13 Ch12 Classic linear regressions

13.1 Summary

1. **General steps**

   (a) Model: $E(R^{ei}) = (\alpha_i +)\ \beta_i' \lambda$

   (b) *Estimate* parameters. $\hat{\alpha}, \hat{\beta}, \hat{\lambda}$.

   (c) *Standard errors* of parameter estimates. $\sigma(\hat{\alpha}), \sigma(\hat{\beta}), \sigma(\hat{\lambda})$

   (d) *Test* the model. $\hat{\alpha}' V^{-1} \hat{\alpha}$ "too big?"

   (e) *Evaluate, diagnose the model.* Is the SML "too flat?" Do betas drive out characteristics? Does σ^2, β^2 matter? Is $\hat{\lambda}$ reasonable?

   (f) *Test* one model vs. another. Can we drop a factor, e.g. sml?

2. **Time series**

$$R_t^{ei} = \alpha_i + \beta_i f_t + \varepsilon_{it}$$

   (a) Procedure: Just run this, $t = 1, 2, ...T\ \forall i$.

   (b) Estimates and standard errors:

      i. $\hat{\alpha}, \hat{\beta}, \sigma(\hat{\alpha}, \hat{\beta})$ from OLS (OLS/GMM) formulas.

      ii. $\hat{\lambda} = E_T(f); \sigma(\hat{\lambda}) = \sigma(f)/\sqrt{T}$ (iid) or GMM

   (c) Test: Econometric issue is the joint distribution of intercepts $\hat{\alpha}_i$ from N regressions $cov(\hat{\alpha}, \hat{\alpha}')$. Then the statistic is

$$\hat{\alpha}' cov(\hat{\alpha}, \hat{\alpha}')^{-1} \hat{\alpha}$$

   iid

$$\hat{\alpha}' cov(\hat{\alpha})^{-1} \hat{\alpha} = T \left[1 + \bar{f}' \Sigma_f^{-1} \bar{f}\right]^{-1} \hat{\alpha}' \Sigma^{-1} \hat{\alpha} \tilde{\ } \chi_N^2$$

   iid normal "GRS Test."

$$\frac{T - N - K}{N} \left[1 + \bar{f}' \Sigma_f^{-1} \bar{f}\right]^{-1} \hat{\alpha}' \hat{\Sigma}^{-1} \hat{\alpha} \tilde{\ } F_{N, T-N-K}$$

   (d) GMM. Doing N regressions at once is easy – stack them up, let the S matrix handle the correlation!

$$g_T(b) = \begin{bmatrix} E_T(\varepsilon) \\ E_T(f_t \varepsilon_t) \end{bmatrix} = \begin{bmatrix} E_T(R_t^e - \alpha - \beta f_t) \\ E_T((R_t^e - \alpha - \beta f_t) f_t) \end{bmatrix}$$

$$d = \frac{\partial g_T}{\partial \left[\alpha\ \beta\right]'} = \begin{bmatrix} 1 & E(f_t) \\ E(f_t) & E(f_t^2) \end{bmatrix} \otimes I_N$$

$$S = \sum_{j=-\infty}^{\infty} \begin{bmatrix} E(\varepsilon_t \varepsilon_{t-j}') & E(\varepsilon_t \varepsilon_{t-j}' f_{t-j}) \\ E(f_t \varepsilon_t \varepsilon_{t-j}') & E(f_t f_{t-j} \varepsilon_t \varepsilon_{t-j}') \end{bmatrix}$$

$$var \begin{pmatrix} \hat{\alpha} \\ \hat{\beta} \end{pmatrix} = \frac{1}{T} d^{-1} S d^{-1\prime}$$

$$\hat{\alpha}' var(\hat{\alpha})^{-1} \hat{\alpha} \tilde{\ } \chi_N^2$$

(e) Reduces to standard formulas for ε iid and independent of f. But why do that?

3. **Cross section**

(a) Method: First run time series to get β, then run

$$E(R^{ei}) = \gamma + \beta_i \lambda + \alpha_i$$

$i = i, 2, ...N$ in the cross section.

(b) Econometric issues:

i. β are generated regressors.

ii. α are correlated across assets

(c) Classic iid formulas with no intercept $\gamma = 0$

$$\begin{aligned}\sigma^2(\hat{\lambda}_{OLS}) &= \frac{1}{T}\left[(\beta'\beta)^{-1}\beta'\Sigma\beta(\beta'\beta)^{-1}\left(1+\lambda'\Sigma_f^{-1}\lambda\right)+\Sigma_f\right] \\ \sigma^2(\hat{\lambda}_{GLS}) &= \frac{1}{T}\left[\left(\beta'\Sigma^{-1}\beta\right)^{-1}\left(1+\lambda'\Sigma_f^{-1}\lambda\right)+\Sigma_f\right]\end{aligned}$$

$$\begin{aligned}cov(\hat{\alpha}_{OLS}) &= \frac{1}{T}\left(I-\beta(\beta'\beta)^{-1}\beta'\right)\Sigma\left(I-\beta(\beta'\beta)^{-1}\beta'\right)\left(1+\lambda'\Sigma_f^{-1}\lambda\right) \\ cov(\hat{\alpha}_{GLS}) &= \frac{1}{T}\left(\Sigma-\beta(\beta'\Sigma^{-1}\beta)^{-1}\beta'\right)\left(1+\lambda'\Sigma_f^{-1}\lambda\right)\end{aligned}$$

$$\hat{\alpha}'cov(\hat{\alpha},\hat{\alpha}')^{-1}\hat{\alpha}\,\tilde{}\,\chi^2_{N-K-1}.$$

$$\hat{\alpha}'_{GLS}cov(\hat{\alpha}_{GLS})^{-1}\hat{\alpha}_{GLS} = T\left(1+\lambda'\Sigma_f^{-1}\lambda\right)\hat{\alpha}_{GLS}\Sigma^{-1}\hat{\alpha}_{GLS}\,\tilde{}\,\chi^2_{N-K}$$

(d) Classic iid formulas with an intercept, OLS

$$X = \begin{bmatrix} 1_N & \beta \end{bmatrix}$$

$$\sigma^2\left(\begin{bmatrix}\hat{\gamma} \\ \hat{\lambda}\end{bmatrix}\right) = \frac{1}{T}\left[(X'X)^{-1}X'\Sigma X(X'X)^{-1}\left(1+\lambda'\Sigma_f^{-1}\lambda\right)+\begin{bmatrix}0 & 0 \\ 0 & \Sigma_f\end{bmatrix}\right]$$

$$cov(\hat{\alpha}) = \frac{1}{T}\left(I-X(X'X)^{-1}X'\right)\Sigma\left(I-X(X'X)^{-1}X'\right)\left(1+\lambda'\Sigma_f^{-1}\lambda\right)$$

(e) GMM approach

$$ag_T = \begin{bmatrix} I_N & & \\ & I_N & \\ & & \beta'\left(\Sigma^{-1}\right)\end{bmatrix}\begin{bmatrix} E_T\left(R^e - a - \beta f\right) \\ E_T\left[\left(R^e - a - \beta f\right)f\right] \\ E_T\left[\left(R^e - \beta\lambda\right)\right] = E_T(\alpha)\end{bmatrix} = 0$$

note $\hat{\alpha} =$ last N rows of $g_T(a, \beta, \lambda)$.

$$\hat{\alpha}'cov(\hat{\alpha}\hat{\alpha})^{-1}\hat{\alpha}$$

(f) Fact: if the factor is a return, and including the factor as a test asset, GLS cross-section = time-series. If returns are iid normal, also = ML.

(g) Fact: "Efficient GMM" is not GLS when returns are not iid.

4. Test comments

(a) Run $\hat{\alpha}' cov(\hat{\alpha})^{-1}\hat{\alpha}$ tests for cross sections! (but don't necessarily pay too much attention)

(b) Don't test a model by $\lambda = 0$. Don't test for one model vs. another by $\lambda_2 = 0$. $E(f_2) = \alpha_2 + \beta_{21}E(f_1)$ is the right test

5. **Fama MacBeth** (For asset pricing)

(a) Run TS to get betas.

$$R_t^{ei} = a_i + \beta_i' f_t + \varepsilon_t^i \quad t = 1, 2...T \text{ for each } i.$$

(b) Run a cross sectional regression *at each time period*,

$$R_t^{ei} = \beta_i' \lambda_t + \alpha_{it} \quad i = 1, 2, ...N \;\; \text{for each } t.$$

$$R_t^{ei} = \gamma_t + \beta_i' \lambda_t + \alpha_{it} \quad i = 1, 2, ...N \;\; \text{for each } t.$$

(c) Then, *Estimates* of λ, α are the *averages across time*

$$\hat{\lambda} = \frac{1}{T}\sum_{t=1}^{T}\hat{\lambda}_t; \;\; \hat{\alpha}_i = \frac{1}{T}\sum_{t=1}^{T}\hat{\alpha}_{it}$$

(d) *Standard errors* use $\sigma^2(\bar{x}) = \sigma^2(x)/T$

$$\sigma^2(\hat{\lambda}) = \frac{1}{T} var(\hat{\lambda}_t) = \frac{1}{T^2}\sum_{t=1}^{T}\left(\hat{\lambda}_t - \hat{\lambda}\right)^2$$

$$cov(\hat{\alpha}) = \frac{1}{T} cov(\hat{\alpha}_t) = \frac{1}{T^2}\sum_{t=1}^{T}\left(\hat{\alpha}_{it} - \hat{\alpha}_i\right)\left(\hat{\alpha}_{jt} - \hat{\alpha}_j\right)$$

This one main point. These standard errors are easy to calculate

(e) *Test*

$$\hat{\alpha}' cov(\hat{\alpha}, \hat{\alpha}')^{-1}\hat{\alpha} \, \tilde{} \, \chi^2_{N-1}.$$

(f) Fact: FMB corrects for α correlated across time. (It's a panel with clustering). FMB does not correct for generated betas. ("Shanken correction")

(g) Fact: FMB assumes errors are uncorrelated over time. It's easy enough to add correlation over time,

$$\sigma^2(\hat{\lambda}) = \frac{1}{T}\sum_j E_T\left[\left(\hat{\lambda}_t - \hat{\lambda}\right)\left(\hat{\lambda}_{t-j} - \hat{\lambda}\right)\right]$$

(h) Fact: When β_{it} are constant over time, FMB = cross section=pooled estimate.