9 Week 5 Asset Pricing Outline

- Objective: Understand models like the CAPM and FF3F. Deeper objective: undertand theory and concepts behind all of asset pricing.
- Equilibrium vs. opportunity.

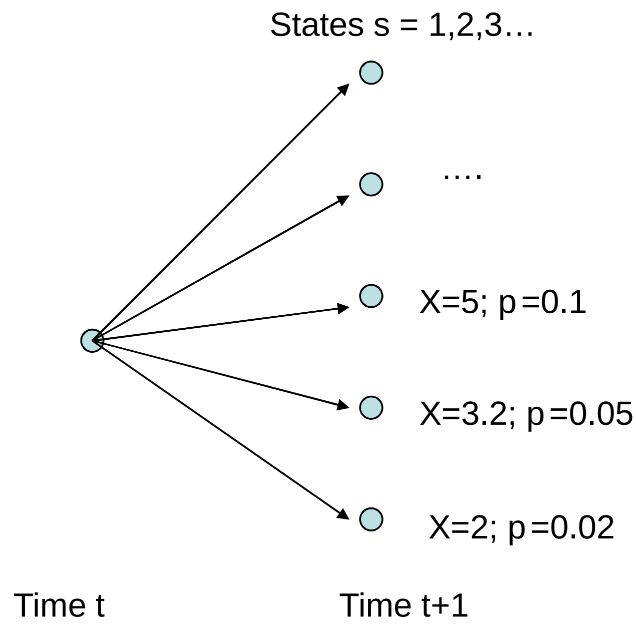
9.1 APT

Portfolios that explain *comovement* of asset returns should be factors to explain *Average returns.*

1. Central trick: how to use a factor model to hedge, exploit an alpha:
$$R^{ei}_{t+1} = \alpha_i + \beta_{i1} f^1_{t+1} + \beta_{i2} f^2_{t+1} + \varepsilon^i_{t+1}.$$
$$\rightarrow E(R^{ei}) = \alpha_i + \beta_{i1} E(f^1) + \beta_{i2} E(f^2)$$
$$R^{ep}_{t+1} = R^{ei} - \beta_{i1} f^1_{t+1} - \beta_{i2} f^2_{t+1} = \alpha_i + \varepsilon^i_{t+1}.$$
$$SR(R^{ep}_{t+1}) = \alpha_i / \sigma(\varepsilon^i)$$
2. This is a deep point. *These factor models tell you how to remove systematic exposures from portfolios to reduce risk; or create "portbable alpha."'*
3. Equilibrium: Traders will buy if the Sharpe ratios are huge.
$$\alpha_i < (\text{max surviving Sharpe}) \times \sigma(\varepsilon^i)$$
4. "Small" residuals, large R^2 should give "small" alphas.
5. Only works when ε is small: for portfolios not individual stocks.
6. Does not explain why the factors get a premium. Why $E(hml)$, $E(smb)$? Relative pricing vs. absolute pricing.

9.2 All of asset pricing theory

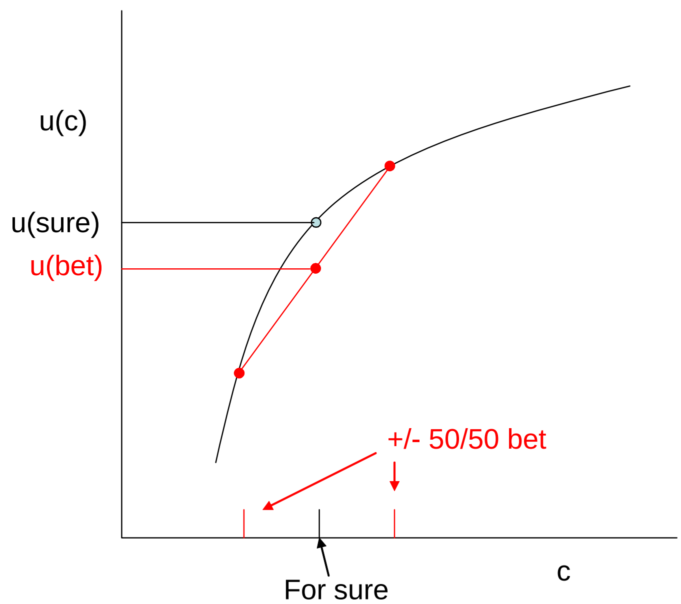
1. Payoffs x_{t+1} tomorrow. (For stocks, $x_{t+1} = p_{t+1} + d_{t+1}$). Value of x_{t+1} at t?



2. Value to who? Utility function captures aversion to *risk* and *delay*

$$U(c_t, c_{t+1}) = u(c_t) + E_t\left[u(c_{t+1})\right]$$

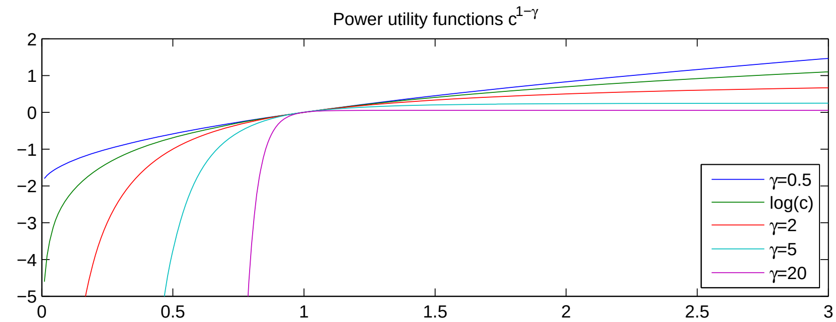
(a) $u(c)$ shape. Concavity $u''(c) < 0$: people dislike *risk*.

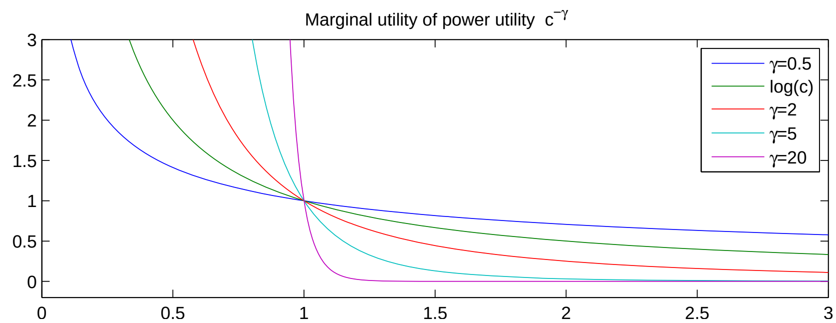


$\beta < 1$ people dislike *delay*.

(b) Power example, γ allows you to vary curvature / risk aversion

$$u(c) = \frac{c^{1-\gamma}}{1-\gamma}; \; u'(c) = c^{-\gamma}$$
$$u(c) = \log(c); u'(c) = c^{-1}$$





3. Valuation

(a) What is x_{t+1} worth (willingness to pay) to a typical investor? Marginal cost = marginal benefit led to.

$$p_t = E_t\left[\beta \frac{u'(c_{t+1})}{u'(c_t)} x_{t+1}\right]$$

(b) We separate this to

$$p_t = E_t\left[m_{t+1}x_{t+1}\right]$$

$$m_{t+1} = \beta\left(\frac{c_{t+1}}{c_t}\right)^{-\gamma} \approx 1 - \delta - \gamma\Delta c_{t+1}$$

m, $u'(c)$ measure "hunger" – higher c means lower m

(c) This is valuation after everyone has invested. To the individual, this tells you how to change *consumption* given prices and returns. But to the economy, prices have to adjust so everyone is happy eating what's available, so this describes how, *asset prices* change given economy-wide consumption.

4. Classic issues in finance (3 lines of algebra from $p = E(mx)$)

(a) Risk free rate

$$\begin{aligned} 1 &= E(mR^f); R^f = 1/E(m) \\ R^f &\approx 1 + \delta + \gamma E_t(\Delta c_{t+1}) \end{aligned}$$

⇒rates should be higher/lower in good/bad times Δc.

(b) Discount for risky assets/projects

$$\begin{aligned} p &= E(mx) = \frac{E(x)}{R^f} + cov(m,x) \\ p &= \frac{E(x)}{R^f} - \gamma cov(x, \Delta c) \end{aligned}$$

Price discount for assets that pay badly in bad times.

(c) Expected returns and covariance/beta

$$\begin{aligned} 0 &= E(mR^e) = cov(m, R^e) + E(m)E(R^e) \\ E(R^e) &= -R^f cov(m, R) = \beta_{R^e,m} \times \lambda_m \\ E(R^e) &\approx \gamma cov(R^e, \Delta c) = \beta_{R^e,\Delta c} \times \lambda_{\Delta c} \\ \lambda_{\Delta c} &\approx \gamma \sigma^2(\Delta c) \end{aligned}$$

i. Expected return depends on beta, tendency to pay badly in bad times/well in good times.

ii. Expected return does not depend on variance $\sigma^2(R^e)$!

iii. "Only systematic risk matters" "idiosyncratic risk does not matter"

$$R^{ei} = \beta_{i.m} m + \varepsilon^i$$

$$var(R^{ei}) = \beta_{i.m}^2 \sigma_m^2 + \sigma_{\varepsilon^i}^2$$

($m = \Delta c$ too)

(d) Mean- variance frontier

i. There is a frontier, all excess returns lie in a cone-shaped mean-standard deviation region. $0 = E(mR^{ei}) \rightarrow$

$$\begin{aligned} \frac{E(R^{ei})}{\sigma(R^{ei})} &= -\frac{\sigma(m)}{E(m)} \rho_{m,R^e} \\ \frac{\|E(R^{ei})\|}{\sigma(R^{ei})} &\leq \frac{\sigma(m)}{E(m)} \approx \gamma \sigma(\Delta c) \end{aligned}$$

ii. No Sharpe ratio can be larger than $\gamma\sigma(\Delta c)$. This justifies the Sharpe ratio limit of the APT.

iii. All frontier assets are perfectly correlated with each other, with m, and spanned by two frontier returns.

(e) Roll theorem. Return on MFV $\Leftrightarrow$ a one-factor model using that return!

$$E(R^e) = \beta_{R^e,R^{mv}} \lambda_{mv} \leftrightarrow R^{emv} \text{ is on the mvf.}$$

i. *Not*: market return on the frontier, investor wants to hold a MV portfolio

ii. *Yes*. Holds even when the CAPM is false, and (say) FF3F is true. (Then $rmrf$ is not on the MVF, and a portfolio of $rmrf, hml, smb$ is on the MVF.)

(f) Predictable returns

$$\begin{aligned} E_t(R_{t+1}) - R_t^f &\approx \sigma_t(R_{t+1})\sigma_t(m_{t+1})\rho_t(R, m_{t+1}) \\ &\approx \gamma\sigma_t(R_{t+1})\sigma_t(c_{t+1})\rho_t(R, \Delta c) \end{aligned}$$

$$\frac{E_t(R_{t+1}) - R_t^f}{\sigma_t(R_{t+1})} = \gamma_{(t?)}\sigma_t(c_{t+1})\rho_t(R, \Delta c)$$

Evidence suggests $\sigma_t(\Delta c_{t+1})$ is not enough, ρ_t is too nebulous – we need time-varying risk aversion γ_t.

(g) *Long lived securities

$$p_t = E_t \sum_{j=1}^{\infty} \beta^j \frac{u'(c_{t+j})}{u'(c_t)} d_{t+j} = E_t \sum_{j=1}^{\infty} m_{t,t+j} d_{t+j}$$

5. Preview: *all* of asset pricing theory is $p = E(mx)$ with something other than consumption growth for m.

9.3 Consumption models and practical application

Figure 1: Annual Excess Returns and Consumption Betas

Plot figure of average annual excess returns on Fama-French 25 portfolios and their consumption betas. Each two digit number represents one portfolio. The first digit refers to the size quintiles (1 smallest, 5 largest), and the second digit refers to the book-to-market quintiles (1 lowest, 5 highest). Annual excess returns and consumption betas are reported in previous table.

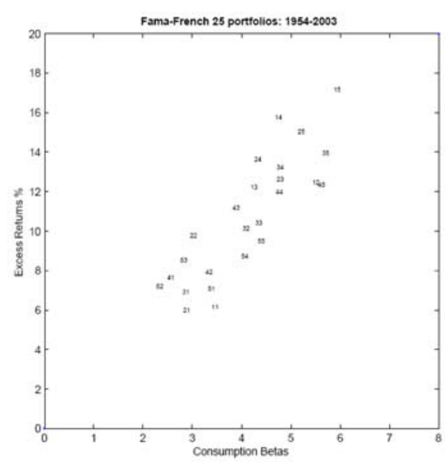
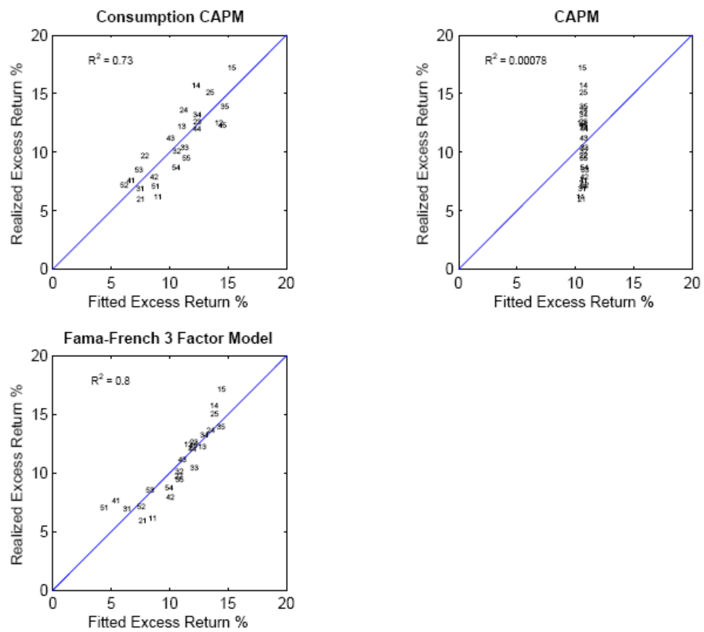


Figure 2: Realized vs. Fitted Excess Returns: FF25 Portfolios

This figure compares realized returns and fitted returns of Fama-French 25 portfolios 1954-2003. Each two digit number represents one portfolio. The first digit refers to the size quintiles (1 smallest, 5 largest), and the second digit refers to the book-to-market quintiles (1 lowest, 5 highest). Three models are compared: CCAPM, CAPM and Fama-French 3 factor model. Models are estimated by using Fama-MacBeth cross-sectional regression procedure Estimation results are reported in previous table.



- Bottom line:
  1. Finally glimmers of it working at 1-year horizons. But consumption is hard to measure, R^2 is low. So,
  2. Interesting for academics, connecting finance to macroeconomics, deep debates about where does hml, smb, umd, rmrf premium come from
  3. Not so useful for workaday application, "does this fund / anomaly beat the value index" (and I don't care why value earns its return

9.4 CAPM and mutifactor models

1. Foundations of CAPM and FF3F models, when we want to go deeper than APT logic.
2. Big picture: using "factors" rmrf, hml, smb, etc. in place of consumption.
3. Math trick

$$m_{t+1} = \beta \frac{u'(c_{t+1})}{u'(c_t)} \approx a - b' f_{t+1} = a - b_1 f^1_{t+1} - b_2 f^2_{t+1}$$

is equivalent to

$$E(R^{ei}) = \beta'\lambda = \beta_{i,1}\lambda_1 + \beta_{i,2}\lambda_2 + ...$$

4. What do we use for f? Idea 1: measures of good/bad times, determinants of consumption (*absolute pricing)*

   (a) CAPM: market return

   $$m_{t+1} = a - bR^m_{t+1}$$

   (b) CAPM also says this is the *only* factor. Multifactor models say "other things matter too."

   (c) Macro: f = labor, other income; investment, unemployment, etc.

   (d) ICAPM: f give news of future investment opportunities (shocks to d/p, interest rates)

   (e) Mimicking portfolio theorem

   $$m_{t+1} = a + \sum_i b_i R^{ei}_{t+1} + \varepsilon_{t+1}$$

   $$R^{ep} = \sum_i b_i R^{ei}_{t+1}$$

   is a single factor,

   $$E(R^{ei}) = \beta_{i,R^{ep}} E(R^{ep})$$

   "Mimicking portfolio for state variables of concern to investors."

   (f) Derivations, important thought: what *can't* be a factor?

5. Portfolio logic for multifactor models. If the average investor wants to get rid of stocks that fall when f falls, independent of what the market is doing, then $E(R^e) = ... + \beta_{R^e,f}\lambda_f$

6. Comments.

   (a) What model you use depends on what you're going to use it for.

      i. "Explanation," behavioral vs. rational debate.

      ii. Manager/strategy evaluation

      iii. Risk management