Return Predictability and Volatility

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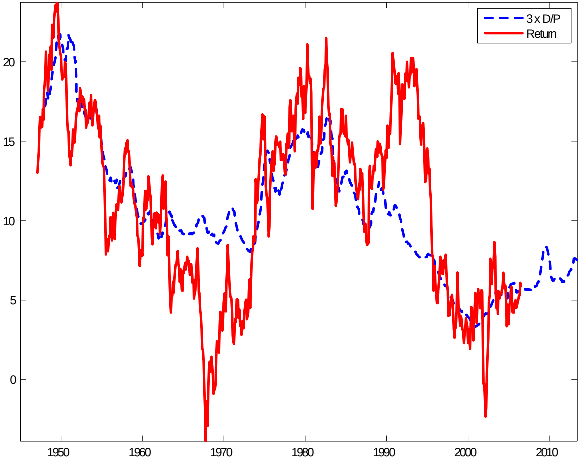
Three facts: 1. Returns are predictable

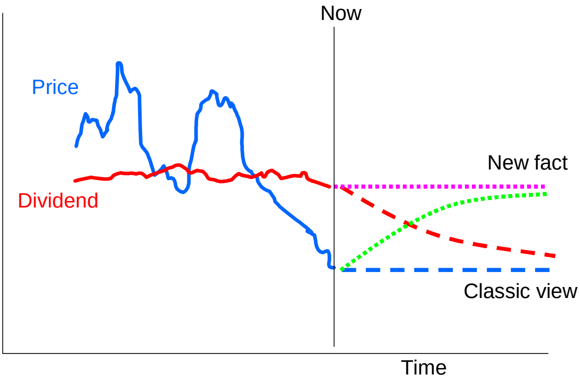
Horizon k	b	$t(b)$	R^2	$\sigma\left[E_t\left(R^e\right)\right]$	$\frac{\sigma[E_t(R^e)]}{E(R^e)}$
1 year	3.8	(2.6)	0.09	5.46	0.76
5 years	20.6	(3.4)	0.28	29.3	0.62



Three facts – 2. Dividend forecasts

- DP does not forecast dividend growth. The *sign* is "wrong"

Horizon k (years)	$R^e_{t\to t+k} = a + b\frac{D_t}{P_t} + \varepsilon_{t+k}$ b	t(b)	R^2	$\frac{D_{t+k}}{D_t} = a + b\frac{D_t}{P_t} + \varepsilon_{t+k}$ b	t(b)	R^2
1	4.0	2.7	0.08	0.07	0.06	0.0001
5	20.6	2.6	0.22	2.42	1.11	0.02



Three facts. 3. Prices seem awfully volatile

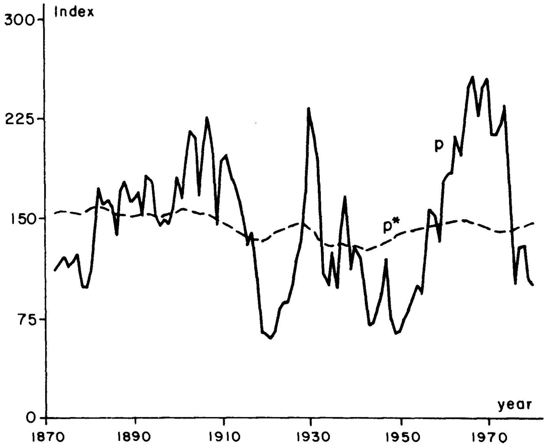


FIGURE 1

Source: Shiller, 1981 AER

Three facts - Shiller equations

$$P_t^* = \sum_{j=1}^{\infty} \frac{1}{R^j} D_{t+j}$$

$$P_t = E_t\left(P_t^*\right) = E_t\left[\sum_{j=1}^{\infty} \frac{1}{R^j} D_{t+j}\right]$$

$$\begin{aligned}
P_t^* &= P_t + \varepsilon_t \\
\sigma^2(P_t^*) &= \sigma^2(P_t) + \sigma^2\left(\varepsilon_t\right) \\
\sigma^2(P_t^*) &> \sigma^2(P_t)
\end{aligned}$$

- Our task: tie all these ideas together. Along the way, develop some very useful tools – dynamic present value identity

Present value identity: One period version

Return and present value identity in logs

$$R_{t+1} = \frac{D_{t+1}}{P_t}$$

$$r_{t+1} = \log(R_t) = d_{t+1} - p_t$$

$$p_t - d_t = (d_{t+1} - d_t) - r_{t+1}$$

$$pd_t = \Delta d_{t+1} - r_{t+1}$$

$$pd_t = E_t(\Delta d_{t+1}) - E_t(r_{t+1})$$

Regression identities: One period version

$$dp_t = r_{t+1} - \Delta d_{t+1}$$

Regression identity.

$$\begin{aligned} r_{t+1} &= b_r\, dp_t + \varepsilon^r_{t+1} \\ \Delta d_{t+1} &= b_d\, dp_t + \varepsilon^d_{t+1} \end{aligned}$$

$$dp_t = \left[b_r\, dp_t + \varepsilon^r_{t+1}\right] - \left[b_d\, dp_t + \varepsilon^d_{t+1}\right]$$

Result:

$$1 = b_r - b_d.$$

$$0 = \varepsilon^r_{t+1} - \varepsilon^d_{t+1}$$

Tie volatility to predictability.

$$b_r = \frac{cov(r_{t+1}, dp_t)}{var(dp_t)}$$

$$var(dp_t) = cov(r_{t+1}, dp_t) - cov(\Delta d_{t+1}, dp_t)$$

- *Which* is it? A:*all* $E(r)$
- Agenda: do this for multiperiod securities.

Campbell-Shiller Return Identity

$$r_{t+1} \approx \rho(p_{t+1} - d_{t+1}) + \Delta d_{t+1} - (p_t - d_t)$$

Derivation. This is just the definition of return:

$$\begin{aligned} R_{t+1} &= \frac{P_{t+1} + D_{t+1}}{P_t} = \frac{\left(1 + \frac{P_{t+1}}{D_{t+1}}\right) \frac{D_{t+1}}{D_t}}{\frac{P_t}{D_t}} \\ r_{t+1} &= \log\left(1 + e^{pd_{t+1}}\right) + \Delta d_{t+1} - pd_t \\ r_{t+1} &\approx \log\left(1 + e^{pd}\right) + \frac{e^{pd}}{\left(1 + e^{pd}\right)}(pd_{t+1} - pd) + \Delta d_{t+1} - pd_t \end{aligned}$$

$$\rho = \frac{1}{1 + D/P} \approx 0.96 \text{ (Annual, with } D/P = 0.04; \text{ P/D=20)}$$

Campbell-Shiller Present Value Formula

- From return identity

$$r_{t+1} \approx \rho(p_{t+1} - d_{t+1}) + \Delta d_{t+1} - (p_t - d_t)$$

Solve forward to present value formula.

$$pd_t \approx \rho \times pd_{t+1} + \Delta d_{t+1} - r_{t+1}$$

$$pd_t \approx \sum_{j=1}^{k} \rho^{j-1} \Delta d_{t+j} - \sum_{j=1}^{k} \rho^{j-1} r_{t+j} + \rho^k \left(pd_{t+k}\right)$$

when $\rho^k \left(pd_{t+k}\right) \to 0$

$$pd_t \approx \sum_{j=1}^{\infty} \rho^{j-1} \Delta d_{t+j} - \sum_{j=1}^{\infty} \rho^{j-1} r_{t+j} = \text{long run } \Delta d \text{ - long run } r$$

- Ex-post; definition of long run return. Also ex ante $E_t(\cdot)$

$$pd_t \approx E_t \sum_{j=1}^{\infty} \rho^{j-1} \Delta d_{t+j} - E_t \sum_{j=1}^{\infty} \rho^{j-1} r_{t+j}$$

- A present value formula.

Volatility

- $$pd_t \approx E_t \sum_{j=1}^{\infty} \rho^{j-1} \Delta d_{t+j} - E_t \sum_{j=1}^{\infty} \rho^{j-1} r_{t+j}$$

P/D ratio moves iff news about Δd or r !

- Run

$$\sum_{j=1}^{\infty} \rho^{j-1} \Delta d_{t+j} = b_d^{lr} dp_t + \varepsilon^d$$

$$\sum_{j=1}^{\infty} \rho^{j-1} r_{t+j} = b_r^{lr} dp_t + \varepsilon^r$$

Identity:

$$1 \approx b_r^{lr} - b_d^{lr}$$

Or, $b = cov(x, y)/var(x)$, so

$$var(p_t - d_t) \approx cov\left\{p_t - d_t, \sum_{j=1}^{\infty} \rho^{j-1} \Delta d_{t+j}\right\} - cov\left\{p_t - d_t, \sum_{j=1}^{\infty} \rho^{j-1} r_{t+}\right.$$

- Volatility = regression coeffs, which must add up. b_r^{lr}, b_d^{lr} *Which* is it?

Volatility facts

Method and horizon	Coefficient $b_r^{(k)}$	$b_{\Delta d}^{(k)}$	$\rho^k b_{dp}^{(k)}$
Direct regression , $k = 15$	1.01	-0.11	-0.11
Implied by VAR, $k = 15$	1.05	0.27	0.22
VAR, $k = \infty$	1.35	0.35	0.00

$$\sum_{j=1}^{k} \rho^{j-1} r_{t+j} = a + b_r^{(k)} dp_t + \varepsilon_{t+k}^{r}$$

- "Rational Bubbles"

$$dp_t = \sum_{j=1}^{k} \rho^{j-1} r_{t+j} - \sum_{j=1}^{k} \rho^{j-1} \Delta d_{t+j} + \rho^k dp_{t+k}$$

$$1 = b_r^{(k)} - b_d^{(k)} + \rho^k b_{pd}^{(k)}$$

- Fama / Shiller debate?

Campbell-Ammer Return Decomposition

$$(E_{t+1} - E_t) : pd_t \approx \sum_{j=1}^{\infty} \rho^{j-1} \Delta d_{t+j} - \sum_{j=1}^{\infty} \rho^{j-1} r_{t+j}$$

$$0 \approx (E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^{j-1} \Delta d_{t+j} - (E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^{j-1} r_{t+j}$$

$$\Delta E_{t+1}(r_{t+1}) = \Delta E_{t+1} \Delta d_{t+1} + \Delta E_{t+1} \sum_{j=1}^{\infty} \rho^{j} \Delta d_{t+1+j} - \Delta E_{t+1} \sum_{j=1}^{\infty} \rho^{j} r_{t+1+j}$$

- $\sigma^2 \left[\Delta E_{t+1}(r_{t+1})\right]$ =About 50% Δd_{t+1}, 50% future returns, 0% future dividends.
- Not inconsistent with $\sigma^2 [dp_t] = 100\%$ future returns.

Vector autoregression

- The basic VAR

$$
\begin{aligned}
r_{t+1} &= b_r\, dp_t + \varepsilon^{r}_{t+1} \\
\Delta d_{t+1} &= b_d\, dp_t + \varepsilon^{d}_{t+1} \\
dp_{t+1} &= \phi dp_t + \varepsilon^{dp}_{t+1}
\end{aligned}
$$

- Estimates, round numbers

	Estimates $\hat{b}, \hat{\phi}$	ε s. d. (diagonal) and correlation. r	Δd	dp
r	0.1	20	+big	-big
Δd	0		14	0 (!)
dp	0.94			15

Vector autoregression

- The basic VAR

$$
\begin{aligned}
r_{t+1} &= b_r dp_t + \varepsilon^r_{t+1} \approx 0.1 \times dp_t + \varepsilon^r_{t+1} \\
\Delta d_{t+1} &= b_d dp_t + \varepsilon^d_{t+1} \approx 0 \times dp_t + \varepsilon^d_{t+1} \\
dp_{t+1} &= \phi dp_t + \varepsilon^{dp}_{t+1} \approx 0.94 \times dp_t + \varepsilon^{dp}_{t+1}; \;\; cov(\varepsilon^d, \varepsilon^{dp}) \approx 0
\end{aligned}
$$

- Identity constrains coefficients and shocks: Really 2 variables, shocks.

$$r_{t+1} \approx -\rho dp_{t+1} + \Delta d_{t+1} + dp_t.$$

$$
\begin{aligned}
b_r &= 1 - \rho\phi + b_d \\
0.1 &= 1 - 0.96 \times 0.94 + 0
\end{aligned}
$$

$$\varepsilon^r_{t+1} \approx -\rho \varepsilon^{dp}_{t+1} + \varepsilon^d_{t+1}.$$

Using the VAR – Connecting long and short horizons

$$
\begin{aligned}
r_{t+1} &= b_r dp_t + \varepsilon^r_{t+1} \\
dp_{t+1} &= \phi dp_t + \varepsilon^{dp}_{t+1}.
\end{aligned}
$$

- Coefficient rises with horizon

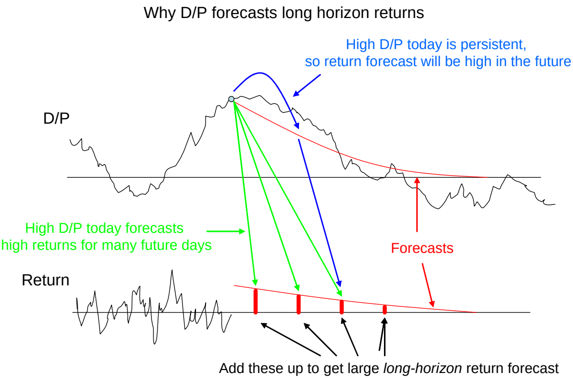
$$
\begin{aligned}
&\Leftrightarrow \quad r_{t+1} + r_{t+2} = b_r(1+\phi)dp_t + (error) \\
&\Leftrightarrow \quad r_{t+1} + r_{t+2} + r_{t+3} = b_r(1+\phi+\phi^2)dp_t + (error)
\end{aligned}
$$

$$
\Leftrightarrow r_{t+2} = b_r\phi dp_t + (error)\,; \;\; r_{t+3} = b_r\phi^2 dp_t + (error)
$$

- Long horizon R^2

$$
\begin{aligned}
R^2_{k=1} &= \frac{b_r^2\sigma^2(dp_t)}{\sigma^2\left(r_{t+1}\right)} \\
R^2_{k=2} &= \frac{b_r^2(1+\phi)^2\sigma^2(dp_t)}{\sigma^2\left(r_{t+1}+r_{t+2}\right)} \approx \frac{b_r^2(1+\phi)^2\sigma^2(dp_t)}{2\sigma^2\left(r_{t+1}\right)} = \frac{(1+\phi)^2}{2}R^2_{k=1}
\end{aligned}
$$

Using the VAR – Connecting long and short horizons



Volatility in the VAR

- Recall

$$\sum_{j=1}^{\infty} \rho^{j-1} \Delta d_{t+j} = b_d^{lr} dp_t + \varepsilon^d$$

$$\sum_{j=1}^{\infty} \rho^{j-1} r_{t+j} = b_r^{lr} dp_t + \varepsilon^r$$

$$1 = b_r^{lr} - b_d^{lr}.$$

- In the VAR

$$b_r^{lr} = \sum_{j=1}^{\infty} \rho^{j-1} \phi^{j-1} b_r = \frac{b_r}{1-\rho\phi}$$

$$1 = \frac{b_r}{1-\rho\phi} - \frac{b_d}{1-\rho\phi} = b_r^{lr} - b_d^{lr} \leftrightarrow b_r = 1 - \rho\phi + b_d!$$

  You can get here much more quickly, but lose interpretation

- Simplified numbers, nice units

$$b_r^{lr} = \frac{0.1}{1 - 0.94 \times 0.96} = 1; \quad b_d^{lr} = 0$$

Impulse-Response Function

- Recall

$$r_{t+1} \approx -\rho dp_{t+1} + \Delta d_{t+1} + dp_t.$$

$$\varepsilon^r_{t+1} \approx -\rho \varepsilon^{dp}_{t+1} + \varepsilon^d_{t+1}.$$

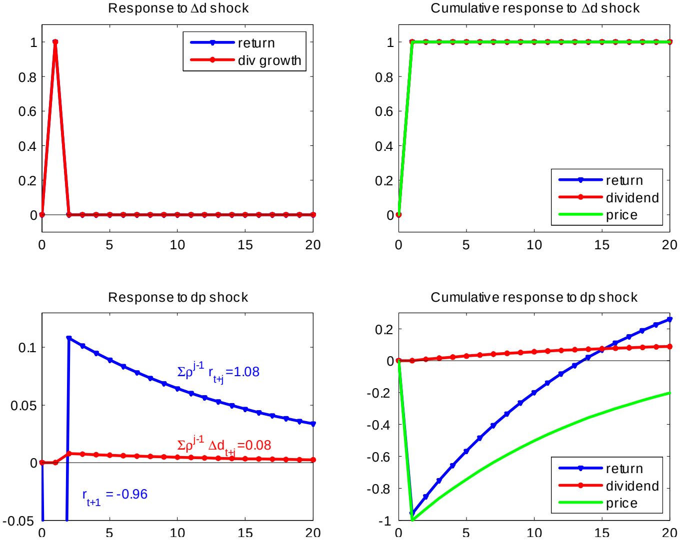
- My choice.

$$\begin{array}{lcl} \Delta d \text{ shock:} & & \left[\begin{array}{ccc} \varepsilon^r_1 & \varepsilon^d_1 & \varepsilon^{dp}_1 \end{array}\right] = \left[\begin{array}{ccc} 1 & 1 & 0 \end{array}\right] \\ \text{“}Er\text{” shock} & : & \left[\begin{array}{ccc} \varepsilon^r_1 & \varepsilon^d_1 & \varepsilon^{dp}_1 \end{array}\right] = \left[\begin{array}{ccc} -\rho & 0 & 1 \end{array}\right] \end{array}$$

Simulate forward

$$\begin{aligned} r_{t+1} &= 0.108 \times dp_t + \varepsilon^r_{t+1} \\ \Delta d_{t+1} &= 0.015 \times dp_t + \varepsilon^d_{t+1} \\ dp_{t+1} &= 0.0937 \times dp_t + \varepsilon^{dp}_{t+1} \end{aligned}$$

Impulse-Response Function

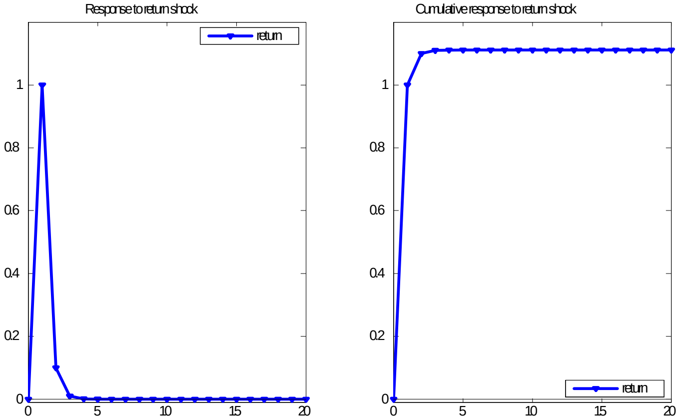


Impulse-Response Function

1. Interpretation: how news about the future changes prices today.
   1.1 ε^d, dividend shock with no *dp* change is a pure expected-cashflow shock with no change in expected returns
   1.2 ε^{dp}, *dp* shock with no change in dividends is (almost) a pure discount-rate shock with no change in expected cashflows.
2. *There is a "temporary component" to stock prices. You need to look at* **both** *prices and dividends to see it.*

Univariate vs multivariate responses

$$r_{t+1} = 0.1 \times r_t + \varepsilon_{t+1}$$



Response function $r_{t+1} = 0.1 \times r_t + \varepsilon_{t+1}$

Similarly, $\sigma^2(r_{t+1} + r_{t+2} + .. + r_{t+k}) \approx k\sigma^2(r)$. Stocks really are not "safer in the long run."

Univariate vs multivariate responses

1. Puzzle: predictable from DP, but nearly a random walk on their own – not "safer in the long run?
2. *The univariate return process implied by the VAR is very close to uncorrelated over time.*

$$\begin{aligned} r_{t+1} &= b_r dp_t + \varepsilon^r_{t+1} \\ r_{t+2} &= b_r \left(\phi dp_t + \varepsilon^{dp}_{t+1} \right) + \varepsilon^r_{t+2} \end{aligned}$$

so

$$\begin{aligned} cov(r_{t+1}, r_{t+2}) &= cov\left[b_r dp_t + \varepsilon^r_{t+1}, b_r \left(\phi dp_t + \varepsilon^{dp}_{t+1} \right) + \varepsilon^r_{t+2} \right] \\ cov(r_{t+1}, r_{t+2}) &= b_r^2 \phi \sigma^2 (dp_t) + b_r cov(\varepsilon^r_{t+1}, \varepsilon^{dp}_{t+1}) \end{aligned}$$

   2.1 Intuition
   2.2 Algebra: if $\rho = \phi$ with $cov(\varepsilon^d, \varepsilon^{dp}) = 0$, then $cov(r_{t+1}, r_{t+2}) = 0$.

More variables

$$R_{t+1} = a + b \times dp_t + cx_t + \varepsilon_{t+1}? \quad !$$

Example ("Discount rates") cay helps to forecast returns!

	Coefficients		t		
Left-hand Variable	dp_t	cay_t	cay_t	R^2	$\sigma\left[E_t(y_{t+1})\right]$ %
r_{t+1}	0.12	0.071	(3.19)	0.26	8.99
Δd_{t+1}	0.024	0.025	(1.69)	0.05	2.80
dp_{t+1}	0.94	-0.047	(-3.05)	0.91	
cay_{t+1}	0.15	0.65	(5.95)	0.43	
$r_t^{lr} = \sum_{j=1}^{\infty} \rho^{j-1} r_{t+j}$	1.29	0.033			0.51
$\Delta d_t^{lr} = \sum_{j=1}^{\infty} \rho^{j-1} \Delta d_{t+j}$	0.29	0.033			0.12

More variables–cay

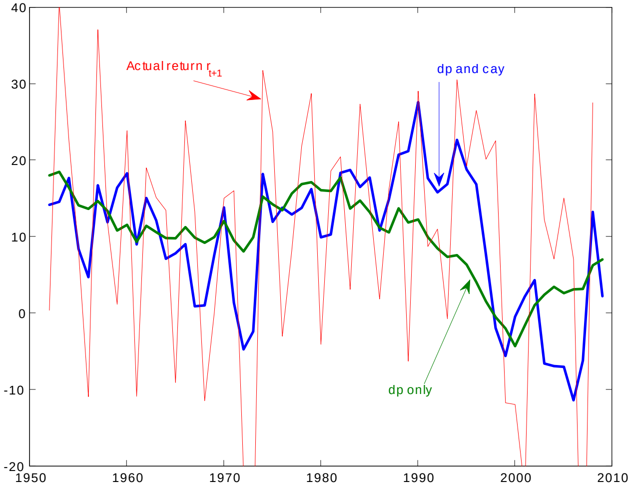


Figure: **Forecast and actual one-year returns.**

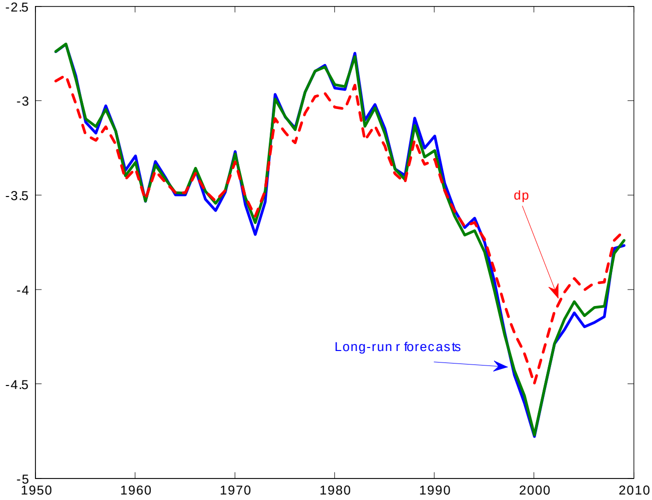
More variables – identities, variance, etc.

$$d_t - p_t \approx E_t \sum_{j=1}^{\infty} \rho^{j-1} \left(r_{t+j} - \Delta d_{t+j} \right).$$

$$\begin{aligned}
\sum_{j=1}^{\infty} \rho^{j-1} r_{t+j} &= a_r + b_r^{lr} \times dp_t + c_r^{lr} \times z_t + \varepsilon_t^r \\
\sum_{j=1}^{\infty} \rho^{j-1} \Delta d_{t+j} &= a_d + b_d^{lr} \times dp_t + c_d^{lr} \times z_t + \varepsilon_t^d
\end{aligned}$$

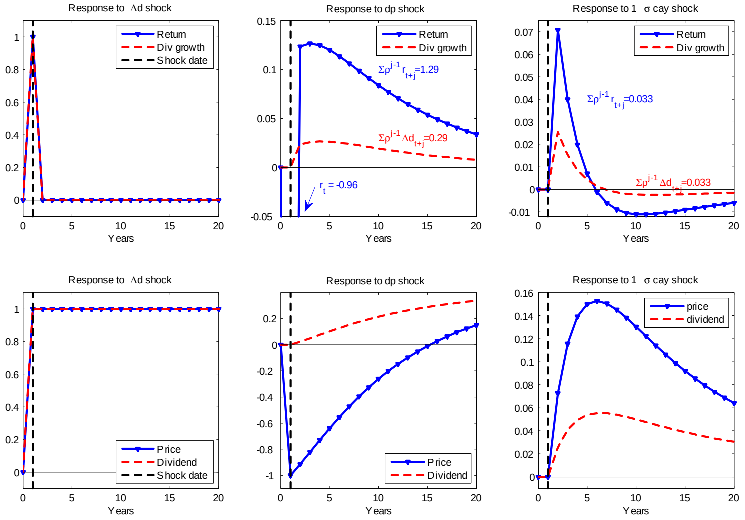
$$\begin{aligned}
b_r^{lr} - b_d^{lr} &= 1 \\
c_r^{lr} - c_d^{lr} &= 0
\end{aligned}$$

More variables – identities, variance, etc.



Plot of $d_t - p_t$, $E_t \sum_{j=1}^{\infty} \rho^{j-1} r_{t+j}$. .

More variables – VAR



Pervasive predictability: a preview

1. More variables:

$$R_{t+1} + a + b(D/P)_t + c \times \text{term}_t + d \times \text{def}_t + f \times \text{I/K}_t + g \times \text{cay}_t + h \times \pi_t$$

2. Individual stocks? (Pay attention)

$$R^i_{t+1} = a + bx^i_t + \varepsilon^i_{t+1}? \leftrightarrow E(R^i_{t+1}) \text{ is higher when } x^i_t \text{ is higher}$$

3. Bonds

$$\begin{aligned} R^{bond}_{t+1} - R^f_t &= a + 1 \times (y^{long}_t - y^{short}_t) + \varepsilon_{t+1} \\ R^f_{t+1} - R^f_t &= a^f + 0 \times (y^{long}_t - y^{short}_t) + \varepsilon^f_{t+1} \end{aligned}$$

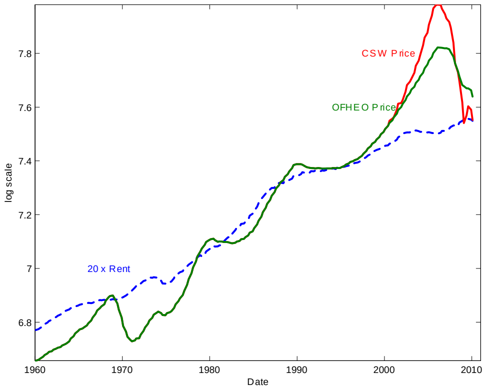
4. Foreign exchange.. $r^{US} = 1\%$, $r^{Eu} = 5\%$ Implication?

$$\begin{aligned} R^{Eu}_{t+1} - r^{\$}_{t+1} &= a + 1 \times (r^{Eu}_t - r^{\$}_t) + \varepsilon_{t+1} \\ \Delta e^{Eu/\$}_{t+1} &= a^e + 0 \times (r^{Eu}_t - r^{\$}_t) + \varepsilon^e_{t+1} \end{aligned}$$

5. Credit spreads do not mean (much) higher chance of default, do mean higher expected return.

Pervasive predictability: a preview

Houses



Houses:	b	t	R^2	Stocks:	b	t	R^2
r_{t+1}	0.12	(2.52)	0.15		0.13	(2.61)	0.10
Δd_{t+1}	0.03	(2.22)	0.07		0.04	(0.92)	0.02
dp_{t+1}	0.90	(16.2)	0.90		0.94	(23.8)	0.91

Pervasive predictability/ time varying risk premiums

- Many questions!
- Do the variables that forecast one thing forecast another?
- What is the *factor structure of expected returns* across markets?
- Correlation with business cycles / financial crises / economic explanation?