Business 35905 **John H. Cochrane**

Problem Set 6

We're going to replicate and extend Fama and French's basic results, using earlier and extended data. Get the 25 Fama French portfolios and factors from the class website (originals available from Ken French's website. http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html You want the monthly 25 portoflios formed on size and book-to-market, value weighted, under U.S. Research Returns Data. Start in 196301 as Fama and French do, but extend to the end of the sample. This lets us see if more recent data changes things, but avoids the early sample in which the CAPM works pretty well!

Subtract the risk free rate rf from the 25 test assets to make them excess returns. The factors are already excess returns.

You may also want to refer to Ch 12 of *Asset Pricing* which covers these regression methods. I added a section to the notes on the class website with all the formulas you need.

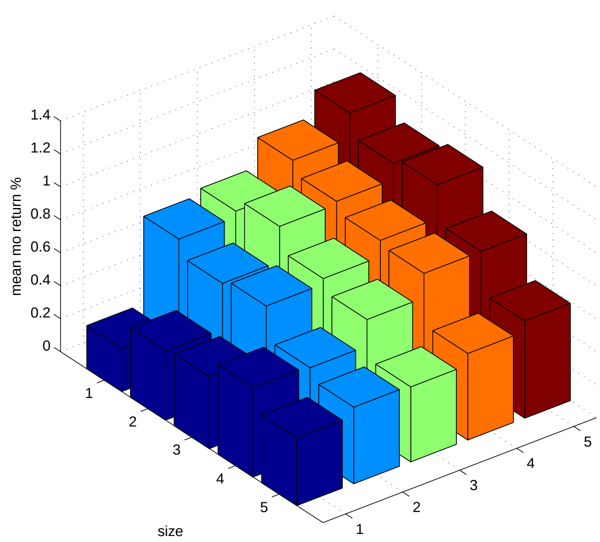
1. Start by documenting mean returns for the FF 25. Here and below, make tables in Fama French format, i.e.

   Mean Return (% per month)

	Low B/M growth				High B/M value
small	.	.	.	.	.
	.	.	.	.	.
	.	.	.	.	.
	.	.	.	.	.
big	.	.	.	.	.

   (You don't have to format tables. disp(reshape(mean(rx)',5,5)') is good enough, then past in a constant-width font or verbatim to tex. Fprintf makes prettier tables)

   The patterns will emerge better if you also make plots of expected excess returns, alphas, and betas i.e.



   I did this with figure; bar3(reshape(mean(rx)',5,5)');ylabel('size'); xlabel('value');

2. Fama and French 1996 we read did not include a documentation of the failures of the CAPM. (The CAPM asks for *single* regression betas, not *multiple* regression betas. The capm betas you find will not be all near one – multiple regressions are different from single regressions.)) Evaluate the CAPM on these 25 portfolios, as follows.

   (a) Run time series regressions of $R_t^{ei} = \alpha_i + \beta_i(rmrf_t) + \varepsilon_t^i \; t = 1, 2, ...T$ for each i. Tabulate $\hat{\alpha}$, $\hat{\beta}$, $t(\hat{\alpha})$.

   (b) Joint alphas.

      i. Calculate the GRS F test and the corresponding χ^2 test for the CAPM and the FF3F models[1]. Report the test statistic, report the 1%, 5% and 10% critical values (if the alphas are really zero, how big a statistic should we see 5% of the time?), and report the probability value of the test statistic (what is the chance that, if alphas are really zero, we see a statistic as large as that found in the data?)

      ii. Calculate also the root mean square alpha $\sqrt{\alpha'\alpha/N} = \sqrt{(\alpha_1^2 + \alpha_2^2 + ... + \alpha_{25}^2)/N}$, and the mean absolute alpha mean(abs(alpha)) $= \frac{1}{25} \|\alpha_1\| + \|\alpha_2\| + ... + \|\alpha_{25}\|$. We're interested in the economic size of alphas as well as the statistical size. Sometimes $\alpha'\Sigma^{-1}\alpha$ is large because Σ is small, not because α is small!

      iii. Make a plot of predicted expected excess return vs. actual average excess return for the ff 25 portfolios. Include the predicted and actual excess return of rmrf and the riskfree rate rf on your plot. Include a 45 degree line.

   (c) Does the CAPM work on these portfolios? Look both at the *statistical* significance – are the ts on the alphas greater than 2 and at the *economic* significance– is there a pattern in average returns, and is that pattern mirrored in the betas so that $E(R^e) = \beta\lambda$ has any hope?

   (In part we're doing this to fill in the absence of a simple CAPM from Fama and French's paper. You should always start by showing the last model does not work on your portfolios. FF did it in an earlier paper. The capm betas you find will not be all near one – multiple regressions are different from single regressions.)

   (d) Run an OLS cross-sectional regression,

      i. using only the test assets (i.e. don't include the market portfolio and risk free rate as test assets), and

      ii. adding the market portfolio ([1,E(rmrf)]) and risk free rate ([0,0]). Allow a free intercept, $E(R^{ei}) = \gamma + \beta_i\lambda + \alpha_i$.

   Make a table. Compute $\hat{\gamma}, \hat{\lambda}, \sigma(\hat{\lambda}), \sqrt{\alpha'\alpha/N}$, and the χ^2 test for all $\alpha = 0$ (report χ^2, 5% critical value, and percent probability value). Compare with the same results from the time-series regression.

      i. Add the cross-sectional fit $\hat{\gamma} + \beta_i\hat{\lambda}$ to your plot of $E(R^{ei})$ vs. beta

      ii. Make a plot of actual $E(R^{ei})$ vs. predicted $\hat{\gamma} + \beta_i\hat{\lambda}$ expected returns for this case.

      iii. Note (not a question for you to answer): You think that nobody would be dumb enough to do this?

[1] Econometric note. The GRS F and χ^2 tests, while traditional, assume that returns are i.i.d., and in the first case, normal. Returns are not i.i.d. or normal; they are heteroskedastic, they display some serial correlation, and they have fat tails. It is easy enough to do GMM tests that surmount these problems, at least asymptotically, so there is no excuse for stopping at these ancient tests! Nonetheless, I won't make you do the GMM tests this week because the problem set is already long enough. However, figure out how to do the tests right before you write papers!

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Table III

Cross-Sectional Regression

This table reports the Fama–MacBeth (1973) cross-sectional regression estimation results for asset pricing model:

$$E[R_{i,t}] = \lambda_0 + \lambda'\beta.$$

Betas are estimated by the time-series regression of excess returns on the factors. Test portfolios are the 25 Fama–French portfolios' annual percentage return from 1954 to 2003. The estimation method is the Fama–MacBeth cross-sectional regression procedure. The first row reports the coefficient estimates ($\hat{\lambda}$). Fama–MacBeth t-statistics are reported in the second row, and Shanken-corrected t-statistics are in the third row. The last column gives the R^2 and adjusted R^2 just below it.

	Constant	Δc	R_m	SMB	HML	log(ME)	log(B/M)	R^2(adj R^2)
Estimate	0.14	2.56						0.73
t-value	0.05	3.89						0.71
Shanken-t	0.02	1.98						
Estimate	11.31		−0.56					0.00
t-value	2.05		−0.09					−0.04
Shanken-t	2.05		−0.08					
Estimate	10.43		−3.26	3.12	5.83			0.80
t-value	2.66		−0.70	1.62	3.11			0.77
Shanken-t	2.37		−0.57	1.03	2.12			
Estimate	11.75	1.58	−3.76	3.00	5.75			0.87
t-value	2.98	3.64	−0.81	1.56	3.07			0.84
Shanken-t	1.95	2.26	−0.50	0.83	1.71			
Estimate	16.20					−0.87	3.46	0.84
t-value	2.95					−1.43	3.00	0.83
Estimate	12.19	0.71				−0.71	2.66	0.86
t-value	2.41	1.62				−1.23	2.12	0.84
Estimate	22.22		−3.80	−0.67	0.96	−1.07	3.04	0.87
t-value	3.50		−0.88	−0.23	0.37	−1.51	2.87	0.84

3. Now for some fun. Repeat, using the sample 193201-196712. (We start in 1932 to avoid some NaNs in portfolio returns). You do not have to repeat tables and plots of alphas and betas. Report only the $E(R^e)$ vs. β plot with fitted lines from TS and CS regressions, and the results for $\hat{\gamma}, \hat{\lambda}, \sigma(\hat{\lambda})$ and α statistics.

   How does the CAPM work on value-sorted portfolios in the earlier period? How to time-series and cross section compare in this sample? (This should just involve rerunning the same code with different start and end dates.) (I have no idea what the deeper meaning of this result is. In my fishing here, I've found that you get "post" results extending back as far as 1947, the usual WWII break in many series, and the "early" results with the CAPM working with samples that extend as far as 1967. As usual detecting breaks like this is hard.)

4. Now, do the same thing for the FF 3F model, replicating the paper, but using the longer 1963-now sample. I.e.

   (a) Run time series regressions

   $$R_t^{ei} = \alpha_i + b_i rmrf_t + h_i hml_t + s_i smb_t + \varepsilon_t^i \quad t = 1, 2, ...T$$

   for each i. Make the same tables of $\hat{\alpha}, t(\hat{\alpha})$, $\hat{b}, \hat{h}, \hat{s}, R^2$. (Basically, you're replicating Table I of Fama and French)

   (b) Make bar or surface graphs like mine above of b, h, s to better see the patterns across assets. Make a plot of actual average returns vs. predicted.

   (c) Do Fama and French's patterns hold up in the longer sample?

   (d) Alphas. Calculate GRS F and χ^2 tests, critical values, probability values, mean square alphas and mean absolute alphas.

5.

(a) Now we can compare the models. Which of the CAPM vs. FF3F model is statistically rejected? Which has larger alphas? What does this tell you about the importance of formal testing in how people change their views about models? (Ok, the last one is a bit of a leading question.)

(b) The difference in *statistical* significance between FF and CAPM is not as large as the difference in the size of the alphas. Why not?

(Hint: is it possible that the α get smaller, but the χ^2 or F test get bigger? If the formula isn't clear, see the bottom of p. 57 of Fama and French. Recall from statistics that we do *not* compare models by seeing which has a better t or χ^2 statistic. Everyone forgets this fact. This problem is designed to help you remember the fact. You can compare models by $\alpha'\Omega^{-1}\alpha$ sorts of statistics, but you have to use the same Ω in the comparison, as in a likelihood ratio test. The error in saying "this model is better than that one because the p values are better" is that you're using a different Ω.)

6. Let's see if we can drop size or hml factors. As explained in the notes, you do *not* just check if $E(smb) = 0$ or $E(hml) = 0$. Do this problem for both the early (1932-1967) and later (1947-now) samples. You'll get different results.

(a) Compute the mean, standard deviation Sharpe ratio, annualized sharpe ratio, and t statistic for the mean of $rmrf$, hml, and smb, to get a sense of all of these numbers.

(b) Check in turn whether you can drop each factor given the other two. Report the mean (equal to alpha), standard deviation, sharpe ratio, annualized sharpe, and t statistic for each of the *orthogonalized* factors, equal to the alpha in a regression of each factor on the others.

(c) Suppose this test comes out that you can in fact drop smb, since $\alpha_s = 0$. You're giving this paper, and a seminar-questioner complains, "but the t statistics on s_i are significant, the R^2 drop a lot, and a test whether smb is jointly significant in all 25 time-series regressions is rejected decisively. You shouldn't drop smb." How do you answer.

(d) If we don't need smb to price assets, should we then drop it from the time-series regression $R_t^{ei} = \alpha_i + b_i rmrf_t + h_i hml_t + s_i smb_t + \varepsilon_t^i$? Or could it be useful even though its inclusion doesn't change alphas?

7. Now, let's find the factors by eigenvalue decomposition. Do an eigenvalue-based factor decomposition of the covariance matrix of the FF25 returns in the 1963-on sample.

(a) Plot the eigenvectors $\sqrt{\lambda}$ to get a sense of where a standard factor analysis would stop. You should find 3 eigenvalues (at most) that stand out of the others.

(b) Plot the loadings of the first four factors, (5 x 5 bar plots are the best way to do this, as above). Do you see an interpretation of the first few factors? (Recall that the columns of Q give "how much does the return of the ith portfolio move if the factor moves" – the *loadings or betas* – and also "how much weight do you put on each return to form the factor portfolio – the *weights*.) (The answer in this data set is more subtle than my claim in lecture that the second and third components just span size and b/m)

(c) Next, I want to think about forming a factor model. Examine or plot the means of all 25 factors and the t statistics of the factor means (t statistics are also proportional to Sharpe ratios). (Look at absolute values because some factors have negative means.) Based on the view that "we should keep factors that have $E(f^*) > 0$ which factors should we keep? Warning – this is another case in which statistical and economic significance give different answers! (The "small" factors in Cochrane-Piazzesi were the first case.)

(d) How does a three-factor model and then a four-factor model based on the eigenvalues work? Plug the first three and then four factors in to your time-series regression program. Let's see if we do better than Fama and French! Check the alpha χ^2 statistic, mean absolute and rmse alphas, and plots of predicted vs acutal mean returns.

You should conclude that Fama and French's procedure is pretty darn close to trying the first three eigenvalues as factors. You should see enough noise that it's not guaranteed that there is a connection between variance and mean. Of course, there is a tremendous amount of information built in here that the portfolios are all sorted by size and b/m first.

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Problem set 6 part II

Give very short, 1-2 sentence answers. Citing page numbers and results from tables is a good idea. This is just a help towards digesting the papers.

Multifactor anomalies

1. Are small stocks necessarily ones with small numbers of employees, small plants, etc.?
2. Can we summarize Fama and French's 3 factor model amount to saying "We can explain the average returns of a company by looking at its size and book/market ratio?""
3. Which gets better returns going forward, stocks that had great past growth in sales, or stocks that had poor past growth in sales?
4. What pattern of betas explains the average returns of stocks sorted on sales growth?
5. Are the s coefficients on sales growth portfolios constant? Can you think of a story to explain them?
6. Which results show the "long-term reversal" effect in average returns best? Which show the "momentum" effect best?
7. Why do the sorts in Table VI stop at month -2 rather than go all the way to the minute the portfolio is formed?
8. Why might the average investor try to avoid holding value stocks, and hence drive up the equilibrium premium (according to Fama and French)?

Dissecting Anomalies

1. FF point out dangers of the common practice of sorting stocks by some variable, and then looking at the average returns of the 1-10 spread portfolio. What don't they like about this practice?
2. How do FF define "tiny" stocks? What fraction of their sample are tiny, and what fraction of market cap do tiny stocks represent? How can the
3. 1656 breakpoints are 20 and 50 percentiles of NYSE, 60% of stocks and 3% of market value. The sample includes amex and nasdaq which have many smaller stocks than NYSE.
4. Are the average returns in Table II adjusted for the three-factor model somehow?
5. Why are the t- statistics for the High-Low portfolio in Table II so much better than for the individual portfolios?
6. It seems we get better returns and higher t statistics the finer we chop portfolios. Can you make anything look good by making 100 portfolios and then looking at the 1-100 spread?
7. Name an anomlay that only seems to work in tiny stocks in Table II.
8. The Profitability sort seems not to work in Table 2.. How did people think it was there? (Hint: 1663 pp2)
9. Explain why the numbers in Table III jump so much between 4 and high.

10. Explain what the top left 4 numbers mean in Table IV.
11. "The novel evidence is that these results [size effect] draw much of their power from tiny stocks" What numbers in Table IV are behind this conclusion?
12. What is a "good" pattern of results in Table IV? Which variables have it, and which do not?
13. Do any of the anomaly variables drive the other ones out in a big multiple regression, or does each seem to give a separate piece of information about expected returns?
14. In the conclusions, FF say "The evidence..is consistent with the standard valuation equation which says that controlling for B/M, higher expected net cashflows...imply higher expected stock returns" and "Holding the current book-to-market ratio fixed, firms with high expected future cash flows must have high expected returns" Isn't this the fallacy that "profitable companies have higher stock returns" , or "confusing good companies with good stocks"?

    Note: If you can't directly answer the following questions from the paper, at least think about what else you need to know in order to figure out the answer.
15. Do these new average returns correspond to new dimensions of common movement across stocks, as B/M and size corresponded to B/M and size factors?
16. What is the highest Sharpe ratio you can get from exploiting one of these anomalies? (Choose any one).
17. What is the highest Sharpe ratio you can get from combining all these anomalies and exploiting them as much as possible?