Business 35905 **John H. Cochrane**

Problem Set 3

Due Friday, week 4

1. How important is the restriction I've been imposing to eliminate lags of returns and dividend yields, and to restrict ourselves to an AR(1) representation? Use $\Delta d_{t+1} = r + \rho dp_{t+1} - dp_t$ so identities hold exactly. This problem guides you through some fairly special (and, I suspect, sample-specific) results, so use the data on the class website.

   (a) Start by verifying the VAR with only dp on the right hand side

   $$\begin{aligned} r_{t+1} &= b_r dp_t + \varepsilon^r_{t+1} \\ \Delta d_{t+1} &= b_d dp_t + \varepsilon^d_{t+1} \\ dp_{t+1} &= \phi dp_t + \varepsilon^{dp}_{t+1} \end{aligned}$$

   produces the expected results, i.e. that a dividend yield shock with no change in dividend growth is an "expected return" shock and a dividend growth shock without a change in dividend yield is largely a permanent "cashflow" shock. Plot at least the responses of r_t,Δd_t, d_t, dp_t,p_t and verify they're each doing what they should.

   (b) Now add a single extra lag of the right hand variables.

      i. Your first instinct might be to run

      $$\begin{aligned} r_{t+1} &= b^0_{r,dp} dp_t + b^1_{r,dp} dp_{t-1} + b_{r,r} r_t + b_{r,d} \Delta d_t + \varepsilon^r_{t+1} \\ \Delta d_{t+1} &= b^0_{d,dp} dp_t + b^1_{d,dp} dp_{t-1} + b_{d,r} r_t + b_{d,d} \Delta d_t + \varepsilon^d_{t+1} \\ dp_{t+1} &= \phi dp_t + \phi^1 dp_{t-1} + b_{dp,r} r_t + b_{dp,d} \Delta d_t + \varepsilon^{dp}_{t+1} \end{aligned}$$

      Why won't this work? (If it's not obvious, try it. Matlab will guide you to the answer)

      ii. Part i will suggest that you need to drop a variable. Try it three ways

      $$\begin{aligned} r_{t+1} &= a + b^0_{r,dp} dp_t + b^1_{r,dp} dp_{t-1} + b^0_{r,r} r_t + \varepsilon^r_{t+1} \\ r_{t+1} &= a + b^0_{r,dp} dp_t + b^0_{r,dp} dp_{t-1} + b^0_{r,d} \Delta d_t + \varepsilon^r_{t+1} \\ r_{t+1} &= a + b^0_{r,dp} dp_t + b^0_{r,r} r_t + b^0_{r,d} \Delta d_t + \varepsilon^r_{t+1} \end{aligned}$$

      and similarly for the other variables. You should note a sensible pattern that some coefficients are the same in these regressions.

      iii. Do the extra variables seem to help? Look at economic as well as statistical significance of the coefficients – coefficients, R^2, etc. (Hint: thinking of $3dp_t - 2dp_{t-1} = 1x_t + 2(dp_t - dp_{t-1})$ may help.)

      iv. Plot and look at the impulse-response functions. Are they strongly affected by the extra lag? Do they help sort through which of the three ways of running the regression is best?

      v. Which of the representations in ii do you like best?

   (c) Repeat, using 2, 3, 4 lags. You will naturally see wiggles emerge in the response functions. In your judgment, is there important structure here that we missed with the simple VAR that only had dp in it? (FYI, the answer to this need not be "no!")

2. Now, we'll repeat some of the same exercise using data starting in 1947 to avoid depression and WWII. You'll see some rather different conclusions and have fun rotating shocks.

(a) As always, start with the VAR with just dp. Any differences in the postwar sample? (Yes, what are they?)

(b) Repeat with a single added lag. Diagnose the substantial change in the response function. Look carefully at the regression coefficients to see if the new patterns make economic sense as well as fishing for t stats.

(c) (Optional. I also used two lags, and verified that there's no significant additional change)

3. You'll see that we no longer get such clear "expected return" and "expected cashflow" shocks. But we can always define such shocks – they just won't be so easily identified as "a shock to dividend yields with no change in dividends" and "a shock to dividends with no shock to dividend yields." Let's do that – *renormalize the impulse response functions to produce responses to an "expected return" shock and to a "cashflow" shock*, as follows.

   (a) Show that impulse responses obey the identity

$$0 = \sum_{j=1}^{\infty} \rho^{j-1} e_{z_{t+1} \to r_{t+j}} - \sum_{j=1}^{\infty} \rho^{j-1} e_{z_t \to \Delta d_{t+j}}$$

   where $e_{z_{t+1} \to r_{t+j}}$ denotes the response of r_{t+j} to responses of returns and dividends to *any* shock z_{t+1}.

   (b) Our simple var had the property that 1) In response to the "expected return" shock at time $t+1$, the response of dividend growth $\sum_{j=1}^{\infty} \rho^{j-1} \Delta d_{t+j}$ was essentially zero. The *current* return shock was $r_{t+1} = -\rho$. The response of the sum of *future* expected returns $\sum_{j=2} \rho^{j-1} r_{t+j}$ was therefore $+\rho = 0.96$. 2) In response to a "dividend growth shock" the response of *current* dividend growth Δd_{t+1} was one, and *future* dividend growth was $\sum_{j=2}^{\infty} \rho^{j-1} \Delta d_{t+j}$ was zero, so the response of long run dividend growth $\sum_{j=1}^{\infty} \rho^{j-1} \Delta d_{t+j}$ was one. In response to this shock, *current* returns also had a response $r_{t+1} = 1$, but future returns had a response of zero.

   In sum, here are properties of the simple VAR responses

Response	"Expected return "	"Cashflow"
dp_{t+1}	1	0
r_{t+1}	-ρ	1
$\sum_{j=2} \rho^{j-1} r_{t+j}$	ρ	0
$\sum_{j=1} \rho^{j-1} r_{t+j}$	0	1
Δd_{t+1}	0	1
$\sum_{j=2}^{\infty} \rho^{j-1} \Delta d_{t+j}$	0	0
$\sum_{j=1}^{\infty} \rho^{j-1} \Delta d_{t+j}$	0	1

   It's easiest to identify new shocks by zero restrictions. With this table in mind, create responses to two new shocks in our system with lags, identified as 1) a shock whose response has $\sum_{j=2} \rho^{j-1} r_{t+j} = 1$ (or ρ, it's up to you which you think is prettier) and $\sum_{j=1}^{\infty} \rho^{j-1} \Delta d_{t+j} = 0$, 2) a shock whose response has $\sum_{j=2} \rho^{j-1} r_{t+j} = 0$ and $\sum_{j=1}^{\infty} \rho^{j-1} \Delta d_{t+j} = 1$. Watch the subscripts! (We need to choose some response to be 1/0 and another to be 0/1. Try some of the others if you don't like my choice.)

   Reminder: you can always take linear combinations of impulse-response functions, and define the new shock as the time-zero value of this linear combination. So you don't actually have to rerun the VAR or re-simulate response functions.

   (c) Plot responses to your new "expected return" and "expected cashflow" shocks. Explain their long-term behavior – which should look a lot like the one-variable case – and their short-term behavior – which may not. Give the combinations of time-$t+1$ dp, dd, r shocks which generate each of “expected return” and “expected cashflow” shocks. Is it a lot different than before? Why?

(d) Point of all this: We can still define “expected return” and “expected cashflow” shocks. In the simple system, we were lucky because, with dividend growth a pure random walk, the long-run zero restrictions corresponded easily to short run zero restrictions. That’s not true with general dynamics, alas. But we still can use long-run restrictions to identify shocks.