Problem Set 3 Answers

Part 1. Reading questions

Fama and French, "Multifactor explanations."

1. In Table I, which kinds of stocks have higher vs. lower average returns?

   A: Value and small. Table 1 panel A

2. Would a stock with strong earnings growth be a "growth" stock by FF's definition? Would a stock with a small number of employees be "small"? (Hint: This is a trick question. How do FF define growth and value?)

   A: Table I caption. Size and value are based on total market value of equity, and ratio of market value to book value. They form portfolios every year based on June values. A stock with strong earnings growth is likely to have high price and hence be a "growth" stock, and a stock with a small number of employees is likely to have a small market capitalization, but not necessarily. These are *not* FF's definitions, which is important. It turns out that these alternative measures do not work!

3. Does the spread in average returns in Table 1A present a puzzle, by itself? (Hint: why might you not just go buy small value stocks based on the evidence of this table?)

   A: It would not be a puzzle if betas where high where expected returns are high. Then the high returns would be compensation for risk. *All puzzles are joint puzzles of average returns and betas.*

4. How are FF's "SMB" and "HML" factors constructed? (one sentence)

   A: See Table 1 caption. Basically as big portfolios of large - small and value - growth firms. There is some criticism of FF that the HML factor equally weights the subcategories, giving it a bias towards small firms.

5. Which gets better returns going forward, stocks that had great past growth in sales over the last 5 years, or stocks that had poor past growth in sales?

   A: Poor – see Table II.

6. Which results show the "long-term reversal" effect in average returns best? Which show the "momentum" effect best? (Table and line)

   A: Table VI, 60-13 since they leave out the momentum part. 12-2 shows momentum best, note it doesn't work so well pre 63

7. It looks like we should all buy value, but we can't all buy value, someone has to hold the growth stocks. If we all try to buy value, the value effect will disappear because we drive up the prices. How to Fama and French address this conundrum? (hint, p. 76, 77)

   A: Read p. 76, 77. There is a vital market-clearing issue *we can't all buy value stocks. The average investor must hold the market portfolio. So if value stocks pay more than growth stocks, something about value stocks must be scaring people away.* FF think that value stocks correlated with a "state variable" such as employment. People know the good returns are there, but don't want to hold stocks that will tank when they lose their jobs. (loosely). Hmm, we should do a week on theory to know what all this means...

FF "Dissecting Anomalies"

1. How do FF define "Microcap" and "small" stocks? What percentage of stocks are "Micro"? What fraction of market value do "micro" stocks comprise? How can the percentile breakpoint that defines tiny be different from the fraction of tiny stocks in the sample?

   A: 1656 or Table 1. The breakpoints are the 20% and 50% percentiles of the NYSE. 60% of stocks are micro, but account for 3% of microcaps. *Most stocks are tiny. Most value is in a few large stocks.* This means that equally weighted portfolios will always be weighted towards really small stocks. The sample includes amex and nasdaq which have many smaller stocks than NYSE, and breakpoints come from NYSE

2. Are the average returns in Table II raw, excess, or adjusted somehow? Do they represent returns, or alphas, or something else?

   A: They are "characteristic-adjusted", explained 1658 below II. sorts. This means, find the portfolio of 25 size/book/market whose size and B/M are closest, and subtract off that return. The text says that true size and book/market alphas gives similar results, though since there are some big alphas (small/growth) separating average returns and betas in the 25, I'm not altogether convinced. OTOH, FF argue that individual-stock hml, smb betas are measured badly and wander over time. Thus, they say, the characteristic is a better measure of beta than beta itself. Anyway, read the table as FF's ideas about alphas after controlling for size and b/m.

3. Which anomalies produce strong average returns for all size groups in Table II? What are the most important numbers in Table II that document your answer?

   A: Read 1662 pp3. Issues, accruals, and momentum. Look at the High-Low number. Look for consistency across 4 size groupings, and consistency across VW and EW results.

4. Explain what the first two rows of MC and B/M columns mean in Table IV.

   A: You're seeing the basic size and B/M effects in expected returns. Larger size means smaller ER, Larger B/M means larger ER. (see bottom 1667.)

5. "The novel evidence is that the market cap (MC) result draws[size effect] much of its power from microcaps." (p. 1667) What numbers in Table IV are behind this conclusion?

   A: This is the disappearance of the size coefficient in the other groups in the top left part of Table IV. Note size is also much weaker post 1979 – when the size effect was published and small stock funds started. (not in this paper)

Cochrane, "Discount rates" p. 1053-1064, and 1098-1099.

1. Figure 6 says expected returns are higher for value portfolios. Does the paper say this is the value puzzle?

   A: No. The puzzle is that betas don't also rise, p. 1058 "The fact that betas do not rise with value is really the heart of the puzzle." All puzzles are joint puzzles of expected returns and betas.

2. What central feature of Figure 6 captures FF's "explanation" of the value puzzle?

   A: The fact that $h \times E(hml)$ lines up with $E(r)$. "Higher average returns do line up well with larger values of the h_i regression coefficient." 1059

3. On p. 1060 I say "Covariance is in a sense Fama and French's central result." What table or set of numbers in Fama and French convey this result?

   A: The large R^2 in Table 1 B

4. What kind of regression does "Discount rates" suggest to provide the same information as FF's Table 1A, in the same way we forecast returns last week? Write a regression equation that implements the "discount rates" idea.

   A: $R^e_{t+1} = a + b \times \log(BEME_{it}) + c \times \log(ME_{it}) + \varepsilon_{t+1}$, top of p. 1062 (The equation with C from the paper is good enough, if you explain C should include BEME and ME)

5. Does cay help to forecast market returns? How about long-run stock returns?

   A: It helps to forecast one year returns a lot, with t = 3.19. But it does not help to forecast long run returns at all.

6. In the final column of Figure 5, which components of the present value identity also change so that cay can help to forecast one-year returns without changing the forecast of the long-run dividend yield

   A: It's mostly long-run returns. There isn't that much effect on long-run dividend growth. (Text, bottom of 1057). The key is that, with the identity around, if cay forecasts r_{t+1} given dp_t it must forecast something else too!

Part II

1. As a reminder, here is the regression

$$\begin{aligned}
r_{t+1} &= a_r + b_r(d_t - p_t) + \varepsilon^r_{t+1} \\
\Delta d_{t+1} &= a_d + b_d(d_t - p_t) + \varepsilon^d_{t+1} \\
(d_{t+1} - p_{t+1}) &= a_{dp} + \phi(d_t - p_t) + \varepsilon^{dp}_{t+1}
\end{aligned}$$

and the thing we're trying to compute.

$$p_t - d_t = k + E_t \sum_{j=1}^{\infty} \rho^{j-1} \Delta d_{t+j} - E_t \sum_{j=1}^{\infty} \rho^{j-1} r_{t+j}$$

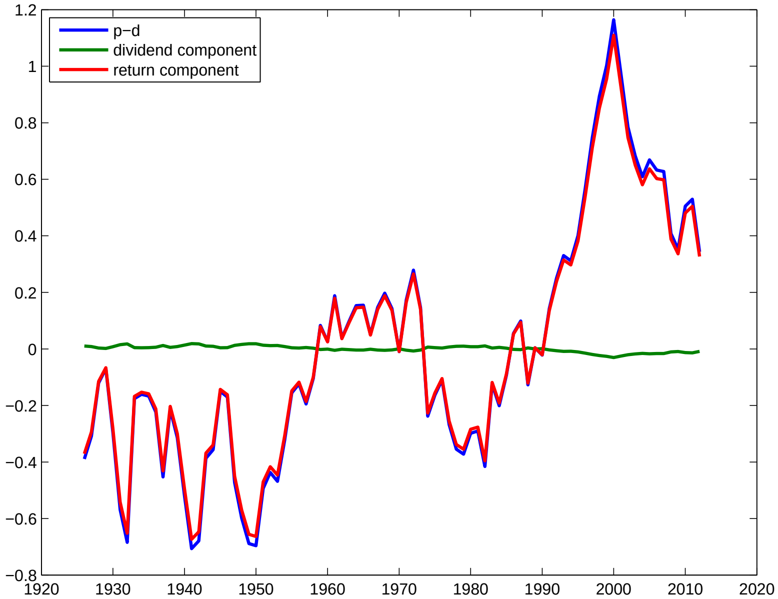
We've done these a few times,

$$\begin{aligned}
E_t \Delta d_{t+1} &= b_d(d_t - p_t) \\
E_t \Delta d_{t+2} &= b_d E_t(d_{t+1} - p_{t+1}) = b_d \phi (d_t - p_t) \\
E_t \Delta d_{t+3} &= b_d E_t(d_{t+2} - p_{t+2}) = b_d \phi^2 (d_t - p_t) \\
E_t \sum_{j=1}^{\infty} \rho^{j-1} \Delta d_{t+j} &= \sum_{j=1}^{\infty} \rho^{j-1} b_d \phi^{j-1} (d_t - p_t) \\
&= b_d \frac{1}{1 - \rho\phi} (d_t - p_t)
\end{aligned}$$

Similarly,

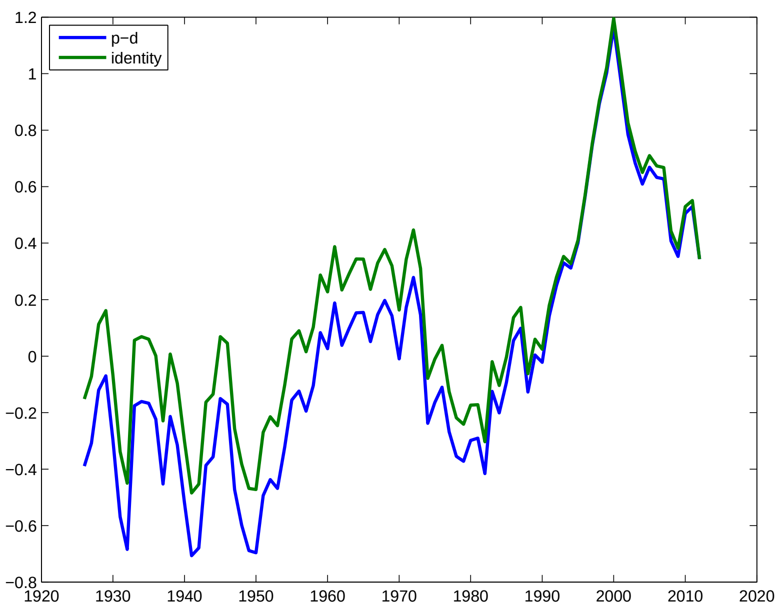
$$E_t \sum_{j=1}^{\infty} \rho^{j-1} r_{t+j} = b_r \frac{1}{1 - \rho\phi} (d_t - p_t)$$

So, we just have to plot $d_t - p_t$, and $-b_d \frac{1}{1-\rho\phi}(d_t - p_t)$ and $b_r \frac{1}{1-\rho\phi}(d_t - p_t)$. (negative sign on b_d to decompose $d - p$, opposite to decompose $p - d$.) Here's my plot. The return component is actually larger than dp. Again, in this sample we predict dividends go the wrong way (high price means low dividends) so returns have to move even more.

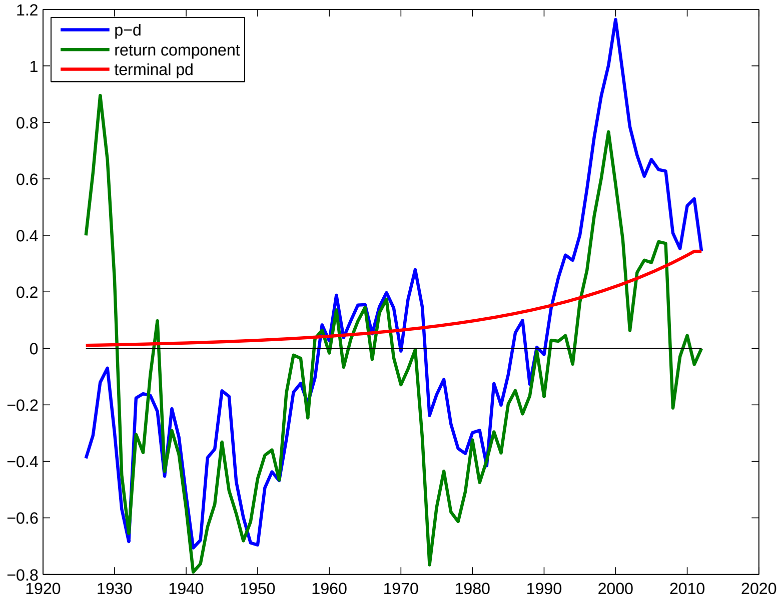


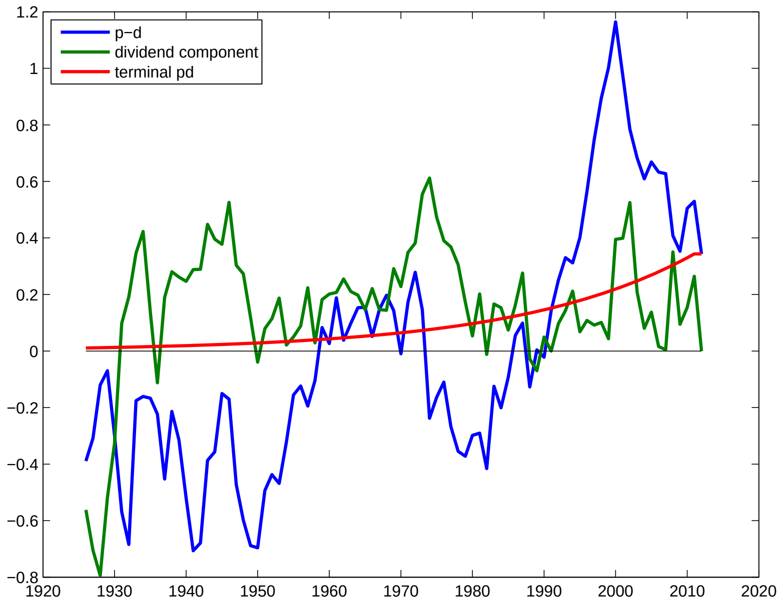
The lines all look the same here since they are only functions of $d-p$ If you had more forecasting variables – say d-p, d-e, and cay together – then these three might forecast returns and dividends differently, so the lines would be more interesting.

2. The identity works pretty well but not perfectly



Now here are the return and dividend terms:





As you can see, even if people forecast so well they actually *know* what dividends will be, ex-post returns correlate much better with dividend yields.

This is as good as it gets – you can't forecast returns better than knowing their ex-post values. In "discount rates" I found that adding cay didn't do much to change the variance decomposition. The top graph shows that nothing will do much to change the variance decomposition!

3. We're looking for the three terms in

$$r_{t+1} - E_t r_{t+1} = (E_{t+1} - E_t)\left(\Delta d_{t+1} + \sum_{j=1}^{\infty} \rho^j \Delta d_{t+1+j} - \sum_{j=1}^{\infty} \rho^j r_{t+1+j}\right)$$

Here we go.

$$r_{t+1} - E_t r_{t+1} = \varepsilon_{t+1}^r$$
$$(E_{t+1} - E_t)\,\Delta d_{t+1} = \varepsilon_{t+1}^d$$

$$\begin{aligned}
\Delta d_{t+2} &= b_d dp_{t+1} + \varepsilon_{t+2}^d = b_d\left[\phi dp_t + \varepsilon_{t+1}^{dp}\right] + \varepsilon_{t+2}^d \\
(E_{t+1} - E_t)\,\Delta d_{t+2} &= b_d \varepsilon_{t+1}^{dp}
\end{aligned}$$

$$\begin{aligned}
\Delta d_{t+3} &= b_d dp_{t+1} + \varepsilon_{t+3}^d \\
&= b_d\left[\phi dp_{t+1} + \varepsilon_{t+2}^{dp}\right] + \varepsilon_{t+3}^d \\
&= b_d\left[\phi^2 dp_t + \phi\varepsilon_{t+1}^{dp} + \varepsilon_{t+2}^{dp}\right] + \varepsilon_{t+3}^d \\
(E_{t+1} - E_t)\,\Delta d_{t+2} &= b_d \phi \varepsilon_{t+1}^{dp}
\end{aligned}$$

Similarly,

$$(E_{t+1} - E_t)\,\Delta d_{t+1+j} = b_d \phi^{j-1} \varepsilon_{t+1}^{dp}$$

so

$$(E_{t+1} - E_t)\sum_{j=1}^{\infty} \rho^j \Delta d_{t+1+j} = b_d \sum_{j=1}^{\infty} \rho^j \phi^{j-1} \varepsilon_{t+1}^{dp} = \frac{\rho b_d}{1-\rho\phi}\varepsilon_{t+1}^{dp}$$

Thus, this term is a constant times the d-p residual, just as in the last case the term was a constant times dp itself. Similarly,

$$(E_{t+1} - E_t)\sum_{j=1}^{\infty} \rho^j r_{t+1+j} = \frac{\rho b_r}{1-\rho\phi}\varepsilon_{t+1}^{dp}.$$

Note these are similar coefficients, just multiplying the *dp* errors rather than *dp*.

In sum, our decomposition is

$$\varepsilon_{t+1}^r = \varepsilon_{t+1}^d + \frac{\rho b_d}{1-\rho\phi}\varepsilon_{t+1}^{dp} - \frac{\rho b_r}{1-\rho\phi}\varepsilon_{t+1}^{dp}$$

Wait a minute...

$$\varepsilon_{t+1}^r = \varepsilon_{t+1}^d + \frac{\rho\,(b_d - b_r)}{1-\rho\phi}\varepsilon_{t+1}^{dp}$$

and we derived from

$$r_{t+1} = -\rho dp_{t+1} + dp_t + \Delta d_{t+1}$$

that

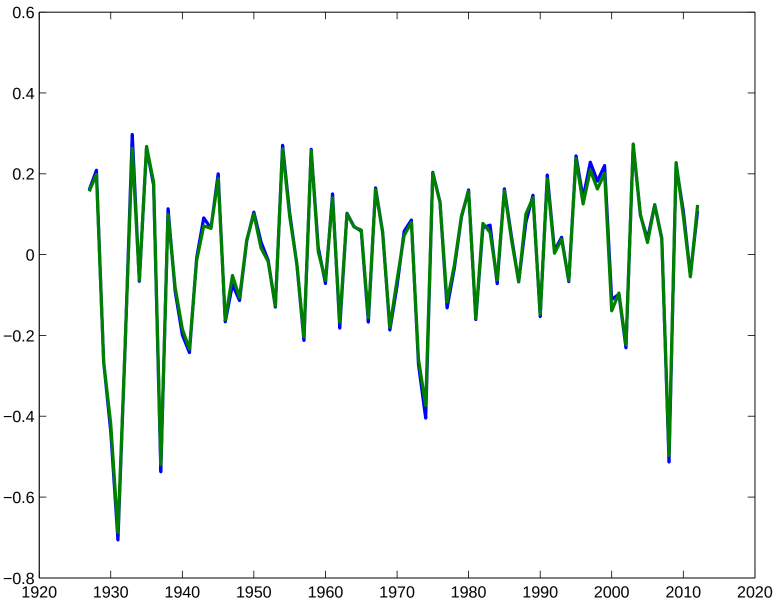
$$b_r - b^d = (1 - \rho\phi)$$

so

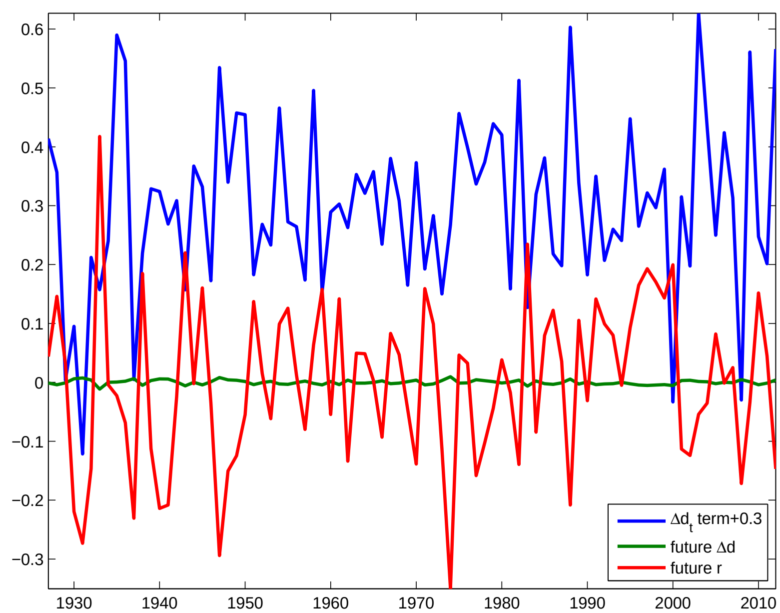
$$\varepsilon_{t+1}^r = \varepsilon_{t+1}^d - \rho\varepsilon_{t+1}^{dp}$$

which also follows from the return identity. *This is just the return identity written in a new and clever way.*

Now, graphs. First let's check the identity, the two sides of (4):



Yes, there are two lines there. This is excellent. Now, let's look at the three terms on the right side. I moved the dividend term up so you could see it on the same graph:

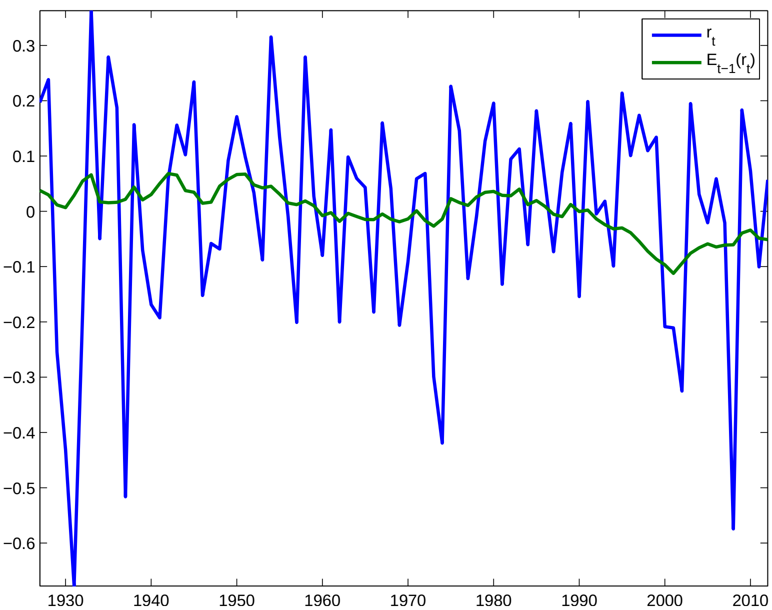


You can see that the volatility of unexpected returns is just about equally split between the volatility of *current* dividend growth, and the volatility of future expected returns – and almost zero volatility of future expected dividend growth. We anticipated this result in our class discussion. The "cashflow shock" moved prices and dividends together, and thus moved returns. It didn't change the *price-dividend ratio* at all, but did move current returns.

This is a decomposition of the variance of *unexpected* returns. If you want a full decomposition of the variance of *returns,* add the variance of expected returns

$$
\begin{aligned}
r_{t+1} &= b_r dp_t + \varepsilon^r_{t+1} \\
\sigma^2(r_{t+1}) &= \sigma^2(b_r dp_t) + \sigma^2(\varepsilon^r_{t+1})
\end{aligned}
$$

The next plot (I did not ask for this, just FYI) plots returns and expected returns. We know that the R^2 is low, meaning that almost all of the variance of *returns* comes from the variance of *unexpected* returns. So we didn't really leave much out. But if you were confused about the "unexpected" part above, this brings it back to the variance of returns – which now has four components: expected returns, unexpected dividend growth, changes in expected future dividend growth and changes in future expected returns.



4)

$$\varepsilon^r_{t+1} = \varepsilon^d_{t+1} + \frac{\rho b_d}{1-\rho\phi}\varepsilon^{dp}_{t+1} - \frac{\rho b_r}{1-\rho\phi}\varepsilon^{dp}_{t+1}$$

$$var(\varepsilon^r_{t+1}) = cov(\varepsilon^r_{t+1}, \varepsilon^d_{t+1}) + \frac{\rho b_d}{1-\rho\phi}cov(\varepsilon^r_{t+1}, \varepsilon^{dp}_{t+1}) - \frac{\rho b_r}{1-\rho\phi}cov(\varepsilon^r_{t+1}, \varepsilon^{dp}_{t+1})$$

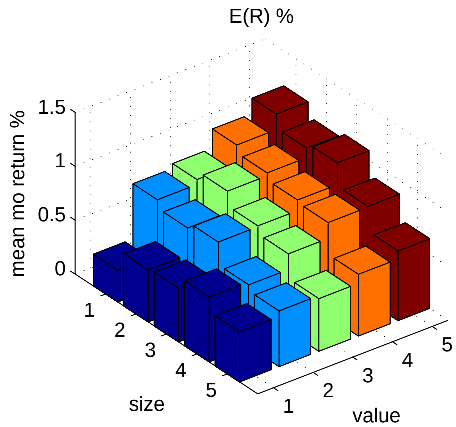
```
regression coefficients br bd and bdp
 0.0966 0.0027 0.9361
100 x var of r, cov(r,d), cov(r,futured) cov(r,futurer)
 3.8214 1.9352 -0.0493 1.7936
as percent var (r)
 100.0000 50.6414 -1.2890 46.9345
```

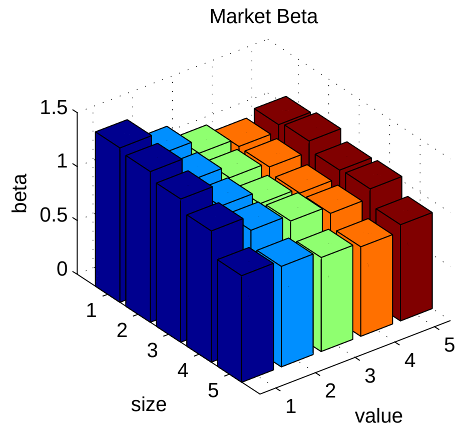
Part III

Here are my results. First, the CAPM in tabular format. You can see that means rise to the northeast as for FF. Capm betas *do* vary. They are not all one. The size pattern is ok, they rise to the north. Alas the rise to the north is more pronounced in the growth portfolios where the returns do not rise to the north than it is for the value portfolios where they do. The value pattern is wrong, the betas rise to the northwest not the northeast. Thus, the spread in alphas across portfolios is larger than the spread of mean returns across portfolios. Alphas are composed of mean returns going one way and betas going the opposite way. The CAPM R2 are in the 60–80% range which is typical for large portfolios. There are lots of alpha t stats above 2

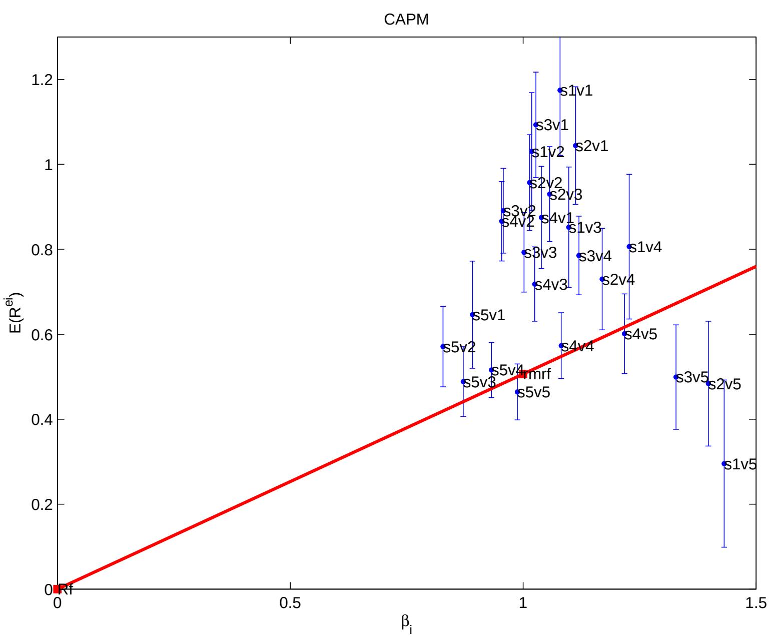
```
Data sample
     196301.00     201311.00

Time series regression results
As in FF all results are in boxes with size and book to market
mean return
          0.30          0.81          0.85          1.03          1.17
          0.48          0.73          0.93          0.96          1.04
          0.50          0.79          0.79          0.89          1.09
          0.60          0.57          0.72          0.87          0.87
          0.46          0.52          0.49          0.57          0.65
CAPM betas
          1.43          1.23          1.10          1.02          1.08
          1.40          1.17          1.06          1.01          1.11
          1.33          1.12          1.00          0.96          1.03
          1.22          1.08          1.02          0.95          1.04
          0.99          0.93          0.87          0.83          0.89
CAPM alphas
         -0.43          0.18          0.30          0.51          0.63
         -0.22          0.14          0.39          0.44          0.48
         -0.17          0.22          0.29          0.41          0.57
         -0.02          0.03          0.20          0.38          0.35
         -0.04          0.04          0.05          0.15          0.19
T on CAPM alphas
         -2.18          1.08          2.09          3.72          4.10
         -1.52          1.15          3.54          3.93          3.47
         -1.41          2.36          3.05          4.06          4.62
         -0.16          0.33          2.27          4.09          2.90
         -0.55          0.68          0.58          1.60          1.54
CAPM R2
          0.64          0.63          0.67          0.64          0.62
          0.75          0.76          0.75          0.73          0.68
          0.80          0.83          0.79          0.75          0.69
          0.85          0.87          0.82          0.78          0.71
          0.88          0.87          0.79          0.72          0.62
```

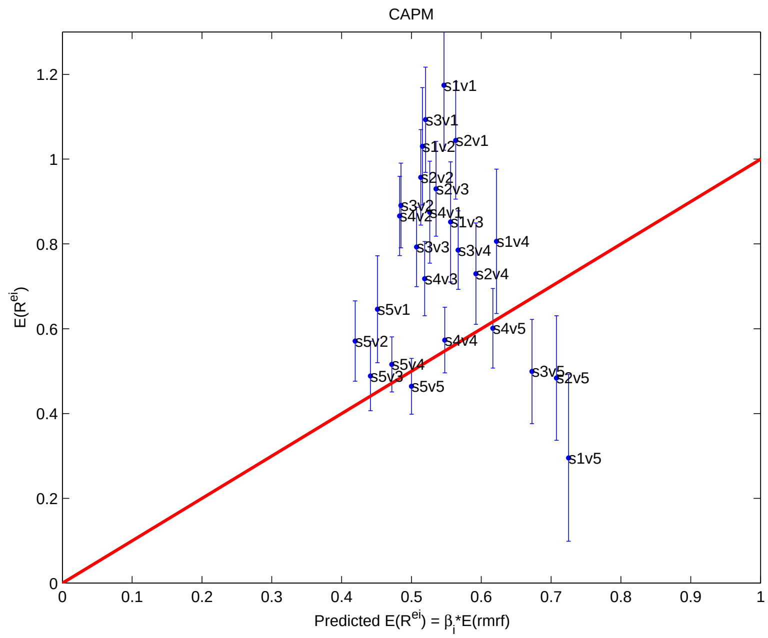
The same thing in pictures: You see FF's pattern of mean returns – increasing to small and value, except in the growth category. You can see that mean returns are not related to betas. If anything the growth companies have higher betas and lower returns. Notice these betas are larger than FF's, and the betas you will find below. Single regression betas are different from multiple regression betas because hml and smb are not exactly uncorrelated with $rmrf$.





I also made expected return vs. beta and expected return vs. beta*market premium plots, the latter can be compared with the 3 factor model: (I added standard error bars. These are the standard errors of alpha. This was not required on the problem set.)





"s1" means small and "s5" means big. "v1" means value and "v5" means growth. These graphs show that there is sort of a blob with some hope for the CAPM, except for the size effect among deep growth firms. As you go from s1v5 (large growth) to s5v5 (small growth) betas rise but average excess returns decline, and these points are way off the line. In the bar graphs, you could see mean returns declining as you go "back." But the blob is way too much of a blob for a successful theory.

Value isn't a puzzle of average returns, it's a puzzle of lack of betas. It's perfectly natural that "value" stocks have high average returns, because it's perfectly natural that they are extra risky. The puzzle is that the beta is not here. That's particularly obvious in the lower plot. The puzzle is that when the market declines, value stocks *don't* get hammered. To "explain" the value puzzle, you have to explain this lack of beta, not the unusual expected returns. All the behavioral stories are looking in the wrong place.

c) Here are the numbers in case you want to compare. The graphs are much more insightful

```
Data sample
     193201.00     196212.00

Time series regression results
As in FF all results are in boxes with size and book to market
mean return
           1.50          1.35          1.89          1.89          2.23
           1.23          1.63          1.59          1.76          2.02
           1.42          1.33          1.52          1.58          1.71
           1.10          1.31          1.44          1.49          1.70
           0.98          0.91          1.29          1.27          1.56
CAPM betas
           1.80          1.84          1.61          1.51          1.68
           1.19          1.36          1.40          1.43          1.65
           1.30          1.15          1.31          1.31          1.69
           0.96          1.15          1.18          1.40          1.75
           0.94          0.89          1.06          1.35          1.36
```

CAPM alphas

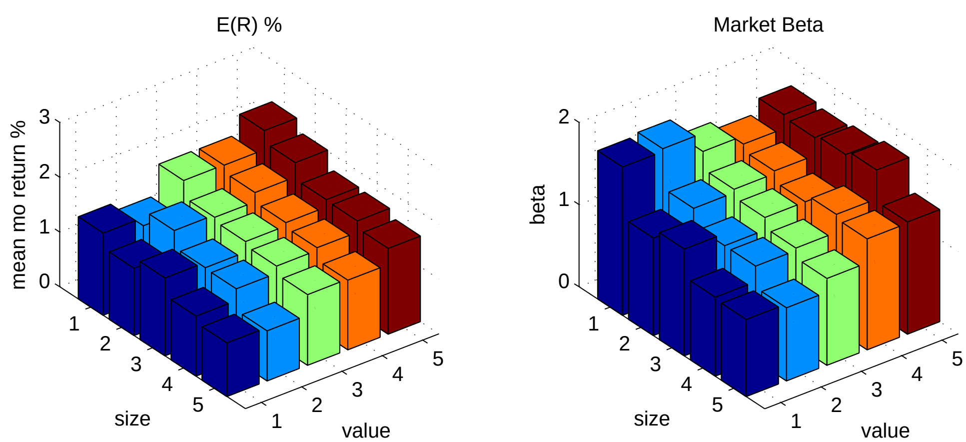
-0.45	-0.63	0.15	0.25	0.41
-0.05	0.17	0.08	0.21	0.24
0.02	0.08	0.11	0.17	-0.11
0.06	0.07	0.17	-0.02	-0.19
-0.04	-0.05	0.15	-0.19	0.09

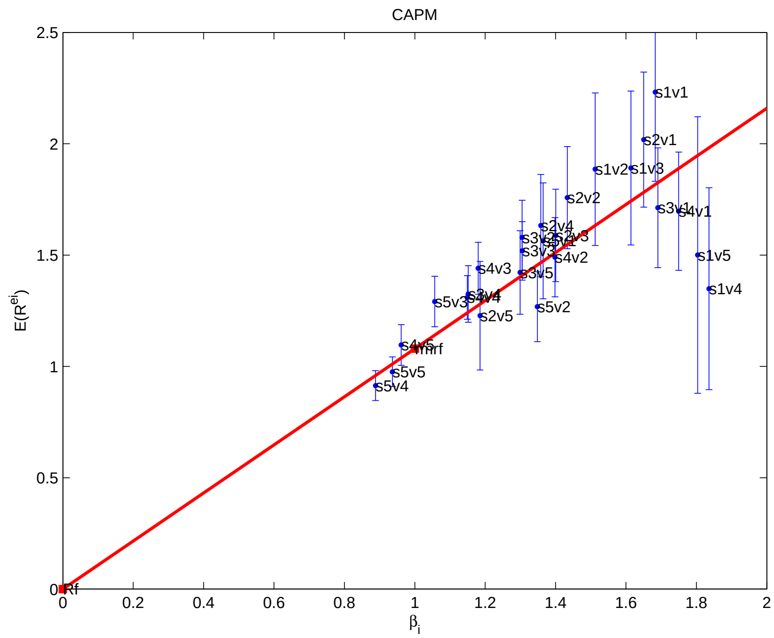
T on CAPM alphas

-0.72	-1.40	0.43	0.74	1.03
-0.21	0.72	0.37	0.92	0.78
0.10	0.64	0.83	1.02	-0.42
0.63	0.70	1.42	-0.11	-0.72
-0.53	-0.68	1.33	-1.20	0.35

CAPM R2

0.47	0.64	0.70	0.68	0.65
0.72	0.79	0.83	0.81	0.76
0.84	0.90	0.91	0.87	0.81
0.92	0.94	0.92	0.87	0.82
0.95	0.95	0.90	0.89	0.75





The CAPM works quite well in the earlier sample. What changed? The pattern of expected returns is the same. In fact, the small growth anomaly is "better" in the earlier sample too, in that small stocks earn more than large stocks even in the extreme growth bucket. *The betas changed.* Once again, the value puzzle is all about the betas, not really the average return! The big anomalies are still the small growth portfolios, s1v4 and s1v5, the back row on the left. They don't have great returns, but they have huge betas. Here the puzzle is "too much" beta.