash in advance models typically lead to an equivalent representation as a nonseparable money in the utility function. In order to consume tomorrow, you must hold money overnight from today to tomorrow, and thereby consumption tomorrow gains an extra cost, the foregone interest.

With constant consumption as in our example, nonseparable utility means that changes in real money growth change the nominal interest rate, and thereby change the price level dynamics that result from (A3.8).

We can now have multiple *stable* equilibria around the steady state, so we have indeterminacy even without worrying about transversality conditions or the zero interest rate state. The phase diagram Figure 19.1 can cut from above rather than below. (Obstfeld (1984) makes this point in a delightfully concise 5 page paper. Why don't journals publish papers like this any more?)

Suppose

$$u(c_t, m_t) = -c_t^{-a} m_t^{-b}$$

Now,

$$u_c(c_t, m_t) = a c_t^{-a-1} m_t^b$$

$$u_m(c_t, m_t) = b c_t^a m_t^{-b-1}$$

[1]

$$\frac{d\Lambda_t}{\Lambda_t} = \frac{d\left[e^{-\delta t} u_c(c_t, m_t)\right]}{e^{-\delta t} u_c(c_t, m_t)}$$

$$= -\delta dt + \frac{c_t u_{cc}}{u_c} \frac{dc_t}{c_t} + \frac{m_t u_{cm}}{u_c} \frac{dm_t}{m_t} \text{ (+Ito terms).}$$

so the first-order conditions give

$$\frac{c_t b}{m_t a} = 1 - \beta \left(\frac{c_{t+1}}{c_t}\right)^{-a-1} \left(\frac{m_{t+1}}{m_t}\right)^{-b} \frac{P_t}{P_{t+1}}.$$

In equilibrium with money growth μ and endowment $y = 1$, then,

$$\frac{b}{m_t a} = 1 - \beta \left(\frac{m_{t+1}}{m_t}\right)^{-b} \frac{P_t}{P_{t+1}}.$$

$$\frac{b}{a}\frac{1}{m_t} = 1 - \frac{1}{(1+\delta)(1+\mu)} \left(\frac{m_{t+1}}{m_t}\right)^{1-b}.$$

$$\frac{m_{t+1}}{m_t} = \left[(1+\delta)(1+\mu)\left(1 - \frac{b}{a}\frac{1}{m_t}\right)\right]^{\frac{1}{1-b}}.$$

The steady state is

$$1 = (1+\delta)(1+\mu)\left(1 - \frac{b}{a}\frac{1}{m}\right).$$

In terms of the steady state,

$$\frac{m_{t+1}}{m_t} = \left[\frac{1 - \frac{b}{a}\frac{1}{m_t}}{1 - \frac{b}{a}\frac{1}{m}}\right]^{\frac{1}{1-b}}.$$

Near the steady state,

$$\frac{m_{t+1}}{m_t} \approx 1 + \frac{d}{dm_t}\left\{\left[\frac{1 - \frac{b}{a}\frac{1}{m_t}}{1 - \frac{b}{a}\frac{1}{m}}\right]^{\frac{1}{1-b}}\right\}_{m_t = m} (m_t - m).$$

$$\frac{m_{t+1}}{m_t} \approx 1 + \frac{b}{a}\frac{1}{1-b}\frac{(m_t - m)}{m^2}.$$

The coefficient multiplying the last term is negative for

$$b > 1.$$

In that case, dynamics are stable around the steady state, giving multiple stable equilibria that unquestionably satisfy the transversality condition!

A CES specification is also useful as it lets us separate intertemporal substitution from the interest elasticity of money demand, but maintaining the useful proportionality of money to nominal income. If

$$u(c_t, M_t/P_t) = \frac{\left[c_t^\rho + \theta(M_t/P_t)^\rho\right]^{\frac{1-\gamma}{\rho}} - 1}{1-\gamma}$$

then we have

$$u_c(y_t, M_t/P_t) = \left[y_t^\rho + \theta(M_t/P_t)^\rho\right]^{\frac{1-\gamma-\rho}{\rho}} y_t^{\rho-1}$$

$$= \left[1 + \theta\left(\frac{M_t}{P_t y_t}\right)^{\rho}\right]^{\frac{1-\gamma-\rho}{\rho}} y_t^{-\gamma}$$

so asset prices are driven by

$$\frac{\Lambda_{t+1}}{\Lambda_t} = \beta \frac{\left[1 + \theta\left(\frac{M_{t+1}}{P_{t+1} y_{t+1}}\right)^{\rho}\right]^{\frac{1-\gamma-\rho}{\rho}}}{\left[1 + \theta\left(\frac{M_t}{P_t y_t}\right)^{\rho}\right]^{\frac{1-\gamma-\rho}{\rho}}} \left(\frac{y_{t+1}}{y_t}\right)^{-\gamma}.$$

A monetary distortion modifies the standard power utility formula. Growth in real money balances accompanies higher real interest rates, as does growth in consumption. With constant real money $M/(Py)$, we have the usual power utility formula with risk aversion γ.

The money first-order condition is

$$u_m(y_t, M_t/P_t) = \left[y_t^{\rho} + \theta\left(\frac{M_t}{P_t}\right)^{\rho}\right]^{\frac{1-\gamma-\rho}{\rho}} \theta\left(\frac{M_t}{P_t}\right)^{\rho-1}$$

so the money demand function (19.16) becomes

$$\frac{\theta\left(\frac{M_t}{P_t}\right)^{\rho-1}}{y_t^{\rho-1}} = \frac{i_t}{1+i_t}$$

$$M_t = P_t y_t \left(\frac{1}{\theta}\frac{i_t}{1+i_t}\right)^{\frac{1}{\rho-1}}.$$

Here too we have a unit income elasticity and an interest elasticity governed by the separate parameter ρ.

A3.3 Uniqueness in cash in advance models

This section adds to the treatment in Section 19.5.5 on multiple equilibria in cash in advance models.

Woodford (1994) studies a cash in advance economy with cash and credit goods, so it has an interest-elastic money demand. He shows that a constant money growth policy typically leaves multiple equilibria, and always does so if money supports negative inflation at the interest rate and a zero rate, but an interest rate peg can have a unique equilibrium, even if the rate is pegged at zero. He concludes that the Friedman rule can be supported by an interest rate peg, but not a money growth target.

With the advantages of hindsight, Woodford's result is deeply fiscal. Woodford's money growth policy is financed by lump sum taxes or transfers, rebating all seigniorage to households. In the money growth model, Woodford specifies that households receive a nominal transfer equal to money printing, $H_t = M_t - M_{t-1}$ (p. 350). For the interest rate target, Woodford specifies instead that "the government chooses a deterministic (and later "exogenous") sequence $\{h_t\}$ for *real* net transfers to the private sector." With the advantages of hindsight, we recognize the fiscal theory of monetary policy at work in the latter case, and a passive fiscal policy in the former. Woodford's core result, then, is that an active fiscal policy can overturn Sargent and Wallace (1975) indeterminacy of interest rate pegs. Indeed, (p. 378):

> The failure of homogeneity in (3.3) [the result that an interest rate peg leads to determinate inflation] does depend upon the specification of fiscal policy here; in particular, any process $\{M_t\}$ satisfying (3.1)-(3.2) can be made to be an equilibrium consistent with an interest rate peg if net transfers$\{H_t\}$ are assumed to vary with the sunspot state in the way necessary to satisfy the intertemporal budget constraint. On the other hand, the kind of fiscal policy specification required to preserve homogeneity is a special one; the particular case considered here (real net transfers constant over time and unaffected by the path of nominal variables) is simple to analyze but is hardly the only kind of specification for which the intertemporal budget constraint causes the equilibrium conditions to be inhomogeneous.

and p. 373,

> For the increase or decrease in the money supply that would be necessary to accommodate a given change in the current price level carries with it a change in the net indebtedness of the government to the private sector, which will affect the budget constraints of consumers and so have a real effect on the economy... It is this second source of inhomogeneity that is relevant for the representative consumer economy considered here.

Followed by footnote 19,

> Leeper (1991) similarly obtains determinacy due to the intertemporal budget constraint in the case of a variety of types of interest rate policies, in the context of a linear model.

If this is not terribly clear, keep in mind this paper was written just around the time Leeper's foundational "active and passive" Leeper (1991) was published, and long before Woodford's "fiscal requirements for price stability," Woodford (2001).

Chapter Four

Money in the utility function in continuous time

It's easy to get hung up on the timing conventions of discrete time models. For that reason, it is usually simpler and less confusing in the end to work with these models in continuous time.

The utility function is

$$\max E \int_{t=0}^{\infty} e^{-\delta t} u(c_t, M_t/P_t) dt.$$

The present value budget constraint is

$$\frac{B_0 + M_0}{P_0} = \int_{t=0}^{\infty} e^{-\int_{s=0}^{t} r_s ds} \left[c_t - y_t + s_t + (i_t - i_t^m) \frac{M_t}{P_t} \right] dt$$

where

$$r_t = i_t - \frac{dP_t}{P_t}$$

and s denotes real net taxes paid, and thus the real government primary surplus. This budget constraint is the present value form of

$$d(B_t + M_t) = i_t B_t + i_t^m M_t + P_t(y_t - c_t - s_t).$$

Introducing a multiplier λ on the present value budget constraint, we have

$$\frac{\partial}{\partial c_t} : e^{-\delta t} u_c(t) = \lambda e^{-\int_{s=0}^{t} r_s ds},$$

where (t) means $(c_t, M_t/P_t)$. Differentiating with respect to time,

$$-\delta e^{-\delta t} u_c(t) + e^{-\delta t} u_{cc}(t) \frac{dc_t}{dt} + e^{-\delta t} u_{cm}(t) \frac{dm_t}{dt} = -\lambda r_t \ e^{-\int_{s=0}^{t} r_s ds}$$

where $m_t \equiv M_t/P_t$. Dividing by $e^{-\delta t} u_c(t)$, we obtain the intertemporal first-order condition:

$$-\frac{c_t u_{cc}(t)}{u_c(t)} \frac{dc_t}{c_t} - \frac{m_t u_{cm}(t)}{u_c(t)} \frac{dm_t}{m_t} = (r_t - \delta)\, dt. \tag{A4.1}$$

The first-order condition with respect to M is

$$\frac{\partial}{\partial M_t} : e^{-\delta t} u_m(t) \frac{1}{P_t} = \lambda e^{-\int_{s=0}^{t} r_s ds} (i_t - i_t^m) \frac{1}{P_t}$$

$$e^{-\delta t} u_m(t) = e^{-\delta t} u_c(t) (i_t - i_t^m)$$

$$\frac{u_m(t)}{u_c(t)} = i_t - i_t^m. \tag{A4.2}$$

The last equation is the usual money demand curve.

Thus, an equilibrium $c_t = y_t$ satisfies

$$\frac{-c_t u_{cc}(t)}{u_c(t)} \frac{dc_t}{c_t} - \frac{m_t u_{cm}(t)}{u_c(t)} \frac{dm_t}{m_t} = -\delta dt + \left(i_t - \frac{dP_t}{P_t} \right) dt \tag{A4.3}$$

$$\frac{u_m(t)}{u_c(t)} = i_t - i_t^m \tag{A4.4}$$

$$\frac{B_0 + M_0}{P_0} = \int_{t=0}^{\infty} e^{-\int_{s=0}^{t} r_s ds} \left[s_t + (i_t - i_t^m) \frac{M_t}{P_t} \right] dt. \tag{A4.5}$$

A4.1 CES functional form

I use a standard money in the utility function specification with a CES functional form,

$$u(c_t, m_t) = \frac{1}{1-\gamma} \left[c_t^{1-\theta} + \alpha m_t^{1-\theta} \right]^{\frac{1-\gamma}{1-\theta}}.$$

I use the notation $m = M/P$, with capital letters for nominal and lowercase letters for real quantities.

This CES functional form nests three important special cases. Perfect substitutes is the case $\theta = 0$:

$$u(c_t, m_t) = \frac{1}{1-\gamma} \left[c_t + \alpha m_t \right]^{1-\gamma}.$$

The Cobb-Douglas case is $\theta \longrightarrow 1$:

$$u(c_t, m_t) \longrightarrow \frac{1}{1-\gamma} \left[c_t^{\frac{1}{1+\alpha}} m_t^{\frac{\alpha}{1+\alpha}} \right]^{1-\gamma}. \tag{A4.6}$$

The monetarist limit is $\theta \to \infty$:

$$u(c_t, m_t) \to \frac{1}{1-\gamma} \left[\min\left(c_t, \alpha m_t\right) \right]^{1-\gamma}.$$

I call it the monetarist limit because money demand is then $M_t/P_t = c_t/\alpha$, i.e. $\alpha = 1/V$ is constant, and the interest elasticity is zero. The separable case is $\theta = \gamma$:

$$u(c_t, m_t) = \frac{1}{1-\gamma} \left[c_t^{1-\gamma} + \alpha m_t^{1-\gamma} \right].$$

In the separable case, u_c is independent of m, so money has no effect on the intertemporal substitution relation, and hence on inflation and output dynamics in a new-Keynesian model under an interest rate target. Terms in $(\theta - \gamma)$ or $(\sigma - \xi)$ with $\sigma = 1/\gamma$ and $\xi = 1/\theta$ will characterize deviations from the separable case, how much the marginal utility of consumption is affected by money.

With this functional form, the derivatives are

$$u_c = \left[c_t^{1-\theta} + \alpha m_t^{1-\theta}\right]^{\frac{\theta-\gamma}{1-\theta}} c_t^{-\theta}$$

$$u_m = \left[c_t^{1-\theta} + \alpha m_t^{1-\theta}\right]^{\frac{\theta-\gamma}{1-\theta}} \alpha m_t^{-\theta}.$$

Equilibrium condition (A4.4) becomes

$$\frac{u_m(t)}{u_c(t)} = \alpha\left(\frac{m_t}{c_t}\right)^{-\theta} = i_t - i_t^m. \tag{A4.7}$$

The second derivative with respect to consumption is

$$\frac{u_{cc}}{u_c} = (\theta-\gamma)\frac{1}{\left[c_t^{1-\theta} + \alpha m_t^{1-\theta}\right]} c_t^{-\theta} - \theta c_t^{-1}$$

$$-\frac{c u_{cc}}{u_c} = -\frac{(\theta-\gamma)c_t^{1-\theta} - \theta\left[c_t^{1-\theta} + \alpha m_t^{1-\theta}\right]}{\left[c_t^{1-\theta} + \alpha m_t^{1-\theta}\right]}$$

$$-\frac{c u_{cc}}{u_c} = \frac{\gamma c_t^{1-\theta} + \theta\alpha m_t^{1-\theta}}{c_t^{1-\theta} + \alpha m_t^{1-\theta}}$$

$$-\frac{c u_{cc}}{u_c} = \gamma\frac{\left[1 + \frac{\theta}{\gamma}\alpha\left(\frac{m_t}{c_t}\right)^{1-\theta}\right]}{\left[1 + \alpha\left(\frac{m_t}{c_t}\right)^{1-\theta}\right]}.$$

The cross derivative is

$$\frac{m u_{cm}}{u_c} = (\theta-\gamma)\frac{\alpha m_t^{1-\theta}}{c_t^{1-\theta} + \alpha m_t^{1-\theta}}$$

$$= (\theta-\gamma)\frac{\alpha\left(\frac{m_t}{c_t}\right)^{1-\theta}}{1 + \alpha\left(\frac{m_t}{c_t}\right)^{1-\theta}}.$$

or, using (A4.7)

$$\frac{m u_{cm}}{u_c} = (\theta-\gamma)\frac{\left(\frac{m_t}{c_t}\right)(i_t - i_t^m)}{1 + \left(\frac{m_t}{c_t}\right)(i_t - i_t^m)}.$$

A4.1.1 Money demand

Money demand (A4.7) can be written

$$\frac{m_t}{c_t} = \left(\frac{1}{\alpha}\right)^{-\xi}(i_t - i_t^m)^{-\xi}. \tag{A4.8}$$

where $\xi = 1/\theta$ becomes the interest elasticity of money demand, in log form, and α governs the overall level of money demand.

The steady state obeys

$$\frac{m}{c} = \left(\frac{1}{\alpha}\right)^{-\xi} (i - i^m)^{-\xi}. \tag{A4.9}$$

so we can write money demand (A4.8) in terms of steady state real money as

$$\frac{m_t}{c_t} = \left(\frac{m}{c}\right) \left(\frac{i_t - i_t^m}{i - i^m}\right)^{-\xi}, \tag{A4.10}$$

avoiding the parameter α. (Throughout, numbers without time subscripts denote steady state values.)

The product $\frac{m}{c}(i - i^m)$, the interest cost of holding money, appears in many subsequent expressions. It is

$$\frac{m}{c}(i - i^m) = \left(\frac{1}{\alpha}\right)^{-\xi} (i - i^m)^{1-\xi}.$$

With $\xi < 1$, as interest rates go to zero this interest cost goes to zero as well.

A4.1.2 Intertemporal Substitution

The first-order condition for the intertemporal allocation of consumption (A4.3) is

$$-\frac{c_t u_{cc}(t)}{u_c(t)} \frac{dc_t}{c_t} - \frac{m_t u_{cm}(t)}{u_c(t)} \frac{dm_t}{m_t} = -\delta dt + (i_t - \pi_t)\, dt$$

where $\pi_t = dP_t/P_t$ is inflation. This equation shows us how, with nonseparable utility, monetary policy can distort the allocation of consumption over time, in a way not captured by the usual interest rate effect. That is the central goal here. In the case of complements, $u_{cm} > 0$ (more money raises the marginal utility of consumption), larger money growth makes it easier to consume in the future relative to the present, and acts like a higher interest rate, inducing higher consumption growth.

Substituting in the CES derivatives,

$$\gamma \frac{1 + \frac{\theta}{\gamma}\alpha \left(\frac{m_t}{c_t}\right)^{1-\theta}}{1 + \alpha\left(\frac{m_t}{c_t}\right)^{1-\theta}} \frac{dc_t}{c_t} - (\theta - \gamma) \frac{\alpha\left(\frac{m_t}{c_t}\right)^{1-\theta}}{1 + \alpha\left(\frac{m_t}{c_t}\right)^{1-\theta}} \frac{dm_t}{m_t} = -\delta dt + (i_t - \pi_t)\, dt$$

and using (A4.7) to eliminate α

$$\gamma \frac{1 + \frac{\theta}{\gamma}\left(\frac{m_t}{c_t}\right)(i_t - i_t^m)}{1 + \left(\frac{m_t}{c_t}\right)(i_t - i_t^m)} \frac{dc_t}{c_t} - (\theta - \gamma) \frac{\left(\frac{m_t}{c_t}\right)(i_t - i_t^m)}{1 + \left(\frac{m_t}{c_t}\right)(i_t - i_t^m)} \frac{dm_t}{m_t} = -\delta dt + (i_t - \pi_t)\, dt. \tag{A4.11}$$

We can make this expression prettier as

$$\gamma \frac{dc_t}{c_t} + (\theta - \gamma) \frac{\left(\frac{m_t}{c_t}\right)(i_t - i_t^m)}{1 + \left(\frac{m_t}{c_t}\right)(i_t - i_t^m)} \left(\frac{dc_t}{c_t} - \frac{dm_t}{m_t}\right) = -\delta dt + (i_t - \pi_t)\, dt.$$

Re-expressing in terms of the intertemporal substitution elasticity $\sigma = 1/\gamma$ and interest elasticity of money demand $\xi = 1/\theta$, and multiplying by σ,

$$\frac{dc_t}{c_t} + \left(\frac{\sigma - \xi}{\xi}\right) \frac{\left(\frac{m_t}{c_t}\right)(i_t - i_t^m)}{1 + \left(\frac{m_t}{c_t}\right)(i_t - i_t^m)} \left(\frac{dc_t}{c_t} - \frac{dm_t}{m_t}\right) = -\delta\sigma dt + \sigma (i_t - \pi_t)\, dt. \tag{A4.12}$$

We want to substitute interest rates for money. To that end, differentiate the money demand curve

$$\frac{m_t}{c_t} = \left(\frac{m}{c}\right)\left(\frac{i_t - i_t^m}{i - i^m}\right)^{-\xi}$$

$$\frac{m_t}{c_t}\left(\frac{dm_t}{m_t} - \frac{dc_t}{c_t}\right) = -\xi\left(\frac{m}{c}\right)\left(\frac{i_t - i_t^m}{i - i^m}\right)^{-\xi} \frac{d(i_t - i_t^m)}{i_t - i_t^m}$$

$$\left(\frac{dc_t}{c_t} - \frac{dm_t}{m_t}\right) = \xi \frac{\frac{m}{c}}{\frac{m_t}{c_t}}\left(\frac{i_t - i_t^m}{i - i^m}\right)^{-\xi} \frac{d(i_t - i_t^m)}{i_t - i_t^m}.$$

Substituting,

$$\frac{dc_t}{c_t} + \left(\frac{\sigma - \xi}{\xi}\right) \frac{\left(\frac{m_t}{c_t}\right)(i_t - i_t^m)}{1 + \left(\frac{m_t}{c_t}\right)(i_t - i_t^m)} \left(\xi \frac{\frac{m}{c}}{\frac{m_t}{c_t}}\left(\frac{i_t - i_t^m}{i - i^m}\right)^{-\xi} \frac{d(i_t - i_t^m)}{i_t - i_t^m}\right) = -\delta\sigma dt + \sigma (i_t - \pi_t)\, dt$$

$$\frac{dc_t}{c_t} + (\sigma - \xi)\frac{m}{c} \frac{1}{1 + \left(\frac{m_t}{c_t}\right)(i_t - i_t^m)} \left(\frac{i_t - i_t^m}{i - i^m}\right)^{-\xi} d(i_t - i_t^m) = -\delta\sigma dt + \sigma (i_t - \pi_t)\, dt.$$

With $x_t = \log c_t$, $dx_t = dc_t/c_t m$, approximating around a steady state, and approximating that the interest cost of holding money is small, $\left(\frac{m}{c}\right)(i - i^m) \ll 1$, we obtain the intertemporal substitution condition modified by interest costs,

$$\frac{dx_t}{dt} + (\sigma - \xi)\frac{m}{c}\frac{d(i_t - i_t^m)}{dt} = \sigma (i_t - \pi_t). \tag{A4.13}$$

In discrete time,

$$E_t x_{t+1} - x_t + (\sigma - \xi)\left(\frac{m}{c}\right)\left[E_t\left(i_{t+1} - i_{t+1}^m\right) - (i_t - i_t^m)\right] = \sigma (i_t - E_t \pi_{t+1}).$$

For models with monetary control, one wants an IS curve expressed in terms of the monetary aggregate. From (A4.12), with the same approximations and

$\tilde{m} = \log(m)$,

$$\frac{dx_t}{dt} + \left(\frac{\sigma - \xi}{\xi}\right)\left(\frac{m}{c}\right)(i - i^m)\left(\frac{dx_t}{dt} - \frac{d\tilde{m}_t}{dt}\right) = \sigma\left(i_t - \pi_t\right)dt. \tag{A4.14}$$

In discrete time,

$$(E_t x_{t+1} - x_t) + \left(\frac{\sigma - \xi}{\xi}\right)\left(\frac{m}{c}\right)(i - i^m)\left[(E_t x_{t+1} - x_t) - E_t\left(\tilde{m}_{t+1} - \tilde{m}_t\right)\right] = \sigma\left(i_t - \pi_t\right). \tag{A4.15}$$

A4.1.3 A Hamiltonian Approach

Obstfeld (1984) presents this model cleanly, using the standard continuous time, Hamiltonian approach to optimization. The objective is

$$\max \int_0^\infty e^{-\delta t}\left[u(c_t) + v(m_t)\right] dt$$

where $m_t = M_t/P_t$ is real money holdings. The constraint is

$$\dot{m}_t = (y_t - c_t - s_t) - m_t \frac{\dot{P}_t}{P_t}. \tag{A4.16}$$

(Nominal money piles up when income is greater than consumption less net tax payments.

$$dM_t = (y_t - c_t - s_t)P_t dt.$$

Use the definition $m_t \equiv M_t/P_t$ take the time derivative.) The current value Hamiltonian is

$$H = u(c_t) + v(m_t) + \mu\left[(y_t - c_t - s_t) - m_t \frac{\dot{P}_t}{P_t}\right].$$

The first-order conditions are therefore

$$\begin{aligned}
\frac{\partial H}{dc} &= 0 : u'(c_t) = \mu_t \\
-\frac{\partial H}{dm} &: \ \dot{\mu}_t - \delta\mu_t = -v'(m_t) + \mu_t \frac{\dot{P}_t}{P_t} \\
\frac{\partial H}{d\mu} &= 0 : \dot{m}_t = (y_t - c_t - s_t) - m_t \frac{\dot{P}_t}{P_t} \\
\lim_{t\to\infty} e^{-\delta t} m_t \mu_t &= 0.
\end{aligned} \tag{A4.17}$$

Substituting out μ, we can write the familiar money demand conditions.

$$\frac{v'(m_t)}{u'(c_t)} = -\frac{\dot{\mu}_t}{\mu_t} + \delta + \frac{\dot{P}_t}{P_t} = i_t.$$

When consumption is constant, $\dot{\mu} = 0$, the risk free rate is $r = \delta$. If we add a risk free real investment, then $r_t^f = \delta - \dot{\mu}_t/\mu_t$. Either way, the right-hand side equals the nominal interest rate. (A4.17) becomes the transversality condition, which says that the discounted real value of money may not grow faster than the interest rate.

$$\lim_{t\to\infty} e^{-\delta t} u'(c_t) m_t = 0.$$