MacCready Theory in Wave

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1. The idea

MacCready theory applies to wave lift just like any other lift, if you apply it right. The big lesson: In wave lift, you should fly a good deal faster than you would for equivalent thermal lift.

In classic thermal lift, the thermals drift with the wind, so MacCready theory is unaffected by wind speed. You fly as fast as you can through the air, the fact that the ground is moving a different way is unimportant.

In wave lift, the lift is anchored to the ground. One way to incorporate this fact, thinking relative to the air, is to think of wave lift as lift that not only lifts you up, but transports you through the air at the windspeed in the upwind direction. So it's effectively stronger than lift which just transports you up. The other way to incorporate this fact, thinking relative to the ground, is that your glider is effectively lower performance, by the effect of wind speed. Going upwind to wave lift is just like flying through no wind to thermal lift, in a glider whose polar is shifted to the left by the amount of the windspeed.

Figure 1 illustrates. Here i have graphed (blue) the polar of a dry ASG 29, and the conventional MacCready calculation for a 2 knot thermal. It tells you to fly about 70 knots through still air, and to adapt to lift and sink along the way with a Mc 2 setting. Now, if you're flying up a 50 kt wind through still air to 2 knot wave, that is the same thing as shifting the polar to the left by 50 knots, as shown in red. The same classic MacCready calculation applies to this lower performance glider – and now you see you choose a point much further to the right on the polar. The point on the red, shifted polar is the same as the red marked point on the blue polar. One way to think of this is, fly 105 knots rather than 70 knots. But you have to adapt to lift and sink too, so a better way to think of this situation is that you use a much higher MacCready setting – about 8 in this rather extreme example – and adapt to lift and sink using that higher setting.

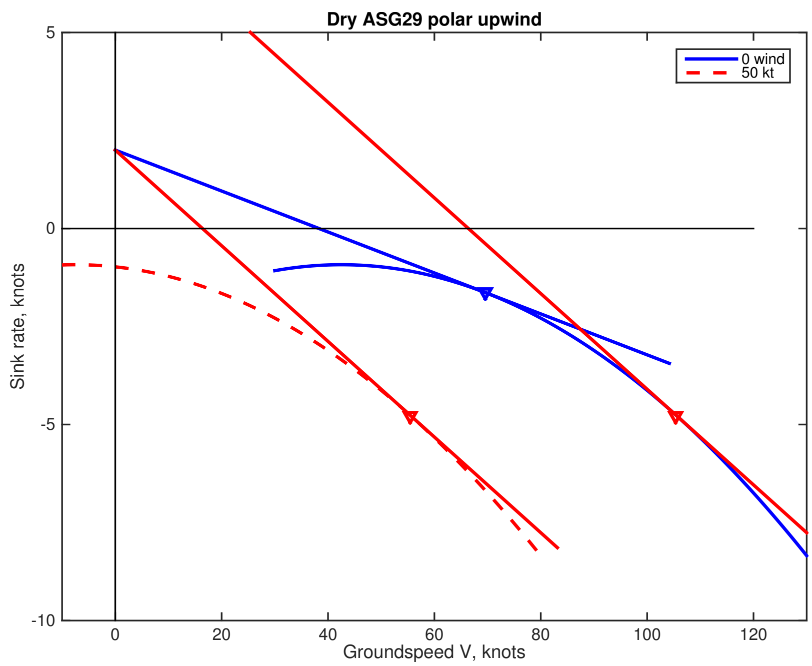


Figure 1: MacCready calculation for flying upwind to 2 knot wave lift. Blue: still air polar and MacCready speed. Red: In wave.

Figure 2 illustrates the crosswind situation. A crosswind still reduces your groundspeed for given airspeed. But rather than groundspeed = airspeed - wind speed, now it's groundspeed = $\sqrt{\text{airspeed}^2 - \text{windspeed}^2}$. The groundspeed polar is shifted less to the left. It's also distorted. If you're going close to the windspeed, going faster helps a lot. If you're going a lot more than the windspeed, going faster doesn't help as much. So the groundspeed polar is now flattened as well as shifted to the left. But you can see the same principle applies. Now, rather than use a Mc 2 setting, flying 70 knots, you use a Mc 4 setting and fly about 85 knots, achieving about 70 over the ground.

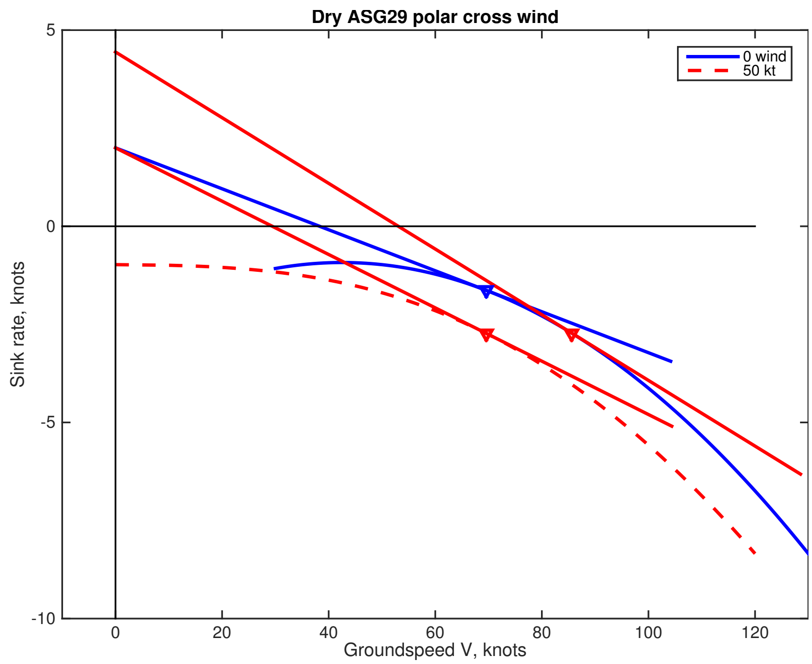


Figure 2: MacCready calculation for flying crosswind to 2 knot wave lift. Blue: still air polar and MacCready speed. Red: In wave.

How important is it to fly the right speed? Figures 4 and **??**show achieved speeds as a function of how fast you fly. The usual lessons apply. MacCready calculations have a smooth top, so flying exactly the right speed isn't terrible, But being grossly off can hurt a lot.

The achieved speeds seem low, but remember, the scenario is important – you fly still air to the wave lift, either straight upwind, or straight crosswind. In reality, your achieved speeds are much higher, because you usually are flying straight through reduced lift. You still use the same (higher) Mac Cready settings, but you will be flying much more

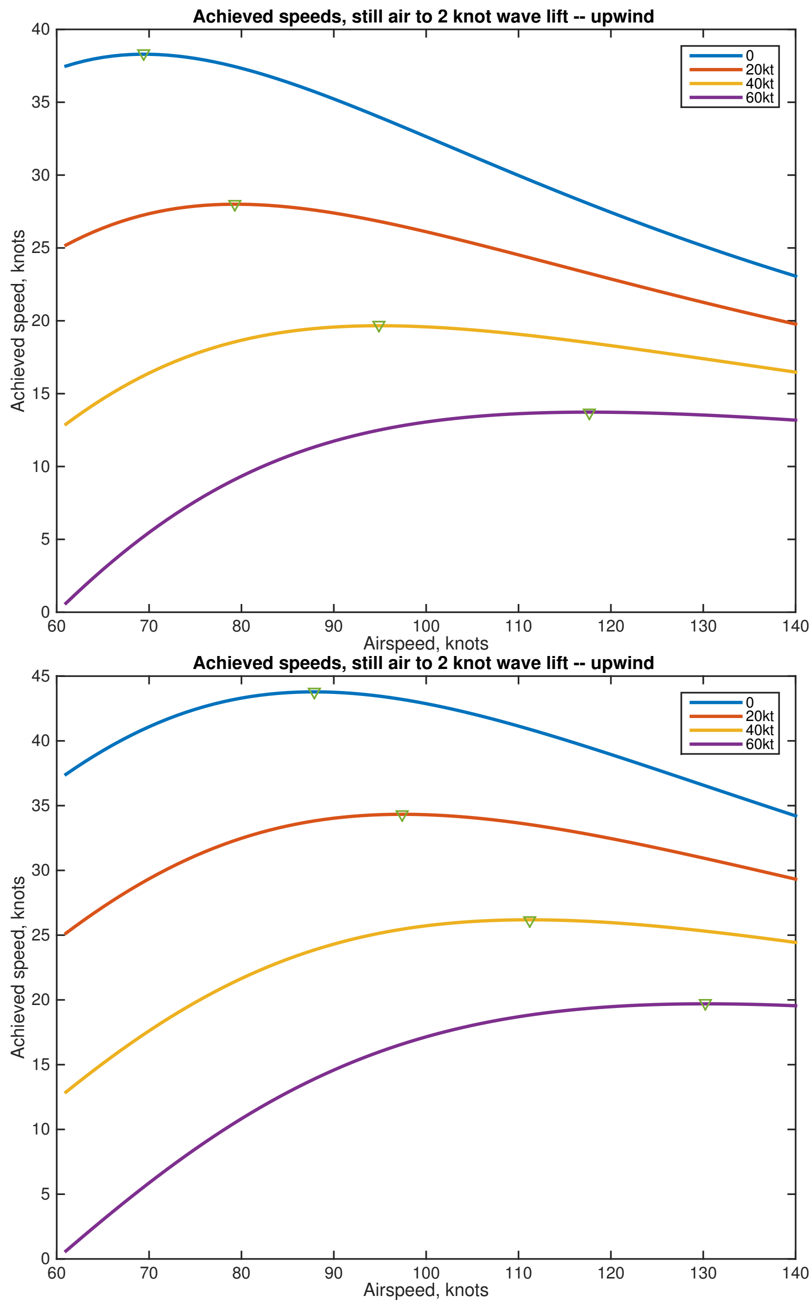
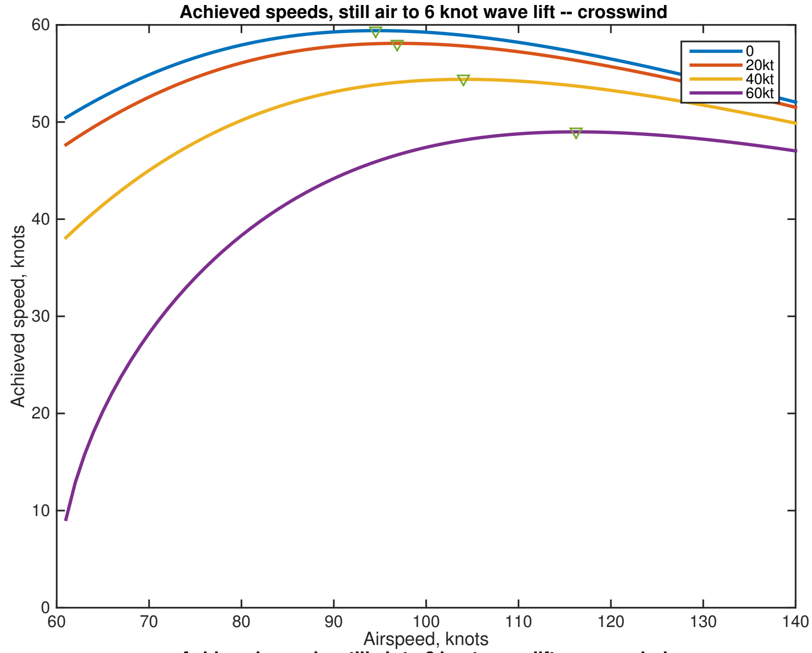


Figure 3: Achieved speed for flying upwind to wave lift, for different airspeeds. Top: dry. Bottom: Wet



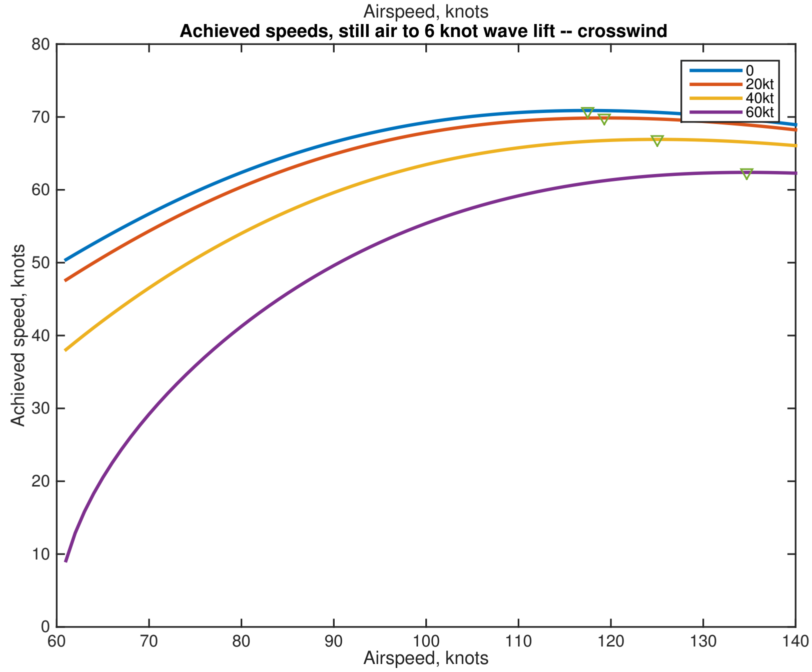


Figure 4: Achieved speed for flying upwind to wave lift, for different airspeeds. Top: dry. Bottom: Wet

slowly.

The Figures show the same calculations for a wet glider. The MacCready settings to use don't change much, as we will see below (another good reason to think about MacCready settings rather than airspeeds) but the airspeeds go up 10 knots, and the achieved speeds go up a lot too. We should be flying with water (and antifreeze) in wave.

2. Tables

Now, just how fast should we go? Table 1 shows speed to fly and (better) MacCready settings for different wind speeds and wave lift values, if you're flying upwind to wave. For example the top row shows that in 2.0 knot lift, you should fly 69 knots and Mc 2.0 in still air. but this rises to flying 95 knots and Mc 6.1 if you're flying upwind to that 2 knot lift. Flying upwind, it's worth flying a *lot* faster!

Table 2 shows speed to fly and (better) MacCready settings for different speeds and wave lift values, if you're flying crosswind to wave.

The effect is reduced, but still substantial, especially for strong wind and weak lift. For example, in 2 knot lift and 40 knot crosswind, you should now fly 79 knots and Mc 3.4. That's more than 69 knots and Mc 2 of still air, but not nearly as drastic as 95 knots and Mc 6.1 going upwind.

For 6 knot lift and 40 knot wind, you speed up from 95 to 104, and Mc 6 to Mc 7.8. Overall, you roughly speed up 5 knots for 20 mph crosswind and 10 knots for 40 mph crosswind and 20 knots for 60 mph crosswind – it's not linear.

In both tables, flying with water ballast increases actual speeds a lot. However, it doesn't have that much effect on MacCready settings. The increase in MacCready setting is a bit muted when flying with water, since the speed of the glider is greater relative to wind-speed. But most of the faster speed to fly with water is captured by the faster speed

Table 1: Speeds to fly and Mc settings to fly upwind to wave lift

Upwind Speed to fly

	Dry				Wet			
	Wind speed				Wind speed			
Lift	0	20	40	60	0	20	40	60
2.0	69	79	95	118	88	97	111	130
4.0	83	95	111	133	104	115	130	149
6.0	95	107	125	146	117	130	146	165

Upwind Mc setting

	Dry				Wet			
	Wind speed				Wind speed			
Lift	0	20	40	60	0	20	40	60
2.0	2.0	3.4	6.1	10.7	2.0	3.2	5.1	8.1
4.0	4.0	6.0	9.3	14.6	4.0	5.6	8.0	11.6
6.0	6.0	8.5	12.4	18.1	6.0	8.0	10.9	14.8

recommended by the same MacCready setting.

The same precautions apply as with standard MacCready theory. Most of all, these calculations assume that altitude is not a problem. If flying faster will put you out of the bottom of the wave, obviously, you fly more slowly, and you stop to take some weak lift along the way.

Table 2: default

Crosswind Speed to fly

	Dry				Wet			
	Wind speed				Wind speed			
Lift	0	20	40	60	0	20	40	60
2.0	69	72	79	94	88	90	96	106
4.0	83	85	93	106	104	106	111	122
6.0	95	97	104	116	117	119	125	135

Crosswind Mc Setting

	Dry				Wet			
	Wind speed				Wind speed			
Lift	0	20	40	60	0	20	40	60
2.0	2.0	2.3	3.4	5.8	2.0	2.2	2.9	4.4
4.0	4.0	4.4	5.7	8.2	4.0	4.3	5.1	6.7
6.0	6.0	6.4	7.8	10.4	6.0	6.3	7.2	8.9

3. Math

$V_g =$ groundspeed, $V_w =$ windspeed, $V =$ airspeed. Thus, upwind

$$V_g = V - V_w$$

and crosswind

$$V_g = \sqrt{V^2 - V_w^2}$$

Let $T =$ time, $T_g =$time spent gliding, $T_{cl} =$time climbing, h = height of climb,$x =$ distance, $M =$ climb rate. Let $S(V)$ be the glider sink rate or polar. We find the achieved speed V_a by:

$$V_a = \frac{MV_g}{M + S(V)}.$$

Derivation:

$$\begin{aligned}
T &= T_g + T_{cl} \\
T &= T_g + h/M \\
T &= T_g + T_g S(V)/M \\
T &= T_g \left(1 + S(V)/M\right) \\
T &= \frac{x}{V_g}\left(1 + S(V)/M\right) \\
\frac{1}{V_a} &= \frac{T}{x} = \frac{1 + S(V)/M}{V_g}
\end{aligned}$$

The optimal speed to fly comes from maximizing achieved speed – minimizing time:

$$\begin{aligned}
&\min_{\{V\}} \frac{M + S(V)}{MV_g} \\
\frac{S'(V)}{MV_g} &= \frac{M + S(V)}{MV_g^2}\frac{dV_g}{dV} \\
M + S(V) &= S'(V)V_g\frac{dV}{dV_g}
\end{aligned}$$

Graphically, the usual MacCready calculation remains valid with a glider of effectively less performance

$$M + S(V_g) = V_g\frac{dS(V)}{dV}\frac{dV}{dV_g} = V_g\frac{dS(V_g)}{dV_g}$$

The upwind case:

$$\begin{aligned}
M + S(V) &= S'(V)V_g \\
V_g &= V - V_w
\end{aligned}$$

$$M + S(V) = S'(V)\,(V - V_w)$$

Use the quadratic polar

$$S(V) = a + bV + cV^2.$$

Plugging in the polar,

$$M + a + bV + cV^2 = (b + 2cV)\,(V - V_w)$$

$$M + a = cV^2 - (b + 2cV)\,V_w$$

$$0 = cV^2 - 2cV_wV - M - a - bV_w$$

$$V = V_w + \sqrt{V_w^2 + (M + a + bV_w)/c}$$

This is the calculated speed to fly in still air.

To communicate this via a MacCready setting, thus generalizing to flight through lift and sink, note the conventional $V_w = 0$ case, the still-air speed V is given by the MacCready setting M^*

$$V = \sqrt{(M^* + a)\,/c}.$$

Thus we can communicate a speed to fly through still air V as its equivalent still air MacCready setting M^*

$$M^* = cV^2 - a.$$

You can describe ”fly V” as ”fly M^*” rather than the true lift M

In the crosswind case,

$$\begin{aligned} M + S(V) &= S'(V) V_g \frac{dV}{dV_g} \\ V &= \sqrt{V_g^2 + V_w^2} \\ \frac{dV}{dV_g} &= \frac{V_g}{\sqrt{V_g^2 + V_w^2}} = \frac{V_g}{V} \end{aligned}$$

$$\begin{aligned} M + S(V) &= S'(V) \frac{V_g^2}{V} \\ M + S(V) &= S'(V) \frac{V^2 - V_w^2}{V} \end{aligned}$$

Plugging in the polar,

$$M + S(V) = S'(V) \frac{V^2 - V_w^2}{V}$$

$$M + a + bV + cV^2 = (b + 2cV) \frac{V^2 - V_w^2}{V}$$

$$M + a + bV + cV^2 = (b + 2cV) \left(V - \frac{V_w^2}{V} \right)$$

$$M + a = cV^2 - b\frac{V_w^2}{V} - 2cV_w^2$$

$$0 = cV^3 - \left[2cV_w^2 + M + a \right] V - bV_w^2$$

Alas, we have to solve that for V solve numerically.