Business 35904 **John H. Cochrane**

Problem Set 5

1. Suppose there is a single excess return R^e, and an information variable z_t which can take two values, $z_t = 1, 2$ with equal probability. Thus, $E_t(R^e_{t+1}) = E(R^e_{t+1}|z_t)$ and $\sigma_t(R^e_{t+1}) = \sigma(R^e_{t+1}|z_t)$ take on different values in each of the z_t states. We will also consider the unconditional moments, i.e. $E(R^e_{t+1}) = E\left[E(R^e_{t+1}|z_t)\right]$, etc.

   (a) What is the payoff space generated by portfolios of R^e? Complete and correct the following, $\underline{X} = \left\{c \times R^e_{t+1},\ c \in ....\right\}$

   (b) Find R^{e*}. Hint: $\underline{X} = \underline{R}^e$ so you're looking for an element of the set described in a that has the right properties. Verify that your $R^{e*} \in \underline{R}^e$, and that it satisfies the defining properties

   $$E\left(R^{e*}R^e\right) = E(R^e)$$

   for both conditional $E(\cdot|z)$ and unconditional $E(\cdot)$ moments. (I.e. you don't need a different R^{e*} for the conditional vs. unconditional mean-variance frontier.)

   (c) Suppose $E(R^e_{t+1}|z_t = 1) = E(R^e_{t+1}|z_t = 2) = 8\%$, but $\sigma(R^e_{t+1}|z_t = 1) = 16\%$; $E(R^e_{t+1}|z_t = 2) = 24\%$. Is R^e_{t+1} on the conditional mean-variance frontier in state 1? In state 2? Is it on the unconditional mean-variance frontier?

   (d) If not, exhibit a better return with $E(R^e) = 8\%$. (Hint: use the Hansen-Richard representation of the mean-variance frontier for excess returns). Express your return by its investments in R^e in each state of nature.

   (e) Draw frontiers $E(R^e_{t+1}|z_t)$ vs. $\sigma(R^e_{t+1}|z_t)$ for $z_t = 1, 2$ and $E(R^e_{t+1})$ vs. $\sigma(R^e_{t+1})$. (The two conditional frontiers can fit on the same graph. Draw a separate graph for the unconditional frontier.) Locate R^e_{t+1}, R^{e*}_{t+1} and, if you found a better return in part c, that return, on all three graphs. (It will help to start with a summary table of all the necessary means and standard deviations.)

   (Note: most of the numbers come out easily, but you need a calculator for some of them.)

2. (a) The expected return- beta model is $E(R^{ei}) = \beta_i\lambda$. This is true by construction – pick any λ, then just define $\beta_i = E(R^{ei})/\lambda$. What went wrong with this logic – how does the model have any testable content?

   (b) The CAPM is

   $$E(R^i) = R^f + \beta_{i,m}\left[E(R^m) - R^f\right]$$

   You can difference the left hand variables, to produce a CAPM with excess returns defined as one risky asset return minus another rather than one risky asset return minus the risk free rate,

   $$E(R^i - R^j) = \beta_{i-j,m}\left[E(R^m) - R^f\right]$$

   (Covariance is a linear operator, so $\beta_{i,m} - \beta_{j,m} = \beta_{i-j,m}$.) Can we can do this on the right hand side too, or must the market premium really be relative to the riskfree rate? Can we write

   $$E(R^{ei}) = \beta_{i,R^m-R^k}E(R^m - R^k)?$$

   (Hint: the answer is "maybe, but only if R^k has special properties.." For this problem, assume a risk free rate is traded.)

3. Do the problems 1-3, in Chapter 8, *Asset Pricing*, p. 146. Notes:

(a) Question 3: To make this question interesting, suppose that returns are *not* iid, that there are observable variables z_t so that $E(R|z_t)$ and $\sigma(R|z_t)$ vary over time.

4. Do problem 1 in Chapter 9 p 183.