Habits

Basic idea:

- Consumption and habit picture.
- *Slow moving* is key. Note much macro is now using one period habit, a bad idea.
- One, business cycle related time varing risk aversion unites a lot of behavior we've studied. dp forecasts, price volatility, etc.
- The Fall 2008 crash looks a lot like habit-induced rising risk aversion to me!
- Is "habit" the mechanism? "leverage" or "irreversible durable goods" behaves the same way
- Also a laboratory for thinking about issues. Such as macro linearizations, conditional vs unconditional models, etc.
- A lot easier than long run risks, EZ, etc.
- Proud reverse-engineering. what must model be to produce the world we see? No difference between functional form and numbers!

Model

$$E\sum_{t=0}^{\infty}\delta^{t}\frac{(C_t-X_t)^{1-\gamma}-1}{1-\gamma}. \tag{10}$$

$$S_t\equiv\frac{C_t-X_t}{C_t}.$$

More s is good times. Less s is bad times.

$$\eta_t\equiv-\frac{C_t u_{cc}(C_t,X_t)}{u_c(C_t,X_t)}=\frac{\gamma C(C-X)^{-\gamma-1}}{(C-X)^{-\gamma}}=\frac{\gamma C}{(C-X)}=\frac{\gamma}{(\frac{C-X}{C})}=\frac{\gamma}{S_t}.$$

Not risk aversion! $rra=V_{WW}/(WvW)$ tells you bets on wealth. $W\neq C$ in general. (more later)

How does consumption adapt to habit? Like $X_t=\sum\phi^j C_{t-j}=\phi X_{t-1}+C_t$

$$s_{t+1}=(1-\phi)\bar{s}+\phi s_t+\lambda\left(s_t\right)\left(c_{t+1}-c_t-g\right). \tag{11}$$

ϕ, g and $\bar{s}$ are parameters. $\lambda\left(s_t\right)$ the *sensitivity function.*

a) use log s to keep S always positive!

b) λ allows chs in m, which we know we need. (s is state variable, m(s)). Reverse engineer it below. (what must λ be to produce the world we see)

Technology

$$\Delta c_{t+1} = g + v_{t+1}; \quad v_{t+1} \sim i.i.d.\, \mathcal{N}(0, \sigma^2). \tag{12}$$

Marginal utility

Habit is external, marginal utility is

$$u_c(C_t, X_t) = (C_t - X_t)^{-\gamma} = S_t^{-\gamma} C_t^{-\gamma}.$$

If *internal,* forward looking terms, $u_x(t+j)\partial X_{t+j}/\partial C_t$. In the end add nothing but complication (later) Convenience.

$$M_{t+1} \equiv \delta \frac{u_c(C_{t+1}, X_{t+1})}{u_c(C_t, X_t)} = \delta \left(\frac{S_{t+1}}{S_t} \frac{C_{t+1}}{C_t} \right)^{-\gamma}.$$

$$\begin{aligned} M_{t+1} &= \delta G^{-\gamma} e^{-\gamma(s_{t+1} - s_t + v_{t+1})} \\ s_{t+1} - s_t &= (1-\phi)(s_t - \bar{s}) + \lambda(s_t) v_{t+1}. \\ M_{t+1} &= \delta G^{-\gamma} e^{-\gamma[(\phi-1)(s_t - \bar{s}) + (1+\lambda(s_t))v_{t+1}]}. \end{aligned}$$

Note "amplificatin" for one period, a shock to c moves S as well. Thus m moves more. 1 is the consumption, λ is the s movment.

HJ Bound

$$max \frac{E_t(R^e_{t+1})}{\sigma_t(R^e_{t+1})} = \frac{\sigma_t(M_{t+1})}{E_t(M_{t+1})}$$

$$M = e^m; \frac{\sigma(M)}{E(M)} = \frac{\sqrt{E(M^2) - E(M)^2}}{E(M)} = \frac{\sqrt{e^{2\mu + 2\sigma^2} - e^{2\mu + \sigma^2}}}{e^{\mu + \sigma^2/2}} = \sqrt{e^{\sigma^2} - 1}$$

$$\frac{E_t(R^e_{t+1})}{\sigma_t(R^e_{t+1})} = \frac{\sigma_t(M_{t+1})}{E_t(M_{t+1})} = \left(e^{\gamma^2 \sigma^2 [1+\lambda(s_t)]^2} - 1 \right)^{\frac{1}{2}} \approx \gamma\sigma[1 + \lambda(s_t)].$$

Remind you : data say time-varying sharpe ratio, and we want high sharpe in bad times. Thus λ should be declining in s

Risk free rate

$$R^f_t = 1/E_t(M_{t+1}).$$

$$M_{t+1} = \delta e^{-\gamma g} e^{-\gamma[(\phi-1)(s_t - \bar{s}) + (1+\lambda(s_t))v_{t+1}]}$$

$$r^f_t = -\ln(\delta) + \gamma g - \gamma(1-\phi)(s_t - \bar{s}) - \frac{\gamma^2\sigma^2}{2} [1 + \lambda(s_t)]^2.$$

- Intertemporal substitution vs. precautionary saving. Now both time-varying. Low S – desperate to borrow, but also worried about further declines. (now!)

- Choose λ declining in S, and they can offset
- Paper: for rhetorical purposes a constant risk free rate. (Now regret, since people don't read the paragraph that shows to to have a time-varying risk free rate and fb regressions) This already means the square root-1 formula.

$$\begin{aligned} -r^f - \ln(\delta) + \gamma g - \gamma(1-\phi)(s_t - \bar{s}) &= \frac{\gamma^2\sigma^2}{2}\left[1+\lambda(s_t)\right]^2. \\ \frac{\sqrt{2}}{\gamma\sigma}\sqrt{\gamma g - r^f - \ln(\delta) - \gamma(1-\phi)(s_t - \bar{s})} - 1 &= \lambda(s_t) \end{aligned}$$

Sensitivity function

Some other pretty considerations, lead us to restrict $\bar{s}$ and the other parameters

$$\bar{S} = \sigma\sqrt{\frac{\gamma}{1-\phi}},$$

$$\lambda(s_t) = \frac{1}{\bar{S}}\left[\sqrt{1-2\left(s_t - \bar{s}\right)} - 1\right]$$

$$s_{\max} \equiv \bar{s} + \frac{1}{2}\left(1 - \bar{S}^2\right).$$

[Show or plot figure 1 of λ]

Properties

- Rf constant works,

$$\begin{aligned} r_t^f &= -\ln(\delta) + \gamma g - \gamma(1-\phi)(s_t - \bar{s}) - \frac{\gamma^2\sigma^2}{2}\left[1+\lambda(s_t)\right]^2 \\ &= -\ln(\delta) + \gamma g - \gamma(1-\phi)(s_t - \bar{s}) - \frac{\gamma^2\sigma^2}{2}\left[(1/\bar{S})\sqrt{1-2\left(s_t-\bar{s}\right)}\right]^2 \\ &= -\ln(\delta) + \gamma g - \gamma(1-\phi)(s_t - \bar{s}) - \frac{\gamma^2\sigma^2}{2}\left[\frac{1-\phi}{\sigma^2\gamma}\left(1-2\left(s_t-\bar{s}\right)\right)\right] \\ &= -\ln(\delta) + \gamma g - \gamma(1-\phi)(s_t - \bar{s}) + \gamma\left(1-\phi\right)\left(s_t-\bar{s}\right) - \frac{\gamma}{2}\left(1-\phi\right) \\ r_t^f &= -\ln(\delta) + \gamma g - \frac{\gamma}{2}\left(1-\phi\right) = -\ln(\delta) + \gamma g - \left(\frac{\gamma}{\bar{S}}\right)^2\frac{\sigma^2}{2} \end{aligned}$$

- Note generalization

$$r_t^f = r_0^f - B\left(s_t - \bar{s}\right).$$

$$\bar{S} = \sigma\sqrt{\frac{\gamma}{1-\phi-B/\gamma}}.$$

Then you get time varying interest rates *and* FB regressions, full set of risk premiums! (But perfectly correlated with equity premium) We took it out. big mistake!

- Notice how we "distinguish intertemporal substitition from risk aversion"! γ (low, 2) governs ies, whle γ/S (high, $\bar{S}\approx 0.05$) governs precautionary savings (here) and risk aversion (in HJ bounds, etc). *you don't need state-nonseparable utility to distinguish ies from ra, this works just fine.*

- $$\begin{aligned}\frac{E_t(R^e_{t+1})}{\sigma_t(R^e_{t+1})} &= \frac{\sigma_t(M_{t+1})}{E_t(M_{t+1})} = \left(e^{\gamma^2\sigma^2[1+\lambda(s_t)]^2}-1\right)^{\frac{1}{2}} \approx \gamma\sigma[1+\lambda(s_t)] \\ &= \frac{\gamma\sigma}{\bar{S}}\sqrt{1-2\left(s_t-\bar{s}\right)}\end{aligned}$$

  Again, $\gamma/\bar{S}$ controls this, like precautionary savings. Also you see how it rises as s declines, just as we hoped it would.

Simulation and calibration

$$\frac{P_t}{C_t}(s_t) = E_t\left[M_{t+1}\frac{C_{t+1}}{C_t}\left(1+\frac{P_{t+1}}{C_{t+1}}(s_{t+1})\right)\right]$$

- Parameters

- Steady state spc distribution. Note that it is left skewed. We plot other variables as functions of this state variable, its AR(1) and steady state then tell you how other things evolve.

- P/C Figure 3 Almost linear, not quite. Thus, pc AR is almost exactly that of S. PC reveals S in this model

- Figure 4, 5, 6. Conditional mean variance and sharpe. Note conditional variance *is* higher in recessions, just not as much as conditional means, so Sharpe ratios also rise. Conditional variance higher now!

- Simulated data Table 2. Note variance of return and p-d – driven all by 1.5 variance of consumption. risk aversion amplifies.

- Autocorrelations and cross correlations. Table 3, 4. Part of what the paper does is show that all these statistics reflect time varying risk aversion. Note the absolute vs level, this is a sign of garch.

- Table 5,6, long horizon regressions and variance decompositions

- Figure 7,8. Conditional capms, correlation etc. At any date, a one shock model. Yet unconditional correlations are smaller. And more when you time aggregate.

- Table 7. Really cool. Notice the are correlations about the same as in the data! Notice the apparent cross-correlation from returns to consumption growth!

- Table 8. The static CAPM is a better approximation than the true consumption CAPM. (emphasize that there is an exact conditional consumption capm driving the data). Why? In low S times, both returns and M are more sensitive to a consumption shock.

- History plots.

Long run equity premium

$$M_{t,t+k} = \delta^k \left(\frac{S_{t+k}}{S_t} \frac{C_{t+k}}{C_t} \right)^{-\gamma}.$$

In one period S moves one for one with C, and "amplifies".

As we go to longer horizons, S and C become uncorrelated. Thus "fear of recession" not "fear of consumption decline" become separate events, and "fear of recession" is stronger.

But S is stationary. Why aren't we back to consumption in the long run?

Answer $S^{-\gamma}$ is not stationary! Fear of occasional deeper and deeper recessions builds with horizon.

Macro and nonstochastic analysis

$$r_t^f = -\ln(\delta) + \gamma g - \gamma(1-\phi)(s_t - \bar{s}) - \frac{\gamma^2 \sigma^2}{2} \left[1 + \lambda(s_t)\right]^2,$$

Don't leave off the second term! If you do, why isn't Rf varying a lot?

Also $E(\Delta c) = 1.89\%$, $r = 0.94\%$. Infinite prices!

Internal vs. external habit is not a big deal

Figure 10

$$MU_t = \frac{\partial U_t}{\partial C_t} = (C_t - X_t)^{-\gamma} - E_t \left[\sum_{j=0}^{\infty} \delta^j \left(C_{t+j} - X_{t+j}\right)^{-\gamma} \frac{\partial X_{t+j}}{\partial C_t} \right]$$

The Point: Asset pricing only depends on *ratios* of marginal utility. If the "internal" effect just raises all marginal utilities, it has no effect at all.

However, internal vs. external makes a huge difference for policy and welfare analysis. This is not the first time that observationally equivalent models have radically different policy and welfare implications.

Example:

Suppose habit accumulation is linear, and there is a constant riskfree rate or linear technology equal to the discount rate, $R^f = 1/\delta$. The consumer's problem is then

$$\max \sum_{t=0}^{\infty} \delta^t \frac{(C_t - X_t)^{1-\gamma}}{1-\gamma} \quad s.t. \sum_t \delta^t C_t = \sum_t \delta^t e_t + W_0; \;\; X_t = \theta \sum_{j=1}^{\infty} \phi^j C_{t-j}$$

The first order conditions are

$$MU_t = E_t\left[MU_{t+1}\right]$$

In the external case, marginal utility is simply

$$MU_t = (C_t - X_t)^{-\gamma}.$$

The solution then obeys

$$(C_t - X_t)^{-\gamma} = E_t\left(C_{t+1} - X_{t+1}\right)^{-\gamma}.$$

In the internal case, marginal utility is

$$MU_t = (C_t - X_t)^{-\gamma} - \theta\sum_{j=1}^{\infty}\delta^j\phi^j E_t\left(C_{t+j} - X_{t+j}\right)^{-\gamma}$$

The sum measures the habit-forming effect of consumption. Now, guess the same solution as for the external case,

$$(C_t - X_t)^{-\gamma} = E_t\left[(C_{t+1} - X_{t+1})^{-\gamma}\right].$$

and plug in to the right hand side. All the$(C_{t+j} - X_{t+j})^{-\gamma}$ terms collapse and we just have

$$MU_t = (C_t - X_t)^{-\gamma}\left[1 - \theta\sum_{j=1}^{\infty}\delta^j\phi^j\right] = \left(1 - \frac{\theta\delta\phi}{1-\delta\phi}\right)(C_t - X_t)^{-\gamma} = \left(1 - \frac{\theta\delta\phi}{1-\delta\phi}\right)MU_t^{\text{external}}.$$

We find that the internal marginal utility is simply proportional to external marginal utility. This expression satisfies the first order condition $MU_t = E_t MU_{t+1}$, so that fact confirms the guess. *Ratios* of marginal utility are the same, so allocations and asset prices are completely unaffected by internal vs. external habit in this example.

A challenge

1. Is the model really isomorphic to leverage? All the stories we told about this fall are "leveraged investors have to dump their securities as markets fall." Does this really work? (Do the investors who provide the leverage just jump in? Or do we need some kind of market segmentation to keep them out.?)

2. Of course, this leads to the question, why are there people with different leverage in the first place? In a market with different risk aversions, and complete markets we aggregate, so the idea that less risk averse will be the more levered will not add up to time-varying risk aversion. But what happens with different risk aversions and incomplete markets?

Verdelhan

Intuition: S is low in bad times. The model is set up so the interest rate r is also low in bad times. Then r is low (relative to foreign) when risk premiums are high for anything.

Well, why is euro decline and dollar rise bad for US investors? Here we use complete markets and $M^f_{t+1} = M^d_{t+1} Q_{t+1}/Q_t$. Q_t is Euros/dollar. Thus, if c^d_{t+1} rises, M^d_{t+1} falls, Q_{t+1} must rise – foreign currency pays off when consumption is already good. *The Dollar depreciates in good times.*

(Just read the paper, it's the same as CC)

$$\bar{S} = \sigma \sqrt{\frac{\gamma}{1-\phi-B/\gamma}}.$$

$B < 0$ so interest rates rise[1] in S

$$r_t^f = -\ln(\delta) + \gamma g - \frac{1}{2}\sigma^2 \left(\frac{\gamma^2}{\bar{S}^2}\right) - B\left(s_t - \bar{s}\right)$$

(3): Complete Markets!

$$\frac{Q_{t+1}}{Q_t} = \frac{M^*_{t+1}}{M_{t+1}}$$

p. 8 "risk premium" defined as ex post return,

$$r^e_{t+1} = \Delta q_{t+1} + r^*_t - r_t$$

This is cool. :

$$r_t = -\log E_t M = -E_t m - \frac{1}{2} var_t(m)$$

$$E_t \Delta q_{t+1} = E_t m^*_{t+1} - E_t m_{t+1} = -r^*_t + r_t - \frac{1}{2} var_t(m^*_{t+1}) + \frac{1}{2} var(m_{t+1})$$

Thus, p. 9:

$$E_t r^e_{t+1} = \frac{1}{2} var(m_{t+1}) - \frac{1}{2} var_t(m^*_{t+1})$$

[1]

$$\begin{aligned}
r_t^f &= -\ln(\delta) + \gamma g - \gamma(1-\phi)(s_t - \bar{s}) - \frac{\gamma^2\sigma^2}{2}\left[1+\lambda(s_t)\right]^2 \\
&= -\ln(\delta) + \gamma g - \gamma(1-\phi)(s_t - \bar{s}) - \frac{\gamma^2\sigma^2}{2}\left[(1/\bar{S})\sqrt{1-2\left(s_t-\bar{s}\right)}\right]^2 \\
&= -\ln(\delta) + \gamma g - \gamma(1-\phi)(s_t - \bar{s}) - \frac{\gamma^2\sigma^2}{2}\left[\frac{1-\phi-B/\gamma}{\sigma^2\gamma}\left(1-2\left(s_t-\bar{s}\right)\right)\right] \\
&= -\ln(\delta) + \gamma g - \gamma(1-\phi)(s_t - \bar{s}) - \frac{\gamma}{2}\left(1-\phi-B/\gamma\right) + \gamma\left(1-\phi-B/\gamma\right)\left(s_t-\bar{s}\right) \\
&= -\ln(\delta) + \gamma g - \frac{\gamma}{2}\left(1-\phi-B/\gamma\right) - B\left(s_t-\bar{s}\right) \\
&= -\ln(\delta) + \gamma g - \frac{\gamma^2\sigma^2}{2}\frac{\left(1-\phi-B/\gamma\right)}{\sigma^2\gamma} - B\left(s_t-\bar{s}\right) \\
&= -\ln(\delta) + \gamma g - \frac{1}{2}\sigma^2\left(\frac{\gamma^2}{\bar{S}^2}\right) - B\left(s_t-\bar{s}\right)
\end{aligned}$$

Basically, a risk premium. When our risk premium is higher than theirs the expected return is higher.

$$var_t(m_{t+1}) = \frac{\gamma^2\sigma^2}{\bar{S}^2}[1 - 2(s_t - \bar{s})]$$

p.10

$$E_t(r^e_{t+1}) = \frac{\gamma^2\sigma^2}{\bar{S}^2}(s^*_t - s_t) = \gamma\frac{(1-\phi)}{B}(r_t - r^*_t)$$

There we go!

Intuition (p.10) From $Q_{t+1}/Q_t = M^*_{t+1}/M_{t+1}$

$$\Delta q_{t+1} = (t) + \gamma\left[1 + \lambda(s_t)\right](\Delta c_{t+1} - g) - \gamma\left[1 + \lambda(s^*_t)\right]\left(\Delta c^*_{t+1} - g\right)$$

As above, when Δc is positive, Δq rises "As a result, when the domestic economy receives a negative consumption growth shock, ..the exchange rate depreciates, lowering domestic returns on foreign bonds."

There is more to it though. $\lambda(s)$ means that in bad times, a domestic shock has a larger effect on the exchange rate. Thus the exchange rate has a *time-varying beta* – beta is large in bad times and low in good times. The beta is always a bit positive, but needs to overcome the $1/2\sigma^2$ effect in expected returns.

Doubts

1. Is the sign right? Does the dollar *depreciate* if the US has relatively good news? I thought high output, high interest rates and *strong* dollar are correlated in the data.

2. Related, what do exchange rates do in an *incomplete* market?

3. Are we just goofing around with $1/2\sigma^2$ effects? Does the UIP also work in levels of interest rates, not logs? The crucial step was $r = -E(m) - 1/2\sigma^2(m)$. What if we look at $R = 1/E(m)$? My impression is that UIP also works in levels. Perhaps writing the model in continuous time is the answer.

Other utility functions

Big picture: The equity premium and other puzzles don't really depend on the shape of the utility function. In continuous time, it's all local anyway, only u'' andu' matter

$u(c,x) = v(c) + w(x)$ then $u_c(c,x) = v_c(c)$ and nothing changes. This is the usual defense of using $u(c)$ and nds consumption ignoring durables.

Thus we need $u_c(c,x)$ to depend on x in a meaningful way.

- Most end up being

$$M_{t+1} = \left(\frac{X_{t+1}}{X_t}\right)\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}$$

with stationary X.

1. This fact means we essentially have a two-factor model. Recall

$$m = a - bf_1 - b_2 f_2 \leftrightarrow E(R) = \beta_1\lambda_1 + \beta_2\lambda_2$$

   That's a good thing. I think in the end the bottom line of Fama French etc. is that another "recession" or "liquidity" factor is important to price assets beyond the market. These models give us another "recession" factor beyond consumption growth per se, as most of the variables tend to be indicators for recessions.

- Nonseparable *across goods* or *including leisure* An example (Yogo)

$$u(C,D) = \left[(1-\alpha)C^{1-\frac{1}{\rho}} + \alpha D^{1-\frac{1}{\rho}}\right]^{\frac{1}{1-\frac{1}{\rho}}}.$$

1. We can write this in the usual form[2]

$$u_C(C,D) = \left[(1-\alpha) + \alpha\left(\frac{D}{C}\right)^{1-\frac{1}{\rho}}\right]^{\frac{1}{\rho-1}} (1-\alpha)C^{\frac{1}{\rho}}.$$

$$\begin{aligned} M_{t+1} &= \left(\frac{(1-\alpha)+\alpha\left(\frac{D_{t+1}}{C_{t+1}}\right)^{1-\frac{1}{\rho}}}{(1-\alpha)+\alpha\left(\frac{D_t}{C_t}\right)^{1-\frac{1}{\rho}}}\right)^{\frac{1}{\rho-1}} \left(\frac{C_{t+1}}{C_t}\right)^{\frac{1}{\rho}} \\ &= \left(\frac{X_{t+1}}{X_t}\right)^{\frac{1}{\rho-1}} \left(\frac{C_{t+1}}{C_t}\right)^{\frac{1}{\rho}} \end{aligned}$$

2. Note: If you do this, you also have the int*ra* temporal condition $u_c/u_D = p_c/p_d$ to worry about. More information is good – this can really pin down preference parameters, since there is no uncertainty about this first order condition. On the other hand, more information really constrains your fishing expedition. (See Pakos. In his case, trends in C vs D really said a lot about the preference parameters.)

---

[2]

$$\begin{aligned} u_C(C,D) &= \frac{1}{1-\frac{1}{\rho}}\left[(1-\alpha)C^{1-\frac{1}{\rho}} + \alpha D^{1-\frac{1}{\rho}}\right]^{\frac{1}{1-\frac{1}{\rho}}-1}(1-\alpha)\left(1-\frac{1}{\rho}\right)C^{-\frac{1}{\rho}} \\ &= \left[(1-\alpha)C^{1-\frac{1}{\rho}} + \alpha D^{1-\frac{1}{\rho}}\right]^{\frac{1}{\rho-1}}(1-\alpha)C^{-\frac{1}{\rho}} \\ &= \left[(1-\alpha)C^{1-\frac{1}{\rho}} + \alpha D^{1-\frac{1}{\rho}}\right]^{\frac{1}{\rho-1}}(1-\alpha)\left(C^{-\frac{\rho-1}{\rho}}\right)^{\frac{1}{\rho-1}} \\ &= \left[(1-\alpha)C^{1-\frac{1}{\rho}}C^{-(1-\frac{1}{\rho})} + \alpha C^{-(1-\frac{1}{\rho})}D^{1-\frac{1}{\rho}}\right]^{\frac{1}{\rho-1}}(1-\alpha)C^{\frac{1}{\rho}} \\ &= \left[(1-\alpha)+\alpha\left(\frac{D}{C}\right)^{1-\frac{1}{\rho}}\right]^{\frac{1}{\rho-1}}(1-\alpha)C^{\frac{1}{\rho}}. \end{aligned}$$

3. A second example (Schneider-Piazzesi-Tuzel, also Lustg et al): Consumption is nonseparable across consumption and housing

$$u(c, H)$$

and then we recover

$$m_{t+1} = \beta \left(\frac{C_{t+1}}{C_t}\right)^{-\frac{1}{\sigma}} \left(\frac{\alpha_{t+1}}{a_t}\right)^{\eta}$$

α =housing services /total consumption, which varies withe the business cycle.

   (a) However, you don't *have* to do this. Standard index-number theory means that it's perfectly ok to form an aggregate of consumption goods that enter nonseparably in the utility function, and then treat them as a single good. Going back to basics like this implies you don't like something about index number construction.

- Nonseparable *over time.* Durables is a classic example

$$U = \sum_t \beta^t u(k_t);\ k_{t+1} = (1-\delta)k_t + c_{t+1}.$$

$$U = \sum_t \beta^t u\left(\sum_{j=0}^{\infty} (1-\delta)^j c_{t-j}\right).$$

where c_t denotes purchases. Then

$$\frac{\partial U}{\partial c_t} = u_c(t) + \beta\left(1-\delta\right) u_c\left(t+1\right) + ...$$

This can be physical or psychological.

- Habits

$$U = \sum_t \beta^t u\left(c_t - \theta \sum_{j=0}^{\infty} \phi^j c_{t-j}\right).$$

$$\frac{\partial U}{\partial c_t} = u_c(t) - \phi u_c(t+1) - \theta\phi u_c(t+2)...$$

1. Notice the key difference is the sign. for durability, consumption today raises utility tomorrow, for habits consumption today lowers utility tomorrow.
2. Note: There is a tendency to get weird interest rate/quantity behavior unless you are clever.
3. As in CC, if you rewrite this with a state variable such as X/C or S, again it is often of the form $M = X\left(C_{t+1}/C_t\right)^{-\gamma}$

- Nonseparable *across states.* Expected utility is separable.

$$Eu(c) = \sum_s \pi(s) u\left[c(s)\right]$$

Instead, recursive utility

$$U_t = \left((1-\beta)c_t^{1-\rho} + \beta\left[E_t\left(U_{t+1}^{1-\gamma}\right)\right]^{\frac{1-\rho}{1-\gamma}}\right)^{\frac{1}{1-\rho}}.$$

1. If $\rho = \gamma$ this reduces to power utility

$$\begin{aligned}
U_t &= \left((1-\beta)c_t^{1-\rho} + \beta\left[E_t\left(U_{t+1}^{1-\gamma}\right)\right]\right)^{\frac{1}{1-\rho}}. \\
U_t^{1-\gamma} &= (1-\beta)c_t^{1-\gamma} + \beta\left[E_t\left(U_{t+1}^{1-\gamma}\right)\right]. \\
V_t &= c_t^{1-\gamma} + \frac{\beta}{1-\beta}\left[E_t\left(V_{t+1}\right)\right]. \\
V_t &= E_t \sum \delta^j c_{t+j}^{1-\gamma}
\end{aligned}$$

2. Two periods

$$\begin{aligned}
U_{t+1} &= \left((1-\beta)c_{t+1}^{1-\rho}\right)^{\frac{1}{1-\rho}}. \\
&= (1-\beta)^{\frac{1}{1-\rho}} c_{t+1} \\
U_{t+1}^{1-\gamma} &= (1-\beta)^{\frac{1-\gamma}{1-\rho}} c_{t+1}^{1-\gamma}.
\end{aligned}$$

$$\begin{aligned}
U_t &= \left((1-\beta)c_t^{1-\rho} + \beta\left[E_t\left((1-\beta)^{\frac{1-\gamma}{1-\rho}} c_{t+1}^{1-\gamma}\right)\right]^{\frac{1-\rho}{1-\gamma}}\right)^{\frac{1}{1-\rho}} \\
&= (1-\beta)^{1-\rho}\left(c_t^{1-\rho} + \beta\left[E_t\left(c_{t+1}^{1-\gamma}\right)\right]^{\frac{1-\rho}{1-\gamma}}\right)^{\frac{1}{1-\rho}} \\
\max W_t &= c_t^{1-\rho} + \beta\left[E_t\left(c_{t+1}^{1-\gamma}\right)\right]^{\frac{1-\rho}{1-\gamma}}
\end{aligned}$$

   (a) The point then is that we have a different power for substitution across time (ρ) than across states (γ). This is the sense of a different EIS and Risk aversion

   (b) "Non expected utility?" "Preference for early resolution of uncertaity?" I guess you can't see these in a two period model. Exercise: do a three period model to see these effects?

3. **Central Results.** See Appendix to *Financial Markets and Real Economy* for proofs.

   (a)

$$m_{t+1} = \beta\left(\frac{c_{t+1}}{c_t}\right)^{-\rho}\left(\frac{U_{t+1}}{\left[E_t\left(U_{t+1}^{1-\gamma}\right)\right]^{\frac{1}{1-\gamma}}}\right)^{\rho-\gamma}.$$

   Again, see the usual pattern: consumption growth to one power (IES here) times something else.

(b) Substitute the return on wealth portfolio for the utility index

$$
\begin{aligned}
m_{t+1} &= \left[\beta\left(\frac{c_{t+1}}{c_t}\right)^{-\rho}\right]^{\theta}\left(\frac{1}{R^W_{t+1}}\right)^{1-\theta} \\
\theta &= \frac{1-\gamma}{1-\rho}
\end{aligned}
$$

1. Epstein Zin: this makes the model "work" like CAPM. (Back when they wrote, the CAPM worked!)
2. But...this really must be the entire wealth portoflio,

$$W = E_t \sum m_{t,t+j} c_{t+j}$$

including human capital, and including all consumption goods!

(c) Substitute future consumption for the utility index. If $\rho = 1$, and lognormal homoskedastic Δc

$$(E_{t+1} - E_t)\ln m_{t+1} = -\gamma\left(E_{t+1} - E_t\right)\left(\Delta c_{t+1}\right) + (1-\gamma)\left[\sum_{j=1}^{\infty} \beta^j \left(E_{t+1} - E_t\right)\left(\Delta c_{t+1+j}\right)\right]$$

1. News about future consumption is an extra "factor." This form is currently hot – Banasl, Yaron; Hansen Heaton Li, etc.
2. Wait a minute–wasn't news already in the ICAPM? A: Yes, but with power utility, good news about the future is already reflected in today's consumption. Here, news about the future matters *above and beyond* the way it is reflected in today's consumption. Loosely (?) today's consumption responds via the ies, but news matters to asset pricing via the risk aversion coefficient.
3. Modeling strategy: posit a process in which there news about the future. Then x today can enter as a factor if x today has news about long run consumption growth. It's easy to do in calibration models.
4. JC doubts: it is very hard to independently measure long-run consumption growth properties. And it is very dangerous to infer the long run from short run models. For example, both an AR(1) and a MA(1) can capture the first order correlation of consumption growth, but have drastically different implications for the long run.
5. A deeper doubt. If news about the long run, above and beyond today's consumption, really is the key extra state varaible, asset pricing will be a very amorphous field. This looks a lot like "sentiment."