12.1 GMM Formula summary

1. Model moments, true value b_0

$$g(b_0) = E\left[f(x_t, b_0)\right] = E\left[u_t\right] = E\left[(m_{t+1}(b_0)x_{t+1} - p_t\right] = 0$$

2. Sample moments:

$$g_T(b) = E_T\left[f(x_t, b)\right];\ E_T(\cdot) = \frac{1}{T}\sum_{t=1}^{T}\cdot$$

3. GMM estimator $\hat{b}$

$$a_T g_T(\hat{b}) = 0$$

4. Standard errors

$$\sqrt{T}(\hat{b} - b_0) \to N(0, \left(ad\right)^{-1}aSa'(ad)^{-1\prime})$$

where

$$\begin{aligned} d &= \frac{\partial g(b)}{\partial b'} \\ a &= plim\ a_T \\ S &= \sum_{j=-\infty}^{\infty} E\left[f(x_t, b_0)f(x_{t-j}, b_0)'\right] = \sum_{j=-\infty}^{\infty} E_t\left[u_t u_{t-j}'\right] \end{aligned}$$

5. Variance of moments:

$$\sqrt{T}g_T(\hat{b}) \to N(0, \left[I - d(ad)^{-1}a\right]S\left[I - d(ad)^{-1}a\right]').$$

$$\begin{aligned} \text{var}\left[g_T(b_0)\right] &= \frac{1}{T}S; \\ \text{var}\left[g_T(\hat{b})\right] &= \frac{1}{T}\left[I - d(ad)^{-1}a\right]S\left[I - d(ad)^{-1}a\right]' \end{aligned}$$

This can be used to test individual moments, or for χ^2 tests for joint significance. In particular, g_T

$$g_T' var(g_T)^{+} g_T \tilde{} \chi^2_{N-K}$$

6. Efficient GMM. Use any efficient first stage to estimate S, then use

$$a = d'S^{-1}$$

with this choice,

$$\begin{aligned} var(\hat{b}) &= \frac{1}{T}(d'S^{-1}d)^{-1} \\ cov(g_T) &= \frac{1}{T}\left[S - d(d'S^{-1}d)^{-1}d'\right] \end{aligned}$$

$$g_T' cov(g_T)^{+} g_T = T g_T' S^{-1} g_T = T J_T \tilde{} \chi^2_{\#mom-\#par}$$

7. Minimization approach

$$\hat{b}_1 = \min_b g_T(b)' W g_T(b)$$

This is a choice of a_T :

$$\left\{ \frac{\partial g_T(b)'}{\partial b} W \right\} g_T(b) = a_T g_T(b) = 0$$

Second stage/efficient

$$\hat{b}_2 = \min_b g_T(b)' S^{-1} g_T(b)$$

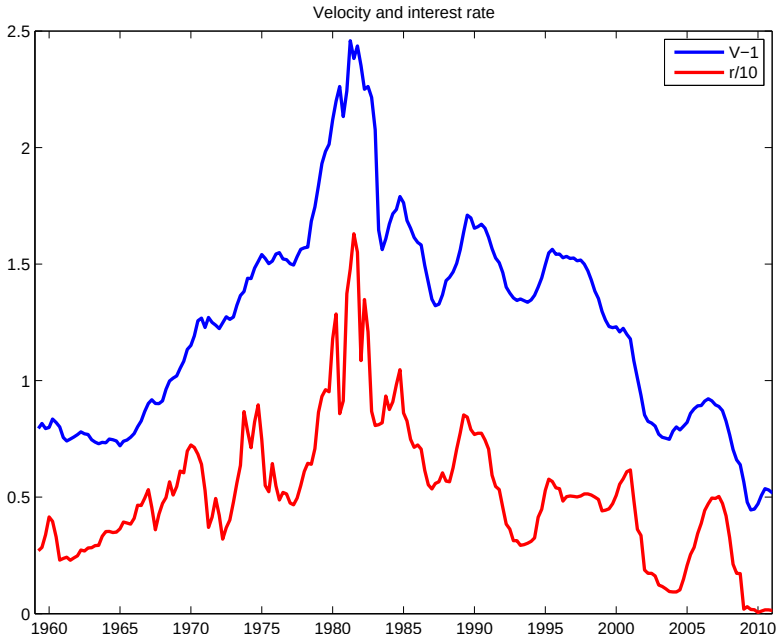
First order condition: this is the efficient GMM estimator

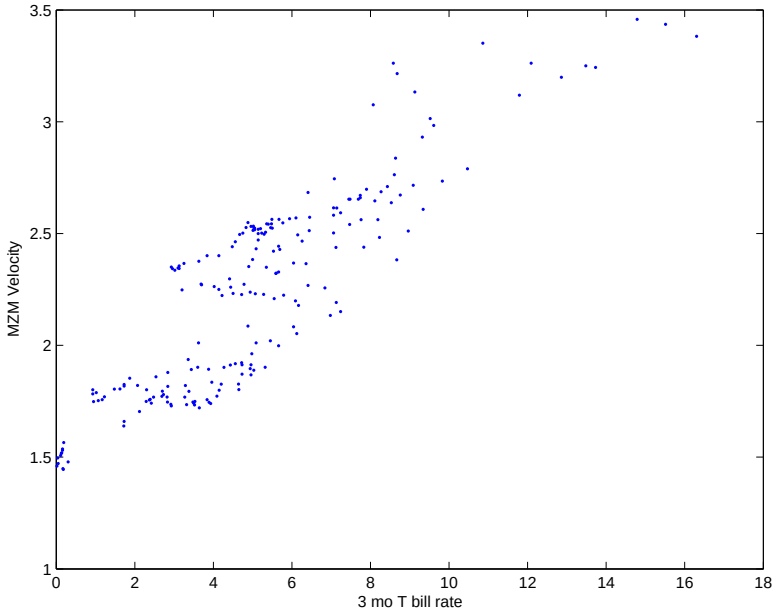
$$\left\{ \frac{\partial g_T(b)'}{\partial b} S^{-1} \right\} g_T(b) = d' S^{-1} g_T(b) = 0$$

Then, we can write the second stage test as

$$T J_T = T \min_{\{b\}} \left[g_T(b)' S^{-1} g_T(b) \right] \tilde{} \chi^2_{\#mom-\#par}$$

12.2 OLS GMM example



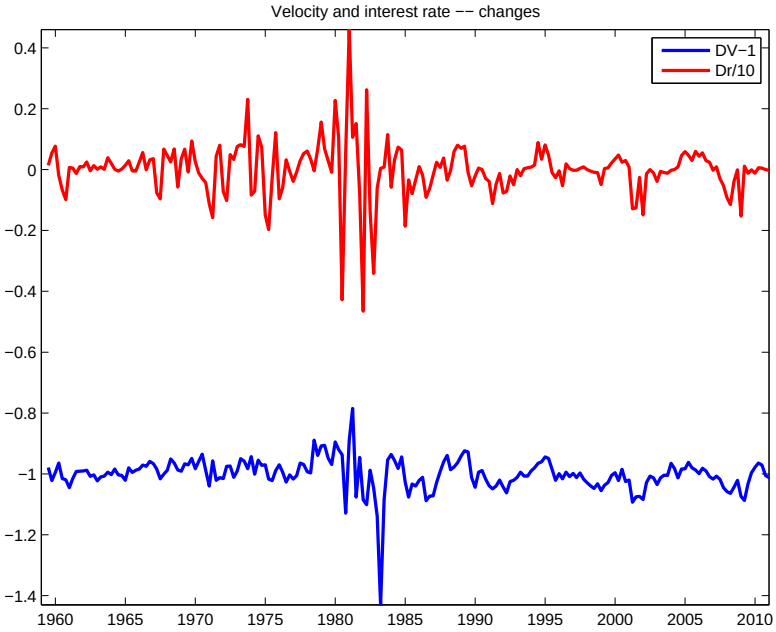


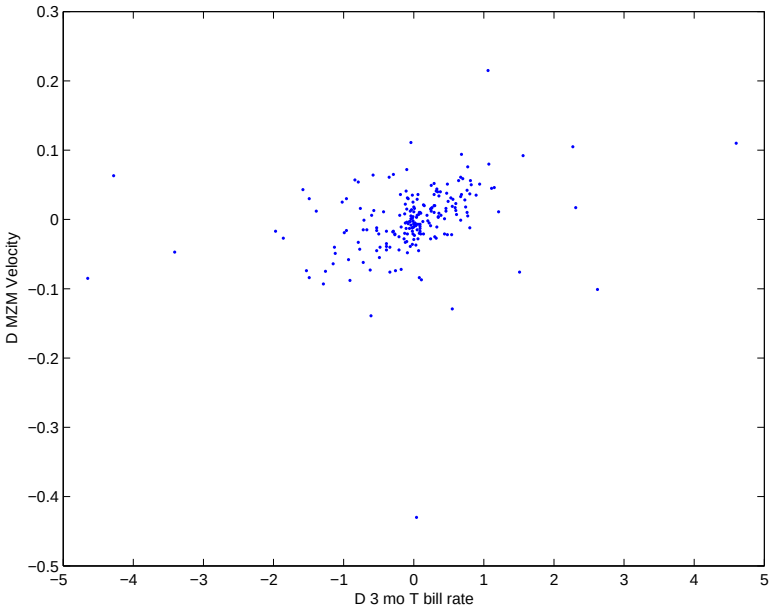
				raw		nonpara.		parametric	
	b	R^2	ρ	se	t	se	t	se	t
levels	0.136	0.77	0.86	0.004	30.44	0.008	17.7	0.016	8.73
$x_t - \hat{\rho} x_{t-1}$	0.043	0.25	0.51	0.010	4.52				
$x_t - x_{t-1}$	0.018	0.09	0.31	0.005	3.37				

$$\begin{aligned} V_t &= a + b r_t + \varepsilon_{t+1} \\ V_t - \hat{\rho} V_{t-1} &= a + b\left(r_t - \hat{\rho} r_{t-1}\right) + \left(\varepsilon_{t+1} - \hat{\rho}\varepsilon_t\right) \\ V_t - V_{t-1} &= a + b\left(r_t - r_{t-1}\right) + \left(\varepsilon_{t+1} - \varepsilon_t\right) \end{aligned}$$

nonparametric: Newey-West, 5 years

parametric: $(X'X)^{-1} X'\Omega X (X'X)^{-1}$


Velocity and interest rate −− changes
DV−1
Dr/10
0.4
0.2
0
−0.2
−0.4
−0.6
−0.8
−1
−1.2
−1.4
1960
1965
1970
1975
1980
1985
1990
1995
2000
2005
2010


D MZM Velocity
D 3 mo T bill rate
0.3
0.2
0.1
0
−0.1
−0.2
−0.3
−0.4
−0.5
−5
−4
−3
−2
−1
0
1
2
3
4
5