Business 35904 **John H. Cochrane**

Problem Set 9

More tests

Rev up your program from last week (or use mine). Start by repeating the CAPM test, using the full sample from 193201. In all these cases you need only keep the time series and cross section with free constant, factors not priced; the iid standard errors, the χ^2 test statistic, and the alpha diagnostics. For the CAPM, here are my answers. (Don't fret if your answers are close but not exact.)

```
capm
Using sample from    193201  to   201107
                gamma  lambda  chi2/F      5%      1%     % p
 Time Series
       E(f)     0.00    0.70
   iid se       0.00    0.17
   iid t chi2   0.00    4.13   89.82   37.65   44.31 3.06e-007

 Cross section, free constant, factors not priced
    estimate     0.24    0.63
       iid se    0.33    0.36
  iid t, chi2    0.74    1.72   70.19   35.17   41.64 0.000114

alpha diagnostics
               rmse a mean|a|
   ts            0.27    0.22
   cs            0.22    0.18
```

Repeat these exercises using a) the Fama-French 3 factor model, and b) a Fama-French two factor model that uses only rmrf and hml. In each case, complement your tables with plots. Since there are multiple factors, we can't do $E(R^{ei})$ vs. β^i. Instead, we have to make graphs of $E(R^{ei})$ vs. "predicted" $E(R^{ei}) = (\gamma_0) + b_i\lambda_m + h_i\lambda_h + s_i\lambda_s$, etc. Make one graph for the time series estimate, and one for the cross-sectional estimate. Include the test assets, the factors, and the "predicted" line (which is 45 degrees). Compare the results as follows

1. Is the TS estimate of the Fama French 3 factor model rejected? How does its χ^2 value (which, recall is based on alphas) compare with the CAPM? Does it look like a much better model by this measure?

2. Compare the mean absolute alphas and mean squared alphas with those of the TS estimate of the CAPM. Does the FF3F model look better here? If this paints a different picture than part a, explain why. (The graph will help.)

3. Now, look at the cross sectional regression. You should start by reminding yourself how the CAPM time series and cross section compare. Then

   (a) Compare the TS and CS FF3F regression.

   (b) Compare the cross sectional regression result for CAPM and FF3F model. (You will also compare the cross-sectional vs. time series results for each of CAPM and FF3F to understand why this comes out differently. You compared CAPM time series and cross section last week.)

4. Now, compare FF2F model and CAPM, FF3F model. Can we drop SMB?

   (a) In the FF3F model is λ_s economically and statistically significant?

   (b) Does the FF2F model produce a much worse χ^2 statistic, or much larger alphas than the FF3F model? (Hint: Is it possible for the FF2F time series estimate to produce smaller alphas than FF3F? The cross sectional estimate?)

5. You should come to the conclusion from d, that these statistics are not the right statistics for asking the question, can we drop *smb* from the list of factors.

   (a) Show that the following all are equivalent, and correct ways of asking whether we can drop a factor from our understanding of expected returns: (this is a representation question, not a distribution theory question. Keep it short, I'm only looking for a one to two line argument.)

      i. $b_s = 0$ in $m = 1 - b_m rmrf - b_h hml - b_s smb;\ 0 = E(mR^e)$

      ii. $b_s = 0$ in $E(R^e) = cov(R^e, rmrf)b_m + cov(R^e, hml)b_h + cov(R^e, smb)b_s$

      iii. $\lambda_s = 0$ in $E(R^e) = b\lambda_m + h\lambda_h + s\lambda_s$ where the b, h, s are single regression coefficients (not multiple)

      iv. $\alpha_s = 0$ in $smb_t = \alpha_s + b_s rmrf_t + h_s hml_t + \varepsilon_t$

      v. $E(smb_t^*) = 0$ where smb^* is the orthogonalized factor $smb_t^* = smb_t - (b_s rmrf_t + h_s hml_t)$

   (b) Do a proper test. Can we drop smb from rmrf, smb, hml? Can we drop hml? Do we need smb in addition to rmrf? Do we need hml in addition to rmrf? (You only need to do one, not all the variants above.)

   (c) Even if it fails this test, might you want to keep sml for other purposes?