

5 Week 3. Fama-French and the cross section of stock returns – overheads

5.1 Fama and French "Multifactor Anomalies"

1. Big Questions
2. CAPM,

$$E(R^{ei}) = \beta_i \lambda \quad (+\alpha_i)$$

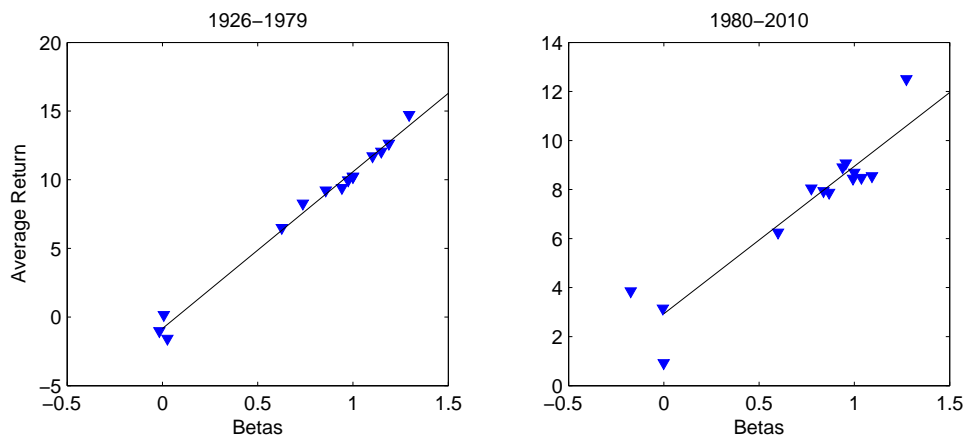
- (a) β_i are defined from *time series regressions*

$$R_t^{ei} = \alpha_i + \beta_i R_t^{em} + \varepsilon_t^i;$$

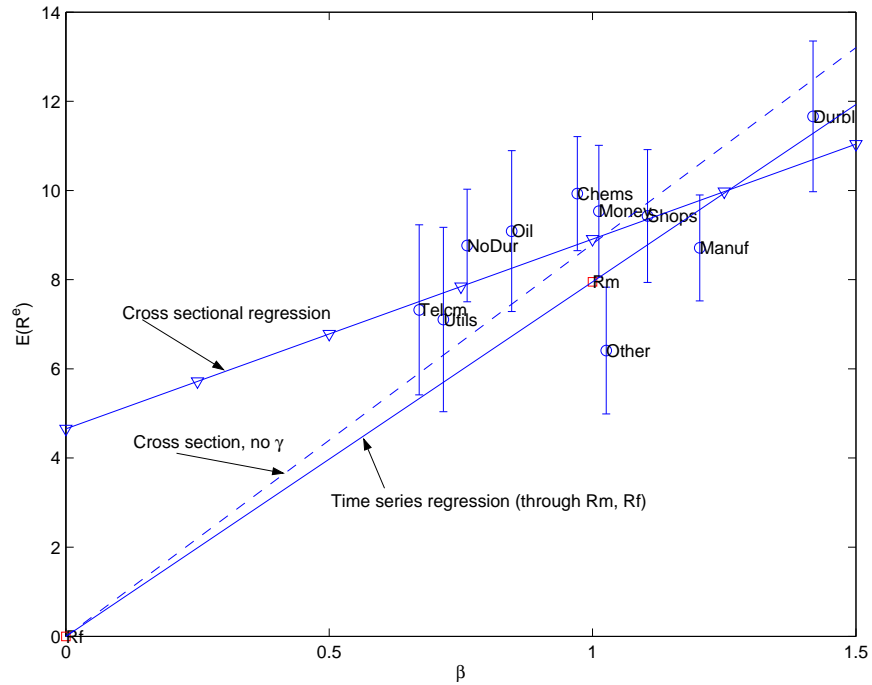
$$(R_t^{ei} = R_t^i - R_t^f)$$

- (b) What we do: see if attractive opportunities $E(R^{ei})$ have higher β_i .
3. Evidence: The CAPM worked great and still does on many assets.

(a) From "Discount Rates" The CAPM works great on size portfolios.

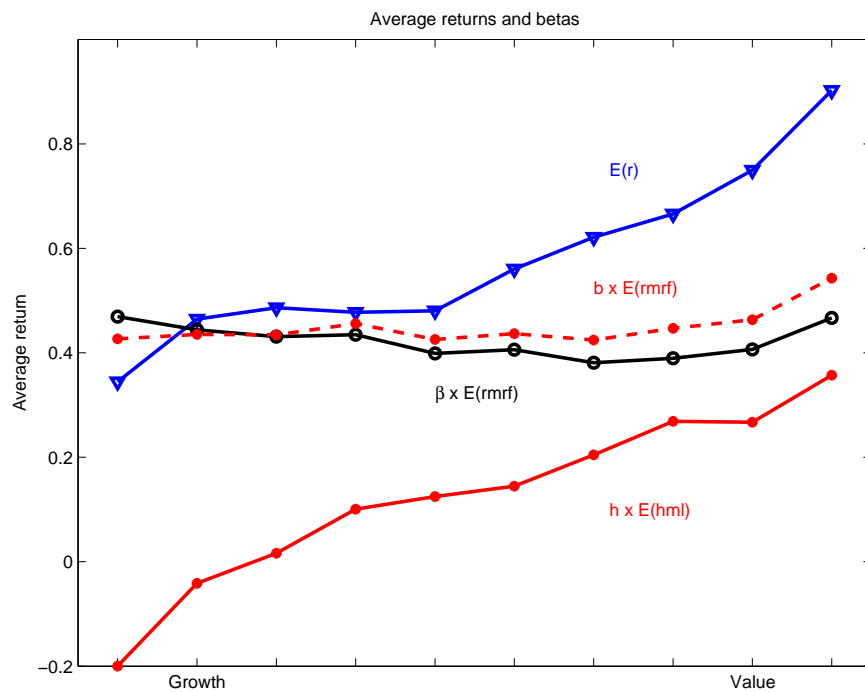


4. CAPM Example 2: industry portfolios



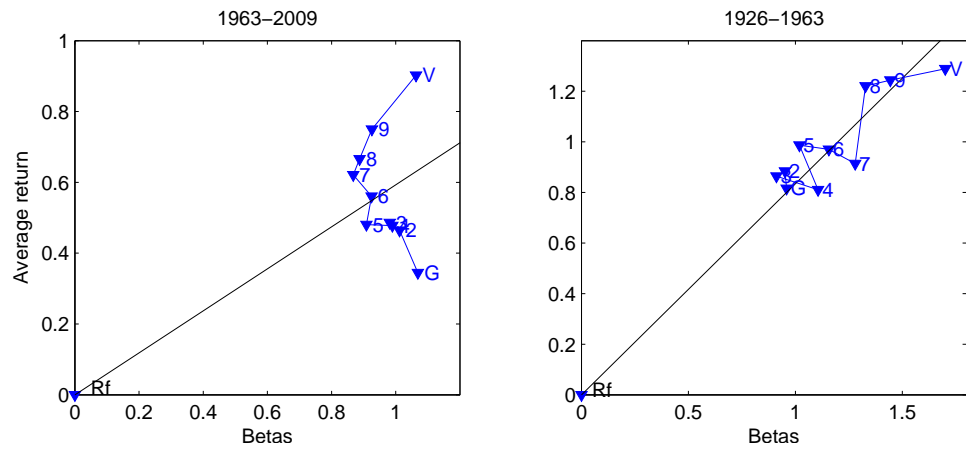
5. The Value Puzzle

- (a) FF. Ok for size, industry, beta portfolios. What about book/market? Do low prices mean high returns *across stocks*?
- (b) Facts: There is a big spread in average returns. But market beta is a disaster. *Puzzle depends on average returns and betas!* From "Discount rates"



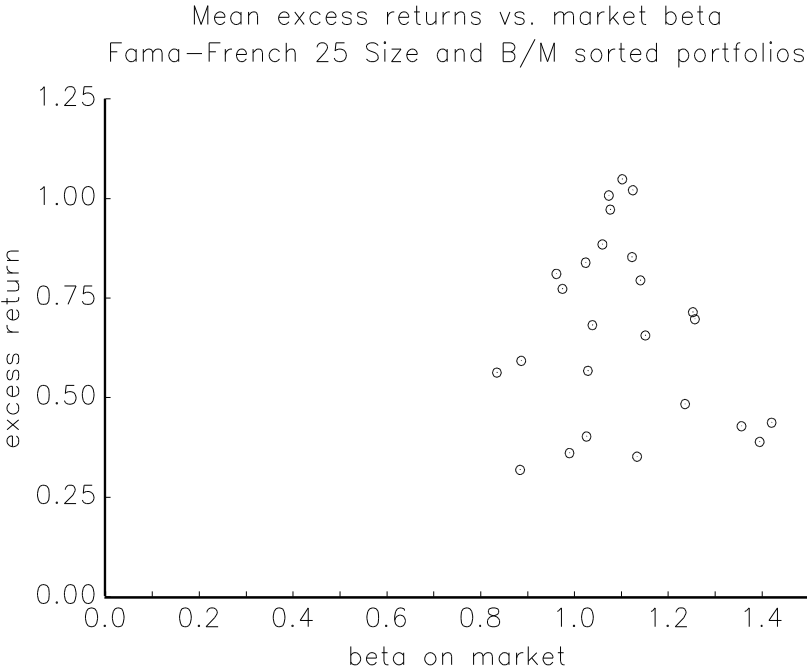
Average returns and betas for Fama - French 10 B/M sorted portfolios. Monthly data 1963-2010.

(c) Also in "Discount Rates"

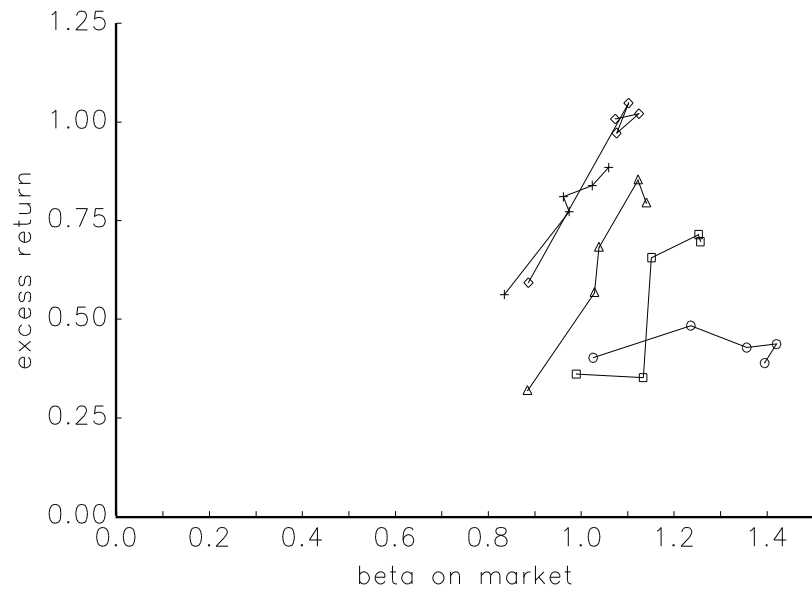


Value effect before and after 1963.

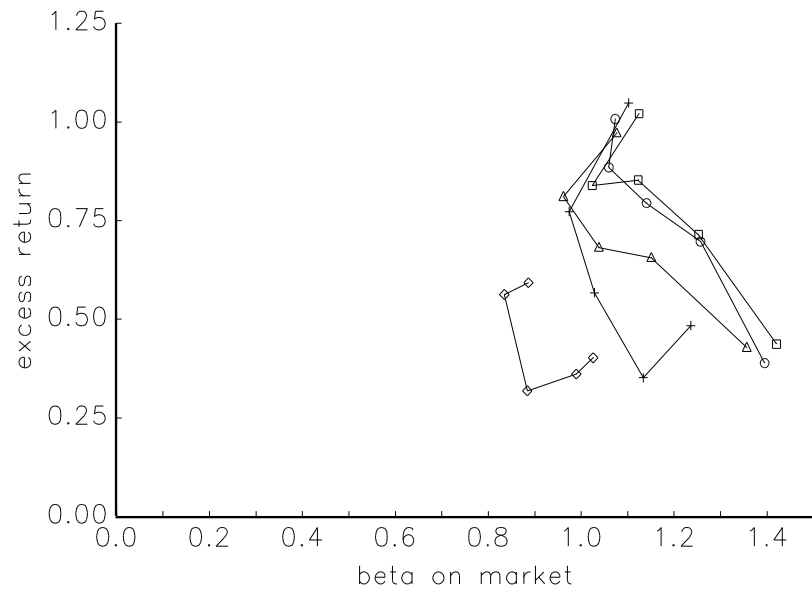
(d) Value From *Asset Pricing*



Mean excess returns vs. market beta
lines connect changing SIZE within B/M categories



Mean excess returns vs. market beta
lines connect changing B/M within SIZE categories



6. Fama-French solution:

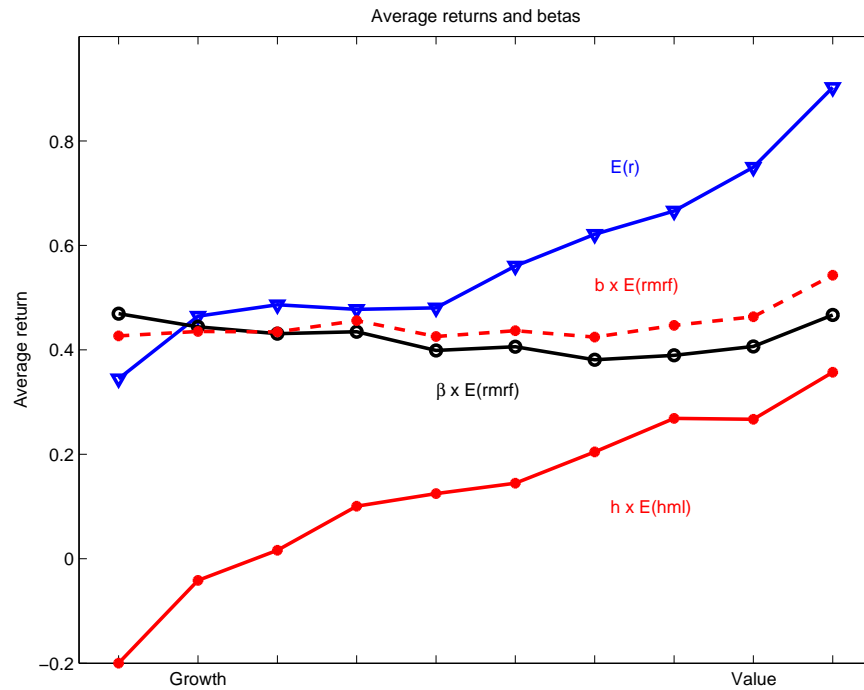
- (a) Run *time series regressions* that include additional *factors* (portfolios of stocks) SMB, HML

$$R_t^{ei} = \alpha_i + b_i R_t^{em} + s_i SMB_t + h_i HML_t + \varepsilon_t^i; \quad t = 1, 2, \dots, T \text{ for each } i = 1, 2, \dots, N.$$

- (b) Look across stocks

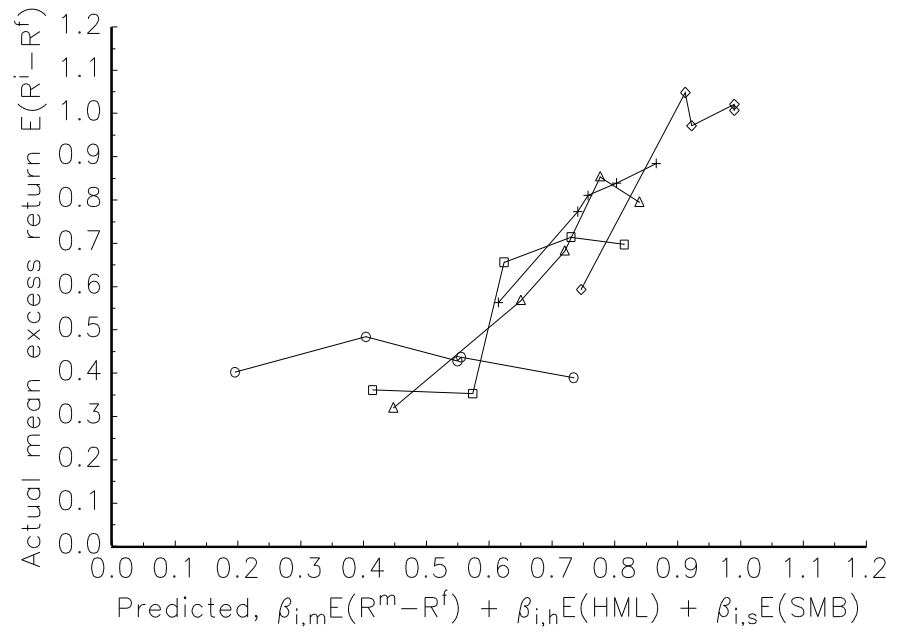
$$E(R^{ei}) = \alpha_i + b_i E(R^{em}) + s_i E(SMB) + h_i E(HML)$$

- (c) Result from “Discount rates.”

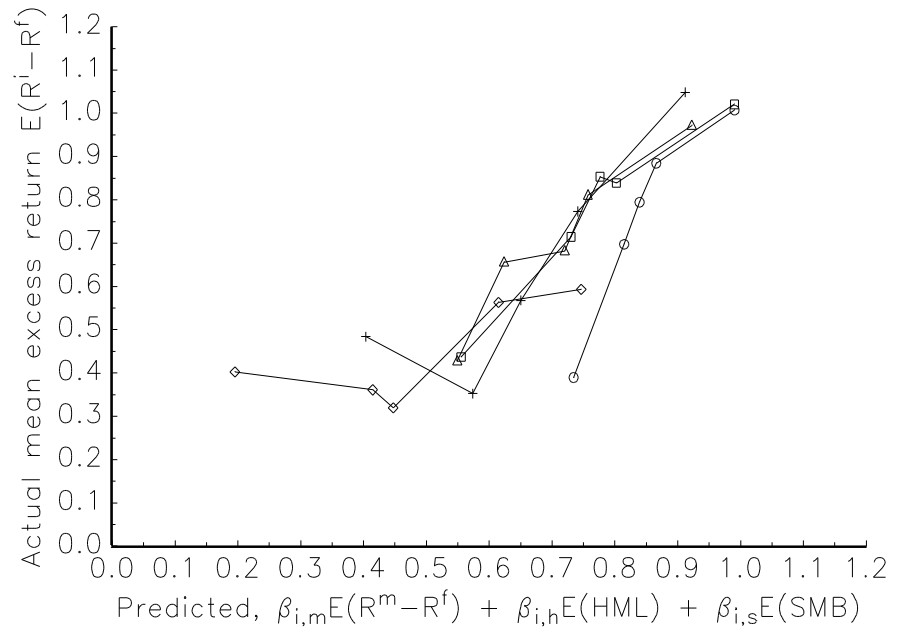


(d) 25 portfolios from *Asset Pricing*

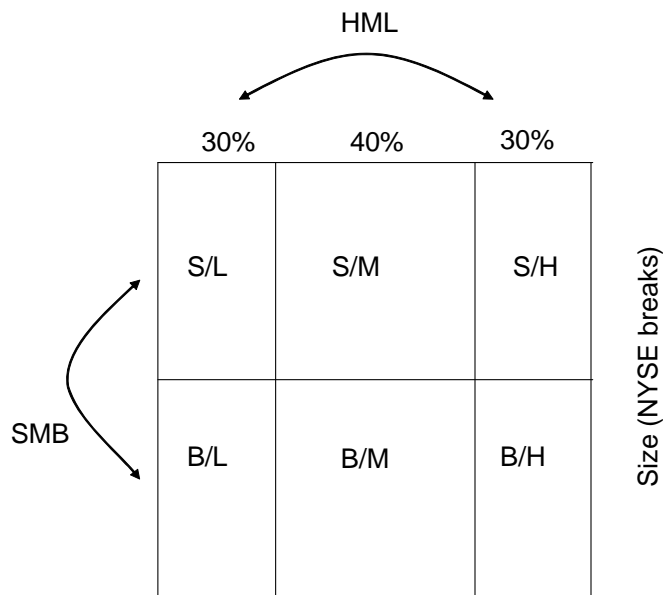
Mean excess return vs. predicted by 3 factor model
lines connect changing size within B/M categories



Mean excess return vs. predicted by 3 factor model
lines connect changing BM within size categories



7. Fama-French paper:



Book/market (NYSE breaks)

$$HML = (S/H + B/H)/2 - (S/L+B/L)/2$$

$$SMB = (S/L + S/M + S/H)/3 - (B/L + B/M + B/H)/3$$

- (a) Run *time series regressions* that include additional *factors* (portfolios of stocks) SMB, HML

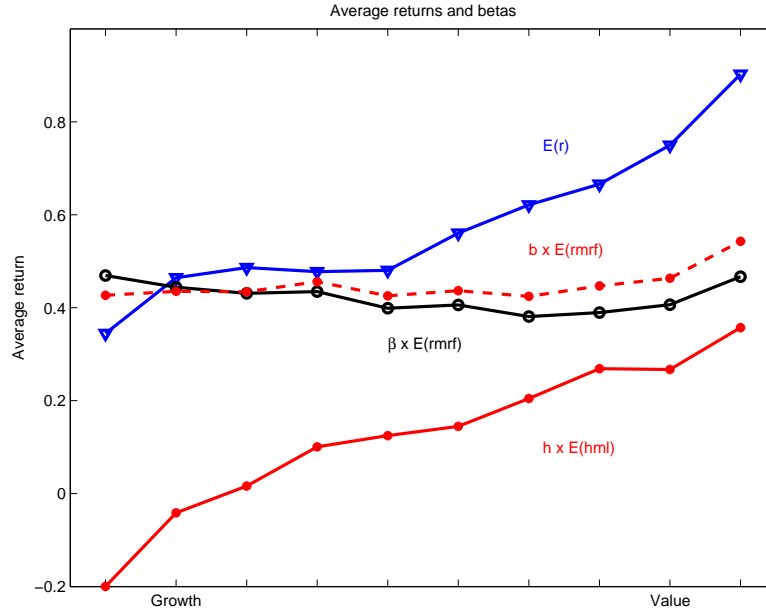
$$R_t^{ei} = \alpha_i + b_i R_t^{em} + s_i SMB_t + h_i HML_t + \varepsilon_t^i; \quad t = 1, 2 \dots T \text{ for each } i = 1, 2 \dots N.$$

- (b) Look across stocks at *the cross-sectional implication of this time-series regression* (Take E of both sides again):

$$E(R^{ei}) = \alpha_i + b_i E(R^{em}) + s_i E(SMB) + h_i E(HML)$$

This works pretty well (α not big) except for small growth.

- (c) “Discount Rates” one stop summary again. Now look at the sum of red solid and red dashed lines. $E(r) = b \times E(rmr f) + h \times E(hml)$.



Average returns and betas for Fama - French 10 B/M sorted portfolios. Monthly data 1963-2010.

8. FF

- (a) See FF Table 1. In depth!
- (b) What's wrong with $E(R^{ei}) = (size_i)\lambda_s + (b/m_i)\lambda_{B/M}$? “How you behave” vs. “who you are”
- (c) Understand the difference between “explaining returns” (time-series regression) and “explaining average returns” (cross-sectional relation between average return and beta)!
- (d) The main point is to produce a robust model that explains *other* anomalies. That is what the CAPM did for many years. See Sales, long term reversal. Not momentum

9. Do we really need the smb portfolio? Smb makes it a better model of *returns*, doesn't help much on *average returns*, and improves precision.

- (a) Example: Suppose the CAPM works add a beta-hedged industry portfolio.

$$R_t^{eI} = \alpha_I + \beta_I R_t^{em} + \varepsilon_{It}$$

$$R_t^{eI*} = R_t^{eI} - \beta_I R_t^{em}$$

Now run

$$R_t^{ei} = \alpha_i + \beta_i R_t^{em} + \gamma_i R_t^{e*I} + \varepsilon_{it}$$

- i. $\gamma_i > 0$, R^2 improves, t statistics improve, $\sigma(\varepsilon_i)$ decreases. The model of *variance* improves
- ii.

$$E(R_t^{ei}) = \beta_i E(R_t^{em}) + \gamma_i E(R_t^{e*I}) = \beta_i E(R_t^{em}) + \gamma_i 0$$

The model of *mean* is unchanged.

- (b) This is roughly true. FF keep SMB because it is so useful to explain the *variance* of size-sorted portfolios.
10. Is it a tautology to “explain” 25 B/M, size portfolios by 2 B/M, size portfolios? (No, why?)
) → Other sorts.
11. Where does FF come from?
- (a) ICAPM: “State variables of concern to investors” Suppose people don’t want stocks that fall especially (more than others) in recessions.
- (b) APT: “Minimalist interpretation.” Suppose $R^2 = 1$,

$$R_t^{ei} = b_i r_{mrf_t} + h_i h_{ml_t} + s_i smb_t + 0$$

→

$$E(R_t^{ei}) = b_i E(r_{mrf_t}) + h_i E(h_{ml_t}) + s_i E(smb_t)$$

- (c) Practice: like the CAPM for digesting anomalies.
12. A big picture for “dissecting anomalies” and the whole question of multivariate forecasts:

$$dp_t \approx E_t \sum_{j=1}^{\infty} \rho^j r_{t+j} - E_t \sum_{j=1}^{\infty} \rho^j \Delta d_{t+j}$$

dp reveals to us market expectations.

- (a) How can z_t help?
- (b) z_t can predict *both* r and Δd . z_t can predict r_{t+1} and r_{t+j} in opposite directions.
- (c) Fama and French “Dissecting anomalies:” This is why additional “cashflow forecast” anomaly variables help to forecast returns.
- (d) “Discount rates” the cay experiment turns out to forecast the time path of returns.

13. Regressions summary.

- (a) Forecasting

$$R_{t+1}^{em} = a + bx_t + \varepsilon_{t+1}; \quad t = 1, 2, \dots, T$$

- (b) The “market model” of returns (return variance)

$$R_t^{ei} = \alpha_i + \beta_i R_t^{em} + \varepsilon_t^i; \quad t = 1, 2, \dots, T \text{ for each } i$$

- (c) FF’s three-factor model of returns (return variance)

$$R_t^{ei} = \alpha_i + b_i rmr f_t + h_i hml_t + s_i smb_t + \varepsilon_t^i; \quad t = 1, 2, \dots, T \text{ for each } i$$

- (d) The CAPM model of mean returns. (We implicitly run this when we look at expected return vs. beta. We will run this “cross-sectional regression” explicitly soon.)

$$E(R_t^{ei}) = \beta_i \lambda_m + \alpha^i; \quad i = 1, 2, \dots, N$$

- (e) The slope coefficient in d should equal the mean market return (since its beta is one) λ_m should = $E(R^{em})$, so we sometimes force that in the implicit cross sectional “regression”

$$E(R_t^{ei}) = \beta_i E(R^{em}) + \alpha^i; \quad i = 1, 2, \dots, N$$

- (f) Fama and French. They do option e. They are implicitly running a cross sectional regression with the slopes equal to means of the factors. Table 1 is just data for this regression

$$E(R_t^{ei}) = b_i E(rmr f_t) + h_i E(hml_t) + s_i E(smb_t) + \alpha_i; \quad i = 1, 2, \dots, N$$

- (g) The cross-sectional characteristic regression. Rather than Table 1A, FF dissecting anomalies and discount rates describe mean returns by a characteristic regression

$$E(R_t^{ei}) = a + bE[\log(B/M_{it})] + cE[\log(ME_{it})] + \varepsilon^i; \quad i = 1, 2, \dots, N$$

more generally with C_i a vector of characteristics

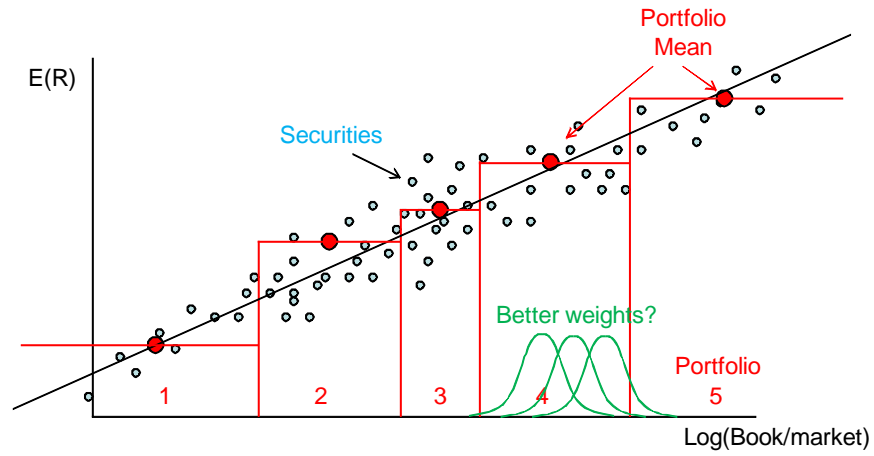
$$E(R_t^{ei}) = a + bC_i; \quad \varepsilon^i; \quad i = 1, 2, \dots, N$$

- (h) The characteristic regression is the same thing as a forecasting regression. (Note sometimes there are fixed effects, a_i or a_t)

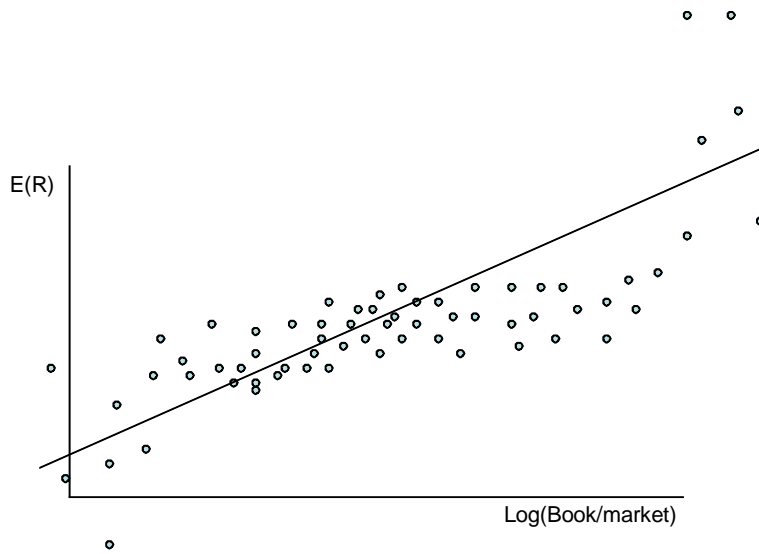
$$\begin{aligned} R_{t+1}^{ei} &= a + b \log(B/M_{it}) + c \log(ME_{it}) + \varepsilon^i; \quad t = 1, 2, \dots, T \quad i = 1, 2, \dots, N \\ R_{t+1}^{ei} &= a + bC_{it} + \varepsilon_{t+1}^i \end{aligned}$$

5.2 Fama and French “Dissecting Anomalies”

1. The relationship between portfolios and cross-sectional regressions



Sorted portfolios and cross-sectional regressions.



A warning on OLS equally-weighted cross-sectional regressions