

Problem Set 7

We're going to do GMM. The model is

$$0 = E_t \left[\left(\frac{c_{t+1}}{c_t} \right)^{-\gamma} R_{t+1}^e \right]$$

Get consumption and return data from the class website. (Yes, it's good for you to learn to go get data, but not this week. Also, I want to make sure that we are at least making the same mistakes in data transformations so that the task is only to learn the right lessons from GMM.) The program I used to make the data is attached here, and the raw data is also on the class website FYI.

I use annual returns and annual quarter to quarter consumption data. The problem with standard tests is that consumption is the *average* for the quarter and returns are point to point. So, do you line up the average consumption for april+may+june / average consumption jan+feb+mar with returns from Jan1 to mar 1? Or from mar 31 to June 31? It's not clear. If you get this wrong, you destroy the test. If consumption and returns are uncorrelated, the model totally fails – no beta. If consumption and returns iid (a fair assumption) and the correlation between R_{t+1} and Δc_{t+1} is strong, the correlation between R_{t+1} and Δc_{t+2} or Δc_t is zero. You completely destroy the model if you get the timing off. One answer is to do it in continuous time and get the time aggregation right. Not for us. So instead, we'll do jan+feb+mar of year 2, divided by jan+feb+mar of year 1, and divide by feb year 2 / feb year 1 stock return and hope we've got it about right.

Here's what you get to do:

1. Find the mean and correlation matrix of consumption growth and the three excess returns. Is consumption growth positively correlated with the returns? Which ones have greater correlation? This will be important below.
2. Plot $E \left[\left(\frac{c_{t+1}}{c_t} \right)^{-\gamma} R_{t+1}^e \right]$ for γ from 0 to 100, for $R^e = \text{rmrf}, \text{hml}, \text{and smb}$. The equity premium puzzle should show up in your γ values. Two of the three lines look “right” though they disagree on γ , but that may be sampling error. One looks “wrong.” What's wrong? This will be important in figuring out what's going on later.
3. Now perform a first-stage (identity matrix) and second-stage GMM estimate of γ for the following sets of moment conditions. (Obviously, you should write a function once, which accepts arbitrary asset returns.) At each stage present
 - (a) your estimate $\hat{\gamma}$,
 - (b) standard error $\sigma(\hat{\gamma})$,
 - (c) overidentifying restrictions test. Give the χ^2 statistic and probability value.
 - (d) Also compare the alphas to the expected returns – in this case compare discounted expected returns $g_T = E(mR^e)$ to actual $E(R^e)$. Include standard errors of g_T so you get a sense whether alphas are meaningful or not. (Report annual percent units.)
 - (e) Present and interpret $a = d'W =$ which moments GMM is setting to zero. (the numerical values of a are small, so normalize them to something larger. Of course $1000 \times ag_T(\hat{b})$ also = 0..)

Do this for

- (a) market rmrf alone
- (b) hml alone
- (c) smb alone
- (d) market and hml
- (e) market and smb

Explain what's going on with your estimates. Some give weird answers, some don't. Of course a lot of the lesson here is "things that can go wrong with GMM."

Hints:

The d matrix is

$$d = \frac{\partial}{\partial \gamma} E \left[\left(\frac{c_{t+1}}{c_t} \right)^{-\gamma} R_{t+1}^e \right] = \frac{\partial}{\partial \gamma} E \left[e^{-\gamma \log(c_{t+1}/c_t)} R_{t+1}^e \right] = -E \left[\log \left(\frac{c_{t+1}}{c_t} \right) e^{-\gamma \log(c_{t+1}/c_t)} R_{t+1}^e \right]$$

You can use the fact that under the null

$$E_t (m_{t+1} R_{t+1}^e) = 0 \rightarrow E [(m_{t+1} R_{t+1}^e) (m_t R_t^e)] = 0$$

to simplify the S matrix a lot. Read the textbook hints on constructing S matrices. In particular, take out the mean of mR . Yes, under the null the mean is zero, but this is a covariance matrix so force that in sample.

You can use the matlab search to $\min g_T(b)' W g_T(b)$, or the matlab equation solver to solve $\partial g_T(b) / \partial b' W g_T(b) = 0$. However, I encourage you to do it yourself – this will let you diagnose problems better. The easiest way is a "divide and conquer" grid search. Try 10 or 100 values of γ , locate the general range of the minimum/solution; try a finer grid, etc. For one dimensional problems this is faster and much more robust than matlab's derivative-based methods.

How I created the data:

```
% ps5_gmm program for problem set 5, GMM
% new version for new problem set 2011

close all;
clear all;
qstart = 1; % which quarter is starting quarter. 1 2 3 4.
offset = 1; % which month for returns 0 = jan 1 = feb 2 = march.

tester = 0;      /* verbose output on search */
gamtol = 1E-6;  /* band of uncertainty for gamma */

convt = 0;      /* make eps files to import into solutions? */
printdata = 1;  % print data for students

x = load('pop.txt'); % this is updated 2011
pop = x(3:3:size(x,1),4); % convert to quarterly by taking end of quarter value
popdates = x(3:3:size(x,1),1:3); % convert to quarterly by taking end of quarter value

% new consumption data
x = load('pcend.txt');
cnd = x(:,4);
x = load('pcesv.txt');
cs = x(:,4);
cdates = x(:,1:3);
x = load('pctetpi.txt');
plev = x(:,4);
c = (cnd+cs)./plev; % real consumption
c = c(1:end-3,:); % lose the last 3 quarters because pop stops in 2010
c = c./pop;

% create annual consumption growth from quart to quart.
dca = c(qstart+4:4:end-4+qstart)./c(qstart:4:end-4+qstart-4);
plotdate = (1948:2010)';

% stock returns and risk free rate
x = load('ff_factors.txt');
ffdates = x(247:end-7,1); % 47:1 to 2010:12
ff_m = x(247:end-7,2:4); % rrmf smb hml
rf_m = x(247:end-7,5);

% create cumulative returns
ff_cum = cumprod(1+ff_m/100+(rf_m*[1 1 1]/100));
rf_cum = cumprod(1+rf_m/100);

% create annual returns from quart to quart with offset;
rfa = rf_cum(13+offset:12:end-2+offset)./rf_cum(1+offset:12:end-2+offset-12);
ffa = ff_cum(13+offset:12:end-2+offset, :)./ff_cum(1+offset:12:end-2+offset-12, :) - rfa*[1 1 1]; % now o
```

```
%data for students
if printdata
    fid = fopen('ps5_data_new.txt','w');
    fprintf(fid,'%10.9s %10.9s %10.9s %10.9s %10.9s %10.9s \n','date','dc','rmrf','smb','hml','rf');
    fprintf(fid,'%10.0i %10.6f %10.6f %10.6f %10.6f %10.6f \n',[plotdate dca ffa rfa]);
    fclose(fid);
end;
```